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VARIABLE THICKNESS APPROACH FOR FINDING MINIMUM LAMINATE THICKNESS AND INVESTIGATING EFFECT OF DIFFERENT DESIGN VARIABLES ON ITS PERFORMANCE

The performance of majority engineering systems made of composite laminates can be improved by increasing strength to weight ratio. Variable thickness approach (VTA), in discrete form, used in this study is capable of finding minimum laminate thickness in one stage only, instead of two stage methodology defined by other researchers, with substantial accuracy for the given load conditions. This minimum required laminate thickness can be used by designers in multiple ways. Current study reveals that effectiveness of VTA in this regard depends on ply thickness increment value and number of plies. Maximum Stress theory, Tsai Wu theory and Tsai Hill theory are used as constraints, while ply angles, ply thicknesses and number of plies in discrete form are used as design variables in current simulation studies. Optimization is carried out using direct value coded genetic algorithm. The effect of design variables such as ply angles, ply thicknesses and number of plies in discrete form on optimum solution is investigated considering Uniform Thickness Approach (UTA) and Variable Thickness Approach (VTA) for various load cases.

Nomenclature

E_{11} elastic modulus in longitudinal direction

E_{22} elastic modulus in transverse direction

G_{12} in plane shear modulus

γ_{12} major Poisson's ratio

S_{Lc} compressive strength in longitudinal direction

S_{Lt} tensile strength in longitudinal direction

S_{Tc} compressive strength in transverse direction

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S_{Tt} tensile strength in transverse direction
S_{Lts} shear strength
θ ply orientation angle
t thickness of each ply
ρ mass density
a length of laminate
b width of laminate

1. Introduction

Composite laminate plate is a structural element made up of multiple fiber reinforced polymer (FRP) laminas/plies connected together to provide required engineering properties for an application (Fig. 1).

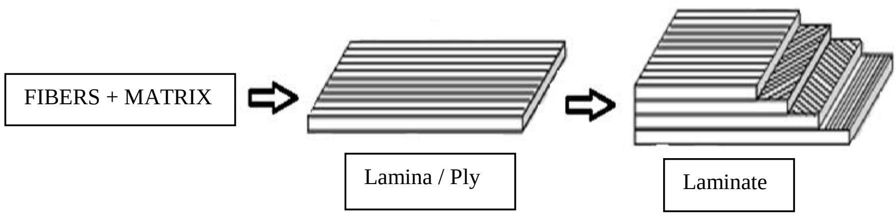

Fig. 1. Constituents of a composite laminate

Composite laminates play a major role in various application segments such as aerospace engineering, automotive technology, marine engineering, civil engineering, etc. The main advantage of composite laminates is their low weight associated with high stiffness and strength along the direction of the reinforcement. The properties of composite laminate depend on various design variables such as total number of laminas, stacking sequence, ply angles, thickness of each ply, etc. The composite laminate can be tailor-made by using a combination of these design variables to suit the application under consideration.

A composite laminate subjected to in plane loads N_{xx}, N_{yy} and N_{xy} is shown in Fig. 2. In the figure, X, Y and Z denotes global coordinate system of the composite laminate, while 1 and 2 represent local coordinate system for individual lamina. Axis 1 of the local coordinate system is along the length of the fiber and axis 2 is perpendicular to local axis 1. Ply angle θ is the angle between fiber and the longitudinal axis of the plate.

In any structural design application, the aim of design is to maximize strength to weight ratio for achieving maximum material performance in terms of durability of structure, fuel economy, cost, etc. In order to satisfy this major design requirement, many researchers [1–22] have taken weight minimization objective during design optimization of composite laminate for different loading conditions. Efforts taken

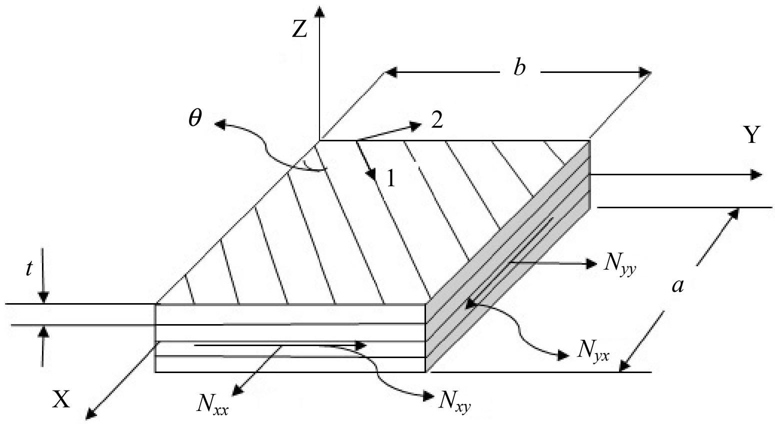

Fig. 2. Global and local coordinate systems for composite laminate

by different researchers in the field of design optimization of composite laminates are described in [23].

