

Technical Notes

Optimization of Parameters for a Damped Oscillator Excited by a Sequence of Random Pulses

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The paper is another step in discussion concerning the method of determining the distributions of pulses forcing vibrations of a system. Solving a stochastic problem for systems subjected to random series of pulses requires determining the distribution for a linear oscillator with damping. The goal of the study is to minimize the error issuing from the finite time interval. The applied model of investigations is supposed to answer the question how to select the parameters of a vibrating system so that the difference between the actual distribution of random pulses and that determined from the waveform is as small as possible.

Keywords: random pulses, oscillator with damping, vibroacoustics.

1. Introduction

The problems of theoretical and applied mechanics in the broad sense are the subject of numerous studies carried out currently at the *Faculty of Mechanical Engineering and Robotics of the AGH University of Science and Technology in Cracow*. The Faculty develops the most up-to-date domains of mechanics including vibroacoustics which combine the problems of the theory of linear vibrations (KASPRZYK, 2011; WICIAK, TROJANOWSKI, 2014), engineering dynamics, theoretical acoustics (SNAKOWSKA, JURKIEWICZ, 2010; RDZANEK *et al.*, 2011), and technical acoustics (PIECHOWICZ, 2011; BARAŃSKI, WSZOŁEK, 2013). Research is carried out in the areas connected with the problems of propagation of the noise produced by road traffic, wind power plants (KASPRZAK *et al.*, 2014; WSZOŁEK *et al.*, 2014), and industry, especially ducted systems with flow (JURKIEWICZ *et al.*, 2012). These studies are aimed at reducing vibroacoustic noise to a minimum and application of vibroacoustic signals in assessment and control of technological processes (RACZKA *et al.*, 2013; SIBIELAK *et al.*, 2013; KONIECZNY *et al.*, 2013). Vibroacoustic diagnostics also includes studies regarding the theory of vibration processes occurring in vibrating systems as well as studies concerning semi-active and active methods of reduction of vibrations (WICIAK, 2007; TROJANOWSKI, WICIAK, 2010). The study presented in this

paper is a continuation of the aforementioned tradition of combining science and technology, since the development of a mathematical model was inspired by an attempt to construct a measuring device that would be able to detect a sequence of both large and small dust particles hitting a sensor in the transported dust stream. The model of investigations connected with analysis of vibrations forced by stochastic pulses can be placed between the studies directed at solving technical problems and those carried out in other academic centres, namely, the analyses closely connected with mathematics, using probabilistic and statistical methods in the theory of vibrations (ROBERTS 1972; TYLIKOWSKI, MARKOWSKI, 1986; IWANKIEWICZ, NIELSEN, 1992; KOYLUOGLU *et al.*, 1994; DIPAOLA, VASTA, 1997; SOBIECHOWSKI, SOCHA, 2000; HUANG *et al.*, 2000). In the present paper we deal with application of problems of the linear vibrations theory to vibroacoustic diagnostics, where the vibration signal, that is, the deflection of the system from its balance position, will be used for assessment and execution of technological processes.

2. Distribution of random pulses acting on a vibrating system as a function of its motion

Let us consider a one-dimensional of damped oscillator (JABŁOŃSKI, OZGA, 2006) given by the equation

