Central European Journal of Economic Modelling and Econometrics

Examination of Seasonal Volatility in HICP for Baltic Region Countries: Non-Parametric Test versus Forecasting Experiment

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Submitted: 11.09.2016, Accepted: 13.02.2017

Abstract

The aim of this paper is to examine the problem of existing seasonal volatility in total and disaggregated HICP for Baltic Region countries (Denmark, Estonia, Latvia, Finland, Germany, Lithuania, Poland and Sweden). Using nonparametric tests, we found that in the case of m-o-m prices, including *fruit*, *vegetables*, and *total HICP*, the homogeneity of variance during seasons is rejected. Based on these findings, we propose an exponential smoothing model with periodic variance of error terms that capture the repetitive seasonal variation (in conditional or unconditional second moments). In a pseudo-real data experiment, the short-term forecasts (nowcasting) for the considered components of inflation were determined using different specifications of considered models. The forecasting performance of the models was measured using one of the scoring rules for probabilistic forecasts called logarithmic score. We found instead that while the periodic phenomenon in variance was statistically significant, the models with a periodic phenomenon in variance of error terms do not significantly improve forecasting performance in disaggregated cases and in the case of *total HICP*. The simpler models with constant variance of error term have comparative forecasting (nowcasting) performance over the alternative model.

Keywords: HICP, seasonal volatility, exponential smoothing, nowcasting, predictive distribution, logscore

JEL Classification: C32, C53, E31, E37

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1 Introduction

Models with periodic autocovariance phenomenon are used in macroeconomics. Many monthly or quarterly macroeconomic time series exhibit seasonality in variance. The most popular model is a generalization of ARMA models applied to a periodic case (i.e., the so-called periodic ARMA model - PARMA in short). This generalization assumes time-varying periodic coefficients of AR and MA parts. Therefore, under appropriate regularity conditions, the PARMA model is a model with a periodic autocovariance function. The theoretical and practical background concerning models with a periodic phenomenon in variance can be found in Parzen and Pagano (1979), Osborn and Smith (1989), Franses and Boswijk (1996), Franses and Paap (2004), Franses and Ooms (1997), Franses and Dijk (2005), Bollerslev and Ghysels (1996), Burridge and Taylor (2001) and many other studies. In addition, the models with seasonal volatility were considered by Doshi *et al.* (2011), Frank *et al.* (2010), and many others.
Recently, the problem of seasonal volatility in inflation for Turkey was considered in Berument and Sahin (2010). The results presented is this paper suggest the presence of seasonality in the conditional variance of inflation. Therefore, the authors recommend using the models with seasonal conditional volatility. The problem of inflation volatility has also been considered by many others researchers. A literature review concerning this issue can be found in Berument and Sahin (2010).
In Novales and Fruto (1997), Wells (1997) and Herwartz (1999), the authors show the advantage of the so-called periodic models over non-periodic models in forecasting. In Herwartz (1999), the author shows that periodic models improve the in-sample fit considerably, but this model can involve a loss in ex-ante forecasting relative to non-periodic models. In Wells (1997), the author shows that out-of-sample forecasting tests may indicate that models with periodic phenomenon are sometimes encompassed by non-periodic cases. The performance of Periodic Autoregressive Models in forecasting was considered in Osborn and Smith (1989). The periodic time series models were also considered in Pagano (1978), Tiao and Grupe (1980) and Osborn (1991).
The graphical procedures for detecting periodicity in the autocovariance function were considered in Hurd and Gerr (1991) in a nonparametric counterpart. Following this article, the extension to a almost periodic case was considered in Lenart (2011) with a subsampling application. The idea of a graphical procedure was examined in Lenart (2016) in the case of an m-dependent almost periodically correlated time series, where the application to economic data was also considered. In Lenart *et al.* (2008), the subsampling methodology was used to construct an asymptotically consistent test to detect periodicity in an autocovariance function.
This paper addresses two main questions. The first is whether the periodic phenomenon in the second moment of disaggregated HICP is present. The second is whether we can improve nowcasting of HICP components and *total HICP* by assuming periodicity in variance of error terms in models. To answer these questions, we

consider disaggregated HICP (month over month, in short m-o-m) with exponential smoothing methods (see Hyndman *et al.* (2008)).
Using disaggregate data for Polish inflation is not new and was considered in Mazur (2015), where it was shown that VAR models with restrictions for disaggregated CPI perform better in forecasting (using probabilistic scoring rules) then for aggregate CPI. In Hałka and Kotłowski (2014), the authors examined the influence of output gap to disaggregate Polish inflation. Lenart and Leszczyńska-Paczesna (2016) used market prices for disaggregated inflation to forecast Polish inflation.
This paper is organized as follows. In section 2, the data are presented with adequate preparation. Section 3 contains an examination of the periodic phenomenon in first and second moment of considered HICP components. The parametric models are presented in section 4. The empirical results concerning performance of considered models in forecasting (nowcasting) are contained in the last section.

2 Data presentation and preparation

We consider monthly HICP m-o-m from Jan. 1997 to Dec. 2015. Based on *Classification of individual consumption by purpose*, we consider the disaggregation of m-o-m HICP. Note that seasonality in inflation volatility is obvious in many HICP components. Unfortunately, there are several problems related to the modelling of seasonal volatility. As a first step, we would like to shed light on this problem by making a connection to possible seasonal volatility modelling.
Let us consider price of Education m-o-m (see Figures 1a-1b, on the left) in the period and set of countries under consideration. One can note that in this case the seasonal pattern is obvious in most considered countries, but the dynamic of this pattern changes over time (see for example, Estonia and Latvia). It can be noticed that there is one or few fixed months during each year with a significant peak in price, while for the rest of the time during year the change in price equals zero or is close to zero percent. For example, in the case of Estonia, Latvia, Poland and Finland the peak in price occurs roughly in September or October (after summer holidays). In addition, seasonal volatility can be suggested here. Therefore, the variance of this price process can be divided into two groups during year: one corresponding to observed changes in September and October and the second corresponding to the rest of the months during the year. Summing up, the dynamics in the first and the second moment is not easy to model jointly with possible seasonal volatility.
As the second problematic example, let us consider passenger transport by air m-o-m (see Figures 1a-1b, on the right). In this case, it is not obvious if there is significant seasonal pattern in such data for each considered country. For example, in the case of Lithuania, Poland and Sweden the amplitude changes significantly over time and therefore the joint modelling for such a process that includes rising amplitude, seasonal pattern and seasonal volatility is not an easy task.
The goal of this paper is not to examine seasonal volatility for all possible price

Figure 1a: HICP (2015 = 100) - monthly data (monthly rate of change) from Jan. 1997 to Dec. 2015 for education and passenger transport by air.

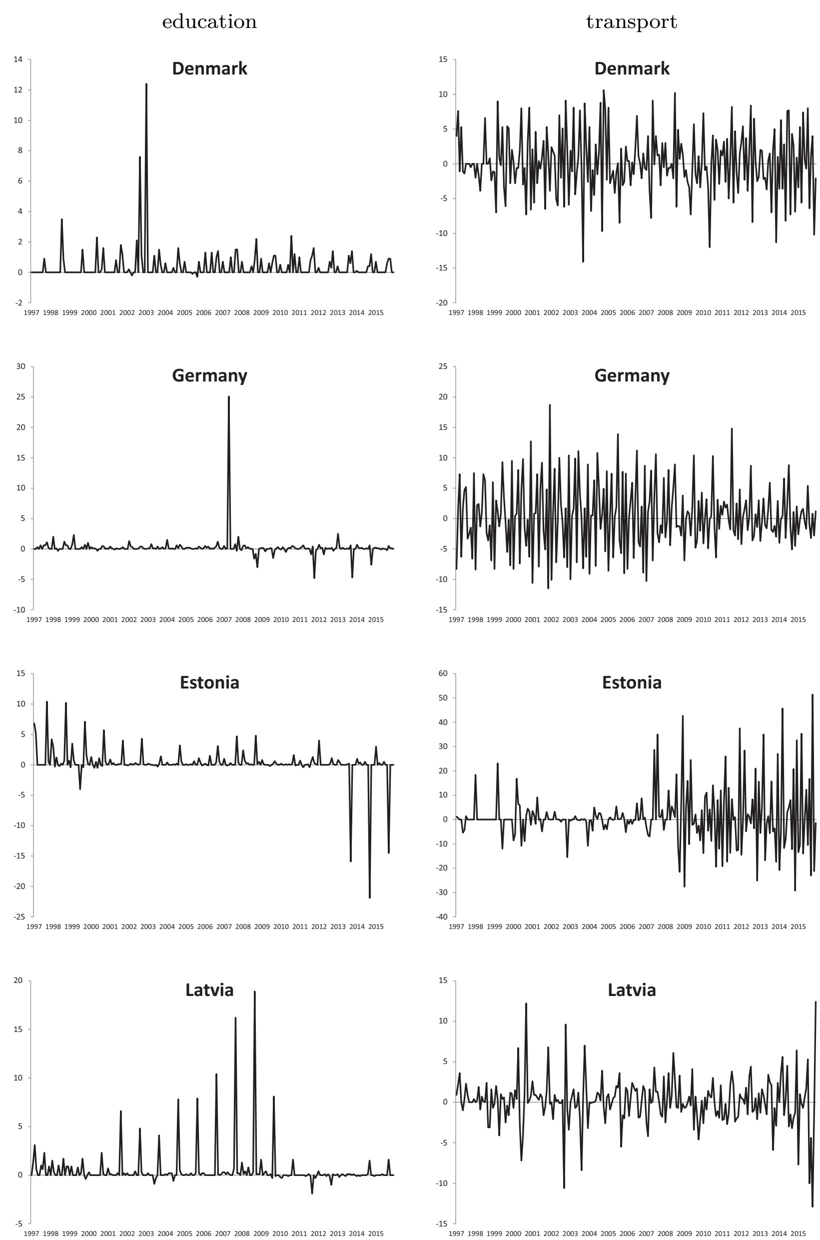
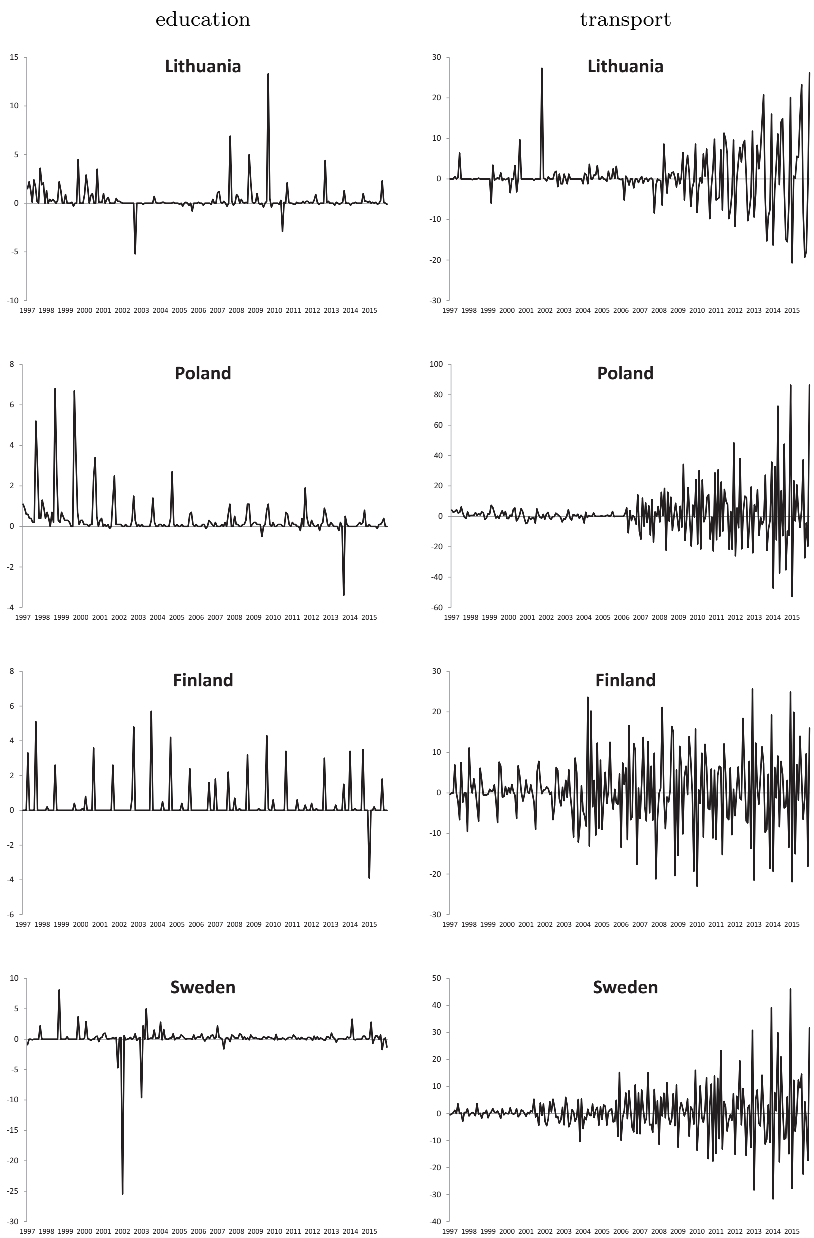


Figure 1b: HICP (2015 = 100) - monthly data (monthly rate of change) from Jan. 1997 to Dec. 2015 for education and passenger transport by air.



processes. In addition, we are not interested in examining all subcomponents where seasonality of variance can be significant. Rather, we focus on some typical examples where seasonal volatility can be diagnosed. More precisely, we consider price for *fruit*, *vegetables*, *clothing and footwear*, and for the *remaining part* of *total HICP*. This disaggregation is considered in Table 1 with weight in *total HICP*, range statistics and used labels. We use such disaggregation since those variables have the potential of exhibiting seasonality in volatility, which follows from the fact that the unexpected price changes can be determined by the weather (especially for *fruit* and *vegetables*), which has a different influence on this price during seasons. In the same way, the length of seasons can cause price dynamics for clothing and footwear. Note that the joint weight for these tree components is approximately 10% of *total HICP*.

Table 1: Range statistics and weights.