Conflicting nature of properties and multiple variables involved in the problem motivates the use of optimization methods for design of composite laminate. Comparison of various optimization methods reported by different researchers shows that genetic algorithm is the most popular and suitable method for design optimization of composite laminates [24].

A composite laminate can be designed using two approaches; Uniform Thickness Approach (UTA) and Variable Thickness Approach (VTA). In UTA, all the laminas in a laminate will have same thickness, while in VTA the laminas in the laminate may have same or different thicknesses. The use of ply thickness as discrete variable is rarely observed [1, 13] so far in the available literature. Majority researchers have accepted UTA based laminate design, while the VTA is rarely used to design a laminate because of manufacturing difficulty and mathematical complexity. Comparison of both the approaches yields that the number of design variables in VTA becomes greater than the design variables in UTA. Moreover, the nature of variables, i.e., ply angle and ply thickness is different. Generally, ply angle is an integer number and ply thickness is a real number.

Few of the researchers [1, 13, 14, 16] have used VTA for design analysis of composite laminates, but with different perspective. Minimization of weighted sum of deflection and weight of composite laminate subjected to normal loading using Tsai Wu theory as constraint is proposed in [13]. Fiber angles and layer thicknesses, both in discrete form are considered as design variables in the simulation study and optimization is carried out using binary coded genetic algorithm. Conventional binary-coded genetic algorithm has limitations in catching exact incremental value of design variables commonly used in engineering applications. One of the ways to overcome this difficulty is to use the integer-coded genetic algorithm for design optimization of composite laminates [3]. Direct value-coded genetic algorithm is more simplified and efficient way to overcome limitations of binary-coded genetic algorithm. This methodology has been successfully implemented for weight minimization of conventional 0°, ±45°, 90° composite laminates subjected to Tsai Wu, Puck and maximum stress theory as constraints considering UTA [4]. Yet, effective utilization of direct value-coded genetic algorithm, in generic form for UTA as well as VTA is not observed in available literature.

Few of the researchers [14, 16] used two-level optimization strategy to determine minimum required thickness of laminate subjected to lateral and in plane loading based on Tsai Hill criteria. In the first stage of analysis, fiber angles in discrete form are treated as design variables, while in the second stage, layer thicknesses in continuous form are used as design variables. Even though, two-level strategy is accurate in prediction of minimum laminate thickness, it is time consuming because of use of ply thickness as continuous design variable and two stages of optimization. At the same time, two-level strategy also involves complex mathematical calculations. VTA, used for the same purpose and presented in the current study, is simple to understand and possess substantial accuracy.

To check the failure of the laminate while minimizing its weight, researchers working in this field have proposed various criteria (theories), which may have some minor or major weaknesses. Authors of [25, 26] compared and assessed different leading theories for predicting failure in composite laminates under complex states of stress against experimental evidence through few selected test cases.

In the present study, first ply failure criterion is adopted to predict failure of laminate. The laminate is considered as failed when any single ply of the laminate fails. Among all these theories, Maximum Stress theory, Maximum Strain theory, Tsai Hill theory and Tsai Wu theory are the most preferred theories for predicting failure of laminate and used by most of the researchers [1–7, 16–18, 21, 22, 27, 28] for in plane loading conditions. Out of these four failure theories, Maximum Stress theory (MS), Tsai Wu theory (TW) and Tsai Hill theory (TH) are considered and applied individually as constraints in the current study as these theories are strength based.

A brief description of these theories is given below. Detail description of these theories and mechanics of composites is available in [29].

1.1. Maximum stress theory

According to this failure theory, a composite lamina will fail, when any one of the principal stresses developed in the lamina reaches its limiting value. The limiting values are defined by the respective strengths. The lamina will fail, if

$$\begin{aligned} \sigma_{11} &= S_{\text{LC}} \quad \text{or} \quad \sigma_{11} = S_{\text{Lt}}, \\ \sigma_{22} &= S_{\text{TC}} \quad \text{or} \quad \sigma_{22} = S_{\text{Tt}}, \\ \tau_{12} &= S_{\text{Lts}}, \end{aligned}$$

where σ_{11} and σ_{22}, are normal stresses developed in direction 1 and 2, respectively, while τ_{12} is the shear stress developed in plane 1–2 for individual lamina.

