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Bayesian Stochastic Frontier Analysis of Economic Growth and Productivity Change in the EU, USA, Japan and Switzerland

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Abstract

The paper discusses Bayesian productivity analysis of 27 EU Member States, USA, Japan and Switzerland. Bayesian Stochastic Frontier Analysis and a two-stage structural decomposition of output growth are used to trace sources of output growth. This allows us to separate the impacts of capital accumulation, labour growth, technical progress and technical efficiency change on economic development. Since estimates of the growth components are conditioned upon model parameterisation and the underlying assumptions, a number of possible specifications are considered. The best model for decomposing output growth is chosen based on the highest marginal data density, which is calculated using adjusted harmonic mean estimator.

Keywords: stochastic frontier analysis, Bayesian inference, productivity analysis, economic growth decomposition

JEL Classification: C11, C23, O47, O52

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1 Introduction

The concept of a frontier analysis (FA) was first coined by Koopmans (1951) and Debreu (1951). Their works presented the theoretical basis later used by Farrell (1957) in his pioneering work on efficiency analysis of the US agriculture industry. Today, among many other fields of application, FA is used as a tool for macro-scale productivity analyses. The idea is to use the concept of a production frontier in order to compare entire economies as producing a mutually comparable product (e.g., GDP) using a set of production factors (e.g., physical capital and labour) under a common technology (see, e.g., Growiec, Pajor, Pelle and Prędki, 2011; Growiec 2012a,b; Makieła 2009, 2012; or Fried, Lovell and Schmidt, 2008; for a lengthy list of applications). In such a model, economic growth (i.e., the increase in GDP from one period to another) is caused either by accumulation of production factors (IC), or by increased productivity (TFP change, labelled PC hereafter). This is a well-known framework, which summarizes what can be seen as a one-stage decomposition of output growth. However, one may also want to further decompose changes in TFP (PC) into technical efficiency (EC) and technical progress (TC) contributions. This framework has been first used in the context of a frontier analysis by Färe, Grosskopf, Norris and Zhang (1994), who have used Data Envelopment Analysis (DEA) to analyse economic growth of selected countries. Later, Koop, Osiewalski and Steel (1999) have proposed a Bayesian approach to derive components of output growth. More recently, however, researchers' attention has turned to investigating the impact of capital accumulation on economic growth, especially in the context of the EU Member States (see, e.g., Salinas-Jiménez, Alvarez-Ayuso and Delgado-Rodriguez, 2006; or Makieła 2012 in the context of EU15). Hence, one may also want to decompose the IC component in order to analyse impact of each production factor separately.

Many researchers seem to prefer DEA as a tool for macro-scale productivity analysis nowadays (see, e.g., Färe, Grosskopf and Margaritis, 2006; Margaritis, Fare and Grosskopf, 2007; Growiec 2012a,b). The argument is that, being a nonparametric approach, DEA does not require imposing any structure on the production frontier. However, in DEA the production frontier is estimated as a piece-wise linear function which significantly constraints the analysis.

The above limitations give way for Bayesian Stochastic Frontier Analysis (BSFA hereafter). BSFA has several advantageous features distinctive to Bayesian inference. First, being a stochastic approach, it is less affected by outlying observations and nuisance in the data (Fried, Lovell and Schmidt, 2008). Simar and Zelenyuk (2011) have proposed a stochastic class of DEA estimators which mitigate this problem. See also Prędki (2012) and Kuosmanen and Kortelainen (2012) for other propositions of bridging the gap between parametric and non-parametric methods. Second, it allows us to obtain exact small sample results, which is of particular importance in relatively small macroeconomic datasets. Third, we can easily impose economic regularity conditions on the production function and test for additional restrictions (e.g. constant returns to scale) or possible model simplifications. Fourth, if a proper model is chosen, parametric methods yield more information. Model choice, however, is often made *ad hoc*, which raises concerns because the final results are conditioned not only on the data but on the parametric specification as well. This makes Bayesian approach even more appealing as it allows us to formally compare competing model specifications and to choose the best model on the basis of model probabilities given the data. In particular much progress has been made recently to overcome numerical issues that have been troubling the harmonic mean estimator (HME) used for purposes of model comparisons. Lenk (2009) shows that the so-called pseudo-bias of HME can be corrected if we take into account differences between numerical representations of prior and posterior supports. Osiewalski and Osiewalski (2013) propose practical way of computing Lenk's correction in highly dimensional space of parameters and latent variables while Pajor and Osiewalski (2013) refine Lenk's concept on improving the performance of computed harmonic mean estimator.

The aim of this work is to use Bayesian Stochastic Frontier Analysis (BSFA) in order to, first, select the best model given the data, and then use it to trace changes in economic growth patterns among 27 EU Member States, USA, Japan and Switzerland in 1996-2010. In doing so, this paper builds on previous work by Makieła (2009) extending it twofold. First, the output decomposition methodology proposed by Koop, Osiewalski and Steel (1999) in the context of BSFA is extended by additional decomposition of IC component. This allows us to separately trace changes in capital and labour contributions to economic growth. Second, unlike Makieła (2009) who builds his analytical conclusions around only one model we take full advantage of the Bayesian approach and consider a set of 42 plausible model specifications. Following Makieła (2012) we use the Bayesian criterion for choosing the optimal model under equal prior probabilities – the highest marginal data density. Unlike in Makieła (2012), however, the harmonic mean estimator is corrected for pseudo-bias. Since this has not been done before in the context of BSFA, we report on our findings from such pseudo-bias correction. Also, unlike Makieła (2012) we use Lindley-type testing, which allows us to obtain some additional insights into model parameterisation dilemma.

The data used in the analysis come from AMECO database, supervised by Directorate-General for Economics and Finance of the European Commission. These are GDP in mld in Purchasing Power Standard (PPS) in 2000 constant prices, net capital stock in mld PPS in 2000 constant prices and total number of hours worked annually in a given country (in thousands). The paper summarizes some of the key aspects of a more extensive study from the author's Ph.D. dissertation and it is structured as follows. BSFA models used in the study are presented in Section 2. Next, in Section 3 we provide details on structural decomposition methodology and, in Section 4, describe the method used for choosing the optimal model. Decomposition results, obtained using the best model, are discussed in Section5 and concluding remarks are summarized in Section 6.

2 BSFA models considered in the study

Let Y_{it}, K_{it} and L_{it} be levels of production, capital and labour respectively in i-th country ($i = 1, \ldots$, N) in t-th period ($t = 1, \ldots, T$), and lowercase letters y_{it}, k_{it} and l_{it} indicate their natural logs. The general model takes the following form

$$y_{it} = h_t\left(k_{it}, l_{it}; \beta\right) + v_{it} - u_{it} \tag{1}$$

where $h_t(\cdot)$ is the log form of a production function (possibly time-varying according to changing technology), v_{it} are independent normally distributed variables with zero mean and an unknown precision σ^{-2}, and u_{it} are independent identically distributed nonnegative variables reflecting inefficiency, i.e., technical efficiency is $r_{it} = \exp\left(-u_{it}\right)$ where $0 < r_{it} \leq 1$, and $r_{it} = 1$ is maximum efficiency; v_{it} and u_{js} are stochastically independent for any i, t, j and s. The parametric specifications are

1. M1: Cobb-Douglas production function (labelled CD)

$$h_t\left(k_{it}, l_{it}; \beta\right) = h\left(k_{it}, l_{it}; \beta\right) = \beta_0 + \beta_1 k_{it} + \beta_2 l_{it} \tag{2}$$

2. M2: Cobb-Douglas production function with time trend (labelled CDt)

$$h_t\left(k_{it}, l_{it}; \beta\right) = \beta_0 + \beta_1 k_{it} + \beta_2 l_{it} + \beta_3 t \tag{3}$$

3. M3: Cobb-Douglas production function with linear trend in each parameter of the function (labelled CD-LT)

$$h_t\left(k_{it}, l_{it}; \beta_t\right) = \beta_{t0} + \beta_{t1} k_{it} + \beta_{t2} l_{it}$$

where $\beta_{ta} = \dot{\beta}_a + t\ddot{\beta}_a$ $(a = 0, 1, 2)$. This formula can be rearranged as

$$h_t\left(k_{it}, l_{it}; \beta\right) = \dot{\beta}_0 + \dot{\beta}_1 k_{it} + \dot{\beta}_2 l_{it} + t\left(\ddot{\beta}_0 + \ddot{\beta}_1 k_{it} + \ddot{\beta}_2 l_{it}\right) \tag{4}$$