Country	Denmark	Germany	Estonia	Latvia	Lithuania	Poland	Finland	Sweden
Range statistics (in percentage points)								
Fruit	11.1	19.7	22.1	30.3	31.5	52.8	30.6	14.4
Vegetables	20.5	27.3	45.6	51	48.1	51.5	36.2	25.7
Clothing and footwear	30.3	12	10.4	18.2	18.8	7.1	19.6	30
Remaining part	1.7	2.3	3.4	4.3	4.5	3.9	2.7	1.8
Average weight (in % in period 1997-2015)								
Fruit	0.84	1.03	1.3	1.59	1.46	1.31	1.23	1.1
Vegetables	1.41	1.22	1.86	2.34	1.81	2.16	1.59	1.57
Clothing and footwear	5.75	5.99	7.64	6.31	7.54	5.19	5.68	6.79
Remaining part	92	91.76	89.2	89.76	89.19	91.34	91.5	90.54

To clarify, we introduce the following notations. By T we denote the *total HICP* m-o-m (the weighted sum of F-*fruit*, V-*vegetables*, C-*clothing and footwear* and R-*remaining part*). Hence, $T_t = w_{F\,t}F_t + w_{V\,t}V_t + w_{C\,t}C_t + w_{R\,t}R_t$, where $w_{F\,t}, w_{V\,t}, w_{C\,t}, w_{R\,t}$ are time-varying weights such that (for each t) $w_{F\,t} + w_{V\,t} + w_{C\,t} + w_{R\,t} = 1$. Note that taking into account the variability of considered variables and weights (see range statistics and weights in Table 1), those variables are important drivers of *total HICP*, especially drivers of seasonal patterns of *total HICP*.
The seasonal pattern in variables F, V and C is recognized (without any statistical justification) for most countries (see Figure 2a-3b). In the case of *clothing and footwear*, the amplitude of the seasonal pattern rises significantly in the case of Germany, Estonia, Latvia, Lithuania and Poland, while for *fruit* and *vegetables* this pattern seems to be more stable over time. In addition, in most cases the data exhibit cyclical fluctuations much longer than one year. This effect is clearly visible in the case of *clothing and footwear* for Poland. We are not interested in examining such fluctuations in this study; however, to adjust such cyclical fluctuations, we use a known linear filter, centered moving average with period 12 (in short 2×12MA). Let $L_{2\times 12\mathrm{MA}}(B)$ be the operator corresponding to 2×12MA filtration:

Figure 2a: HICP (2015 = 100) - monthly data (monthly rate of change) from Jan. 1997 to Dec. 2015 for fruit and vegetables.

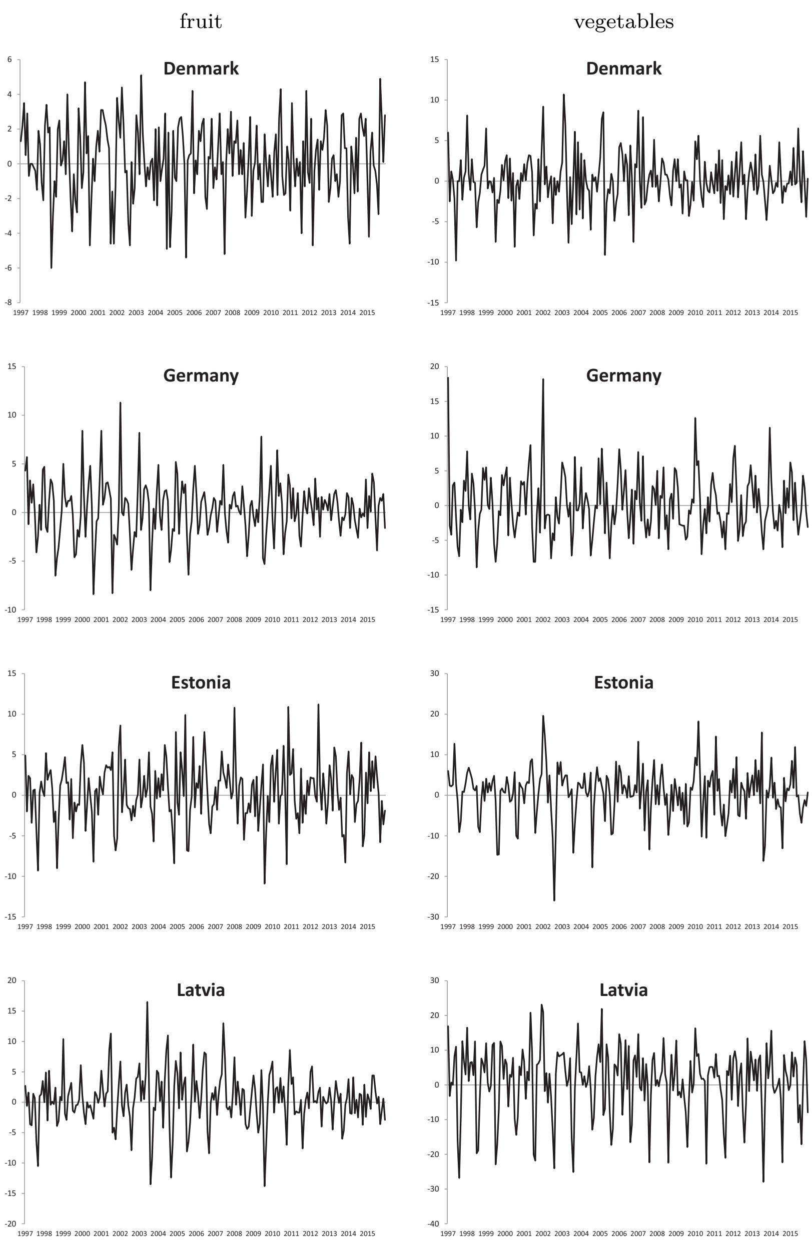


Figure 2b: HICP (2015 = 100) - monthly data (monthly rate of change) from Jan. 1997 to Dec. 2015 for fruit and vegetables.

fruit vegetables

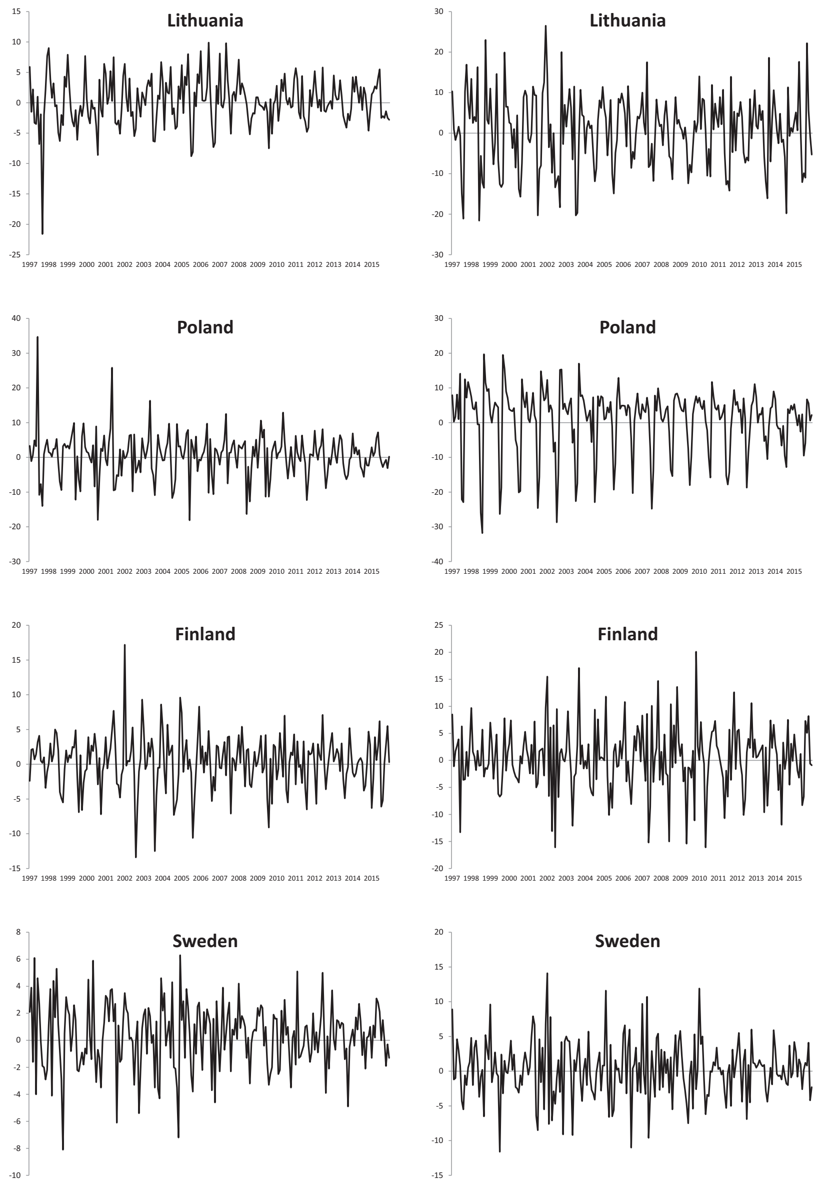


Figure 3a: HICP (2015 = 100) - monthly data (monthly rate of change) from Jan. 1997 to Dec. 2015 for clothing and footwear and the remaining part.

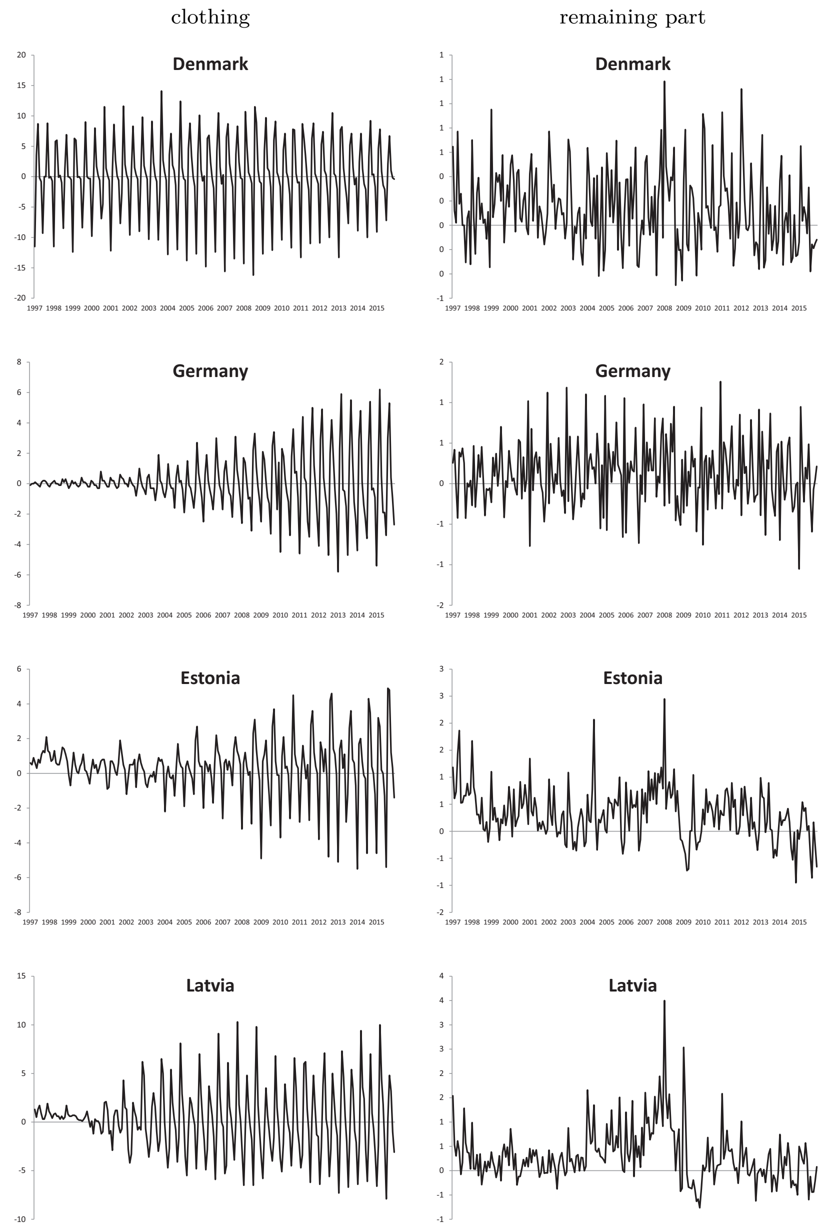
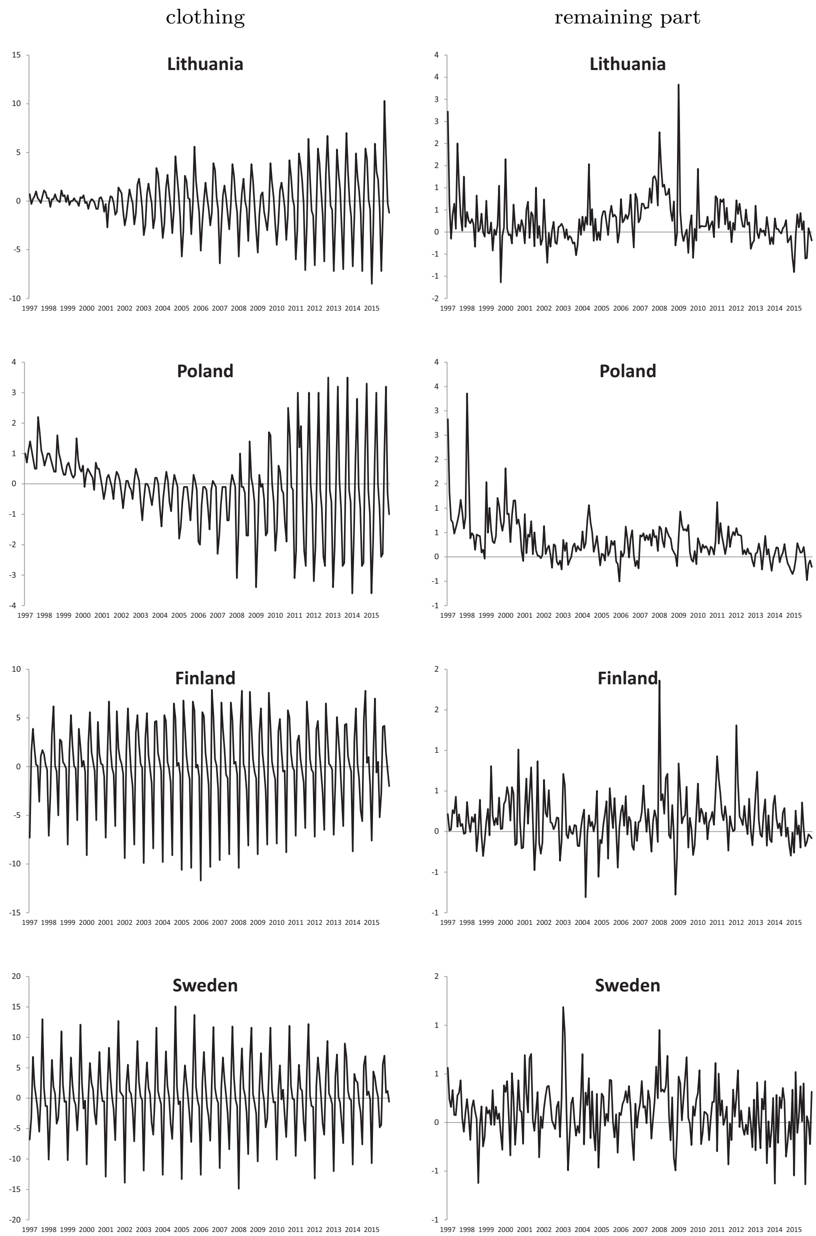