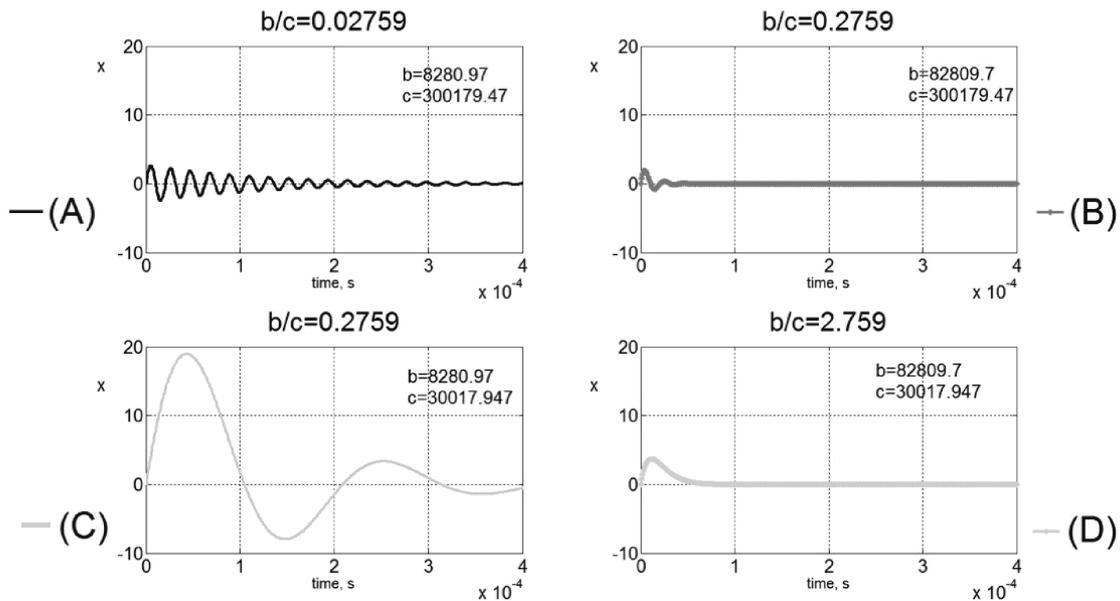


Fig. 1. Vibrations of four systems with parameters b and c given in the diagram forced by a pulse of the value $\eta_1 = 845\,778.47\text{ s}^{-1}$.

$$\frac{d^2x}{dt^2} + 2b\frac{dx}{dt} + a^2x = f(t), \quad (1)$$

with the following initial conditions:

$$x(0) = 0 \quad \text{and} \quad x'(0) = 0. \quad (2)$$

In the mathematical model (JABŁOŃSKI, OZGA, 2008; 2009) used for determination of distributions of random pulses forcing vibrations of a one-dimensional physical system (JABŁOŃSKI, OZGA, 2010; JABŁOŃSKI *et al.*, 2011), the force $f(t)$ is defined by two stochastically independent variables, random amplitudes η_i , and random instants of time t_i at which the pulses occur:

$$f(t) = \sum_{t_i < t} \eta_i \delta(t - t_i), \quad (3)$$

where $\delta(t - t_i)$ are Dirac distributions at the time t_i . The time intervals $(t_i - t_{i-1})$ between the subsequent pulses with the exponential distribution $F(\tau)$, namely:

$$F(\tau) = \begin{cases} 1 - \exp(-\lambda\tau) & \text{if } \tau \geq 0, \\ 0 & \text{if } \tau < 0. \end{cases} \quad (4)$$

The constant λ is the pulse rate (JABŁOŃSKI, OZGA, 2012), η_i is a sequence of independent identically distributed random variables with finite expectation and it assumes a finite number of values $\{\eta_1, \eta_2, \dots, \eta_i\}$ with probabilities $p_i = p(\eta_i)$.

With the assumptions defined above the function of vibrations (JABŁOŃSKI, OZGA, 2013) for a one-dimensional physical system influenced by $f(t)$ has the form

$$x(t) = \frac{1}{c} \sum_{0 < t_i < t} \eta_i \exp(-b(t - t_i)) \sin(c(t - t_i)), \quad (5)$$

where the damping coefficient b [s^{-1}] and the frequency $c = \sqrt{a^2 - b^2}$ [s^{-1}] are parameters of the vibrating system.

In earlier studies (OZGA, 2014), while analyzing a finite time interval, the differences between the distribution $\bar{p}_i(t)$ determined from the function of vibrations and the distribution imposed in the simulation p_i are the smaller, the stronger is the damping coefficient b . In this work the problem of seeking the optimum *parameters* of a system was complemented with an analysis of the influence of a modified frequency of vibrations c on the errors in the determined distributions. Simulations were conducted for four harmonic oscillators with three different values of the quotient b/c (Fig. 1) labeled with A, B, C, and D.

The parameters c , b , and η_i from the case A applied in the simulations discussed in this paper correspond, within the framework of electro-mechanical analogies to a system consisting of an RLC circuit with inductivity $L = 5\text{ mH}$, capacity $C = 0.2\text{ nF}$, and a voltage source (OZGA, 2013). The parameters of oscillators B, C, and D were selected according to the needs of the simulation.

