Comparing simulation results of a structure defined mathematical model of aircraft

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Abstract: This paper is focused on creation, accuracy and simulation of 2-parameter control of a mathematical model for motion of aircraft in a flying simulator. We are discussing many of important advances in applied aircraft modeling. Modelling on various computer architectures (central, parallel, distributed) has an impact on a structure of a simulator system of aircraft. The way of description of a numerical method and its accuracy, a shared memory system and a distributed memory system, is an important part. Necessary accuracy of implemented simulation methods, an analytical approach to definition of mathematical models, and corresponding simulation implementation architectures are presented in the article.

Key words: accuracy of integration, structure defined systems, control modules, shared and distributed computing, modelling and simulation

1. Introduction

Mathematical models are mostly used in natural sciences (physics, chemistry, earth science) and engineering disciplines (computer science, biological science, genetic engineering), as well as in social sciences. Creation, design, visualization and simulation of a continuous mathematical model of aircraft motion in a flying simulator was described in the published paper [11]. Design, integration accuracy and simulation of 1-parameter control of a distributed mathematical model for motion of aircraft in a flying simulator was described in the published paper [12].

A mathematical model of motion of aircraft can be simulated using central computer architecture, which can be based on single-processor systems. A structure defined mathematical model of aircraft can be simulated using central and parallel computer architecture. Parallel computer architecture can be based on multiprocessors; each processor is of multicore architecture. Multi-processor systems are computationally more powerful than such systems compared to central computer architecture, see [7].

As it is known from the specialized literature, the creation of mathematical models for motion of aircraft consists of the following phases: definition of a physical base for creation of...
a mathematical model – and selection of notation of a mathematical model of motion. The Laplace transformation, computation of parameters of aircraft for a selected flight phase, determination and computation of coefficients, numerical integration, programming and simulation on a computer, etc. are used in our procedure suitably.

Results of the document “Visualization of Aircraft Longitudinal-Axis Motion” – [11] will be used in this paper for the aforementioned purposes. The effort invested to the creation of mathematical models of motion in the abovementioned document is aimed at improvement and extension of a field of view in simulation of mathematical models on different types of computer architecture.

2. Description of aircraft mathematical models

Such unstable characteristics as aero elasticity impact, fuel density, changing geometry of aircraft and some other parameters support complexity of their design. Mathematical models of motion interact with intervention of pilot’s control of aircraft and real equipment responds to the pilot’s interventions, data on equipment is observed by the pilot [11]. We can use a continuous simulation method to solve differential equations in mathematical models of motion of aircraft created this way; aerospace engineers often use Newton’s laws of motion in design and creation of a mathematical model of motion of aircraft and relationships of equations are described by differential operators [12].

According to 3-rd Newton’s Law: to every action there is always an equal opposite reaction, or the mutual action of two bodies upon each other is always directed to opposite parts [5]. The basic system of equations has the form [1]:

$$\dot{x}_i + f_i(x_1, \ldots, x_n, u_1, \ldots, u_m; \xi_1, \ldots, \xi_p) = 0, (i = 0, 1, \ldots, p) \tag{1}$$

where: $x_1, \ldots, x_n$ are the object coordinates, $u_1, \ldots, u_m$ are the elements of control, $\xi_1, \ldots, \xi_p$ are the failure functions. Troubles of simulation and synchronization of mathematical models on computers and appropriation of using a linear model of motion are discussed in the part “Mathematical Models of Aircraft and Physical Bases of Mathematical Models” [11]. Precision between results of transient response of nonlinear mathematical models if compared to linear mathematical models from the point of view of human precision can be neglected [17].

The following items: $a^{H_1} \Delta H, a^{H_2} \Delta H, a^{H_m} \Delta H$, for flight speed $\Delta V$ and other parameters of aircraft motion, seem to be the weakest couples for flight height in a linear model of aircraft motion. The equations of a mathematical model of motion of aircraft have two control parameters, the form [10]:

$$\Delta V + a^V \Delta V + a^\alpha \Delta \alpha + a^\theta \Delta \theta = a^M \Delta M,$$

$$\Delta \theta + a^V \Delta V + a^\alpha \Delta \alpha + a^\theta \Delta \theta = a^M \Delta M,$$

$$\Delta \omega_c + a^V \Delta V + a^\alpha \Delta \alpha + a^\theta \Delta \theta + a^\omega_c \Delta \omega = a^\mu \Delta \mu,$$

$$\Delta \omega = \omega_c, \Delta \theta = \Delta \theta + \Delta \alpha. \tag{2}$$
Equations for numerical integration of a mathematical model