Figure 3b: HICP (2015 = 100) - monthly data (monthly rate of change) from Jan. 1997 to Dec. 2015 for clothing and footwear and the remaining part.



$$L_{2\times 12\mathrm{MA}}(B) = (B^{-6}+2B^{-5}+2B^{-4}+\ldots+2B^{-1}+2+2B+\ldots+2B^4+2B^5+B^6)/24,$$

where $B^j X_t = X_{t-j}$ and X_t is a filtered time series. It should be emphasized that in the case of a periodically correlated time series X_t (see Gladyshev (1961)) with period $T = 12$ (in short PC(12)) the time series after the following filtration $\tilde{L}_{2\times 12\mathrm{MA}}(B) = (1 - L_{2\times 12\mathrm{MA}}(B))X_t$ has a constant mean function. This means that time series $\tilde{L}_{2\times 12\mathrm{MA}}(B)X_t$ is also PC(T) with the same shape of the mean function as series X_t (see Theorem 1 in Appendix).
To understand the problem of seasonal volatility, at the first step we present *fruit*, *vegetables*, *remaining part* and *total HICP* (after 2×12MA filtration) in different months of the period: July 1997 - June 2015 (see Figures 4a-4d). The estimated standard deviation signals the periodic phenomenon of variance in many cases for *fruit* (see Germany and Poland) and *vegetables* (see Poland). Taking into account the corresponding confidence intervals for unknown standard deviations, seasonal volatility can be an important assumption during modelling that helps to improve forecasting power. Furthermore, the periodicity in levels (seasonal effect) is not obvious in *remaining part*. In the case of *clothing and footwear*, the similar simple visual examination is not interesting since the amplitude of the seasonal pattern is clearly time varying during the examined period in most countries. In the case of the *remaining part* and *total HICP*, the seasonal volatility can also be supported by the data (see the case of Lithuania).
Taking into account the above visual observation, in the next section, we examine in detail the potential existence of a seasonal volatility pattern in considered disaggregation and *total HICP*. Note that for *fruit*, *vegetables*, *clothing and footwear* and *total HICP*, the seasonal pattern in levels is obvious, and therefore procedures and tests are not applied in this case. Note that *remaining part* is a mixture of some prices that exhibit a seasonal pattern (seasonal products or services). Therefore, for the component *remaining part*, the seasonal pattern is not obvious and should also be examined.

3 Testing the periodic phenomenon in disaggregated HICP

In this section, we consider two effects: seasonality in volatility and seasonality in mean. We consider both visual justifications (a graphical method based on usual periodogram) and formal statistical testing procedures. In doing so, we assume that analysed or transformed time series is periodically correlated with natural period $T = 12$ (PC(12) in short). Formally, this means that the mean function $E(X_t) = \mu(t)$ and the autocovariance function $\mathrm{cov}(X_t, X_{t+\tau}) = B(t, \tau)$ exists and is a periodic function with the period 12. In such cases these functions have the following Fourier

Figure 4a: From left in columns: fruit and vegetables (m-o-m, after 2 × 12MA filtering) during each year (in period July 1997 to June 2015). The price during each year (grey lines, left axis); sample mean (dark line, left axis); sample standard deviation (grey dotted line, right axis) and corresponding 95% confidence bounds (black doted lines, right axis). Confidence bounds were obtained under iid and normality assumptions in sample.

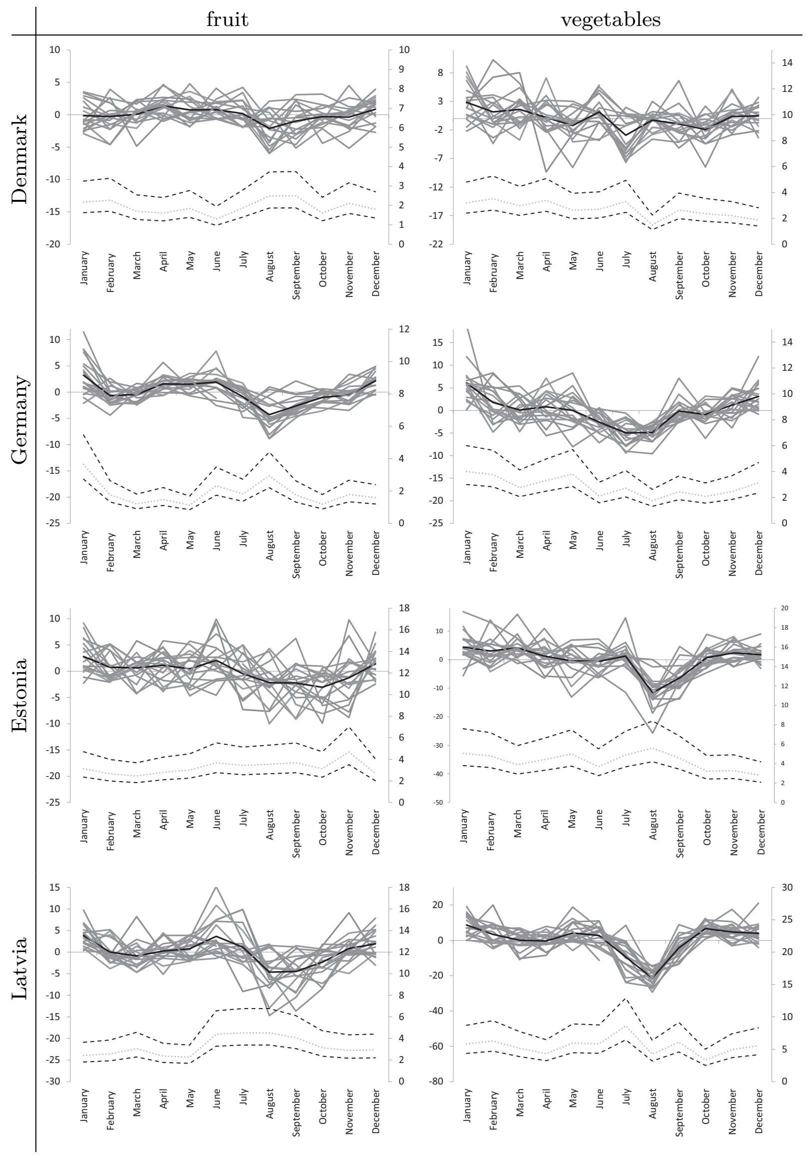


Figure 4b: From left in columns: remaining part and total HICP (m-o-m, after $2 \times$ 12MA filtering) during each year (in period July 1997 to June 2015). The price during each year (grey lines, left axis); sample mean (dark line, left axis); sample standard deviation (grey dotted line, right axis) and corresponding 95% confidence bounds (black doted lines, right axis). Confidence bounds were obtained under iid and normality assumptions in sample.

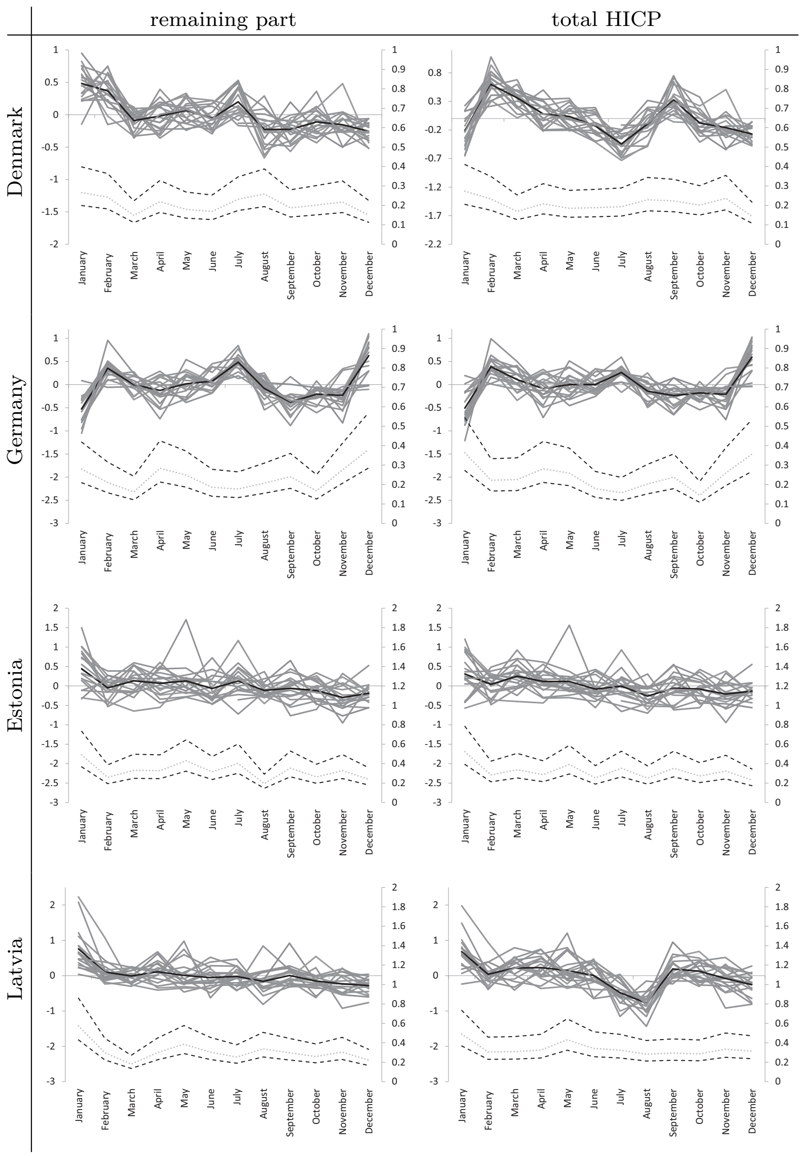


Figure 4c: From left in columns: fruit and vegetables (m-o-m, after 2 × 12MA filtering) during each year (in period July 1997 to June 2015). The price during each year (grey lines, left axis); sample mean (dark line, left axis); sample standard deviation (grey dotted line, right axis) and corresponding 95% confidence bounds (black doted lines, right axis). Confidence bounds were obtained under iid and normality assumptions in sample.

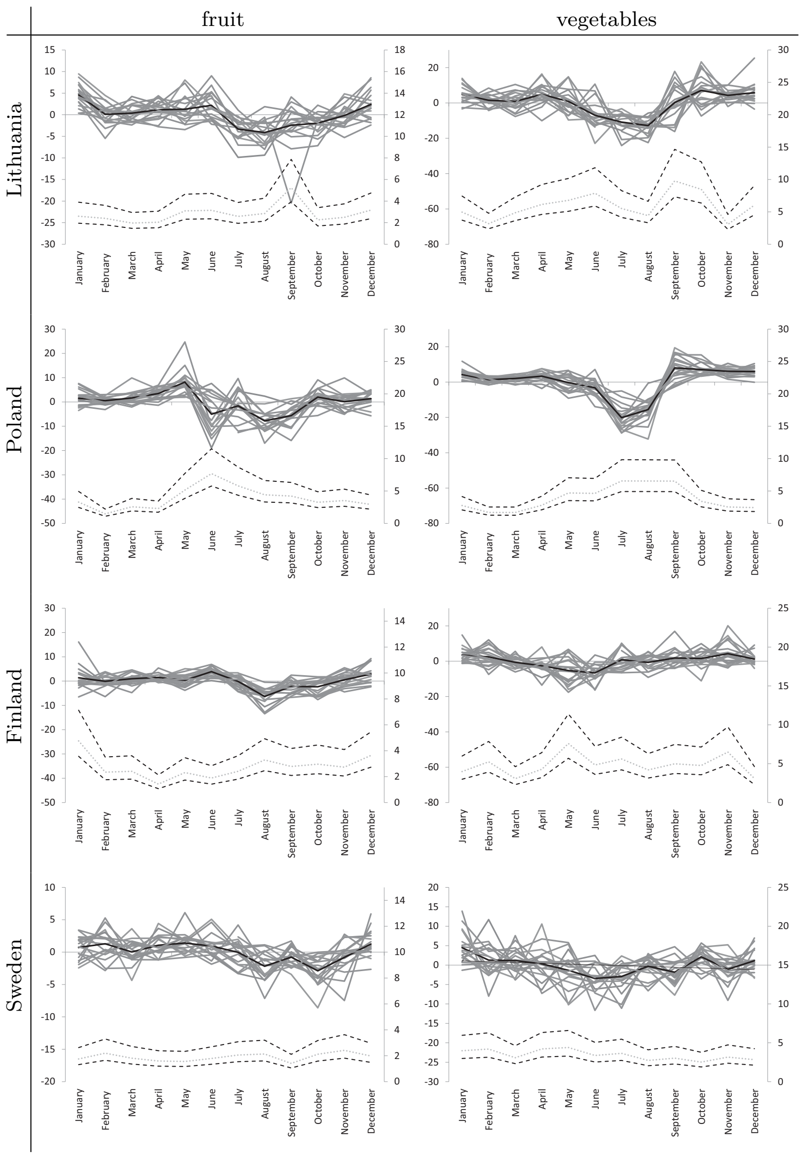
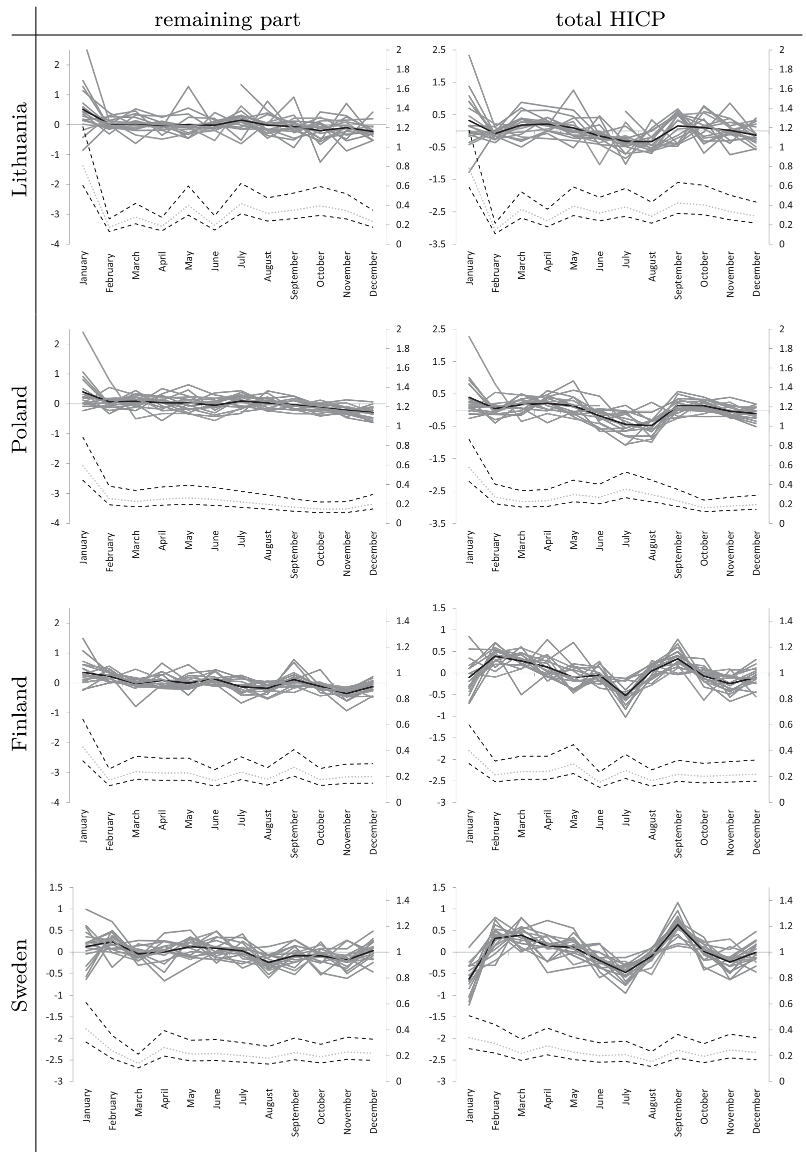


Figure 4d: From left in columns: remaining part and total HICP (m-o-m, after 2×12MA filtering) during each year (in period July 1997 to June 2015). The price during each year (grey lines, left axis); sample mean (dark line, left axis); sample standard deviation (grey dotted line, right axis) and corresponding 95% confidence bounds (black doted lines, right axis). Confidence bounds were obtained under iid and normality assumptions in sample.



representations:

$$\mu(t) = \sum_{\psi \in \Psi_{12}} m(\psi) e^{i\psi t},$$

$$B(t,\tau) = \sum_{\psi \in \Psi_{12}} a(\psi,\tau) e^{i\psi t}, \qquad \tau \in \mathbb{Z}$$

where $\Psi_{12} = \{k\pi/6 : \; k = 0, 1, \ldots, 11\}$ and $m(\psi)$, $a(\psi,\tau)$ are the Fourier coefficients.