3. Computer simulations

In simulations conducted in MATLAB environment, in the first step we generate pseudo random excitations $f(t)$. The intervals between subsequent pulses $\Delta_i = (t_i - t_{i-1})$ are randomized according to the formula

$$\Delta_i = -\frac{1}{\lambda} \ln(1 - \text{rand}). \quad (6)$$

The pseudorandom variable *rand* reaches values drawn from the standard uniform distribution on the open interval (0,1). In all simulations which are considered in this paper, the pseudorandom variable η_i assumes three values: $\eta_1 = 845778.47 \text{ s}^{-1}$, $\eta_2 = \eta_1/2$, and $\eta_3 = \eta_1/10$. Due to the stochastic character of the phenomenon these pulses are randomized in the form of two distributions Φ_i which were chosen in such a way that all pulses should occur and in both cases $E(\eta)$ should have the same value 447416.8 s^{-1} . These distributions are:

$$\begin{aligned} \Phi_1: p_1 = 0.33, p_2 = 0.33, p_3 = 0.34; \\ \Phi_2: p_1 = 0.25, p_2 = 0.51, p_3 = 0.24. \end{aligned}$$

Due to the fact that on the basis of observation it is not possible to foresee $x(t)$ at any time instant, all the considerations below compare the systems *A*, *B*, *C*, and *D*, which vibrate under the influence of pulses generated in MATLAB environment occurring at the same time t_i and having the same amplitude η_i . In this way we analyze the signals received in identical conditions and on the same time interval. However, the differences are significant – under the influence of a pulse η_i , depending on parameters *c* and *b* of the system, some vibrations of the system expire decay faster than others (Fig. 2).

Describing roughly the method of determining the distributions, one should first determine the stochastic moments $m_i(t)$, $i = 1, 2, 3$, from the waveform $x(t)$ of the harmonic oscillator

$$m_i(t) \cong \frac{1}{[t/h]} \sum_{kh < t} x^i(kh), \quad (7)$$

where x is the deflection of the system from the balance position, $m_i(t)$ is the i -th stochastic moment of the random variable x , and h is the interval of time between successive measurements equal to 10^{-6} s .

The determined values of stochastic moments m_n are put in the Eqs. (8) in order to determine the distributions \bar{p}_i (Fig. 3, Fig. 4)

$$\sum_{i=1}^k \bar{p}_i \left[(m_i(t)m_1(t) - m_{i+1}(t))\eta_i + \sum_{j=1}^n \binom{n}{j} m_{(n-j)}(t)m_1(t)\eta_i^{(j+1)} \frac{C(j+1)}{C(1)c^j} \right] = 0, \quad (8)$$

where k is the number of the sought values of random amplitudes η_i for $i = 1, 2, \dots, k-1$. We obtain an additional equation using (9)

$$\sum_{i=1}^k \bar{p}_i = 1 \quad (9)$$

and for $j > 1$ and for even j

$$C(j) = \frac{j!}{\prod_{r=0}^{j/2-1} ((jb/c)^2 + (2r)^2)} \frac{c}{jb}, \quad (10)$$