1. M4: translogarithmic (translog hereafter) production function (labelled TR)

$$h_t\left(k_{it}, l_{it}; \beta\right) = h\left(k_{it}, l_{it}; \beta\right) = \beta_0 + \beta_1 k_{it} + \beta_2 l_{it} + \beta_3 k_{it}^2 + \beta_4 l_{it}^2 + \beta_5 k_{it} l_{it} \tag{5}$$

2. M5: translog production function with time trend (TRt hereafter)

$$h_t\left(k_{it}, l_{it}; \beta\right) = \beta_0 + \beta_1 k_{it} + \beta_2 l_{it} + \beta_3 k_{it}^2 + \beta_4 l_{it}^2 + \beta_5 k_{it} l_{it} + \beta_6 t \tag{6}$$

3. M6: translog production function with linear trend in each parameter (labelled TR-LT)

$$h_t\left(k_{it}, l_{it}; \beta_t\right) = \beta_{t0} + \beta_{t1} k_{it} + \beta_{t2} l_{it} + \beta_{t3} k_{it}^2 + \beta_{t4} l_{it}^2 + \beta_{t5} k_{it} l_{it}$$

where $\beta_{ta} = \dot{\beta}_a + t\ddot{\beta}_a$ $(a = 0, \ldots, 5)$. The formula can be rearranged as

$$\begin{aligned} h_t(k_{it}, l_{it}; \beta) &= \dot{\beta}_0 + \dot{\beta}_1 k_{it} + \dot{\beta}_2 l_{it} + \dot{\beta}_3 k_{it}^2 + \dot{\beta}_4 l_{it}^2 + \dot{\beta}_5 k_{it} l_{it} + \\ &+ t(\ddot{\beta}_0 + \ddot{\beta}_1 k_{it} + \ddot{\beta}_2 l_{it} + \ddot{\beta}_3 k_{it}^2 + \ddot{\beta}_4 l_{it}^2 + \ddot{\beta}_5 k_{it} l_{it}) \end{aligned} \tag{7}$$

1. M7: translog production function with quadratic trend in each parameter (labelled TR-QT):

$$h_t\left(k_{it}, l_{it}; \beta_t\right) = \beta_{t0} + \beta_{t1} k_{it} + \beta_{t2} l_{it} + \beta_{t3} k_{it}^2 + \beta_{t4} l_{it}^2 + \beta_{t5} k_{it} l_{it}$$

where $\beta_{ta} = \dot{\beta}_a + t\ddot{\beta}_a + t^2 \dddot{\beta}_a$ $(a = 0, \ldots, 5)$. Like in TR-LT this can be rearranged as follows

$$\begin{aligned} h_t\left(k_{it}, l_{it}; \beta\right) &= \dot{\beta}_0 + \dot{\beta}_1 k_{it} + \dot{\beta}_2 l_{it} + \dot{\beta}_3 k_{it}^2 + \dot{\beta}_4 l_{it}^2 + \dot{\beta}_5 k_{it} l_{it} + \\ &+ t(\ddot{\beta}_0 + \ddot{\beta}_1 k_{it} + \ddot{\beta}_2 l_{it} + \ddot{\beta}_3 k_{it}^2 + \ddot{\beta}_4 l_{it}^2 + \ddot{\beta}_5 k_{it} l_{it}) + \\ &+ t^2(\dddot{\beta}_0 + \dddot{\beta}_1 k_{it} + \dddot{\beta}_2 l_{it} + \dddot{\beta}_3 k_{it}^2 + \dddot{\beta}_4 l_{it}^2 + \dddot{\beta}_5 k_{it} l_{it}) \end{aligned} \tag{8}$$

Formulas (2-8) can be summarized as $h_t\left(k_{it}, l_{it}; \beta\right) = x_{it}'\beta$ where vector x_{it} is the element of $\mathbf{X}$ and contains the list of arguments appropriate to the given production function specification in (2-8). This work also considers two most widely used distributions of inefficiency term, i.e., exponential and half-normal (Greene, 2008). Thus, we have two "types" of SFA models: normal-exponential (labelled EXP hereafter) and normal-half-normal (labelled NHN hereafter). The full Bayesian EXP-type model is

$$\begin{aligned} & f_N\left(\beta | b, C^{-1}\right) f_G\left(\sigma^{-2} | 0.5 n_0, 0.5 a_0\right) f_G\left(\lambda^{-1} | 1, -\ln(r_0)\right) \cdot \\ & \cdot \prod_{i=1}^{N} \prod_{t=1}^{T} f_N\left(y_{it} | x_{it}'\beta - u_{it}, \sigma^2\right) f_G\left(u_{it} | 1, \lambda^{-1}\right) \end{aligned} \tag{9}$$

where $f_N(.|w, Z)$ is a normal density function with mean w and covariance matrix Z, $f_G(.|w, z)$ is a gamma density function with mean $\frac{w}{z}$ and variance $\left(\frac{w}{z}\right)^2$. The model structure is similar to Koop, Osiewalski and Steel (1999); the difference is in the prior on β. I set $n_0 = a_0 = 10^{-6}$ which leads to a quite flat distribution for σ^{-2} with mean 1 and variance $2 \cdot 10^6$. The r_0 parameter refers to prior median efficiency and it means that we give equal prior chances that technical efficiency of a given country is either greater or smaller than r_0. We set r_0 as 0.6, 0.75 or 0.875. Thus, each parametric specification of an EXP model is estimated for three different settings of r_0. To allow for cross-model comparability b and C^{-1} parameters have been calibrated so that the prior on β shares the following properties in all models: i) average elasticities of capital and labour have prior means 0.5 with prior standard deviation 0.2, ii) neutral technical change has prior mean 0.02 and prior standard deviation 0.01. Economic regularity conditions (nonnegative factor elasticities of production) are imposed through inequalities appropriate to the given parametric specification. It is important to note that for translog models prior standard deviations for factors and scale elasticities (at their means) are slightly higher than in Cobb-Douglas. This is necessary because otherwise the prior covariance matrix is not invertible. Such difference in the prior structure favours Cobb-Douglas functions and may constitute a decision problem if marginal data densities are higher there than in translogs. Fortunately, as we learn later it is not the case here. The model is too complex to analytically derive marginal posterior distributions of its parameters. We can, however, draw from their conditional posterior distributions, which are

$$\begin{aligned}
p\left(\beta|y,X,u,\lambda^{-1},\sigma^{-2}\right) &\propto f_N^J\left(\beta|C_*^{-1}\left[Cb+\sigma^{-2}X'(y+u)\right],C_*^{-1}\right)\\
p\left(\sigma^{-2}|y,X,u,\lambda^{-1},\beta\right) &\propto f_G\left(\sigma^{-2}|\tfrac{n_0+NT}{2},\tfrac{1}{2}\left[a_0+(y+u-X\beta)'(y+u-X\beta)\right]\right)\\
p\left(u|y,X,\lambda^{-1},\sigma^{-2},\beta\right) &\propto f_N^{NT}\left(u|X\beta-y-\sigma^2\lambda^{-1},\sigma^2\cdot I_{NT}\right)I\left(u\in R_+^{NT}\right)\\
p\left(\lambda^{-1}|y,X,u,\sigma^{-2},\beta\right) &\propto f_G\left(\lambda^{-1}|NT+1,\textstyle\sum_{n=1}^{N}\sum_{t=1}^{T}u_{it}-\ln\left(r_0\right)\right)
\end{aligned} \tag{10}$$

where $C_*^{-1}=(C+\sigma^{-2}X'X)^{-1}$ and J is the number of elements in β. Based on the formulas above one can approximate characteristics of the joint and marginal posterior distributions using Gibbs sampler.