3.1 Testing the seasonality in levels for remaining part

To examine seasonality in details we consider variable R_t and three transformations: $R_t - R_{t-1}$ (first difference), $R_t - R_{t-12}$ (seasonal difference) and $\tilde{L}_{2\times 12\mathrm{MA}}(B)R_t$ (series after trend subtraction using 2×12MA filtering). Note that from theoretical point of view, only in the case of seasonal difference are the fluctuations connected with seasonal mean function removed.
In this part, we use a formal statistical test for seasonal effect. We use test for seasonal fluctuations proposed by Lenart and Pipień (2013). This non-parametric test is based on a general almost periodically correlated (APC) class of time series and subsampling methodology. For greater reliability, we use different significance levels in the testing procedure ($\alpha \in \{8\%, 5\%, 2\%\}$) and different values of the block length ($b \in \{2.5\sqrt{n}, 3\sqrt{n}, 3.5\sqrt{n}\}$) in quantile evaluation using subsampling methodology. Note that parameter b can change the empirical results. The results of the testing procedure are included in Table 2. For most countries, the results clearly support the existence of a seasonal pattern at each of the test's significance levels, a different length of the block b and considered variable R_t and transformed variables $R_t - R_{t-1}$ and $\tilde{L}_{2\times 12\mathrm{MA}}(B)R_t$. Only in the case of Lithuania and Poland, and in some cases for Sweden, do the results depend on block length or transformed variable. In the case of seasonal differences, the test statistics are below the critical value uniformly at significance level and length of the block for each country. Summing up, the presented test supports the existence of a seasonal pattern in considered series R_t in most countries under consideration.

3.2 Testing the presence of seasonal volatility

We consider the problem of periodic variance using a formal statistical test based on Lenart (2013) and graphical test based on a usual periodogram. We analyse the data after 2×12MA filtration with seasonal mean subtraction and data after a seasonal difference. As mentioned in the previous section, the periodic phenomenon in the mean and autocovariance function is robust to moving average filtration. Our analysis considers two assumptions separately: deterministic and a possible stochastic phenomenon of seasonal fluctuations in the considered data.
Under the deterministic seasonal fluctuations assumption, we estimate the seasonal coefficient $s_1, s_2, \ldots, s_{12}$ using a natural nonparametric estimator (see Hurd and

Table 2: Non-parametric test for seasonality. Italic denotes that test statistics exceed the quantile (we reject the hypothesis that the series does not have a seasonal mean).

Series	Country		Denmark	Germany	Estonia	Latvia	Lithuania	Poland	Finland	Sweden
R_t	value of the test statistics		**3.44**	**4.85**	**3.05**	**4.02**	**3.16**	**2.95**	**2.72**	**2.04**
quantile value	$b = 2.5\sqrt{n}$	$\alpha = 8\%$	*2.01*	*2.6*	*1.95*	*3.3*	*3.12*	*1.92*	*1.93*	*1.48*
		$\alpha = 5\%$	*2.07*	*2.67*	*2.06*	*3.46*	3.2	*2.75*	*2,00*	*1.54*
		$\alpha = 2\%$	*2.25*	*2.87*	*2.16*	*3.82*	3.32	*2.91*	*2.07*	*1.65*
	$b = 3\sqrt{n}$	$\alpha = 8\%$	*2.18*	*2.8*	*1.99*	*3.54*	*2.94*	*1.92*	*1.99*	*1.45*
		$\alpha = 5\%$	*2.2*	*2.83*	*2.11*	*3.6*	*3.01*	*2.79*	*2.04*	*1.47*
		$\alpha = 2\%$	*2.28*	*2.87*	*2.24*	*3.72*	3.27	*2.94*	*2.21*	*1.57*
	$b = 3.5\sqrt{n}$	$\alpha = 8\%$	*2.28*	*3.01*	*2.08*	*3.56*	*2.97*	*1.99*	*2.09*	*1.54*
		$\alpha = 5\%$	*2.32*	*3.14*	*2.11*	*3.74*	*2.99*	*2.6*	*2.14*	*1.59*
		$\alpha = 2\%$	*2.46*	*3.18*	*2.15*	*3.9*	3.28	*2.92*	*2.55*	*1.63*
$R_t - R_{t-1}$	value of the test statistics		**4.38**	**8.02**	**4.06**	**5.43**	**4.27**	**3.51**	**3.41**	**2.4**
quantile value	$b = 2.5\sqrt{n}$	$\alpha = 8\%$	*2.72*	*4.48*	*2.78*	*4.67*	4.3	*2.86*	*2.48*	*2.13*
		$\alpha = 5\%$	*2.78*	*4.65*	*2.82*	*4.89*	4.71	3.71	*2.71*	2.52
		$\alpha = 2\%$	*2.85*	*4.85*	*2.9*	*5.06*	4.73	3.75	*2.75*	2.81
	$b = 3\sqrt{n}$	$\alpha = 8\%$	*2.9*	*4.84*	*2.83*	*4.94*	4.31	*3.02*	*2.55*	2.4
		$\alpha = 5\%$	*2.95*	*4.96*	*2.87*	*5.02*	4.36	3.79	*2.61*	2.47
		$\alpha = 2\%$	*2.98*	*5.02*	*3.09*	*5.26*	4.52	4.05	*2.81*	2.71
	$b = 3.5\sqrt{n}$	$\alpha = 8\%$	*3.06*	*5.2*	*2.9*	*5.05*	*4.22*	*3.11*	*2.63*	2.47
		$\alpha = 5\%$	*3.08*	*5.43*	*2.97*	*5.16*	4.31	3.77	*2.68*	2.69
		$\alpha = 2\%$	*3.19*	*5.58*	*3,00*	*5.27*	4.47	4.11	*3.17*	2.83
$R_t - R_{t-12}$	value of the test statistics		**0.3**	**0.36**	**0.35**	**0.43**	**0.88**	**0.6**	**0.16**	**0.35**
quantile value	$b = 2.5\sqrt{n}$	$\alpha = 8\%$	0.71	0.81	1.32	1.49	1.68	0.93	0.94	0.82
		$\alpha = 5\%$	0.73	0.82	1.38	1.63	1.74	1.52	1.22	0.89
		$\alpha = 2\%$	0.75	0.88	1.53	1.7	1.81	1.61	1.26	0.94
	$b = 3\sqrt{n}$	$\alpha = 8\%$	0.66	0.82	1.38	1.51	1.55	0.86	0.97	0.77
		$\alpha = 5\%$	0.7	0.83	1.41	1.66	1.59	1.31	1.04	0.79
		$\alpha = 2\%$	0.75	0.85	1.54	1.72	1.63	1.42	1.09	0.81
	$b = 3.5\sqrt{n}$	$\alpha = 8\%$	0.62	0.76	1.31	1.42	1.48	0.98	0.81	0.73
		$\alpha = 5\%$	0.64	0.79	1.37	1.48	1.57	1.13	0.86	0.74
		$\alpha = 2\%$	0.67	0.8	1.42	1.56	1.62	1.23	0.93	0.76
$\tilde{L}_{2\times 12MA}(B)R_t$	value of the test statistics		**3.36**	**4.83**	**2.72**	**3.76**	**2.66**	**2.45**	**2.74**	**1.92**
quantile value	$b = 2.5\sqrt{n}$	$\alpha = 8\%$	*1.97*	*2.52*	*1.87*	*3.22*	2.87	*1.79*	*1.9*	*1.4*
		$\alpha = 5\%$	*2.02*	*2.56*	*1.9*	*3.42*	3.25	*1.86*	*2.04*	*1.43*
		$\alpha = 2\%$	*2.07*	*2.68*	*1.94*	*3.51*	3.26	2.77	*2.07*	*1.45*
	$b = 3\sqrt{n}$	$\alpha = 8\%$	*2.17*	*2.8*	*1.96*	*3.47*	2.95	*1.79*	*2,00*	*1.42*
		$\alpha = 5\%$	*2.2*	*2.81*	*2,00*	*3.53*	3.03	*1.91*	*2.06*	*1.43*
		$\alpha = 2\%$	*2.26*	*2.86*	*2.13*	*3.56*	3.15	2.82	*2.15*	*1.48*
	$b = 3.5\sqrt{n}$	$\alpha = 8\%$	*2.27*	*2.92*	*1.97*	*3.39*	2.93	*1.85*	*2.1*	*1.46*
		$\alpha = 5\%$	*2.29*	*3.03*	*2.02*	*3.53*	2.96	*1.92*	*2.13*	*1.49*
		$\alpha = 2\%$	*2.41*	*3.15*	*2.06*	*3.69*	3.02	2.67	*2.17*	*1.52*

Miamee (2007), Chapter 9), which is consistent in the mean square sense. Assuming we have a sample $X_1, X_2, \ldots, X_n$ (after 2×12MA filtration) where $n = 12d$, this estimator takes the form:

$$\hat{s}_{n,j} = \frac{1}{d}\sum_{i=1}^{d} X_{j+(i-1)12}, \tag{1}$$

where $j = 1, 2, \ldots, 12$. Finally, by $Y_1, Y_2, \ldots, Y_n$ we denote the series after subtracting the estimated seasonal coefficients (based on estimated seasonal coefficients (1)). Note that the usual estimator of Fourier coefficients based on subtracted sample is consistent in mean square sense under regularity conditions. Combining this with Theorem 1 (see Appendix) and the results presented in Dudek and Lenart (2017) the testing procedure proposed in Lenart (2013) can be applied to $Y_1^2, Y_2^2, \ldots, Y_n^2$ after constant mean subtraction for examining periodicity in the variance function (under appropriate regularity conditions). Under the same arguments, the periodogram can also be examined.

Figures 6a-6b presents results of a graphical test (usual periodogram) for $Y_1^2, Y_2^2, \ldots, Y_n^2$ in case of *fruit*, *vegetables*, *remain part* and *total HICP*. Observing the periodogram for squared data clearly shows the peaks at seasonal frequencies in the case of *fruit* for Germany, Latvia, and Poland. In the case of vegetables, the predominant peak is observed for Poland. In the case of the *remaining part* and *total HICP*, the predominant peak is observed at the Nyquist frequency for Estonia and Lithuania. Summing up, these observations suggest possible periodic phenomenon in variance function in *fruit*, *vegetables*, *remaining part* and *total HICP*.

In the next step, we consider the stochastic counterpart of the seasonal fluctuations. In such cases, we use standard seasonal difference filtration (instead of subtracting the mean (1)) to remove the seasonal pattern. We employ the general (not restrictive) assumption that after the seasonal difference the obtained series is PC(12). After the seasonal difference, the analysis of periodograms for squared data also supports the periodic phenomenon in the variance function (see Figures 5a-5b). For example, in the case of *fruit* and *vegetables* for Poland, the biggest value of the periodogram is observed for the seasonal frequency $\pi/6$, while in the case of *remaining part* and *total HICP*, the greatest value of the periodogram is observed at Nyquist frequency $\pi/2$.