Numerical integration is applied in calculation of a mathematical model of motion of aircraft as shown below. A state-space model is as follows:

\[ \dot{x}(t) = f(x(t), u(t), t), \]

where: \( x \) is a state vector, \( u \) is an input vector, and \( t \) represents time, with a set of initial conditions:

\[ x_{i_{0}} = x_{0}. \]

Equation (6) represents, in a form of a matrix, a complex aircraft dynamic system of a mathematical model of a flying simulator comprising in a process of simulation of 11 state variable sensors of information, 18 state variables that express situation coordinates of performing elements in the system. They are divided into two halves and the rest is divided into 38 state variables that represent unmeasured noise and sensor failures [1]. Four parts of the mathematical models of motion of aircraft are expressed by the state vector \( n = 4 \) that represents a state matrix. In accordance with the Equation (2), we get [10]:

\[ \dot{x} = Ax + Bu, \]
\[ y = Cx. \]
\[ \Delta \dot{V} + a_4^v \Delta V + a_5^v \Delta a + a_6^v \Delta \theta + a_7^v \Delta H = a_8^v \Delta \delta_m, \]
\[ \Delta \theta + a_4^\theta \Delta V + a_5^\theta \Delta a + a_6^\theta \Delta \theta + a_7^\theta \Delta H = a_8^\theta \Delta \delta_v, \]
\[ \Delta \dot{H} - \sin (\theta_0) \Delta V - \cos (\theta_0) \Delta V = 0; \ \Delta \dot{\omega} = \omega, \ \Delta \theta = \Delta \theta + \Delta \alpha. \]

The items \( \Delta V, \Delta \alpha, \Delta \theta, \Delta \delta_m \) and \( \Delta \delta_v \) are defined in [12]. Transfer functions have the form [10]:
\[ \Delta V(s) = -G_{V/sM}(s)\Delta \delta_m(s) - G_{V/sV}(s)\Delta \delta_v(s), \]
\[ \Delta \alpha(s) = -G_{a/sM}(s)\Delta \delta_m(s) - G_{a/sV}(s)\Delta \delta_v(s), \]
\[ G_{V/sM}(s) = a_4^{sM}(\Delta_{1M}), \ G_{V/sV}(s) = a_4^{sV}(\Delta_{2V}) - a_5^{sV}(\Delta_{3V}), \]
\[ G_{a/sM}(s) = a_4^{sM}(\Delta_{1M}), \ G_{a/sV}(s) = a_4^{sV}(\Delta_{22}) - a_5^{sV}(\Delta_{21}), \]

where \( \Delta_{ij} \) is the sub-determinant of the \( i \)th row and the \( j \)th column. In our described procedure, the first equation of the system (8) is employed to describe the change of the speed \( \Delta V \), also the equation describing displacement of the speed depending on the displacement of throttle engine lever \( G_{V/sM}(s) \) and the displacement of elevator angle \( G_{V/sV}(s) \) are employed. The items of the second equation of system (8) \( G_{a/sM}(s) \) and \( G_{a/sV}(s) \) are employed in [11]. The third row of the system (8) defines how to compute these changes of two parameters for a change of aircraft speed respectively the fourth row of the system (8) defines how to compute these changes of 2-parameters for a change of an aircraft angle of attack.

In our case, flight of the aircraft is steady and without any random interferences (wind, storm, or other outer interferences) [6]. Coefficients \( c_i \) and \( e_j \) represent aerodynamic parameters, see [11] for their computation.