The full Bayesian NHN-type model used in the study is

$$\begin{aligned}
&f_N\left(\beta|b,C^{-1}\right)f_G\left(\sigma^{-2}|0.5n_0,0.5a_0\right)f_G\left(\omega^{-2}|5,10\ln^2(r_0)\right)\cdot\\
&\cdot\prod_{i=1}^{N}\prod_{t=1}^{T}f_N\left(y_{it}|x_{it}'\beta-u_{it},\sigma^2\right)f_N\left(u_{it}|0,\omega^2\right)
\end{aligned} \tag{11}$$

where $n_0=a_0=10^{-6}$ and again we estimate the model setting $r_0=0.6$, 0.75 or 0.875 as prior median; the prior on ω^{-2} is as proposed in van den Broeck, Koop, Osiewalski and Steel (1994). Like in the case of EXP, this model is also very complex, and thus the characteristics of joint and marginal posterior distributions must be approximated numerically, e.g., using the derived conditionals and Gibbs sampler. For β and σ^2 the conditionals do not change in comparison to EXP model. Conditionals for the remaining parameters are

$$\begin{aligned}
p\left(\omega^{-2}|y,X,u,\sigma^{-2},\beta\right) &\propto f_G\left(\omega^{-2}|\tfrac{1}{2}NT+5,\tfrac{1}{2}\cdot\sum_{n=1}^{N}\sum_{t=1}^{T}u_{it}+10\ln^2(r_0)\right)\\
p\left(u|y,X,\omega^{-2},\sigma^{-2},\beta\right) &\propto f_N^{NT}\left(u|\frac{\omega(X\beta-y)}{\omega^2+\sigma^2},\tfrac{\omega^2\sigma^2}{\omega^2+\sigma^2}\right)I\left(u\in R_+^{NT}\right)
\end{aligned} \tag{12}$$

As a result we have two model types (NHN, EXP), seven possible parameterisations of the production function (CD, CDt, CD-LT, TR, TRt, TR-LT, TR-QT) and three settings for the prior median efficiency (0.6, 0.75, 0,875) to choose from. This amounts to 42 different parametric variations. Since the number of models is substantial, each model is given a codename as follows: "model type label"/"parametric specification label" followed by prior median given in brackets; e.g., EXP/TR(0.6) is a normal-exponential model with translog production function and prior median 0.6.
The models have been estimated using Gibbs sampler coded in MATLAB. One million draws were taken discarding initial hundred thousand (burn-in process). During each simulation, convergence of the chain to its limiting stationary distribution was monitored using both sequential plots, cusum paths (Yu and Mykland, 1998), multivariate potential scale reduction factor (Brooks and Gelman, 1998) – MPSRF in short, effective sample size (see, e.g., LeSage and Pace 1999; or notes in Table 1) and I-statistic (Raftery and Lewis, 1992). Convergence diagnostics results for selected models are provided in Table 1. Sequential plots have been primarily used to asses if the burn-in stage is long enough, while cusum plots, MPSRF, effective sample size and I-statistic have been used to analyse samplers' mixing speeds. All simulations stabilize way before the end of their burn-in phase and samplers' mixing speeds are either very good (i.e., highly oscillatory cusum paths with low excursion and hardly any difference in comparison to their benchmark paths), or at least satisfactory in the two most complex NHN models – TR-LT and TR-QT. The study also reports that samplers for EXP models are significantly outperforming NHN samplers in terms of their mixing speeds (see results in Table 1; or examples of cusum path plots in Figure 1). Regardless of the model, the number of burn-in cycles and the number of accepted draws could have been smaller if we were to base our analysis on commonly used statistics such as posterior means and posterior standard deviations. In this case, however, long runs are necessary to acquire precise-enough estimates of marginal data densities for all models, especially those where samplers' mixing speeds are relatively low.

3 Structural decomposition of output growth

The difference in the log of GDP between two corresponding periods t and $t+1$ can be written as (Koop et al., 1999)

$$\Delta y_{t+1} = \frac{1}{2}(x_{i,t+1} + x_{it})'(\beta_{t+1} - \beta_t) + \frac{1}{2}(\beta_{t+1} + \beta_t)'(x_{i,t+1} - x_{it}) + (u_{it} - u_{i,t+1}) \quad (13)$$

where the first term reflects output change due to technical progress (or regress), the second is due to change in production factors, and the third reflects changes to technical efficiency. This allows us to derive three main components of output growth

$$IC_{i,t+1} = \exp\left[\frac{1}{2}(\beta_{t+1} + \beta_t)'(x_{i,t+1} - x_{it}))\right] \quad (14)$$

Table 1: Convergence diagnostics for selected models

Model code	MPSRF	EFF.Samp	I-statistic
NHN/TR-LT(0.6)	1.0436	98589	35.22
NHN/TR-LT(0.75)	1.0044	213360	4.41
NHN/TR-LT(0.875)	1.0033	439836	2.17
NHN/TR-QT(0.6)	1.1654	31953	124.24
NHN/TR-QT(0.75)	1.0703	67355	29.00
NHN/TR-QT(0.875)	1.0081	131787	3.33
NHN/TR(0.6)	1.0033	140342	17.59
NHN/TR(0.75)	1.0065	139389	7.10
NHN/TR(0.875)	1.0005	504528	4.65
NHN/TRt(0.6)	1.0052	187174	18.84
EXP/TR-LT(0.6)	1.0006	598162	1.06
EXP/TR-LT(0.75)	1.0021	542466	1.06
EXP/TR-LT(0.875)	1.0005	238675	1.04
EXP/TR-QT(0.875)	1.0005	643600	1.09
EXP/TR-QT(0.75)	1.0047	494745	1.08
EXP/TR-QT(0.6)	1.0438	409184	1.05
EXP/TR(0.6)	1.0020	49205	4.08
EXP/TR(0.75)	1.0033	147683	2.12

Source: author's calculations using CODA diagnostics Toolbox for MATLAB.

Note: MPSRF stands for multivariate potential scale reduction factor; Eff.samp is effective sample size indicates what sample size would be needed to achieve the same level of uncertainty if the chain was white noise. For example, in normal–half-normal model with TR-LT function and 0.6 prior median (i.e., NHN/TR-LT(0.6) model) a sample of 900 000 is an equivalent of 98 589 draws from a chain that is a multivariate white noise; see, e.g., LeSage and Pace (1999)

$$TC_{i,t+1} = \exp\left[\frac{1}{2}(x_{i,t+1} + x_{it})'(\beta_{t+1} - \beta_t)\right] \tag{15}$$

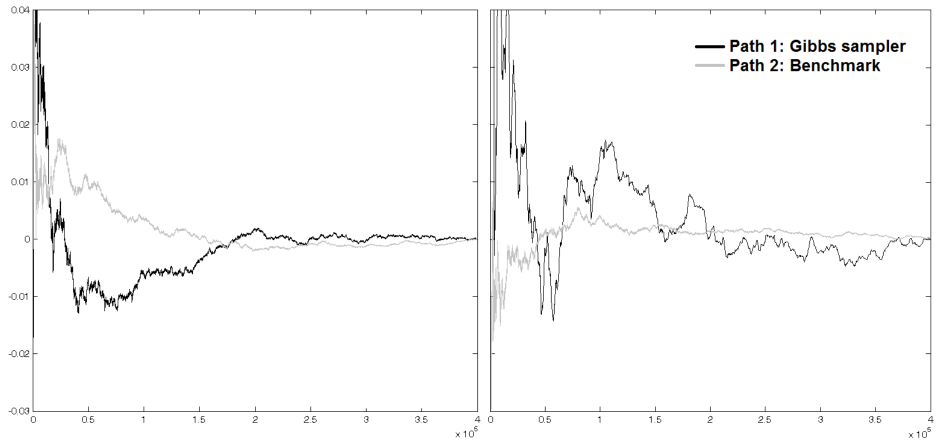
$$EC_{i,t+1} = \exp(u_{it} - u_{i,t+1}) \tag{16}$$

and the joint impact of TC and EC as

$$PC_{i,t+1} = EC_{i,t+1} \times TC_{i,t+1} \tag{17}$$

Formulas (13-17) summarize decomposition methodology introduced by G. Koop, J. Osiewalski and M.F.J. Steel (1999) in the context of BSFA models. The reader should note that output growth breakdown into IC and PC constitutes a one-stage output growth decomposition strategy well known in the literature. In this work, however, we want to investigate all sources of output change separately. Thus, for a two-factor

Figure 1: Convergence diagnostics for models' intercepts; cusum path plot for normal-exponential (on the left) and normal-half normal (on the right); based on TR-QT parametric specification and 400 000 accepted draws; scale: -0.03:0.04; benchmark path is based on draws from normally distributted independent sampler with the same mean and standard deviation as aquired from Gibbs sampler; see Yu and Mykland (1998) for details.