whereas for odd $j > 0$,

$$C(j) = \frac{j!}{\prod_{r=0}^{(j-1)/2-1} ((jb/c)^2 + (2r+1)^2)}. \quad (11)$$

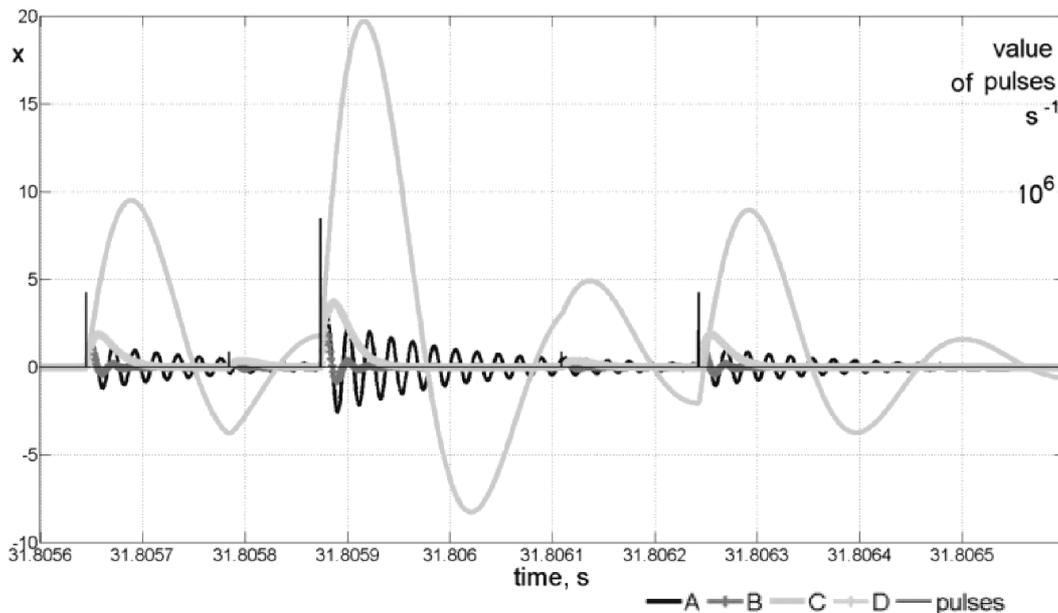


Fig. 2. Vibrations obtained as a result of simulation of four harmonic oscillators labeled *A*, *B*, *C*, and *D* forced by pulses of the values η_1 , η_2 , and η_3 corresponding to $\lambda = 10^3 \text{ s}^{-1}$ and Φ_1 .

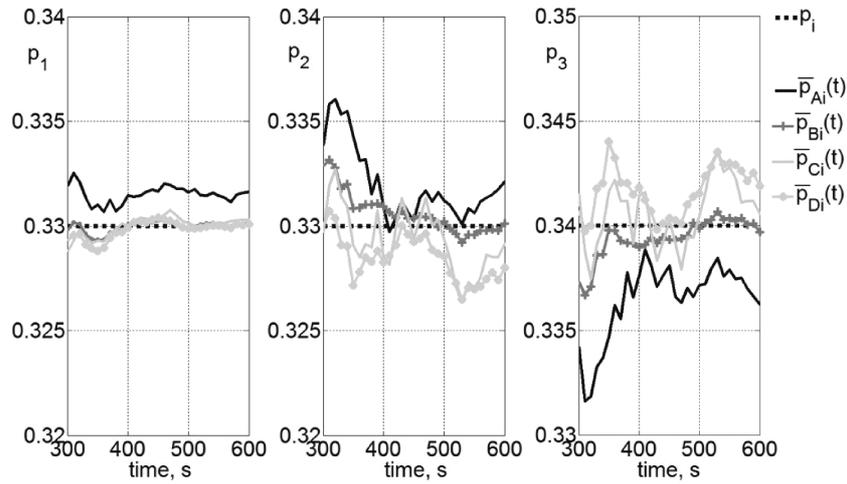


Fig. 3. Probabilities p_i and estimators $\bar{p}_i(t)$ corresponding to $\lambda = 10^3 \text{ s}^{-1}$ and Φ_1 .

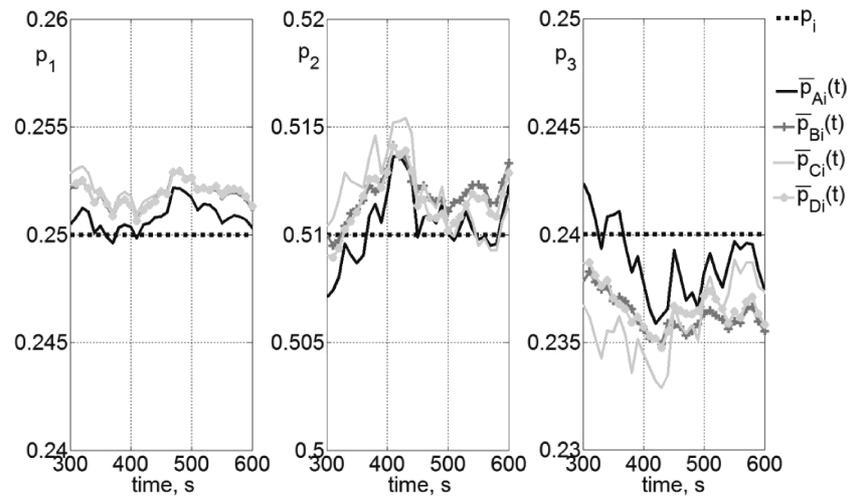


Fig. 4. Probabilities p_i and estimators $\bar{p}_i(t)$ corresponding to $\lambda = 10^2 \text{ s}^{-1}$ and Φ_2 .