1.2. Tsai Wu theory

This failure criterion is based on von Mises yield criterion. According to this theory, the lamina under consideration will fail, when the following condition is satisfied.

$$F_1\sigma_{11} + F_2\sigma_{22} + F_6\tau_{12} + F_{11}\sigma_{11}^2 + F_{22}\sigma_{22}^2 + F_{66}\tau_{12}^2 + 2F_{12}\sigma_{11}\sigma_{22} = 1$$

where: F_1, F_2, F_6, F_{11}, F_{22}, F_{66} and F_{12} are the coefficients which can be calculated using strengths of the lamina in different directions as given below,

$$F_1 = \frac{1}{S_{\text{Lt}}} - \frac{1}{S_{\text{LC}}}, \qquad F_2 = \frac{1}{S_{\text{Tt}}} - \frac{1}{S_{\text{TC}}}, \qquad F_6 = 0,$$

$$F_{11} = \frac{1}{S_{\text{Lt}}S_{\text{LC}}}, \qquad F_{22} = \frac{1}{S_{\text{Tt}}S_{\text{TC}}}, \qquad F_{66} = \frac{1}{S_{\text{Lts}}^2}.$$

The value of F_{12} depends on various principle unidirectional strengths of the laminate as well as biaxial failure stresses. The recommended range for F_{12} in case of composite laminates is,

$$-\frac{1}{2}(F_{11}F_{22})^{0.5} \leqslant F_{12} \leqslant 0.$$

In absence of any experimental data, the lower limit of above equation is preferred as F_{12}.

$$F_{12} = -\frac{1}{2}(F_{11}F_{22})^{0.5}.$$

1.3. Tsai Hill theory

According to the Tsai Hill theory, used by few of the researchers [14],

$$\frac{\sigma_{11}^2}{S_{\text{Lt}}^2} - \frac{\sigma_{11}\sigma_{22}}{S_{\text{Lt}}S_{\text{Tt}}} + \frac{\sigma_{22}^2}{S_{\text{Tt}}^2} + \frac{\tau_{12}^2}{S_{\text{Lts}}^2} = 1.$$

It is observed that most of the research articles [16, 18] use a generalized form of this theory given below for the analysis. The results presented in this article are obtained using different forms of generalized equation of Tsai Hill theory.

According to the generalized form of this theory, the lamina will fail when following condition is satisfied

$$\frac{\sigma_{11}^2}{S_{\text{Lt}}^2} - \frac{\sigma_{11}\sigma_{22}}{S_{\text{Lt}}^2} + \frac{\sigma_{22}^2}{S_{\text{Tt}}^2} + \frac{\tau_{12}^2}{S_{\text{Lts}}^2} = 1.$$

This equation of generalized form is applicable when both σ_{11} and σ_{22} are positive, i.e., for the first stress quadrant. For remaining stress quadrants, the present form is modified as per necessity and applied in the analysis [29].

In this study, theories of failure are used as constraints, which have to be satisfied for safe design. A procedure, based on mechanics of materials, used to find the stresses required to develop constraint equations is explained in the next section.

2. Analysis of composite laminate

The stresses and strains developed in each lamina are calculated using classical lamination plate theory. The geometry of the laminate coordinate systems and loading conditions considered in the analysis are shown in Fig. 2. The additional geometric parameters required in the analysis of laminate are shown in Fig. 3.

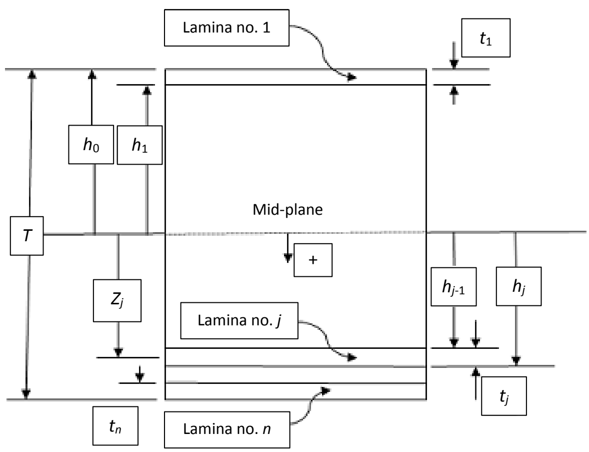

Fig. 3. Additional geometric parameters of laminate

In Fig. 3, t_1, t_j and t_n denote thicknesses of 1^{st}, j^{th} and n^{th} lamina; h_0 denotes distance from the laminate mid-plane to the top of the 1^{st} lamina, while h_1 is the distance from the laminate mid-plane to the bottom of the 1^{st} lamina. h_{j-1} is the distance from the laminate mid-plane to the top of the j^{th} lamina and h_j is the distance from the laminate mid-plane to the bottom of the j^{th} lamina. The total thickness of the laminate is denoted with letter T. Z_j is the distance from the laminate mid-plane to the mid-plane of the j^{th} lamina.