production function formula (14) is now broken down into (Makieła, 2012):

$$\begin{aligned} IC_{i,t+1} &= \exp\left[\Delta \mathrm{f1}_{i,t+1} \cdot El.\mathrm{f1}\left(\frac{1}{2}(x_{i,t+1} + x_{it})\right)\right] \times \\ &\quad \times \exp\left[\Delta \mathrm{f2}_{i,t+1} \cdot El.\mathrm{f2}\left(\frac{1}{2}(x_{i,t+1} + x_{it})\right)\right] \\ &= IC.\mathrm{f1}_{i,t+1} \times IC.\mathrm{f2}_{i,t+1} \end{aligned} \tag{18}$$

where f1 and f2 are the production factors (e.g., capital and labour) and *El*.f1(.), *El*.f2(.) are their elasticity functions. The reader should note that the two-input formula in (18) can be easily extended to incorporate more factors; e.g., a measure of human capital, if such is available. This leads us to a two-stage decomposition where, first, output growth is decomposed into IC and PC, and second (Eq. 17-18), where PC is decomposed into EC and TC components, and IC is decomposed into each factor's contribution. To sum up, the change in GDP level between period t and $t+1$ ($OC_{i,t+1}$) in this study can be summarized as

$$OC_{i,t+1} = IC\_k_{i,t+1} \times IC\_l_{i,t+1} \times TC_{i,t+1} \times EC_{i,t+1} \tag{19}$$

where components on the right side of (19) reflect the impact of capital change, labour change, technical progress and efficiency change on economic growth respectively. To make interpretations more intuitive, indicators from (14-19) are given as percentage changes to the previous year. Thus, a simple transformation $\Delta\%=100\%(\delta\text{-}1)$ is used, where δ is the initial level of an indicator from (14-19). Labels AOC, AIC (AIC_K, AIC_L) and APC (ATC, AEC) denote time-averages for a given country of those indicators.

To conclude this section it is worth noting that model choice has a profound impact on the structure of output growth decomposition. That is, it determines how *in-depth* the output decomposition can be. For this reason when choosing the optimal model a researcher should consider the following. CD model does not allow for technical progress (frontier cannot shift over time) and since factors' elasticities are constant among all observations, changes in the IC component can be caused only by changes in factors' input levels. Effectively this leads to a simple Solow-type decomposition of output growth. CDt model does allow for a technical change which, however, is constant not only through time but across countries as well. CD-LT model allows the technology to change, but only over time and in a linear fashion. TR model allows us to consider technical change more flexibly through factors elasticities, which can vary over time and across countries (if inputs change). It is not possible, however, to distil the effects of technical change from the impact of input change component. TRt model solves this problem only partially because we face the same issue as in CDt model; unrealistic assumption regarding technical change. A full-scale decomposition can be obtained with TR-LT model. Introducing linear trend into each parameter of the translog production function allows the technical change, here seen as gradual change in production technology function parameters over time, to impact each country differently over time (though in a linear fashion). TR-QT model further loosens the restrictions on how technical change can impact a country's growth. In doing so, it allows us to investigate changes in technical progress contribution to a given country's economic growth over time. Other possibilities are also mentioned in the literature. For example Koop, Osiewalski and Steel (2000) propose to extend a simple Cobb-Douglas form by AR(1) autoregressive processes in factor elasticities. This approach is more up to date as regards the contemporary research on time-series analysis. Unfortunately we face considerable numerical problems when applying it in translog models because factor elasticities of production are functions of parameters and data in translogs. Since one of the aims of this research is to compare Cobb-Douglas and translog functional forms, modelling technology change using autoregressive processes is left out of the scope of this study.

4 Choosing the best model

A key aspect of every Bayesian analysis that involves more than one model is the computation of marginal data density, also referred to as integrated likelihood or marginal likelihood. Since this paper uses MCMC methods we can approximate marginal data density (mdd hereafter) for each model using the Harmonic Mean Estimator (HME hereafter) proposed by Newton and Raftery (1994). Though it is by far the most commonly used and universal technique to approximate mdd, it has earned serious critique for being unstable or "over optimistic". For this reason a number of alternatives to HME have been proposed (see, e.g., Newton and Raftery 1994; Gelfand and Dey, 1994; or Chib and Jeliascov, 2001) as well as methods to improve on it (see, e.g., Newton and Raftery, 1994; Raftery, Newton, Satagopan and Krivitsky, 2007; Lenk 2009; Osiewalski and Osiewalski, 2013; Pajor and Osiewalski, 2013). As Lenk (2009) shows, HME is pseudo-biased because the MCMC simulation only visits the area of the parameter space with substantial posterior mass. In other words, it will never visit regions of the parameter space, for which the binary representation of the posterior density is zero. Since BSFA models have highly dimensional spaces of parameters and latent variables, we follow HME correction method proposed by Osiewalski and Osiewalski (2013), and adopt it to BSFA framework.

Table 2 presents marginal data densities (in decimal logs) for all 42 models considered in this study along with model rankings before and after HME's correction. The correction does not change much in terms of overall model ranking for EXP-type models (Spearman rank correlation is 0.9857). It yields, however, some interesting results as regards NHN vs. EXP comparison and how NHN models perform given different levels of prior median.

The biggest correction for HME pseudo-bias is in NHN/TR-QT(0.6) model. Its mdd dropped nearly by 783 orders of magnitude. Seemingly the best NHN models – translogs with low prior median – have turned out to be among the worst after the adjustment. However, similar models with higher prior median remain at the top of both rankings; before and after the adjustment. As a result NHN models' ranking has changed considerably after HME adjustment (Spearman rank correlation is -0,2195). Setting r_0 (prior median) has a huge impact on HME pseudo-bias in NHN models, mostly via Lenk's correction on latent variables. It seems that the more our assumption about prior median is not in line with the information in the data the bigger Lenk's correction on model's latent variables is – and all NHN models are very sensitive to the choice of r_0. At its extreme Lenk's correction in translog models with 0.6 prior median is roughly between 400 and 780 orders of magnitude. On condition that the prior median is well-tuned NHN models still outperform their EXP counterparts after HME adjustment. The differences, however, are much smaller than before HME adjustment.

A good addition to formal model selection based on mdd's is Lindley-type testing. Such tests are based on a classic concept of hypothesis testing of possible model reductions. Though they do not give us information as to what type of model to choose (NHN vs. EXP), or what prior median to set, they can be found helpful in testing possible parametric simplifications of the production function or, e.g., to empirically check for constant returns to scale (CRS restrictions) often assumed in economic growth literature. Furthermore, the tests are relatively easy to compute and can be found more intuitive for practitioners that are not that familiar with formal