Analyzing the distributions presented in Figs. 3 and 4 for two samples received in simulation, we can compare the distances between the curves – the imposed distribution p_i and the distributions determined from the waveform $x(t)$, labeled respectively $\bar{p}_{Ai}(t)$, $\bar{p}_{Bi}(t)$, $\bar{p}_{Ci}(t)$, and $\bar{p}_{Di}(t)$. The differences between $p_i - \bar{p}_{Ai}(t)$, $p_i - \bar{p}_{Bi}(t)$, $p_i - \bar{p}_{Ci}(t)$, $p_i - \bar{p}_{Di}(t)$ depend on time in which we conduct the analysis, the pulse rate, and the amplitude of the pulse for which we determine the distributions. For the strongest pulse η_1 , the differences between the imposed distribution and the calculated one, no matter which oscillator is taken into consideration, are the smallest, and for the weakest pulse η_3 the differences are the largest. These conclusions are confirmed by statistical investigations discussed in the next section of the paper.

4. Statistical investigations

In this chapter we discuss a part of the investigations consisting in performing 100 simulations in both

distributions Φ_1 and Φ_2 for four different values of the pulse rate λ equal to 10, 10^2 , 10^3 , and 10^4 s^{-1} , in order to draw the reader’s attention to the classification of signals from the point of view of their statistical properties. In the study we analyze the measures of central tendency describing the location of the differences $p_i - \bar{p}_i(t)$ as well as the measures of dispersion describing the dispersion of the results for each of the simulations on the time interval from 0 to 900 seconds with a step of 6 seconds. The results of analyses show that regardless of the distribution Φ_i or the pulse rate λ , while analyzing the *mean value*, *median*, *standard deviation*, *kurtosis*, and *skewness* for the histograms from the differences $p_i - \bar{p}_i(t)$ which follow from the simulation, we come to conclusions presented below.

1. The strongest pulses η_1 are determined with the least error, while the weakest η_3 are determined with the largest error (Fig. 5, Fig. 6, Table 1, Table 2).
2. The smallest differences $p_i - \bar{p}_i(t)$ of all pulses η_i were registered for the system with strong damp-

ing, short pulse response, and high values of the modified frequency of vibrations c – oscillator B (Fig. 5, Fig. 6).

- At low intensities λ (Fig. 7), the impact of a single pulse or a series of subsequent pulses is so high that the standard deviation σ read from the histograms

may not decrease with the passage of time as one could expect, for instance:

$$\sigma(p_{A1} - \bar{p}_{Ai}(180\text{ s})) > \sigma(p_{A1} - \bar{p}_{Ai}(120\text{ s}))$$

and

$$\sigma(p_{B2} - \bar{p}_{B2}(840\text{ s})) > \sigma(p_{B2} - \bar{p}_{B2}(780\text{ s})).$$