### 3.1. A structure defined mathematical model of an aircraft in a simulator

For design of a mathematical model, we apply a matrix form in state space described in the previous section. We should express – the first equation from Equation (6) as follows [13]:

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4
\end{bmatrix} =
\begin{bmatrix}
a_{11} & a_{12} & a_{13} & a_{14} \\
a_{21} & a_{22} & a_{23} & a_{24} \\
a_{31} & a_{32} & a_{33} & a_{34} \\
a_{41} & a_{42} & a_{43} & a_{44}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{bmatrix}.
\] (9)

Let’s decompose the given system – mathematical models of motion of aircraft into four subsystems, see [12]. The state space is divided into 4 parts:

\[ x = (x_1, x_2, x_3, x_4)^T = (x_{11}, x_{12}, x_{13}, ..., x_{44}), \]

where items \( x_i, x_j \) represent a state vector. If variable \( i \) stands for an order of relevancy \( n \), i.e. the number of the subsystem, then variable \( j \) stands for a sequential number of the item in the given model parts. The mutual relations between the first and second isolated subsystems are described by \( I_{12}(x) \) meaning that the equation of the first and second isolated subsystem is:
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\[ I_{12}(x) = A_{11} \begin{pmatrix} x_1 \\ x_2 \\ 0 \\ 0 \end{pmatrix} = (A_{11}, A_{12}, A_{13}, A_{14})^T \begin{pmatrix} x_1 \\ x_2 \\ 0 \\ 0 \end{pmatrix}. \]  

(11)

The described solution is relatively simple due to description of the analyzed system using Equations (9), see [2] for more information.

4. A mathematical model of speed and angle of attack in a flight simulator

To create a system of differential equations, one must know aerodynamic coefficients, a mathematical model of aircraft systems and other parameters of aircraft [11, 12]. The mathematical model of motion of aircraft in a flight simulator is created by this approach in the Laplace transformation, see [2, 10].

Fig. 1. Block diagram of a mathematical model of control of aircraft motion – speed and angle of attack

The action of an aircraft elevator control stick angle and throttle control stick displacement on aircraft motion – two control parameters (see Fig. 1) are discussed below. The picture shown in [11] represents a block diagram of 1-parameter control of a mathematical model of motion of aircraft in a flying simulator. Figure 1 in this paper represents a block diagram of the already mentioned 2-parameter control of a mathematical model of motion of aircraft in a flying simulator.

4.1. A structure defined mathematical model of aircraft in a simulator

The input values are represented by a change of an aircraft elevator control stick angle and a throttle control stick of the engine. The values are adjusted according to the required ones and are forwarded to the input of the mathematical model of motion of aircraft – speed. The aircraft speed \( \Delta V \) displacement equation defines a change in fuel supply and a change of the angle of an aircraft elevator [10]:

\[ \Delta V(s) = -G_{V_{1/0M}}(s)\Delta \delta_v(s) - G_{V/AV}(s)\Delta \delta_e(s). \]

(12)
where $G_{V/M}(s)$ defines the mathematical model – a transfer function for fuel supply, $\Delta \delta_M(s)$ is the input function for fuel supply in the Laplace transformation, $G_{V/V}(s)$ is the mathematical model – a transfer function for an aircraft elevator, $\Delta \delta_V(s)$ stands for the input function of an aircraft elevator angle in the Laplace transformation. Derivation of equations of a mathematical model of speed increment is shown in the “Visualization of Aircraft Longitudinal-Axis Motion” [12]. Information about the mathematical solution of these equations is known [4]. The items for $l_{12}(x)$ defined by Equation (11) are as follows, where:

$$A = s^4 + 1.134 s^3 + 62.798 s^2 + 28.659 s + 4.093$$

is the same in all equations.

$$A_{11} = \frac{5 s^3 + 1.12 s^2 + 62.782 s + 25.32}{A}, \quad x_1 = \Delta \delta_M(s),$$

(13)

$$A_{12} = -\frac{0.11 \cdot (9.81 s + 620.973) - 0.42 \cdot (-9.81 s - 10.006)}{A}, \quad x_2 = \Delta \delta_V(s).$$

(14)

### 4.2. The angle of attack dependence

From the description in paper [12], we can derive that the equation of the angle of attack displacement defines a change in the angle of an aircraft elevator and a change of fuel supply [10]:

$$\Delta \alpha(s) = -G_{\alpha/M}(s) \Delta \delta_M(s) - G_{\alpha/V}(s) \Delta \delta_V(s).$$

(15)

where the items are defined in [12]. Stability determined by zeroes of a characteristic equation is used as a numerator in the mathematical model of motion of aircraft, see [11]. Next, we define permutation and transformation with regard to Equation (11) and we have coefficients for $l_{21}(x)$:

$$A_{21} = \frac{5 \cdot 0.002 s^3 - 0.252 s - 0.1}{A}, \quad x_1 = \Delta \delta_M(s),$$