In the next part, we apply for *fruit*, *vegetables*, *clothing and footwear*, *remaining part* and *total HICP* (after seasonal difference) the test proposed in Lenart *et al.* (2008). Let X'_t denote the series after the seasonal difference. Elementary calculations shows that under standard assumptions formulated in Lenart *et al.* (2008) this test can be used to examine the periodicity for the autocovariance function of $B_{X'}(t,\tau) = \mathrm{cov}(X'_t, X'_{t+\tau})$. Note that the periodicity of $B_{X'}(t,\tau) = \mathrm{cov}(X'_t, X'_{t+\tau})$ is equivalent to the periodicity of the gross data. Note that for $\tau = 0$ the function $B_{X'}(t,0)$ is the usual variance function. Without loss of generality we assume from now that $\tau \geq 0$. Recall from Lenart *et al.* (2008) that for any nonnegative integer τ our testing problem is as follows:

$$H_0 : B_{X'}(\cdot,\tau) \text{ is not periodic (depends only on } \tau)$$

$$H_1 : B_{X'}(\cdot,\tau) \text{ is periodic with period } T = 12,$$

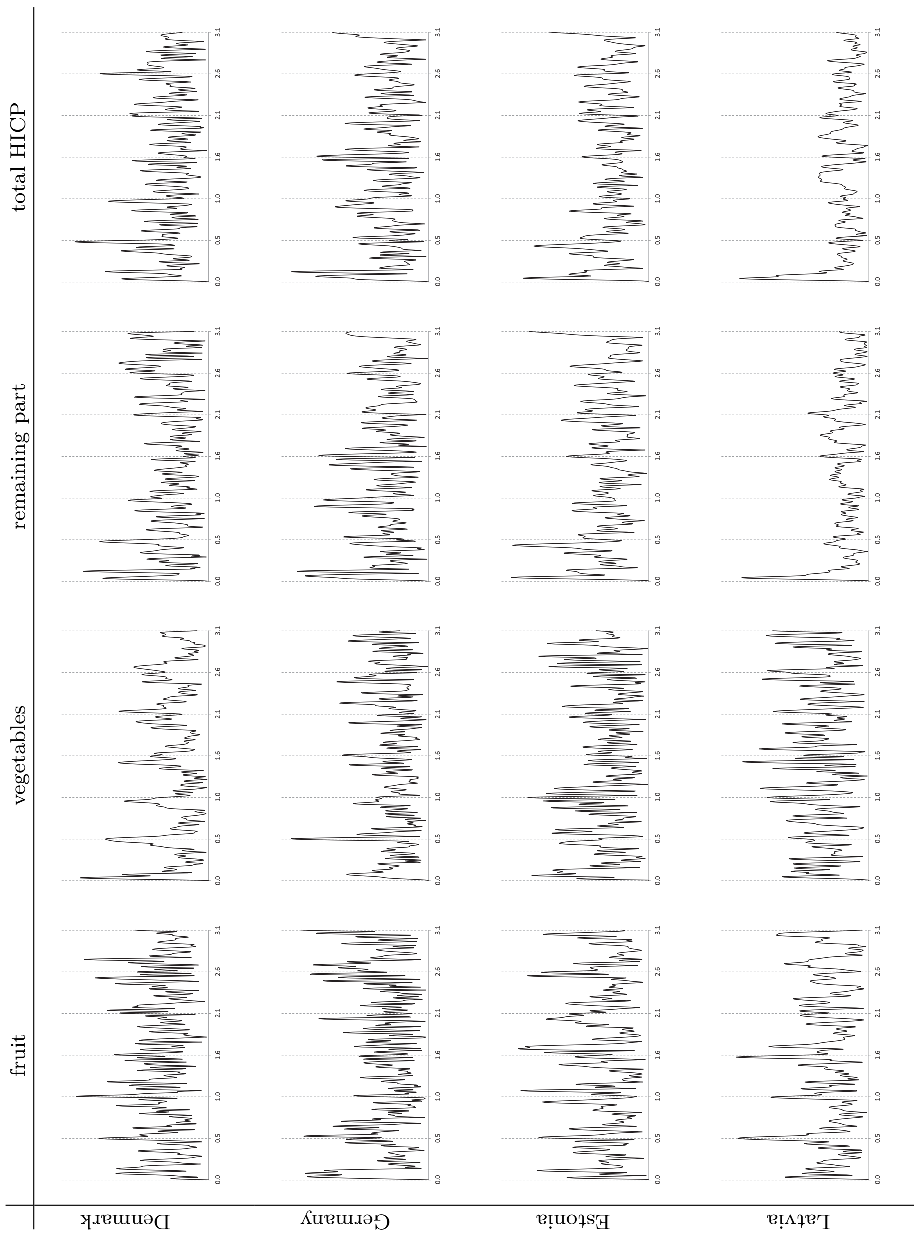


Figure 5a: Periodograms for (mean subtracted) square of seasonal difference of fruit, vegetables, remaining part and total HICP.

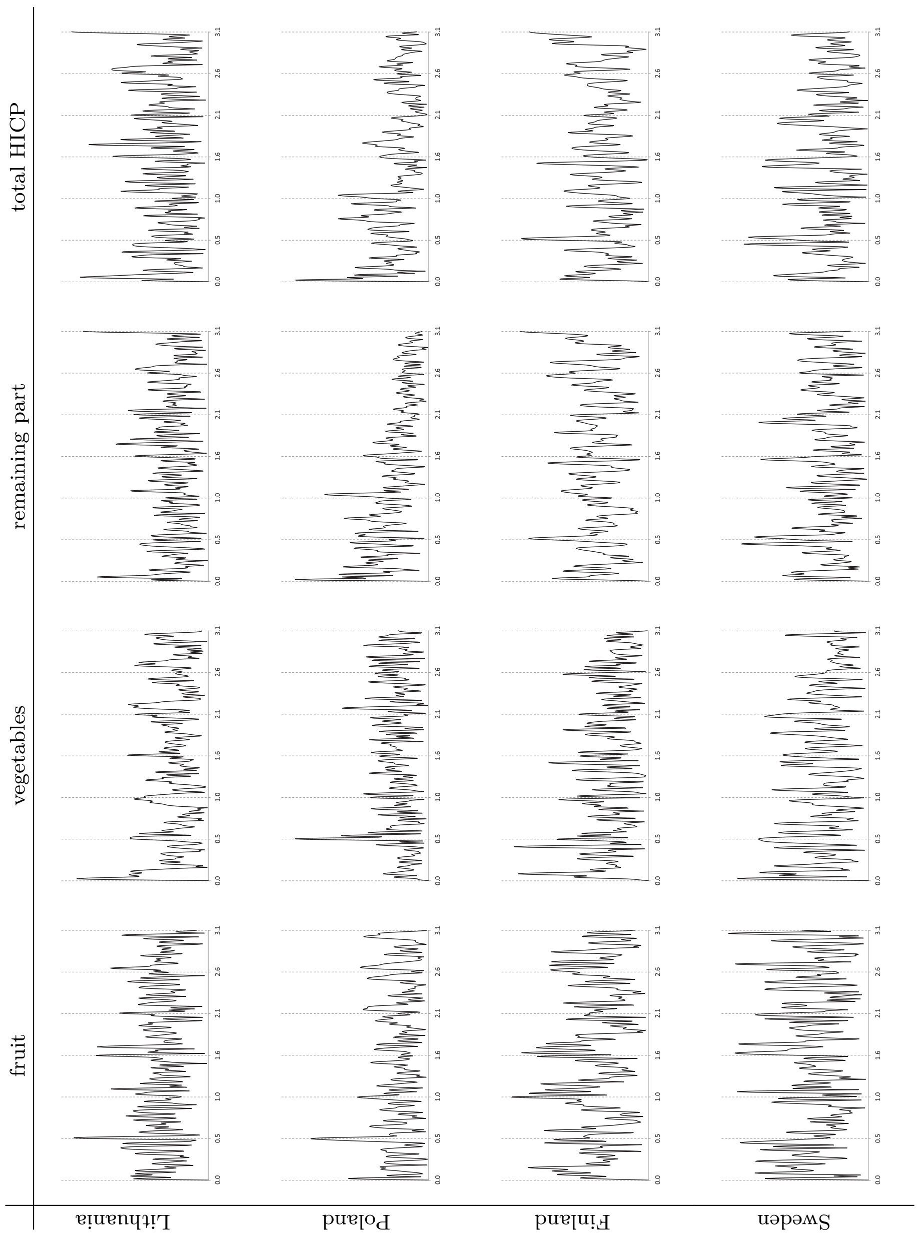
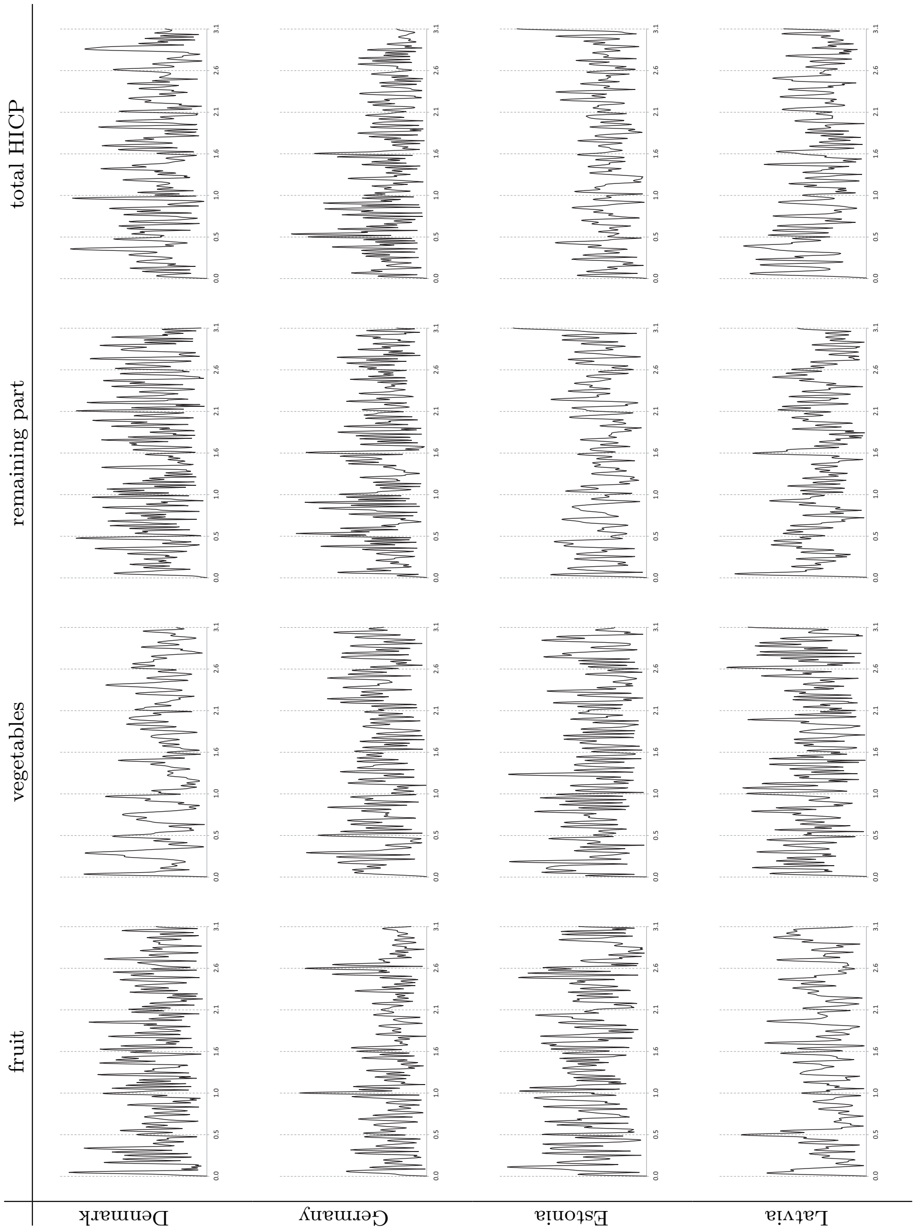


Figure 5b: Periodograms for (mean subtracted) square of seasonal difference of fruit, vegetables, remaining part and total HICP.

Figure 6a: Periodograms for (mean subtracted) squares of fruit, vegetables, remaining and total HICP (after 2 × 12MA and seasonal subtraction by (1)).



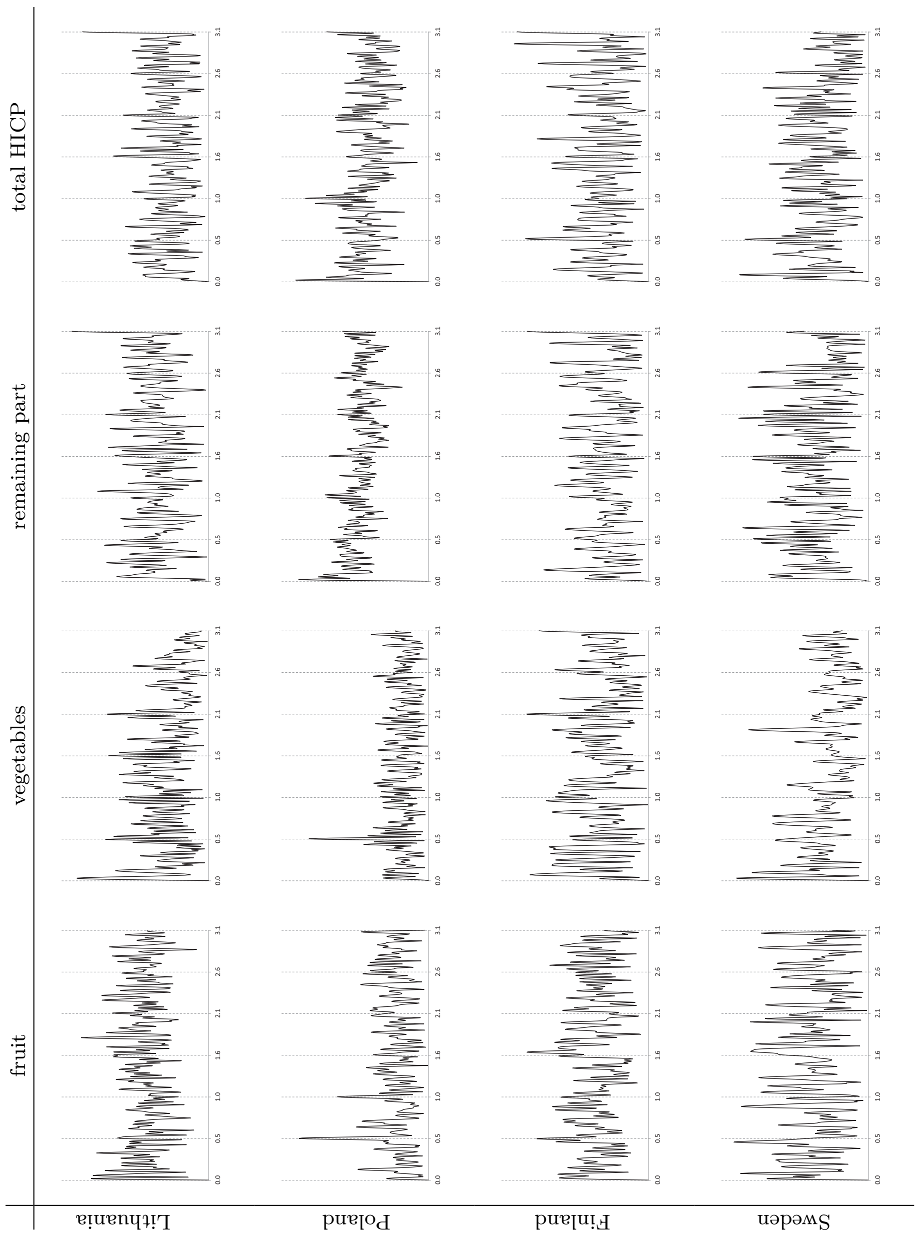


Figure 6b: Periodograms for (mean subtracted) squares of fruit, vegetables, remaining and total HICP (after 2 × 12MA and seasonal subtraction by (1)).

with test statistics:

$$U_n(\tau) = \sqrt{n}\left(\sum_{k=1}^{6} |\hat{a}_n(\lambda_k, \tau)|\right),$$

where $\lambda_k = \frac{2k\pi}{12}$, and for any real λ we have $\hat{a}_n(\lambda, \tau) = \frac{1}{n}\sum_{t=1}^{n-\tau} X'_t X'_{t+\tau} e^{-i\lambda t}$ (for more detail see Lenart *et al.* (2008)). We calculate aggregate test statistics $U_n(\tau)$ only for $\tau = 0$ with quantile obtained by subsampling approach (at standard nominal level $\alpha = 5\%$). Since the optimal length of the block in the subsampling approach is still an open problem we show the results of the testing problem for $b \in \{50, 55, 60, 65, 70\}$. The considered sample length is $n = 204$.