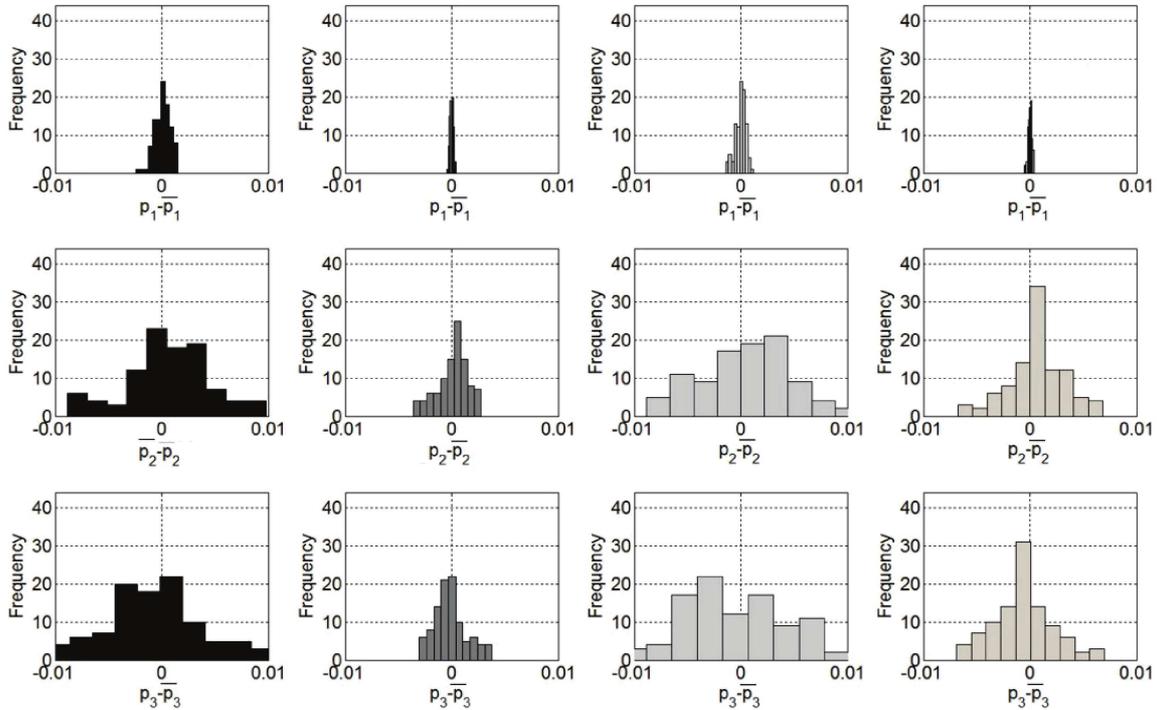


Fig. 5. Histograms of $p_i - \bar{p}_i(600\text{ s})$, where $p_i = p(\eta_i)$, corresponding to $\lambda = 10^2\text{ s}^{-1}$, Φ_2 . Consecutive columns of the panels correspond to cases A, B, C , and D , respectively.

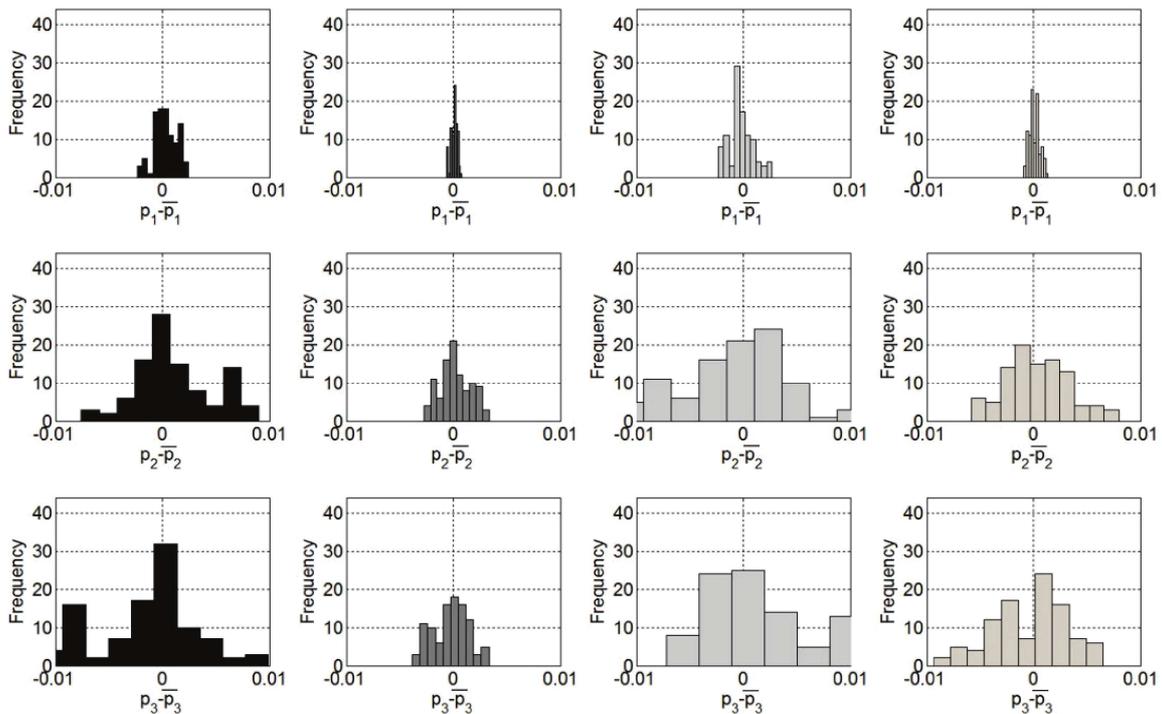


Fig. 6. Histograms of $p_i - \bar{p}_i(900\text{ s})$, where $p_i = p(\eta_i)$, corresponding to $\lambda = 10^4\text{ s}^{-1}$, Φ_1 . Consecutive columns of the panels correspond to cases A, B, C , and D , respectively.