(16)

$$A_{22} = -\frac{0.11 \cdot (s^2 + 0.886 s^2 + 0.0124 s - 2.453) - 0.42 \cdot (s^2 - 0.414 s - 0.025)}{A}, \quad x_2 = \Delta \delta_V(s).$$

(17)

Equations (13) and (16) will use a step change of fuel supply in the Laplace transformation $\Delta \delta_M(s) = 1/s$. Equations (14) and (17) will use a step change of aircraft elevator angle in the Laplace transformation $\Delta \delta_V(s) = 1/s$. The next design of a mathematical model of motion in a flying simulator is conditioned by identification of its stability. Roots of the characteristic equation, denominator Equations (13), (14), (16), respectively, Equation (17), see [11].

### 5. Visualization and simulation of models

Initial or limiting restricting conditions in the given flight phase affect the form of equations of the system depending on for which phase of aircraft motion they are calculated,
says McCormic et al. [15]. Visualization of results has influence on quality simulation, Yauan writes: “simulation attempts to get the information on properties of a real system by means of an experiment, the so-called simulation model” [19]. “Computer simulation of a flying simulator is employed as enlargement or replacement of a mathematical model of aircraft motion for which an analytical solution is difficult or even impossible” – Stevens [18].

Sequential run of a mathematical model program is characterized by equations of simulation of aircraft motion in single computer time in equidistant moments. A disadvantage of this method is a power constraint of a processor that computes the mathematical models of motion, see [8]. For presentation of more accurate simulation results, we need a higher-quality visualization system such as a visualization generator providing artificial surrounding of required quality; this surrounding is a 3-dimensional scene, see Figure 2.

Fig. 2. Principle of a pilot’s activity and its visualization in a projection system

5.1. Application of mathematical models

A simulation problem can be realized using shared memory system (SMS) architecture (OpenMP Control) or distributed memory system (DMS) architecture (MPI Control); they are identified as node computers [14].

According to Equation (12) or (15), the first member represents a transfer function (aircraft mathematical model) of speed displacement with dependence on fuel supply of aircraft with “–” sign, respectively and a transfer function (aircraft mathematical model) of the angle of attack displacement with dependence on fuel supply of aircraft with “–” sign. In a polynomial format of transfer function, we induce the following form for a transfer function of computed speed of displacement from fuel supply in meters per second respectively of the computed displacement of the angle of attack from fuel supply in radians [11]:

\[ \text{Transfer Function} = \frac{\text{Output}}{\text{Input}} \]
If displacement of the speed is considered in Equations (18) and (19), respectively, and this is conditioned by the step of change in fuel supply (unit step), the meaning of the items is defined in [12].

According to Equation (12) or (15) the second member represents a transfer function of speed displacement with dependence on the aircraft elevator angle with “...” sign, respectively, and a transfer function of the angle of attack displacement with dependence on the aircraft elevator angle with “...” sign. In a polynomial format of transfer function, we induce the following form for a transfer function of computed speed displacement from the aircraft elevator in meters per second respectively of the computed displacement of the angle of attack from the aircraft elevator in radians [11]:

$$-G_{\text{v1,3s}}(s)\Delta \delta_{\text{m}}(s) = -\frac{0.002 s^2 - 0.252 s - 0.1 M}{A} s,$$

$$-G_{\text{u1,4s}}(s)\Delta \delta_{\text{u}}(s) = -\frac{0.11 \cdot (9.91 s + 620.973) - 0.42 \cdot (-9.91 s - 10.006) V}{s},$$  

The displacement of elevation is considered in Equation (20) or (21), respectively, and this is conditioned by the step of elevator angle (unit step), the meaning of the items is defined in [12]. The simulation in our solution takes 30 seconds and the intermediate data is sent in periodical time to the processor’s core or node processing recorded simulated data and creating a graphical form of calculated results after the end of simulation.

5.2. Parallel application of mathematical models on a shared memory system

The simulation of a mathematical model of motion described above was also realized on a computer by SMS based on the OpenMP standard that supports parallel programming in C/C++. The presented system is modelled on a personal computer that consists of a CPU Intel Quad Core Q9450 processor with 4 cores, 2.66 GHz each, 2GB RAM DDR3, 1066 MHz.