Table 3 presents the results of the testing procedure. In the case of *fruit* we reject H_0 (for all considered b) for Denmark, Lithuania and Poland, which means that the data support the existence of periodicity in variance at the significance level $\alpha = 5\%$. In the case of *vegetables*, the test statistics clearly exceed the quantile for each considered value of parameter b for all countries except Denmark. For clothing and footwear, we reject the hypothesis assuming constant variance in the case of Denmark, Latvia, Finland and Sweden. Note that for these countries (excluding Latvia), the amplitude of seasonal pattern in *clothing and footwear* is much more stable than in the other countries considered. In case of the *remaining part* and *total HICP*, the test statistics clearly exceeds the critical value in the case of Lithuania.

Summing up, the periodic phenomenon in the second moment was detected using the periodogram observation and formal statistical procedure based on subsampling approach. In the next part, we indicate the models that have a periodic phenomenon in noise variance and assess whether this can help in forecasting for disaggregated and *total HICP*.

4 Exponential smoothing model with periodic noise

We consider the following exponential smoothing model (in state space representation, see Hyndman *et al.* (2008)):

$$\begin{cases} \mathbf{y}_t = \mathbf{W}'\mathbf{x}_{t-1} + \boldsymbol{\epsilon}_t & \text{observation equation} \\ \mathbf{x}_t = \mathbf{F}\mathbf{x}_{t-1} + \mathbf{G}\boldsymbol{\epsilon}_t & \text{state equation,} \end{cases} \tag{2}$$

where $\boldsymbol{\epsilon}_t$ - is gaussian noise with zero mean and time-varying variance-covariance matrix Σ_t^2, $\mathbf{y}_t$ - observations vector, $\mathbf{x}_t$ - $r \times 1$ column of unobserved states. In the next part, we consider two specifications of this model. The difference is that the first specification concerns the flexible seasonal pattern, while the second concerns a fixed seasonality pattern. In both cases, we consider the possible periodic variance of the noise.

Table 3: Test for seasonality of variance. The testing statistics was $U_n(0)$ applied for seasonal difference. Italic denotes that we reject null hypothesis.

		Country	Denmark	Germany	Estonia	Latvia	Lithuania	Poland	Finland	Sweden
Fruit		test statistics	**89.75**	**55.75**	**256.84**	**302.98**	**236.41**	**641.98**	**141.63**	**61.38**
	quantile value	$b = 50$	*21.21*	75.57	278.46	395.98	*209.41*	*612.87*	193.77	66.68
		$b = 55$	*20.71*	70.24	*253.09*	441.15	*181.74*	*591.47*	178.92	67.12
		$b = 60$	*23.56*	64.92	*225.61*	464.43	*179.87*	*542.51*	185.64	71.16
		$b = 65$	*20.87*	61.95	*204.86*	435.39	*162.28*	*500.23*	171.52	70.71
		$b = 70$	*18.69*	57.6	*181.22*	509.57	*162.17*	*491.47*	167.95	65.34
Vegetables		test statistics	**169.2**	**237.27**	**492.31**	**632.94**	**786.81**	**390.45**	**399.95**	**258.76**
	quantile value	$b = 50$	302.56	*175.75*	*296.1*	*428.5*	*693.99*	*211.3*	*304.39*	*212.51*
		$b = 55$	284.15	*167*	*257.32*	*403.84*	*705.51*	*190.82*	*298.93*	*204.11*
		$b = 60$	269.42	*171.3*	*225.3*	*382.86*	*639.66*	*165.31*	*278.68*	*203.18*
		$b = 65$	250.69	*159.3*	*205.45*	*372.23*	*704.44*	*149.3*	*269.27*	*210.11*
		$b = 70$	233.06	*143.73*	*174.98*	*350.59*	*654.2*	*156.08*	*247.54*	*214.41*
Clothing and footwear		test statistics	**73.64**	**5.8**	**11.52**	**16.87**	**14.33**	**6.55**	**38.47**	**43.84**
	quantile value	$b = 50$	*43.37*	7.91	14.26	*16.78*	14.43	7.76	*20.22*	*27.24*
		$b = 55$	*41.05*	7.32	12.95	*15.95*	13.45	7.49	*17.41*	*24.79*
		$b = 60$	*36.06*	7.3	11.65	*16.36*	*12.28*	8.4	*15.01*	*30.11*
		$b = 65$	*34.19*	6.81	*10.61*	*15.68*	*11.44*	7.78	*12.45*	*27.76*
		$b = 70$	*29.98*	6.31	*9.73*	*14.22*	*10.41*	7.23	*10.37*	*24.81*
Remaining part		test statistics	**0.89**	**1.21**	**5.81**	**4.83**	**7.58**	**1.41**	**2.14**	**1.17**
	quantile value	$b = 50$	0.96	1.53	6.59	5.77	*3.23*	1.84	2.45	1.47
		$b = 55$	0.94	1.48	6.25	5.39	*2.77*	1.7	2.28	1.39
		$b = 60$	0.94	1.38	6.08	5.03	*2.33*	1.59	*2.08*	1.28
		$b = 65$	*0.87*	1.29	*5.75*	*4.69*	*1.88*	1.49	*2.07*	1.2
		$b = 70$	*0.86*	*1.18*	*5.31*	*4.82*	*1.46*	*1.32*	2.24	*1.1*
Total HICP		test statistics	**0.99**	**1.22**	**4.84**	**3.51**	**5.47**	**1.45**	**1.7**	**1.05**
	quantile value	$b = 50$	1.11	1.33	5.89	3.54	*2.17*	1.73	1.83	1.59
		$b = 55$	1.05	1.29	5.56	*3.35*	*1.89*	1.67	*1.68*	1.6
		$b = 60$	1.04	*1.18*	5.51	*3.04*	*2.16*	1.53	*1.6*	1.48
		$b = 65$	1.02	*1.1*	5.19	*2.99*	*1.88*	1.49	*1.5*	1.41
		$b = 70$	*0.98*	*1*	*4.75*	*2.93*	*1.63*	*1.35*	1.72	1.35

4.1 Damped level model with additive seasonality and periodic variance of the noise

Let us consider a univariate damped level model with additive seasonality and periodic variance σ_t^2 of the noise ϵ_t with period m:

$$\begin{cases} y_t = \phi l_{t-1} + s_{t-m} + \mu + \epsilon_t \\ l_t = \phi l_{t-1} + \alpha \epsilon_t \\ s_t = s_{t-m} + \gamma \epsilon_t . \end{cases} \tag{3}$$

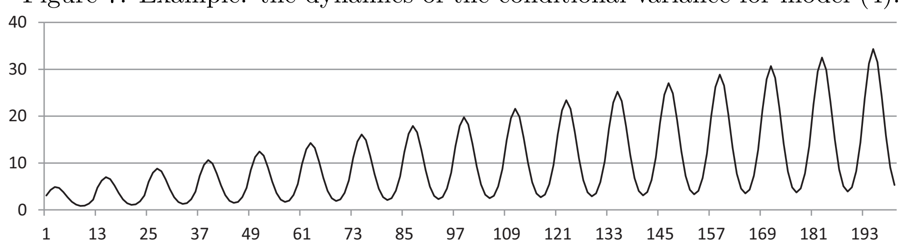
The state space form of this model is:

$$\begin{cases} y_t = \begin{bmatrix} \phi & 0 & \dots & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} l_{t-1} \\ s_{t-1} \\ s_{t-2} \\ \vdots \\ s_{t-m} \\ \mu \end{bmatrix} + \epsilon_t \\ \begin{bmatrix} l_t \\ s_t \\ s_{t-1} \\ \vdots \\ s_{t-m+1} \\ \mu \end{bmatrix} = \begin{bmatrix} \phi & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & \dots & 0 & 1 & 0 \\ 0 & & & & 0 & 0 \\ \vdots & & I_{m-1} & & \vdots & \vdots \\ 0 & & & & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_{t-1} \\ s_{t-1} \\ s_{t-2} \\ \vdots \\ s_{t-m} \\ \mu \end{bmatrix} + \begin{bmatrix} \alpha \\ \gamma \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \epsilon_t. \end{cases} \tag{4}$$

Note that the unconditional mean function does not exist for this model; therefore, this model cannot be periodically correlated. To show the dynamics of the second conditional moment of this model, we consider the following illustrative example.

Example 1. *Let us consider a data generating process of the form (4) with* $\alpha = 0.5$, $\phi = 0.8$, $\gamma = 0.9$ *and periodic variance of the noise* $\sigma_t = 1 + \sin(t\pi/6)/2$. *Note that in such cases, the variance of the noise is a periodic function with period* $m = 12$, *which corresponds to monthly data. Since the unconditional variance does not exist in such case, we show the dynamics of conditional variance for* $t \geq 0$ *under* $\mathcal{F}_0$. *Figure 7 shows the dynamics of conditional variance* $var(y_t|\mathcal{F}_0)$ *for* $t = 1, 2, \dots, 200$ *in the considered example.*

Figure 7: Example: the dynamics of the conditional variance for model (4).



It is clearly visible that conditional variance varies over time. It can be shown that $var(y_t|\mathcal{F}_0)$ *has a linear increase of the value. This property influences predictive distribution, especially for long-term horizons.*

The source of the nonstationarity of the above data generating process is the dynamics of the seasonal component: $s_t = s_{t-m} + \gamma\epsilon_t$. In informal meaning, if the parameter γ is closer to zero, then the increase of conditional variance is weaker and the seasonal pattern is closer to fixed seasonality. Therefore, in the next section, we consider the limiting case when $\gamma = 0$.

4.2 Damped level model with seasonal mean and periodic variance of the noise

Let us consider the univariate damped level model with fixed seasonal mean function ($\gamma = 0$) and periodic variance σ_t^2 of the noise ϵ_t with period $m = 12$:

$$\begin{cases} y_t = \phi l_{t-1} + \mu_{t-m} + \epsilon_t \\ l_t = \phi l_{t-1} + \alpha\epsilon_t \\ \mu_t = \mu_{t-m}. \end{cases} \tag{5}$$

Note that the considered model has periodic unconditional variance iff $|\phi| < 1$. The variance can be easily calculated and has the following form: $var(y_t) = \sum_{k=0}^{\infty} \alpha^2\phi^{2k}\sigma_{t-k}^2$. Using Fourier representation for variance function $\sigma_t^2 = \sum_{\psi\in\Psi_m} m(\psi)e^{i\psi t}$ (where $\Psi_m = \{2k\pi/m : k = 0, 1, \ldots, m-1\}$), we obtain $var(y_t) = \sum_{\psi\in\Psi_m} \left[m(\psi)\frac{\alpha^2 e^{i\psi}}{e^{i\psi} - \phi^2}\right] e^{i\psi t}$, which means that under assumption $|\phi| < 1$, the considered model is periodically correlated with period m (PC(m)).

5 Forecasting exercise

5.1 Detailed model specification

In this part, we consider model (3) and model (4) with a constant and periodic phenomenon in variance of the white noise. Note that model (3) is more flexible in seasonal pattern than model (4), where the fixed seasonal effect is assumed. Generally, by M we denote the models with flexible seasonal pattern (model (3)) and by M′ we denote the models with fixed seasonality pattern (model (4)). To clarify, we introduce the notation $M_{(x_1,x_2,\ldots,x_{12})}$ where the vector $(x_1, x_2, \ldots, x_{12})$ is a sequence of labels that corresponds to variance in months during the year. For example, $(x_1, x_2, \ldots, x_{12}) = (1, 1, \ldots, 1)$ means that there is only one group of value for variance, which means that variance is constant during years, while $(x_1, x_2, \ldots, x_{12}) = (1, 1, 1, 2, 2, 2, 3, 3, 3, 4, 4, 4)$ means that variance has four different values during the year (equal value during each quarter of the year). In this case, we assume constant variance during three consecutive months of each quarter

($\sigma_t = \sigma_{t+1} = \sigma_{t+2}$ for time t corresponding to January, April, July, October). Finally, $(x_1, x_2, \ldots, x_{12}) = (1, 2, \ldots, 12)$ means that the variance can be different in each month.

The number of all possible specifications under assumptions that we have two groups of variance during year equals 2^{11}, while in the case of tree groups, the number of possible models equals 3^{11}. In Trimbur and Bell (2012), the authors introduce an algorithm based on AIC criteria assuming two groups of months in the case of seasonal heteroscedasticity. In this paper, we use simple specifications for the vector $(x_1, x_2, \ldots, x_{12})$, and we do not try to find the best model from all possible specifications. More precisely, we use the models contained in Table 4.

Table 4: Model specifications

Model notation	Short label	Short description	General model formula
$M_{(1,2,\ldots,12)}$	M_1	periodic variance (σ_t^2), $s = 12$	$\begin{cases} y_t = \phi l_{t-1} + s_{t-m} + \mu + \epsilon_t \\ l_t = \phi l_{t-1} + \alpha\epsilon_t \\ s_t = s_{t-m} + \gamma\epsilon_t \end{cases}$
$M_{(1,1,1,2,2,2,3,3,3,4,4,4)}$	M_2	periodic variance (σ_t^2), $s = 12$	
$M_{(1,1,\ldots,1)}$	M_3	constat variance $\sigma_t^2 = \sigma^2$	
$M'_{(1,2,\ldots,12)}$	M'_1	periodic variance (σ_t^2), $s = 12$	$\begin{cases} y_t = \phi l_{t-1} + \mu_{t-m} + \mu + \epsilon_t \\ l_t = \phi l_{t-1} + \alpha\epsilon_t \\ \mu_t = \mu_{t-m} \end{cases}$
$M'_{(1,1,1,2,2,2,3,3,3,4,4,4)}$	M'_2	periodic variance (σ_t^2), $s = 12$	
$M'_{(1,1,\ldots,1)}$	M'_3	constat variance $\sigma_t^2 = \sigma^2$	

For each variable, we use models M_1, M_2, M'_1, and M'_2 with time varying periodic function of variance of error term. The results for models M_3 and M'_3, have constat variance of error term. For each model, we consider four distributions for white noise, i.e., three heavy tailed Student-t distributions with 3,4,5 degrees of freedom and Gaussian distribution.

5.2 Accuracy measure for nowcasting

It is well known that models with good-in sample fit (based on for example information criteria) do not necessary provide good out-of sample accuracy. Therefore, in our experiment, we measure the performance of forecasts using one of the well-known logarithmic score (*logscore*, in short) measures (in nowcasting). This probabilistic measure of accuracy is broadly studied in Gneiting and Raftery (2007) and used in many empirical studies. We use this measure since it is sensitive to the variance of predictive distribution, which is important while data with potential a periodic phenomenon of variance are analysed.