Additionally, for low intensities for the pulse and

$$\eta_1 \sigma(p_{B1} - \bar{p}_{B1}(t)) > \sigma(p_{D1} - \bar{p}_{D1}(t)), \quad \sigma(p_{B3} - \bar{p}_{B3}(t)) < \sigma(p_{D3} - \bar{p}_{D3}(t)).$$

but for weaker pulses η_2 and η_3 the relationships are reversed

$$\sigma(p_{B2} - \bar{p}_{B2}(t)) < \sigma(p_{D2} - \bar{p}_{D2}(t))$$

This means that although the strongest pulse is determined more precisely, the remaining ones are biased with a larger error.

Table 1. Basic statistics corresponding to histograms of $p_i - \bar{p}_i(600 \text{ s})$, $\lambda = 10^2 \text{ s}^{-1}$, Φ_2 .

Statistics	<i>i</i>	Case			
		<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
Mean	1	5.1190×10^{-5}	-2.2000×10^{-7}	-1.1620×10^{-5}	-4.5500×10^{-6}
	2	5.9253×10^{-4}	1.1933×10^{-4}	9.7761×10^{-4}	6.0732×10^{-4}
	3	-6.4375×10^{-4}	-1.1909×10^{-4}	-9.6592×10^{-4}	-6.0273×10^{-4}
Standard deviation	1	7.4935×10^{-4}	1.5673×10^{-4}	5.0549×10^{-4}	1.7824×10^{-4}
	2	3.8564×10^{-3}	1.4565×10^{-3}	4.8441×10^{-3}	2.8022×10^{-3}
	3	4.4298×10^{-3}	1.5428×10^{-3}	5.0882×10^{-3}	2.8696×10^{-3}
Median	1	9.3000×10^{-5}	-1.5000×10^{-5}	4.6500×10^{-5}	8.5000×10^{-6}
	2	6.6900×10^{-4}	3.8250×10^{-4}	1.3300×10^{-3}	5.0750×10^{-4}
	3	-6.0650×10^{-4}	-2.8650×10^{-4}	-1.4450×10^{-3}	-5.6950×10^{-4}
Skewness	1	-4.2739×10^{-1}	2.7144×10^{-2}	-6.1966×10^{-1}	-3.1756×10^{-1}
	2	-1.8468×10^{-1}	-5.6804×10^{-1}	2.4805×10^{-1}	-2.5130×10^{-1}
	3	1.7856×10^{-1}	4.9931×10^{-1}	-2.2059×10^{-1}	2.9918×10^{-1}
Kurtosis	1	3.1574	3.0928	3.3301	3.0170
	2	3.1598	2.9591	2.9311	3.4330
	3	3.1631	3.0240	2.8883	3.4532

Table 2. Basic statistics corresponding to histograms of $p_i - \bar{p}_i(900 \text{ s})$, $\lambda = 10^4 \text{ s}^{-1}$, Φ_1 .

Statistics	<i>i</i>	Case			
		<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
Mean	1	2.6827×10^{-4}	5.6990×10^{-5}	-2.0349×10^{-4}	9.3290×10^{-5}
	2	1.1186×10^{-3}	1.5275×10^{-4}	-3.4724×10^{-4}	3.3904×10^{-4}
	3	-1.3868×10^{-3}	-2.0977×10^{-4}	5.5085×10^{-4}	-4.3229×10^{-4}
Standard deviation	1	1.0841×10^{-3}	3.0540×10^{-4}	1.1843×10^{-3}	4.9184×10^{-4}
	2	3.9104×10^{-3}	1.4346×10^{-3}	5.3374×10^{-3}	3.0309×10^{-3}
	3	4.9081×10^{-3}	1.6926×10^{-3}	6.3831×10^{-3}	3.4921×10^{-3}
Median	1	1.6750×10^{-4}	1.2350×10^{-4}	-3.6250×10^{-4}	4.3000×10^{-5}
	2	5.7300×10^{-4}	-1.1500×10^{-5}	-2.1400×10^{-4}	-1.3050×10^{-4}
	3	-5.6750×10^{-4}	-1.5850×10^{-4}	2.2500×10^{-4}	2.4500×10^{-4}
Skewness	1	-1.1069×10^{-1}	-3.2801×10^{-1}	2.9302×10^{-1}	2.3606×10^{-1}
	2	1.9344×10^{-1}	1.3264×10^{-1}	1.7494×10^{-1}	2.9513×10^{-1}
	3	-1.4594×10^{-1}	-5.2782×10^{-2}	-1.9689×10^{-1}	-3.0584×10^{-1}
Kurtosis	1	2.6717	2.6814	2.9406	2.4178
	2	2.7303	2.3847	3.1724	2.7182
	3	2.8337	2.3998	3.2121	2.6865