Table 2: Convergence diagnostics for selected models

Model code	Prior median	HME	Lenk's correction	Final result	Ranking before correction		Ranking after correction	
					Overall	Within model type	Overall	Within model type
EXP/CD(0.6)	0,6	117,57	-13,27	104,30	36	15	30	14
EXP/CD(0.75)	0,75	115,16	-12,00	103,16	39	18	33	17
EXP/CD(0.875)	0,875	112,94	-11,06	101,88	42	21	36	20
EXP/CDt(0.6)	0,6	118,14	-14,41	103,72	35	14	32	16
EXP/CDt(0.75)	0,75	115,70	-13,20	102,50	38	17	34	18
EXP/CDt(0.875)	0,875	113,36	-12,23	101,14	40	19	37	21
EXP/CD-LT(0.6)	0,6	118,72	-13,60	105,12	34	13	29	13
EXP/CD-LT(0.75)	0,75	115,83	-11,82	104,01	37	16	31	15
EXP/CD-LT(0.875)	0,875	113,07	-10,76	102,31	41	20	35	19
EXP/TR(0.6)	0,6	141,50	-14,93	126,57	20	5	9	4
EXP/TR(0.75)	0,75	136,92	-12,66	124,26	24	8	12	7
EXP/TR(0.875)	0,875	132,93	-10,62	122,31	28	11	17	10
EXP/TRt(0.6)	0,6	141,59	-15,97	125,62	19	4	10	5
EXP/TRt(0.75)	0,75	137,52	-13,97	123,55	23	7	13	8
EXP/TRt(0.875)	0,875	132,58	-12,47	120,10	30	12	20	12
EXP/TR-LT(0.6)	0,6	142,13	-14,97	127,16	18	3	8	3
EXP/TR-LT(0.75)	0,75	137,68	-12,37	125,31	22	6	11	6
EXP/TR-LT(0.875)	0,875	133,20	-11,07	122,13	27	10	19	11
EXP/TR-QT(0.6)	0,6	142,76	-15,49	127,28	16	1	7	2
EXP/TR-QT(0.75)	0,75	142,54	-12,72	129,82	17	2	6	1
EXP/TR-QT(0.875)	0,875	134,36	-11,60	122,76	26	9	16	9
NHN/CD(0.6)	0,6	145,1	-32,50	112,64	15	15	25	13
NHN/CD(0.75)	0,75	132,8	-17,35	115,43	29	18	23	11
NHN/CD(0.875)	0,875	121,8	-11,90	109,90	33	21	27	15
NHN/CDt(0.6)	0,6	151,5	-32,06	119,44	14	14	21	9
NHN/CDt(0.75)	0,75	136,1	-18,68	117,46	25	17	22	10
NHN/CDt(0.875)	0,875	122,1	-13,23	108,83	32	20	28	16
NHN/CD-LT(0.6)	0,6	153,2	-29,98	123,24	13	13	15	7
NHN/CD-LT(0.75)	0,75	139,3	-17,02	122,26	21	16	18	8
NHN/CD-LT(0.875)	0,875	124,7	-11,70	112,99	31	19	24	12
NHN/TR(0.6)	0,6	269,8	-453,33	-183,50	3	3	41	20
NHN/TR(0.75)	0,75	232,2	-121,79	110,41	6	6	26	14
NHN/TR(0.875)	0,875	174,8	-28,40	146,37	9	9	3	3
NHN/TRt(0.6)	0,6	256,0	-418,34	-162,37	4	4	40	19
NHN/TRt(0.75)	0,75	214,9	-120,49	94,41	8	8	38	17
NHN/TRt(0.875)	0,875	169,4	-24,67	144,70	11	11	4	4
NHN/TR-LT(0.6)	0,6	276,6	-400,47	-123,87	1	1	39	18
NHN/TR-LT(0.75)	0,75	221,4	-73,21	148,20	7	7	2	2
NHN/TR-LT(0.875)	0,875	159,1	-18,52	140,61	12	12	5	5
NHN/TR-QT(0.6)	0,6	276,3	-782,66	-506,36	2	2	42	21
NHN/TR-QT(0.75)	0,75	233,1	-109,82	123,29	5	5	14	6
NHN/TR-QT(0.875)	0,875	170,4	-17,50	152,86	10	10	<u>1</u>	<u>1</u>
Spearman rank correlation (before vs. after HME adjustment)								
Within EXP model type		0,9857						
Within NHN model type		-0,2195						
Overall		0,254						

Bayesian model comparison (BMC). The tests used here are Bayesian counterparts of chi-square tests (see, e.g., Marzec and Osiewalski, 2008; or Makieła, 2009). Let γ denote such a subvector of parameters of the full model that $\gamma = \gamma^*$ (e.g., $\gamma = 0$) leads to the model restriction under question (e.g., simplification). Since (for a large enough number of observations) the marginal posterior distribution of γ is approximately normal with mean $E(\gamma|\ y, X)$ and covariance matrix $\mathrm{V}(\gamma|\ \mathrm{y}, \mathrm{X})$, the quadratic form

$$\tau(\gamma;;\ y, X) = [\gamma - E(\gamma|\ y, X)]'\ \mathrm{V}^{-1}(\gamma|\ y, X)\ \ [\gamma - E(\gamma|\ y, X)]$$

has the posterior close to the chi-square distribution with as many degrees of freedom as there are parameters in γ. The test amounts to checking whether the parameter value γ^* lies in the tail of the posterior distribution of γ – if so, the restriction is considered invalid. Table 3 shows a summary of Lindley-type tests on TR-LT and TR-QT functions.

Though the best model is still being chosen based on the highest mdd criterion, there are two added values from performing these tests here. First, it is clear that TR-QT function is a statistically meaningful extension of a simple translog. The tested value, here $\gamma^* = 0$, is far in the tail of the posterior distribution of $\tau(\gamma;;\mathrm{y},\mathrm{X})$ for all simplifications under considerations. This indicates that there are significant temporal dynamics in the data, which favour a more time-flexible parameterisation. Second, global CRS restrictions are not supported by the data.

5 Decomposition of output growth in 27 EU Member States, USA, Japan and Switzerland

Tables 4-5 show posterior characteristics of the main decomposition results for individual countries and the following aggregates:

1. a simple average to indicate average change in those economies that form a given economic entity (e.g. the EU),
2. a weighted average, weighted by each country's average GDP level in the analysed period. This average is used as an indicator of average change in the economic area as a whole, and thus is an indicator of economic change within regions like EU12, EU15 or the EU.

The results are based on NHN/TR-QT(0.875) model which has been selected based on the analysis performed in Section 4. Due to space considerations the paper does not provide decomposition results for all 42 models. It is worth noting, however, that most models considered in the study (including all top-ranked models) gave consistent results as regards output change decomposition. This means that the decomposition outcome is quite robust to parametric specification. As noted in Section 3 more restrictive parametric specifications will limit the detail of the decomposition. However, the results for comparable components from simpler models are in line with those from more complex ones and more substantial differences are found in components' dispersion measures; i.e., simpler models yield higher posterior standard deviations.

Decomposition results show that countries economic development is mostly shaped by changes in input factors, followed by productivity change. Second level decomposition reveals that capital accumulation component is the key ingredient of rapid economic growth in those countries and that efficiency change is the dominant component of productivity change. Moreover, whenever we witness capital-driven rapid economic growth, a country loses on productivity, either through efficiency change or (to a lesser degree) technical change component (correlation between posterior means of APC and AIC_K is -0.87.). Thus it seems that rapid, and driven by capital accumulation economic growth has a negative impact on productivity. For example, EU12 productivity has been dropping on average by 1.39% (0.1%) annually. The notion of capital-driven growth vs. productivity-driven growth may seem counterintuitive at first. The reader should note, however, that this is no more than plain evidence of a relation between equipment and productivity – it takes time to learn the newly acquired capital. Previous research in this regard provides some additional evidence. For example, Makieła (2009) reports that countries such as Portugal, Slovenia and South Korea, which had very high average input change component had also very low average productivity change component.

Productivity change is not influenced by the second component of input change, changes in labour input (correlation between posterior means of APC and AIC_L is 0.23.). Productivity decomposition shows that productivity change is mostly shaped by changes in technical efficiency, which is also the most volatile component over time and accounts for most of growth acceleration and deceleration.

The "old" Member States have had a more balanced yet smaller economic growth. Impact of labour change on economic growth was positive and coupled with increased productivity [by 0.13% (0.06%)], which is probably due to efficiency increase [0.14% (0.13%)]. This impact (of joint positive influence of labour change and efficiency change) is particularly strong in three EU15 countries: Ireland, UK and the Netherlands. Interestingly enough these EU countries are also amongst the biggest beneficiaries (in *per capita* terms) of inexpensive, yet skilled labour force from EU12.

Impact of technical change is more difficult to assess. It appears that the distinction should be made between big and small economies rather than EU12 and EU15. The technical change component of output growth has a positive effect on bigger economies and a negative effect on smaller ones. Due to economic meltdown at the time it is hard to reach firm conclusions about the impact of technology on economic growth. All economies started producing less given their inputs after around 2005 which caused the "best practices" frontier eventually to shift downwards. Thus, though there were some considerable dynamics as regards frontier change over time (indicated by significance

Table 3: Lindley-type tests for different parametric specifications nested in TR-QT and TR-LT