We divide the sample into two parts: a training period (up to Dec. 2010) and a forecasting period (from Jan. 2011 to Dec. 2015). At the first step, the model is estimated using the whole training period, and the first predictive distribution is evaluated. At each following step, the data are actualized by adding the next observation and deleting the last one from the sample. In this way, the length of the sample is constant over time (14-year rolling window). Finally, we collect sixty predictive distributions for each month ahead. Let us denote these predictive density functions by $f_{t+1}(\cdot|M, \mathcal{F}_t)$ for $t = \text{Dec 2010}, \text{Jan 2011}, \ldots, \text{Nov 2015}$ (conditional on $\mathcal{F}_t$). The average logarithmic score is calculated using a simple average $logscore = \frac{1}{60} \sum_{t=\text{Dec 2010}}^{\text{Nov 2015}} \log(f_{t+1}(x_{t+1}|M, \mathcal{F}_t))$, where x_{t+1} is a realization of price dynamic at time $t+1$. This average was also calculated in Lenart and Leszczyńska-Paczesna (2016) as an estimate of unknown logarithmic score measure. The idea of such a measure is that by the law of large numbers, the above mean should be convergent (under appropriate regularity conditions) to $E[\log(f(X))]$, where f is a one-ahead predictive distribution for model (under the assumption that the number of predictive distributions tends to infinity). In the case of normal predictive distribution with mean μ and standard deviation σ, we have $E[\log(f(X))] = \ln(1/(2e\pi\sigma))$. Using a theoretical background for a logarithmic score (see Gneiting and Raftery (2007)) the higher average logarithmic score means that the model seems to be more accurate. Unfortunately, easy calculations show that in the case of periodic variance and gaussian predictive distribution f, the above average (under appropriate regularity conditions) tends to

$$\ln(1/(2e\pi\bar{\sigma})),$$

where $\bar{\sigma}$ is a geometric mean for variance during one season (whole period, for example: one year). It means that the "performance" of models with a periodic phenomenon in different months is averaged. Hence, the standard application of the Diebold-Mariano Test is limited since the sequence of logscore does not seem to satisfy the stationarity assumption (see Diebold and Mariano (1995)). Therefore, all empirical results concerning the average logscore should be interrelated with caution.

5.3 Results of the pseudo-real experiment

In Tables 5-8, the average logscore is presented for each considered specification for variables and countries. Italic denotes that this average logscore is maximal over the 24 considered models. Figures 8a-8b present the results of the test proposed by Amisano and Giacomini (2007). For fixed-country variables, we consider the comparison by pairs between all 24 considered models. Note that 12 models are with flexible seasonal effect (with 8 first models with periodic variance of noise and last 4 with constant variance of error term) while 12 for fixed seasonal effect. The order in models denoted by M and M' is the same as in Tables 5-8. "X" at some position means that using test proposed by Amisano and Giacomini (2007) at significance level 5% the model in column is better then model in row, while "O" means that the model in row is better then the model in column. Dark grey area correspond to case where models with periodic variance are compared with non-periodic case inside

Table 5: Averaged logscore. Italic means maximal average logscore in a set of 24 different specifications for fixed component and fixed country.

Variance type			Periodic variance ($T = 12$)								Constat variance, ($T = 1$)			
Variance specification			$(1, 2, 3, \ldots, 12)$				$(1, 1, 1, 2, 2, 2, 3, 3, 3, 4, 4, 4)$				$(1, 1, 1, \ldots, 1)$			
Error term distribution			N	S(5)	S(4)	S(3)	N	S(5)	S(4)	S(3)	N	S(5)	S(4)	S(3)
Denmark	HICP	M	*0.116*	0.115	0.111	0.101	0.100	0.096	0.089	0.076	0.108	0.101	0.095	0.083
		M′	0.068	0.054	0.046	0.026	0.088	0.082	0.065	0.044	0.099	0.073	0.056	0.043
	Fruit	M	-2.176	-2.197	-2.209	-2.224	-2.313	-2.227	-2.229	-2.238	*-2.166*	-2.191	-2.201	-2.218
		M′	-2.426	-2.371	-2.357	-2.376	-2.356	-2.347	-2.344	-2.348	-2.389	-2.369	-2.364	-2.367
	Vegetables	M	-2.414	-2.444	-2.454	-2.468	*-2.377*	-2.390	-2.398	-2.414	-2.384	-2.383	-2.391	-2.405
		M′	-2.503	-2.510	-2.510	-2.517	-2.401	-2.427	-2.436	-2.446	-2.407	-2.406	-2.422	-2.435
	Clothing	M	-1.793	-1.802	-1.810	-1.828	*-1.744*	-1.793	-1.802	-1.819	-1.894	-1.893	-1.901	-1.915
		M′	-1.943	-1.937	-1.944	-1.959	-1.901	-1.918	-1.924	-1.928	-1.988	-1.956	-1.954	-1.961
	Remaining	M	0.099	0.092	0.085	0.070	0.034	0.043	0.038	0.027	0.046	0.056	0.052	0.043
		M′	0.103	*0.106*	0.104	0.096	0.066	0.081	0.076	0.061	0.088	0.099	0.092	0.060
Germany	HICP	M	-0.065	-0.049	-0.052	-0.061	-0.056	-0.033	-0.033	-0.037	-0.048	*-0.031*	-0.032	-0.037
		M′	-0.241	-0.143	-0.142	-0.169	-0.246	-0.196	-0.183	-0.203	-0.261	-0.214	-0.207	-0.217
	Fruit	M	-2.344	-2.241	-2.247	-2.253	-2.232	-2.223	-2.228	-2.239	-2.193	*-2.189*	-2.195	-2.206
		M′	-2.518	-2.435	-2.439	-2.452	-2.464	-2.418	-2.420	-2.427	-2.459	-2.427	-2.428	-2.435
	Vegetables	M	-2.631	-2.632	-2.641	-2.659	-2.619	-2.624	-2.632	-2.647	-2.598	-2.622	-2.633	-2.652
		M′	-2.628	-2.634	-2.646	-2.661	-2.608	-2.614	-2.620	-2.633	*-2.572*	-2.595	-2.608	-2.632
	Clothing	M	-2.779	-1.918	-1.889	-1.868	-1.791	-1.723	-1.725	-1.730	*-1.557*	-1.678	-1.694	-1.712
		M′	-3.072	-2.597	-2.591	-2.589	-2.611	-2.531	-2.534	-2.544	-2.715	-2.690	-2.695	-2.710
	Remaining	M	-0.080	-0.086	-0.092	-0.103	-0.083	-0.070	-0.072	-0.078	-0.077	*-0.060*	-0.061	-0.066
		M′	-0.253	-0.157	-0.156	-0.159	-0.177	-0.139	-0.144	-0.148	-0.171	-0.136	-0.138	-0.147

Table 6: Averaged logscore. Italic means maximal average logscore in a set of 24 different specifications for fixed component and fixed country.

Variance type			Periodic variance ($T = 12$)								Constat variance, ($T = 1$)			
Variance specification			$(1, 2, 3, \ldots, 12)$				$(1, 1, 1, 2, 2, 2, 3, 3, 3, 4, 4, 4)$				$(1, 1, 1, \ldots, 1)$			
Error term distribution			N	S(5)	S(4)	S(3)	N	S(5)	S(4)	S(3)	N	S(5)	S(4)	S(3)
Estonia	HICP	M	-0.614	-0.455	-0.457	-0.466	-0.434	-0.427	-0.433	-0.445	-0.416	*-0.415*	-0.421	-0.433
		M′	-0.508	-0.460	-0.467	-0.483	-0.437	-0.442	-0.458	-0.474	-0.427	-0.455	-0.465	-0.482
	Fruit	M	-2.764	-2.783	-2.793	-2.804	-2.727	-2.745	-2.752	-2.766	-2.705	-2.722	-2.730	-2.746
		M′	-2.697	-2.716	-2.730	-2.770	-2.676	-2.687	-2.699	-2.725	*-2.657*	-2.67	-2.681	-2.708
	Vegetables	M	-3.161	-3.143	-3.147	-3.153	-3.099	-3.108	-3.112	-3.122	-3.179	-3.154	-3.153	-3.157
		M′	-3.131	-3.140	-3.147	-3.159	*-3.070*	-3.087	-3.109	-3.124	-3.134	-3.127	-3.129	-3.143
	Clothing	M	-1.649	-1.544	-1.542	-1.544	-1.577	*-1.519*	-1.520	-1.524	-1.683	-1.615	-1.608	-1.603
		M′	-2.313	-2.027	-2.016	-2.011	-2.220	-2.006	-1.996	-1.987	-2.354	-2.301	-2.298	-2.292
	Remaining	M	-0.527	-0.466	-0.466	-0.470	-0.447	-0.441	-0.447	-0.460	-0.424	*-0.420*	-0.428	-0.443
		M′	-0.486	-0.451	-0.456	-0.469	-0.431	-0.458	-0.472	-0.495	-0.423	-0.453	-0.464	-0.485
Latvia	HICP	M	-0.475	-0.472	-0.482	-0.494	-0.466	-0.453	-0.455	-0.463	-0.454	-0.434	-0.436	-0.444
		M′	-0.449	-0.418	-0.422	-0.431	-0.422	*-0.379*	-0.38	-0.384	-0.413	-0.379	-0.38	-0.385
	Fruit	M	-2.598	-2.626	-2.636	-2.654	*-2.573*	-2.589	-2.598	-2.616	-2.599	-2.600	-2.609	-2.627
		M′	-2.634	-2.632	-2.639	-2.653	-2.606	-2.614	-2.622	-2.631	-2.639	-2.634	-2.640	-2.655
	Vegetables	M	-3.289	-3.316	-3.328	-3.348	-3.22	-3.253	-3.265	-3.288	-3.204	-3.231	-3.244	-3.265
		M′	-3.292	-3.301	-3.31	-3.327	-3.189	-3.218	-3.232	-3.258	*-3.155*	-3.182	-3.194	-3.216
	Clothing	M	-2.042	-1.883	*-1.847*	-1.887	-2.141	-1.925	-1.937	-1.937	-1.860	-1.863	-1.869	-1.891
		M′	-2.086	-2.017	-2.039	-2.083	-2.506	-2.210	-2.212	-2.207	-2.258	-2.186	-2.182	-2.183
	Remaining	M	-0.478	-0.452	-0.452	-0.458	-0.418	-0.378	-0.379	-0.387	-0.399	-0.344	-0.343	-0.347
		M′	-0.519	-0.423	-0.416	-0.413	-0.382	-0.327	-0.330	-0.341	-0.353	*-0.308*	-0.313	-0.327

Table 7: Averaged logscore. Italic means maximal average logscore in a set of 24 different specifications for fixed component and fixed country.

Variance type			Periodic variance ($T = 12$)								Constat variance, ($T = 1$)			
Variance specification			$(1, 2, 3, \ldots, 12)$				$(1, 1, 1, 2, 2, 2, 3, 3, 3, 4, 4, 4)$				$(1, 1, 1, \ldots, 1)$			
Error term distribution			N	S(5)	S(4)	S(3)	N	S(5)	S(4)	S(3)	N	S(5)	S(4)	S(3)
Lithuania	HICP	M	*-0.417*	-0.447	-0.457	-0.476	-0.516	-0.491	-0.493	-0.499	-0.578	-0.521	-0.520	-0.522
		M′	-0.45	-0.486	-0.498	-0.515	-0.488	-0.500	-0.499	-0.488	-0.565	-0.487	-0.479	-0.486
	Fruit	M	-2.342	-2.351	-2.357	-2.365	-2.359	-2.361	-2.366	-2.378	-2.36	-2.358	-2.363	-2.374
		M′	-2.395	-2.405	-2.41	-2.414	*-2.340*	-2.371	-2.378	-2.397	-2.34	-2.367	-2.378	-2.395
	Vegetables	M	*-3.146*	-3.170	-3.180	-3.200	-3.159	-3.191	-3.200	-3.217	-3.152	-3.177	-3.186	-3.204
		M′	-3.283	-3.309	-3.319	-3.339	-3.381	-3.365	-3.368	-3.378	-3.414	-3.366	-3.367	-3.375
	Clothing	M	-2.198	-1.71	-1.676	-1.674	-1.897	-1.649	-1.632	*-1.616*	-2.105	-1.782	-1.762	-1.747
		M′	-2.935	-2.467	-2.45	-2.424	-2.466	-2.382	-2.382	-2.375	-2.961	-2.740	-2.702	-2.649
	Remaining	M	-0.324	-0.33	-0.337	-0.352	-0.429	-0.366	-0.363	-0.362	-0.507	-0.408	-0.401	-0.395
		M′	-0.220	-0.200	*-0.198*	-0.203	-0.294	-0.216	-0.218	-0.217	-0.381	-0.228	-0.224	-0.220
Poland	HICP	M	-0.163	-0.175	-0.183	-0.197	-0.117	-0.133	-0.141	-0.154	-0.087	-0.116	-0.126	-0.141
		M′	-0.112	-0.152	-0.165	-0.188	-0.086	-0.094	-0.113	-0.131	*-0.072*	-0.112	-0.120	-0.136
	Fruit	M	-2.697	*-2.663*	-2.669	-2.684	-2.757	-2.728	-2.729	-2.735	-2.758	-2.689	-2.688	-2.692
		M′	-2.807	-2.793	-2.802	-2.817	-2.905	-2.91	-2.920	-2.937	-2.883	-2.877	-2.885	-2.895
	Vegetables	M	-2.991	-2.899	-2.893	-2.900	*-2.767*	-2.788	-2.797	-2.815	-2.912	-2.878	-2.879	-2.884
		M′	-3.238	-3.074	-3.068	-3.065	-3.007	-3.020	-3.027	-3.041	-3.203	-3.181	-3.18	-3.179
	Clothing	M	-2.230	-1.041	-0.978	*-0.907*	-2.397	-1.180	-1.086	-0.977	-1.74	-1.118	-1.039	-0.946
		M′	-3.539	-2.035	-2.010	-2.003	-3.93	-2.988	-2.933	-2.828	-2.578	-2.507	-2.550	-2.615
	Remaining	M	-0.017	-0.02	-0.024	-0.034	0.038	0.035	0.027	0.010	0.052	0.038	0.029	0.012
		M′	0.048	0.059	0.055	0.048	0.071	0.065	0.059	0.047	0.078	*0.090*	0.087	0.081

Table 8: Averaged logscore. Italic means maximal average logscore in a set of 24 different specifications for fixed component and fixed country.