In a serial mode, tasks run sequentially on available sources in the nodes. Two-parameter control values of mathematical models – fuel supply and an aircraft elevator angle, represent an input in a block diagram. Two-parameter control simulated values of mathematical models of motion – speed of aircraft (two models) and an angle of attack of aircraft (two models) represent an output in a block diagram. If we use more processor cores of a simulator system in our solution (C1, C2,..., Cn), they communicate with each other by means of an SM, see Fig. 3.

One core (C1) is designed as a central core, the others are computing ones and each of them is calculating only one mathematical model of motion of aircraft. As it results from the expression, the mathematical model defined by Equation (18) A_{11} \cdot x_1 is simulated by the C_1 core, Equation (19) A_{12} \cdot x_2 is simulated by the C_2 core, the mathematical model defined by Equation (20) A_{21} \cdot x_1 is simulated by the C_3 core respectively Equation (21) A_{22} \cdot x_2 is simulated by the C_4 core. Results are shown in Section 5.4.
5.2. Parallel application of mathematical models on a distributed memory system

The Message Passing Interface (MPI) systems provide alternative methods for communication and movement of data among multiprocessors [9]. More information is shown in [12]. The MPICH2 implementation is portable, high-performance implementation of the entire MPI-2 standard and consists of a library of routines that can be called from the program [16].

In such a system, there are $n$ nodes, which consist of a processor $P$ and a local memory $M$. The program of a mathematical model of motion of aircraft is divided into concurrent processes, each is executed in a separate processor, see Figure 4. Distributed architecture was realized as connection of five nodes (one is a central computer, the others are computing nodes), this simulation obtains results faster. Each computing node consists of a personal computer with a CPU Athlon X2 processor with two cores – a single processor system that shares one memory. The processor’s core frequency is 2.6 GHz and the memory size is 2 GB RAM, 1.066 MHz. All nodes are interconnected via a 1 Gbit/s Ethernet card. One node ($N_1$) is designed as a central computer, the others are computing ones and each of them is calculating only one equation of a mathematical model of motion.

5.3. Results of simulation of mathematical models

Fig. 5 shows that the following results were obtained: picture a shows the speed increment depending on fuel supply and it is equal to 31.0192 $[m/s]$, picture b shows a speed increment depending on fuel supply and a steady state is equal to 31.0174 $[m/s]$. Like in Figures 6, 7 and 8.

All numbers mean steady state values, see Table 1 for visualizations. Graphical presentation of a speed increment depending on fuel supply respectively an elevator in Figure 5a
respectively in Figure 5b is identical to the presentation in these figures, see Table 1. It can be seen from graphical presentations and dependences in other figures that the results are identical, see Table 1. For comparison, graphical results can be compared with those in [11] and [12].

![Graph](image1)

Fig. 5. Simulation results of methods solved for Equation 18): shared memory system (a); distributed memory system (b)

![Graph](image2)

Fig. 6. Simulation results of methods solved for Equation (19): shared memory system (a); distributed memory system (b)

![Graph](image3)

Fig. 7. Simulation results of methods solved for Equation (20): shared memory system (a); distributed memory system (b)

The general – accuracy from comparison of simulation results from using the two methods is shown in Table 1, resulting in accuracy less than 0.002.
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6. Conclusions

The paper introduces two main types of architecture for efficient simulation of a mathematical model of motion of aircraft: the SMS (central computing) and the DMS (parallel computing). This helps to overcome physical and architectural limitations of computational power that can be achieved with a single-processor system and tasks run sequentially.

The use of a processor, a faster cache memory, operating memory access, 64-bit computer architecture and a higher transmission capacity are then very suitable for the application. The parallel computing architecture provides a higher transmission capacity and higher speed of computation of simulation of a mathematical model for motion of aircraft in a flight simulator.

The two simulation methods of mathematical models of motion of aircraft are also a combination of both the advantages: efficiency and ease of programming of a shared-memory method and scalability of a distributed-memory method.

The simulation results of 2-parameter control of a mathematical model of motion of aircraft in a flight simulator verify higher accuracy depending on 2-parameter control compared to 1-parameter control of a mathematical model of motion of aircraft in a flight simulator. The aforementioned confirms the influence of an aircraft elevator on a speed displacement and also the influence of a change in fuel supply on an angle of attack displacement.
References