Parametric specification	Degrees of freedom	Normal-half-normal $\tau(g^*)$	Normal-half-normal $Pr\{\tau(g) > \tau(g^*)\|\text{data}\}$	Normal-exponential $\tau(g^*)$	Normal-exponential $Pr\{\tau(g) > \tau(g^*)\|\text{data}\}$
TR-QT (prior median= 0.6)					
TR-LT	6	36.00	2.75E-06	18.98	4.20E-03
TR	12	44.40	1.30E-05	29.58	3.23E-03
TRt	11	44.00	7.27E-06	27.48	3.89E-03
CD	15	844.31	0	140.98	0
CDt	14	711.83	0	135.17	0
CD-QT	9	583.13	0	106.54	0
CD-LT	12	700.94	0	130.54	0
CRS restr	9	674.778	0	131.79	0
TR-QT (prior median = 0.75)					
TR-LT	6	32.60	1.25E-05	19.06	4.06E-03
TR	12	39.67	8.15E-05	29.60	3.21E-03
TRt	11	39.65	4.11E-05	27.52	3.83E-03
CD	15	546.03	0	132.37	0
CDt	14	503.54	0	127.60	0
CD-QT	9	428.29	0	98.92	0
CD-LT	12	500.60	0	122.95	0
CRS restr.	9	518.22	0	123.62	0
TR-QT (prior median = 0.875)					
TR-LT	6	20.84	1.96E-03	19.16	3.90E-03
TR	12	27.60	6.33E-03	29.34	3.51E-03
TRt	11	27.28	4.17E-03	27.45	3.93E-03
CD	15	231.09	0	122.94	0
CDt	14	220.63	0	119.49	0
CD-QT	9	187.59	0	89.77	0
CD-LT	12	216.43	0	114.60	0
CRS restr.	9	222.5824	0	114.01	0
TR-LT (prior median = 0.6)					
TR	6	3.90	**0.69**	9.55	**0.15**
TRt	5	3.90	**0.56**	7.86	**0.16**
CD	9	344.40	0	107.89	0
CDt	8	308.62	0	102.75	0
CD-LT	6	7896.08	0	6403.33	0
CRS restr.	6	391.18	0	118.25	0
TR-LT (prior median = 0.75)					
TR	6	4.68	**0.59**	9.51	**0.15**
TRt	5	4.60	**0.47**	7.84	**0.17**
CD	9	248.05	0	102.09	0
CDt	8	232.45	0	97.75	0
CD-LT	6	7604.27	0	5952.67	0
CRS restr.	6	274.28	0	112.95	0
TR-LT (prior median = 0.875)					
TR	6	7.39	**0.29**	9.42	**0.15**
TRt	5	6.88	**0.23**	7.80	**0.17**
CD	9	185.21	0	96.47	0
CDt	8	173.62	0	93.01	0
CD-LT	6	7275.70	0	4617.47	0
CRS restr	6	261.882	0	107.37	0

Note: CRS restr. indicates testing for constant returns to scale.

of quadratic forms by Lindley-type tests), posterior means of average technical change components are small. What can be said more generally about productivity is that, in small countries productivity change has had its source mainly in efficiency change. In bigger economies influence of technical change is more significant. EU12 economies grew impressively throughout the analysed period. The region's average annual growth rate was around 3.5%, significantly higher than EU15 and other economies. Decomposition results indicate that the capital growth component is the sole driver of economic growth in the region. Remaining components have had a marginal and rather negative impact on economic growth. This comes with small exception of Poland where economy grew also due to labour change (0.03% [0.003]) and technical change [0.38% (0.42%)], and Romania where technical change is also quite likely to have had a positive impact on economic growth [0.11% (0.46%)]. Growth through capital accumulation in EU12 is due to a number of reasons. First, EU12 region is highly undercapitalized in comparison to EU15 and the benchmark economies (USA, Japan, and Switzerland). Capital-to-labour ratio is just 32.7 PPS (Purchasing Power Standard) per hour worked in the region and a country average is 35.6, indicating that smaller economies are just slightly better-off. These ratios, however, are nowhere near the EU15 average, which is 93 for the region and 87.9 for a country average (thus bigger economies are slightly better-off). Second, the rate of capital accumulation in EU12 was the highest while labour input even slightly declined. Normally, one would argue that if labour had grown as well, it would have boosted economic growth even further. Though this may be true we should remember that labour input is already very high in relation to the capital stock in EU12, when compared to EU15 or benchmark economies. As EU12 economies evolve this imbalance is expected to disappear and the fastest way is when labour input remains constant or even declines. Hence, it comes as no surprise that economic growth in those countries is gained primarily through capital accumulation.

Table 6 shows correlation coefficients of the joint posterior distributions of the components of output growth for each country. The results reveal the following correspondences between economic growth components.

First, there is a trade-off effect between technical change and efficiency change (negative posterior correlation between ATC and AEC). Though correlation strength ranges from moderate to high, the direction of this relationship remains stable in all countries considered. Second, negative correlation between APC and AIC_k reiterates what has already been mentioned earlier. Capital accumulation-driven economic growth negatively affects productivity. Fortunately this effect is not that strong. Third, relationship between labour component and productivity is more complex. If labour input grows, labour component is positively correlated with productivity component. However, if labour input declines the correlation coefficient is negative. Fourth, capital component and labour component of economic growth are highly correlated. Moreover the sign of the posterior correlation coefficient is dependent

Table 4: Output growth decomposition results; first level of decomposition; averages by country

Country	Emp. growth rate	Expected growth rate		First level of decomposition				Capital stock to labour (PPS per hour worked) (1996-2010 average)	Capital stock annual growth rate (1996-2010 average)	Labour annual growth rate (1996-2010 average)
		E(AOC)	D(AOC)	E(AIC)	D(AIC)	E(APC)	D(APC)			
Austria	2.05	2.050	0.188	1.818	0.049	0.228	0.191	102.786	2.166	0.310
Belgium	1.90	1.885	0.164	1.706	0.024	0.176	0.162	103.495	1.918	1.039
Bulgaria	3.48	3.476	0.190	4.322	0.125	-0.811	0.219	20.591	4.965	-0.462
Czech Republic	2.75	2.755	0.189	2.920	0.064	-0.160	0.194	51.365	3.582	-0.404
Denmark	1.27	1.304	0.128	1.433	0.018	-0.127	0.127	73.370	1.624	0.599
Estonia	4.75	4.768	0.195	8.025	0.225	-3.015	0.269	30.931	8.700	-1.077
Finland	2.78	2.741	0.188	1.424	0.016	1.298	0.187	77.564	1.601	0.744
France	1.70	1.701	0.185	1.741	0.074	-0.040	0.196	112.217	2.186	0.315
Germany	1.27	1.271	0.185	1.088	0.057	0.181	0.192	103.555	1.446	0.002
Greece	2.58	2.581	0.189	2.017	0.034	0.554	0.188	75.820	2.321	0.737
Hungary	2.44	2.502	0.187	4.598	0.084	-2.004	0.185	35.071	5.420	-0.192
Ireland	4.37	4.363	0.192	4.026	0.065	0.324	0.195	82.756	4.623	1.256
Italy	0.88	0.886	0.184	1.391	0.042	-0.498	0.186	88.257	1.665	0.455
Japan	0.66	0.662	0.185	0.380	0.081	0.281	0.201	78.544	0.967	-1.051
Latvia	4.22	4.243	0.193	7.432	0.217	-2.968	0.264	23.606	8.166	-0.579
Lithuania	4.32	4.337	0.184	5.703	0.140	-1.292	0.218	26.762	6.347	-0.162
Netherlands	2.17	2.162	0.177	1.792	0.038	0.363	0.178	101.789	2.068	0.798
Poland	4.27	4.301	0.174	4.583	0.110	-0.270	0.187	28.093	5.561	0.132
Portugal	1.71	1.714	0.187	2.489	0.041	-0.756	0.186	44.554	2.937	0.136
Romania	2.52	2.525	0.189	4.079	0.151	-1.493	0.229	18.179	5.092	-1.250
Slovakia	4.14	4.167	0.183	5.363	0.090	-1.135	0.188	43.247	6.183	-0.361
Slovenia	3.22	3.252	0.189	5.553	0.079	-2.179	0.188	53.005	6.222	0.151
Spain	2.77	2.773	0.189	3.582	0.065	-0.781	0.192	83.170	4.023	1.999
Sweden	2.60	2.596	0.189	1.474	0.030	1.106	0.188	101.577	1.704	0.585
Switzerland	1.87	1.868	0.187	1.068	0.009	0.791	0.185	97.496	1.163	0.857
United Kingdom	2.26	2.248	0.165	1.856	0.063	0.385	0.175	75.091	2.304	0.328
United States	2.38	2.406	0.164	2.191	0.124	0.211	0.193	83.471	2.946	0.256
Cyprus	3.22	3.219	0.189	3.911	0.058	-0.666	0.190	42.542	4.282	1.815
Luxembourg	4.02	4.026	0.169	4.460	0.052	-0.415	0.171	93.110	5.067	2.781
Malta	2.41	2.412	0.188	2.984	0.057	-0.555	0.191	54.155	3.210	0.672
Average	2.70	2.706	0.032	3.180	0.045	-0.442	0.051	66.872	3.682	0.348
weighted	1.98	1.985	0.072	1.881	0.089	0.102	0.108	84.356	2.469	0.173
EU average	2.82	2.818	0.034	3.382	0.047	-0.546	0.054	64.691	3.903	0.384
EU region	1.99	1.985	0.056	2.058	0.054	-0.071	0.076	86.547	2.488	0.433
EU15 average	2.29	2.283	0.044	2.146	0.043	0.134	0.060	87.941	2.510	0.806
EU15 region	1.81	1.802	0.062	1.774	0.057	0.027	0.083	93.044	2.149	0.514
EU12 average	3.48	3.491	0.056	4.948	0.100	-1.388	0.105	35.629	5.644	-0.143
EU12 region	3.51	3.522	0.080	4.445	0.098	-0.884	0.112	32.703	5.301	-0.243
US+JP+CH	1.64	1.641	0.095	1.204	0.074	0.432	0.117	86.503	1.692	0.021
US+JP+CH reg	1.97	1.985	0.121	1.736	0.118	0.244	0.159	82.567	2.454	-0.038

Note: Definitions for AOC, AIC, APC are provided in Section 3; E(. . .) is posterior mean; D(. . .) is posterior standard deviation.