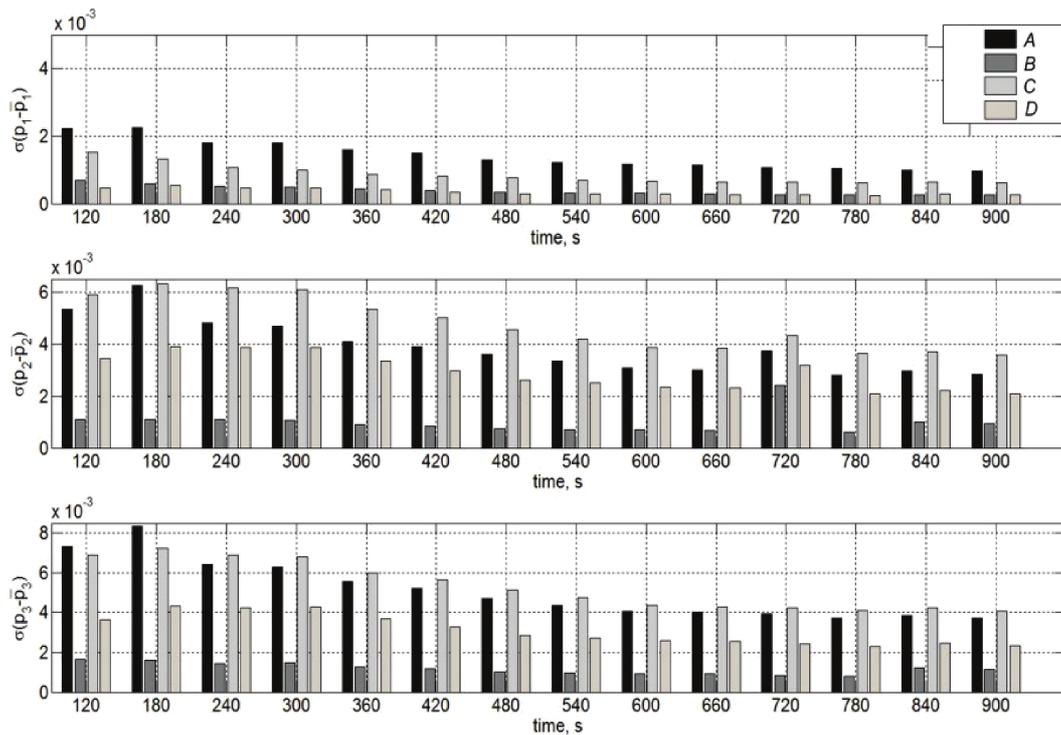


Fig. 7. Standard deviations $p_i - \bar{p}_i(t)$ corresponding to $\lambda = 10 \text{ s}^{-1}$ and Φ_1 .

5. Conclusions

The applied model of investigations was supposed to provide an answer to the question about how to select parameters of a vibrating system so that to eliminate the difference between the actual distribution of random pulses and that determined from the waveform as much as possible. Statistical investigations performed for 100 simulations for each of the two distributions Φ_1 and Φ_2 for four different values of the pulse rate λ equal to 10 , 10^2 , 10^3 , and 10^4 s^{-1} , over the time interval from 0 to 900 seconds with the step of 6 seconds proved that regardless of the distribution Φ_i or the pulse rate λ , the smallest differences $p_i - \bar{p}_i(t)$ of all pulses η_i were registered for the system with strong damping, short pulse response, and high value of the modified frequency of vibrations c represented by oscillator B (Fig. 5, Fig. 6).

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