The stiffness matrix for an individual lamina $\left[\overline{Q}\right]$ is calculated from the material properties as,

$$\left[\overline{Q}\right] = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}. \tag{1}$$

The elements of the stiffness matrix $\left[\overline{Q}\right]$ are calculated as,

$$
\begin{aligned}
\overline{Q}_{11} &= Q_{11}\cos^4\theta + 2(Q_{12}+2Q_{66})\sin^2\theta\cos^2\theta + Q_{22}\sin^4\theta,\\
\overline{Q}_{12} &= Q_{12}(\sin^4\theta+\cos^4\theta) + (Q_{11}+Q_{22}-4Q_{66})\sin^2\theta\cos^2\theta,\\
\overline{Q}_{22} &= Q_{11}\sin^4\theta + 2(Q_{12}+2Q_{66})\sin^2\theta\cos^2\theta + Q_{22}\cos^4\theta,\\
\overline{Q}_{16} &= (Q_{11}-Q_{12}-2Q_{66})\sin\theta\cos^3\theta + (Q_{12}-Q_{22}+2Q_{66})\cos\theta\sin^3\theta,\\
\overline{Q}_{26} &= (Q_{11}-Q_{12}-2Q_{66})\cos\theta\sin^3\theta + (Q_{12}-Q_{22}+2Q_{66})\sin\theta\cos^3\theta,\\
\overline{Q}_{66} &= (Q_{11}+Q_{22}-2Q_{12}-2Q_{66})\sin^2\theta\cos^2\theta + Q_{66}\sin^4\theta + \cos^4\theta,
\end{aligned}
$$

where

$$
Q_{11} = \frac{E_{11}}{1-\gamma_{12}\gamma_{21}}, \qquad Q_{22} = \frac{E_{22}}{1-\gamma_{12}\gamma_{21}}, \qquad \gamma_{21} = \gamma_{12}\frac{E_{22}}{E_{11}},
$$

$$
Q_{12} = Q_{21} = \frac{\gamma_{21}E_{11}}{1-\gamma_{12}\gamma_{21}} = \frac{\gamma_{12}E_{22}}{1-\gamma_{12}\gamma_{21}}, \qquad Q_{66} = G_{12}\,.
$$

The extensional stiffness matrix **A**, coupling stiffness matrix **B** and bending stiffness matrix **D** for laminate are calculated as,

$$
\mathbf{A} = \sum_{j=1}^{n}\left(\overline{Q}_j\right)\left(h_j - h_{j-1}\right), \tag{2}
$$

$$
\mathbf{B} = \frac{1}{2}\sum_{j=1}^{n}\left(\overline{Q}_j\right)\left(h_j^2 - h_{j-1}^2\right), \tag{3}
$$

$$
\mathbf{D} = \frac{1}{3}\sum_{j=1}^{n}\left(\overline{Q}_j\right)\left(h_j^3 - h_{j-1}^3\right). \tag{4}
$$

Above, three matrices are used to calculate the mid plane strains $[\varepsilon^0]$ and curvatures $[K]]$ as given below.

$$
[\varepsilon^0] = [A_1][N] + [B_1][M], \tag{5}
$$

$$
[K] = [C_1][N] + [D_1][M] \tag{6}
$$

where

$$
[\varepsilon^0] = \begin{bmatrix}\varepsilon^0_{xx}\\ \varepsilon^0_{yy}\\ \varepsilon^0_{xy}\end{bmatrix}, \qquad [N] = \begin{bmatrix}N_{xx}\\ N_{yy}\\ N_{xy}\end{bmatrix}, \qquad [M] = \begin{bmatrix}M_{xx}\\ M_{yy}\\ M_{xy}\end{bmatrix},
$$

$$
\begin{aligned}
&[A_1] = [A^{-1}] + [A^{-1}][B][(D^*)^{-1}][B][A^{-1}],\\
&[B_1] = -[A^{-1}][B][(D^*)^{1}],\\
&[C_1] = [B_1]^T,\\
&[D_1] = [(D^*)^{-1}] \quad \text{and} \quad [D^*] = [D] - [B][A^{-1}][B].
\end{aligned}
$$