Table 5: Output growth decomposition results; second level of decomposition; averages by country

Country	Empirical growth rate	Expected growth rate		Second level of decomposition							
		E(AOC)	D(AOC)	E(AIC_K)	D(AIC_K)	E(AIC_L)	D(AIC_L)	E(AEC)	D(AEC)	E(ATC)	D(ATC)
Austria	2.05	2.050	0.188	1.778	0.057	0.039	0.008	0.285	0.260	-0.056	0.157
Belgium	1.90	1.885	0.164	1.574	0.051	0.130	0.027	0.236	0.225	-0.060	0.159
Bulgaria	3.48	3.476	0.190	4.376	0.116	-0.052	0.010	-0.553	0.363	-0.258	0.385
Czech Republic	2.75	2.755	0.189	2.983	0.057	-0.061	0.008	-0.278	0.242	0.118	0.188
Denmark	1.27	1.304	0.128	1.376	0.028	0.056	0.011	-0.068	0.136	-0.059	0.106
Estonia	4.75	4.768	0.195	8.028	0.206	-0.002	0.020	-2.260	0.329	-0.771	0.398
Finland	2.78	2.741	0.188	1.356	0.029	0.067	0.014	1.373	0.211	-0.073	0.108
France	1.70	1.701	0.185	1.664	0.086	0.076	0.012	-0.054	0.331	0.015	0.233
Germany	1.27	1.271	0.185	1.088	0.058	0.000	0.000	0.076	0.281	0.106	0.188
Greece	2.58	2.581	0.189	1.907	0.050	0.107	0.017	0.492	0.219	0.061	0.122
Hungary	2.44	2.502	0.187	4.625	0.081	-0.026	0.003	-2.008	0.298	0.005	0.245
Ireland	4.37	4.363	0.192	3.929	0.089	0.093	0.024	0.458	0.234	-0.133	0.116
Italy	0.88	0.886	0.184	1.276	0.058	0.113	0.016	-0.665	0.231	0.169	0.143
Japan	0.66	0.662	0.185	0.716	0.037	-0.334	0.043	-0.069	0.240	0.351	0.188
Latvia	4.22	4.243	0.193	7.455	0.206	-0.021	0.012	-2.306	0.347	-0.677	0.422
Lithuania	4.32	4.337	0.184	5.713	0.138	-0.009	0.003	-0.819	0.311	-0.477	0.332
Netherlands	2.17	2.162	0.177	1.661	0.061	0.129	0.023	0.368	0.251	-0.004	0.165
Poland	4.27	4.301	0.174	4.552	0.113	0.030	0.003	-0.644	0.416	0.378	0.429
Portugal	1.71	1.714	0.187	2.468	0.043	0.020	0.002	-0.864	0.269	0.110	0.215
Romania	2.52	2.525	0.189	4.324	0.123	-0.235	0.031	-1.598	0.436	0.109	0.462
Slovakia	4.14	4.167	0.183	5.396	0.086	-0.031	0.005	-0.945	0.246	-0.191	0.196
Slovenia	3.22	3.252	0.189	5.548	0.081	0.004	0.002	-1.830	0.222	-0.355	0.147
Spain	2.77	2.773	0.189	3.131	0.130	0.437	0.065	-0.907	0.237	0.128	0.140
Sweden	2.60	2.596	0.189	1.396	0.045	0.076	0.015	1.152	0.254	-0.045	0.155
Switzerland	1.87	1.868	0.187	0.956	0.029	0.111	0.021	0.819	0.235	-0.027	0.141
United Kingdom	2.26	2.248	0.165	1.771	0.075	0.083	0.011	0.124	0.196	0.261	0.149
United States	2.38	2.406	0.164	2.095	0.136	0.093	0.012	-0.091	0.235	0.303	0.230
Cyprus	3.22	3.219	0.189	3.973	0.081	-0.060	0.028	-0.017	0.293	-0.649	0.263
Luxembourg	4.02	4.026	0.169	4.612	0.088	-0.146	0.044	0.032	0.251	-0.446	0.241
Malta	2.41	2.412	0.188	3.042	0.067	-0.057	0.012	0.247	0.369	-0.798	0.348
Average	2.70	2.706	0.032	3.158	0.048	0.021	0.009	-0.347	0.101	-0.096	0.117
weighted	1.98	1.985	0.072	1.844	0.095	0.036	0.006	-0.108	0.147	0.211	0.149
EU average	2.82	2.818	0.034	3.353	0.050	0.029	0.010	-0.415	0.107	-0.131	0.125
EU region	1.99	1.985	0.056	1.968	0.068	0.088	0.015	-0.169	0.148	0.098	0.131
EU15 average	2.29	2.283	0.044	2.058	0.062	0.087	0.019	0.140	0.129	-0.006	0.111
EU15 region	1.81	1.802	0.062	1.668	0.074	0.104	0.017	-0.067	0.173	0.095	0.144
EU12 average	3.48	3.491	0.056	4.994	0.097	-0.044	0.005	-1.105	0.212	-0.286	0.246
EU12 region	3.51	3.522	0.080	4.485	0.093	-0.039	0.005	-1.011	0.295	0.129	0.323
US+JP+CH	1.64	1.641	0.095	1.248	0.071	-0.044	0.005	0.225	0.151	0.206	0.139
US+JP+CH (reg)	1.97	1.985	0.121	1.743	0.117	-0.006	0.001	-0.059	0.196	0.303	0.205

Note: Definitions for AOC, AIC, APC are provided in Section 3; E(. . .) is posterior mean; D(. . .) is posterior standard deviation.

Table 6: Posterior correlations between components of output growth; time averages