Variance type			Periodic variance ($T = 12$)								Constat variance, ($T = 1$)			
Variance specification			$(1, 2, 3, \ldots, 12)$				$(1, 1, 1, 2, 2, 2, 3, 3, 3, 4, 4, 4)$				$(1, 1, 1, \ldots, 1)$			
Error term distribution			N	S(5)	S(4)	S(3)	N	S(5)	S(4)	S(3)	N	S(5)	S(4)	S(3)
Finland	HICP	M	0.059	0.074	0.072	0.065	0.053	0.075	0.073	0.067	-0.007	0.048	0.049	0.046
		M′	0.100	0.118	0.108	0.106	0.096	*0.120*	0.119	0.116	0.039	0.097	0.100	0.102
	Fruit	M	-2.399	-2.361	-2.363	-2.374	-2.326	-2.335	-2.343	-2.357	-2.347	*-2.322*	-2.327	-2.339
		M′	-2.556	-2.512	-2.514	-2.524	-2.461	-2.450	-2.453	-2.462	-2.515	-2.454	-2.455	-2.461
	Vegetables	M	-3.035	-3.040	-3.044	-3.057	-3.058	-3.066	-3.072	-3.085	-3.066	-3.065	-3.070	-3.080
		M′	-2.985	-2.981	-2.987	-3.004	-2.978	-2.986	-2.983	-2.998	*-2.977*	-2.988	-2.994	-3.010
	Clothing	M	-1.912	-1.826	-1.815	-1.821	-1.807	*-1.773*	-1.778	-1.777	-1.884	-1.791	-1.789	-1.791
		M′	-2.119	-1.976	-1.986	-1.99	-2.124	-1.966	-1.965	-1.971	-2.093	-2.030	-2.042	-2.063
	Remaining	M	0.126	0.128	0.124	0.116	0.087	0.111	0.110	0.104	-0.032	0.056	0.057	0.054
		M′	0.136	0.145	0.144	0.139	0.129	0.145	0.144	*0.149*	0.031	0.118	0.114	0.120
Sweden	HICP	M	*-0.012*	-0.015	-0.019	-0.032	-0.032	-0.020	-0.023	-0.030	-0.043	-0.021	-0.021	-0.027
		M′	-0.135	-0.118	-0.125	-0.139	-0.168	-0.143	-0.146	-0.155	-0.180	-0.144	-0.144	-0.147
	Fruit	M	-2.061	-2.082	-2.088	-2.104	-2.047	-2.044	-2.048	-2.057	-2.028	-2.026	-2.031	-2.041
		M′	-2.066	-2.073	-2.085	-2.105	-2.004	-2.034	-2.047	-2.068	*-1.981*	-2.016	-2.029	-2.051
	Vegetables	M	-2.600	-2.596	-2.598	-2.603	-2.574	-2.561	-2.562	-2.568	-2.589	-2.558	-2.558	-2.562
		M′	-2.499	-2.498	-2.499	-2.502	-2.459	*-2.454*	-2.455	-2.460	-2.473	-2.458	-2.459	-2.464
	Clothing	M	-1.828	-1.848	-1.857	-1.870	*-1.775*	-1.800	-1.81	-1.829	-1.925	-1.881	-1.883	-1.892
		M′	-2.094	-2.065	-2.079	-2.097	-2.014	-2.045	-2.058	-2.077	-2.323	-2.229	-2.223	-2.219
	Remaining	M	*0.033*	0.018	0.012	0.002	-0.026	-0.019	-0.022	-0.03	-0.046	-0.016	-0.018	-0.025
		M′	-0.079	-0.103	-0.111	-0.125	-0.163	-0.116	-0.115	-0.118	-0.227	-0.116	-0.113	-0.115

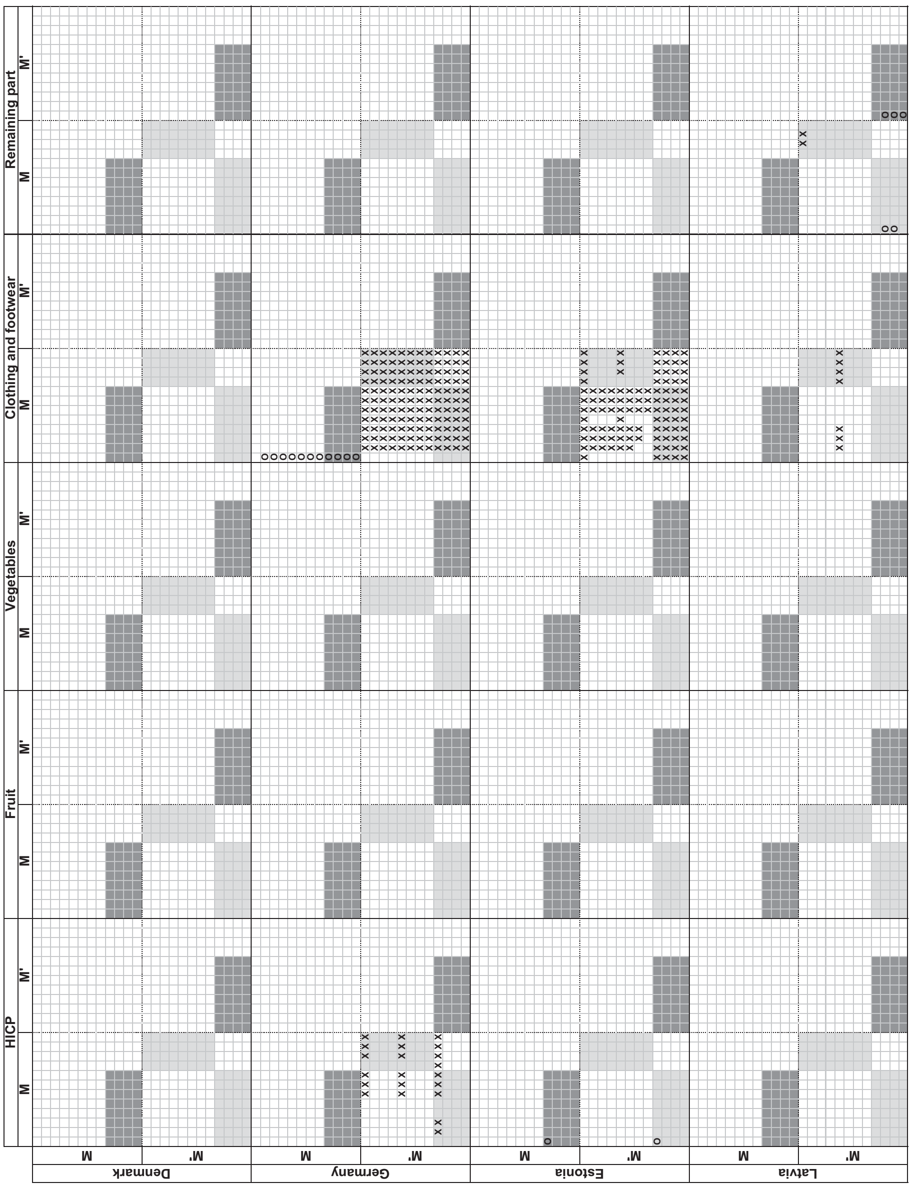


Figure 8a: Results of the test for significance of the difference of logscore proposed by Amisano and Giacomini (2007).

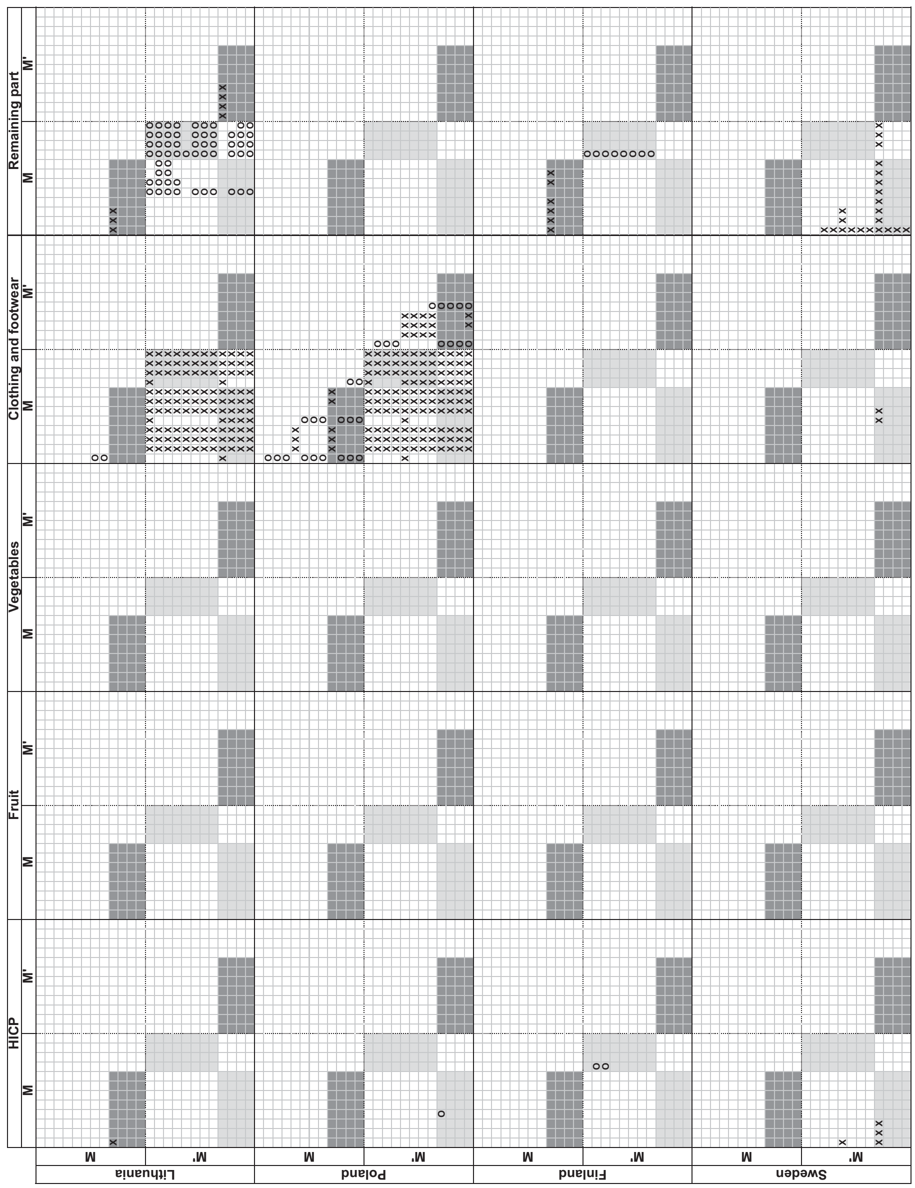


Figure 8b: Results of the test for significance of the difference of logscore proposed by Amisano and Giacomini (2007).

class M or M', while light grey are corresponds to case where models with periodic variance are compared with non-periodic between classes M and M'. Therefore "X" in dark grey area means that model with periodic variance outperform model with constant variance, while "O" in dark grey area means that model with constant variance outperform this with periodic case. In the case of *total HICP*, we have one case where model with periodic variance outperforms the model with constant variance (see the case of Lithuania) and one case where the model with constant variance outperforms the model with periodic variance (case of Estonia). In the case of *fruit* and *vegetables*, no significant differences were detected . For *clothing and footwear* , there are no significant differences in the case of Denmark and Finland. In the case of Latvia and Sweden, some differences were detected but only between groups M and M'. Note that this can be related with regular seasonal patterns in these countries for *clothing and footwear* relative to other countries. In the case of Estonia and Lithuania, the differences are more frequent but recognized only between models from different groups M and M'. In the case of Germany, the model with periodic variance and gaussian noise is uniformly outperformed by all models from group M. In addition, there are significant differences between models in different groups. In the case of Poland, on the dark grey area we observe models with periodic variance, which outperforms these with constant variance and inversely. There are significant differences between models from different groups M and M' and inside each group. In case of *remaining part*, there are no significant differences (or only between groups M and M') in the case of Denmark, Germany, Estonia, Latvia, Poland and Sweden. In the case of Lithuania and Finland, some models with periodic variance outperform models with constant variance in the same group of models (M or M') but not inversely.

Summing up, we cannot find any evidence that models with periodic variance of noise are significantly better compared to models with constant variance. Moreover, we cannot find any evidence supporting the inverse statement. There are only a few cases where models with periodic variance are statistically better or worse.

6 Conclusions

We consider the problem of seasonal volatility in selected components of HICP in Baltic region countries. Proposed disaggregation was arbitrary. Seasonal volatility in the case of *fruit*, *vegetables*, *clothing and footwear*, *remaining part* and *total HICP* was detected by a non-parametric test. Based on these findings, the exponential smoothing models with time-varying periodic variance of error terms was proposed in a forecasting exercise. In this exercise, the main goal was to check forecasting performance for modelling with periodic variance of the noise over alternative models with constant variance. Based on the logscore measure and using the test proposed by Amisano and Giacomini (2007), we do not find evidence that the models with periodic variance of error terms improve forecasting performance over alternative models with

constant variance of error term. The forecasting example shows that exponential smoothing models with flexible seasonal pattern are more adequate than models with fixed seasonality.
This above problem should be extended to another component of HICP jointly with extension to other countries. Note that in the case of Turkey, similar conclusions concerning the presence of seasonal volatility were documented in Berument and Sahin (2010). Moreover, the alternative models should be taken into consideration. In addition, alternative measures of forecast adequacy should be considered with adequate adjustment to the periodic case. The simple proposition is to consider the logscore measure for each season separately, but this proposition involves much longer datasets that can estimate the logscore reasonably.

Acknowledgements

This research was partially supported by the Polish National Science Center based on decision number DEC-2013/09/B/HS4/01945. The author would like to thank an anonymous referee and co-editor for valuable comments on this manuscript.

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Appendix

Theorem 1. *If time series* X_t *is PC(T) with mean function* $\mu_X(t) = \sum_{k=0}^{T/2} a_k \cos(t\lambda_k) + b_k \sin(t\lambda_k)$ *and autocovariance function* $B_X(t,\tau) = \sum_{k=0}^{T/2} a_{k\,\tau} \cos(t\lambda_k) + b_{k\,\tau} \sin(t\lambda_k)$ *(*$\lambda_k = 2k\pi/T$*), then the time series*

$$Y_t = X_t - L_T(B)X_t$$

is also PC(T) with mean function $\mu_Y(t) = \mu_X(t) - a_0$ *and the periodic autocovariance function* $B_Y(t,\tau)$ *given by formula (6.81) in Hurd and Miamee (2007).*

Proof. Elementary calculations (using trigonometric equalities) give $E(L_T(B)X_t) = a_0$. For autocovariance function $R_Y(t,\tau)$ it is sufficient to use formula (6.81) from Hurd and Miamee (2007) and the properties

$$|W(e^{i\lambda_1})W(e^{-i\lambda_2})| \neq 0$$

for $(\lambda_1, \lambda_2) \in (0, 2\pi)^2$, where $W(x) = 1 - L_T(e^{ix})$. The above properties mean that the properties of periodicity in the autocovariance function is robust to filtration given by $1 - L_T(B)$. □