Country	ρ(ATC, AEC)	ρ(APG, AIG)	ρ(APG, AIG_K)	ρ(APG, AIG_L)	ρ(AEC, AIG)	ρ(AEC, AIG_K)	ρ(AEC, AIG_L)	ρ(ATC, AIG)	ρ(ATC, AIG_K)	ρ(ATC, AIG_L)	ρ(AIG_K, AIG_L)
Austria	-0.69	-0.28	-0.28	0.28	-0.50	-0.50	0.51	0.48	0.48	-0.50	-0.99
Belgium	-0.70	-0.14	-0.13	0.13	-0.43	-0.45	0.45	0.46	0.49	-0.51	-0.99
Bulgaria	-0.83	-0.58	-0.58	-0.51	0.40	0.38	0.60	-0.67	-0.65	-0.82	0.85
Czech Rep.	-0.62	-0.35	-0.35	-0.34	0.30	0.30	0.36	-0.70	-0.69	-0.75	0.96
Denmark	-0.47	-0.13	-0.12	0.10	-0.02	0.00	-0.02	-0.13	-0.14	0.15	-0.97
Estonia	-0.73	-0.77	-0.77	-0.67	0.32	0.31	0.38	-0.78	-0.77	-0.77	0.88
Finland	-0.47	-0.14	-0.15	0.15	-0.11	-0.12	0.13	-0.01	-0.01	0.00	-0.98
France	-0.82	-0.41	-0.41	0.40	-0.64	-0.65	0.66	0.58	0.59	-0.61	-0.99
Germany	-0.74	-0.33	-0.33	0.33	-0.48	-0.48	0.48	0.38	0.38	-0.39	-0.99
Greece	-0.52	-0.20	-0.20	0.20	-0.03	-0.02	0.01	-0.26	-0.26	0.28	-0.99
Hungary	-0.79	-0.30	-0.30	-0.23	0.38	0.37	0.53	-0.65	-0.64	-0.79	0.87
Ireland	-0.56	-0.37	-0.37	0.35	-0.42	-0.42	0.43	0.23	0.24	-0.27	-0.98
Italy	-0.60	-0.23	-0.23	0.23	-0.18	-0.18	0.19	-0.02	-0.01	0.00	-0.99
Japan	-0.59	-0.45	-0.45	-0.45	0.15	0.14	0.16	-0.65	-0.63	-0.66	0.98
Latvia	-0.77	-0.76	-0.76	-0.66	0.38	0.37	0.51	-0.78	-0.78	-0.83	0.88
Lithuania	-0.77	-0.62	-0.62	-0.53	0.37	0.36	0.53	-0.72	-0.72	-0.81	0.85
Netherlands	-0.71	-0.24	-0.23	0.23	-0.47	-0.48	0.48	0.46	0.47	-0.49	-0.99
Poland	-0.91	-0.50	-0.50	0.46	0.40	0.41	-0.61	-0.58	-0.59	0.76	-0.90
Portugal	-0.73	-0.22	-0.22	0.20	0.41	0.42	-0.52	-0.66	-0.67	0.77	-0.93
Romania	-0.87	-0.65	-0.64	-0.58	0.35	0.28	0.55	-0.61	-0.55	-0.77	0.85
Slovakia	-0.66	-0.40	-0.40	-0.31	0.25	0.24	0.40	-0.66	-0.64	-0.75	0.83
Slovenia	-0.54	-0.35	-0.34	0.22	0.08	0.08	-0.11	-0.56	-0.56	0.45	-0.82
Spain	-0.59	-0.35	-0.35	0.34	-0.31	-0.32	0.32	0.05	0.07	-0.09	-0.99
Sweden	-0.68	-0.19	-0.19	0.18	-0.42	-0.43	0.43	0.45	0.46	-0.48	-0.99
Switzerland	-0.62	-0.06	-0.06	0.05	-0.23	-0.27	0.27	0.30	0.36	-0.38	-0.99
United Kingdom	-0.52	-0.42	-0.42	0.41	0.07	0.07	-0.08	-0.57	-0.57	0.57	-0.99
United States	-0.67	-0.61	-0.62	0.62	-0.26	-0.25	0.20	-0.28	-0.28	0.34	-0.98
Cyprus	-0.77	-0.31	-0.30	0.23	0.38	0.36	-0.25	-0.64	-0.61	0.43	-0.86
Luxembourg	-0.76	-0.35	-0.31	0.20	0.30	0.20	-0.04	-0.54	-0.41	0.18	-0.89
Malta	-0.86	-0.32	-0.31	0.25	0.41	0.38	-0.15	-0.60	-0.56	0.29	-0.87
Average	-0.69	-0.37	-0.37	0.04	0.02	0.00	0.23	-0.26	-0.24	-0.21	-0.40
weighted	-0.66	-0.46	-0.46	0.32	-0.19	-0.20	0.23	-0.18	-0.17	-0.01	-0.68
EU average	-0.69	-0.37	-0.36	0.04	0.03	0.02	0.23	-0.26	-0.25	-0.21	-0.41
EU region	-0.68	-0.34	-0.34	0.27	-0.24	-0.24	0.28	0.04	0.05	-0.14	-0.87
EU15 average	-0.64	-0.27	-0.26	0.25	-0.21	-0.22	0.23	0.06	0.07	-0.09	-0.98
EU15 region	-0.66	-0.32	-0.32	0.31	-0.31	-0.31	0.31	0.12	0.13	-0.14	-0.99
EU12 average	-0.76	-0.49	-0.49	-0.22	0.34	0.32	0.23	-0.66	-0.65	-0.36	0.29
EU12 region	-0.80	-0.48	-0.48	-0.05	0.35	0.34	0.03	-0.63	-0.62	-0.13	0.12
US+JP+CH	-0.62	-0.37	-0.37	0.07	-0.11	-0.13	0.21	-0.21	-0.18	-0.23	-0.33
US+JP+CH reg	-0.65	-0.57	-0.57	0.36	-0.16	-0.16	0.19	-0.35	-0.35	0.09	-0.52

Note: Definitions for AOC, AIC, AIC_K, AIC_C, APC, ATC and AEC are in Section 3; ρ(a,b) is a posterior correlation coefficient between a and b on whether labour input increases or decreases. When labour change is positive the two components are highly negatively correlated (they force each other out). Hence, if share of capital component in economic growth rises it comes at the expense of labour component and vice-versa. This can be simply explained if we view capital accumulation as an "automatization effect". If one country achieves higher economic growth through capital accumulation (thus by automatizing its macro-scale production) it comes at the expense of labour input (e.g., funds that could have been used to employ more labour).

6 Concluding remarks

Regardless of the model considered, capital accumulation is the main driver of economic growth. Its impact on growth in the studied economies has been on average several times higher than that of technical efficiency change – the second most important component. However, while capital accumulation has had the biggest share in economic growth, efficiency change accounts for most of economic growth dynamics; acceleration and deceleration over time. Labour change component has turned out to be less significant than capital accumulation and even efficiency change. Technical progress has had a marginal impact on economic growth on average. A likely reason for this is that the model has been estimated using data covering the economic crisis that had its origin around the middle of the first decade of the 20th century and every economy has been struggling with it at some point ever since.

To sum up, it should be noted that components of output growth are conditioned upon model choice. Thus, it is important to first select the best parametric specification before proceeding with output growth decomposition. The use of Bayesian Stochastic Frontier Analysis (BSFA) allows us to choose the optimal model not only based on basic conceptual guidelines briefly mentioned at the end of Section 3 but primarily based on information in the data. In particular, model selection exercise performed in the study reports the following findings. First, Cobb-Douglas parameterisation is too restrictive to be used in empirical studies of economic growth. Even though the prior structure of the models slightly favoured Cobb-Douglas functions over translogs (see Section 2), their marginal data densities have turned out to be considerably lower. Second, global constant returns to scale, so often assumed by the theorists of economic growth, are not supported by the data. Third, adjusting HME for pseudo-bias is very important in NHN models and when one wants to compare models of different types, e.g., NHN to EXP. For example in some NHN models, poorly chosen prior median has caused enormous HME pseudo-bias via latent variables leading to an impression that NHN models greatly outperform EXP models. However, much of the difference disappears once HME is adjusted. Fourth, on the one hand NHN models with a well-tuned prior median still considerably outperform EXP. On the other hand, however, EXP models are less dependent on how this hyperparameter is set. In EXP models, HME pseudo-bias has had virtually no impact on model

selection and ranking; quite the opposite in NHN models. This may be due to thicker tails of exponential distribution in comparison to half-normal. The researcher is less penalized for poorly selected prior median than in the case of half-normal distribution. Thus, given the fact that i) EXP models are less dependent on the prior median than NHN models, ii) samplers for EXP models are more efficient (faster mixing speeds, thus fewer runs are required), and that iii) similarly parameterised EXP and NHN models yield similar results, it makes EXP models more appealing to practitioners.
The presence of economic crisis has had its consequence in a downward trend of the production technology frontier at the end of the analysed period. One would argue though that an economic crisis does not affect the technological potential and innovative power of an economy. It might be useful then to model a non-decreasing technology change and analyse how this affects the estimates, and quite possibly model choice, during the time of a crisis. Though it is technically possible to restrict model parameters in such a way, it should be noted that this work follows core literature on SFA and treats technology strictly as the "best practices frontier". That is why if all countries produce less given their inputs over time the production frontier is let to shift downwards – implying diminishing technological potential.
Another possible contribution would be to model GDP, capital and labour using techniques found in time series analysis. This work pre-assumes a causal link between macroeconomic product (represented by GDP), physical capital and labour. However, the very existence of such macroeconomic production function is sometimes disputed in the literature (see, e.g., Growiec, 2012b). Since these processes (GDP, physical capital and labour) are likely to be unit root stochastic processes, it would be interesting to know if they are cointegrated. Should cointegration exist it can be seen as the sought-after macroeconomic production function. However, cointegration analysis for stochastic frontier models has not been developed yet, and using current panel-cointegration techniques can be very misleading.

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