The mid-plane strains and curvatures can produce actual stresses $[\sigma]_j$ and strains $[\varepsilon]_j$ for any j^{th} lamina in global co-ordinate system.

$$
[\varepsilon]_j = \left[\varepsilon^0\right] + Z_j[K], \tag{7}
$$

$$
[\sigma]_j = [\overline{Q}]_j[\varepsilon]_j, \quad \text{where} \quad [\sigma]_j = \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix}_j . \tag{8}
$$

As the first ply failure criterion is based on failure of a single lamina, it is necessary to predict the stresses developed in an individual lamina in local coordinate system. The following relations are used for this purpose.

$$
\sigma_{11} = \sigma_{xx}\cos^2\theta + \sigma_{yy}\sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta, \tag{9}
$$

$$
\sigma_{22} = \sigma_{xx}\sin^2\theta + \sigma_{yy}\cos^2\theta - 2\tau_{xy}\sin\theta\cos\theta, \tag{10}
$$

$$
\tau_{12} = (-\sigma_{xx} + \sigma_{yy})\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta). \tag{11}
$$

These local stress values obtained using equations (9) to (11) for individual laminas are used for constructing all the constraint equations as mentioned in section 1.

3. OptiComp and Genetic Algorithm

OptiComp is a comprehensive optimization module developed using MATLAB for optimal design of composite laminates, which can handle a variety of laminate design problems involving different design objectives, constraints and design variables [2]. OptiComp is developed by both the authors of this article in order to provide a common platform for design optimization of composite laminates. The user can choose a suitable optimization algorithm from a range of algorithms incorporated in OptiComp software module. OptiComp provides a choice of two approaches, namely, Uniform Thickness Approach and Variable Thickness Approach to design a composite laminate. The length and width of the plate, force components, material properties, design variables with increments and limit bounds and maximum number of acceptable plies are to be provided as user input while solving a problem using OptiComp.

For the current study, Genetic Algorithm (GA) is selected as optimization algorithm from the choice of algorithms provided by OptiComp. In the genetic algorithm, the initial random population is improved using genetic operators like selection, crossover and mutation, while selection has been carried out using Roulette wheel method. In the current analysis, the following changes have been made in GA to suit the problem of composite laminate design optimization.

1. Direct value coded representation is used for chromosomes instead of binary-coded representation. This kind of representation facilitates handling of multiple design variables of different nature in discrete form.
2. Direct value-coded chromosome representation can exactly catch increment value for any design variable provided by the user within the limit bounds. This is difficult in binary representation.
3. Single-point crossover function and mutation function are defined in such a manner so as to suit the chromosome representation.

Let θ_L and θ_U be the lower and upper limiting values of the ply angles, while t_L and t_U be the lower and upper limiting values of the ply thicknesses. Let the increment values of ply angles and ply thicknesses within the given limiting bounds be $\Delta\theta$ and Δt, respectively, as provided by the user. The vectors of acceptable values of ply angles (θ_s) and ply thicknesses (t_s) within the given limit bounds can be developed as given below.

$$\theta_s = \{\theta_{1s}, \theta_{2s}, \theta_{3s}, \theta_{4s}, \ldots, \theta_{ns}\},$$

where $\theta_{1s} = \theta_L$, $\theta_{2s} = \theta_L + \Delta\theta$, $\theta_{3s} = \theta_L + 2 \times \Delta\theta, \ldots, \theta_{ns} = \theta_U$.

$$t_s = \{t_{1s}, t_{2s}, t_{3s}, t_{4s}, \ldots, t_{ns}\},$$

where $t_{1s} = t_L$, $t_{2s} = t_L + \Delta t$, $t_{3s} = t_L + 2 \times \Delta t, \ldots, t_{ns} = t_U$.

In OptiComp, the genes of the chromosomes are represented directly by the allowable angle values and ply thickness values randomly selected from acceptable series instead of using classical binary representation. If the laminate is made up of total $2n$ number of plies, then each chromosome will have $2n$ number of elements in VTA. The first n number of elements of the chromosome are the randomly selected ply angle values from the series (θ_s), while the remaining elements are the randomly selected ply thickness values from the series (t_s). The first and second half of the chromosome have given separate treatment during crossover, as shown in Fig. 4.

In mutation, one element of the angle values from the first half and one element of the ply thickness values from the second half are selected from child chromosome on random basis. These elements will be replaced by ply angle and ply thickness value selected randomly from the corresponding acceptable series. The details of crossover and mutation process are given in Fig. 4.

In UTA, all the laminas are having same thickness, so the genes of the chromosomes are represented only by the allowable angle values selected randomly.

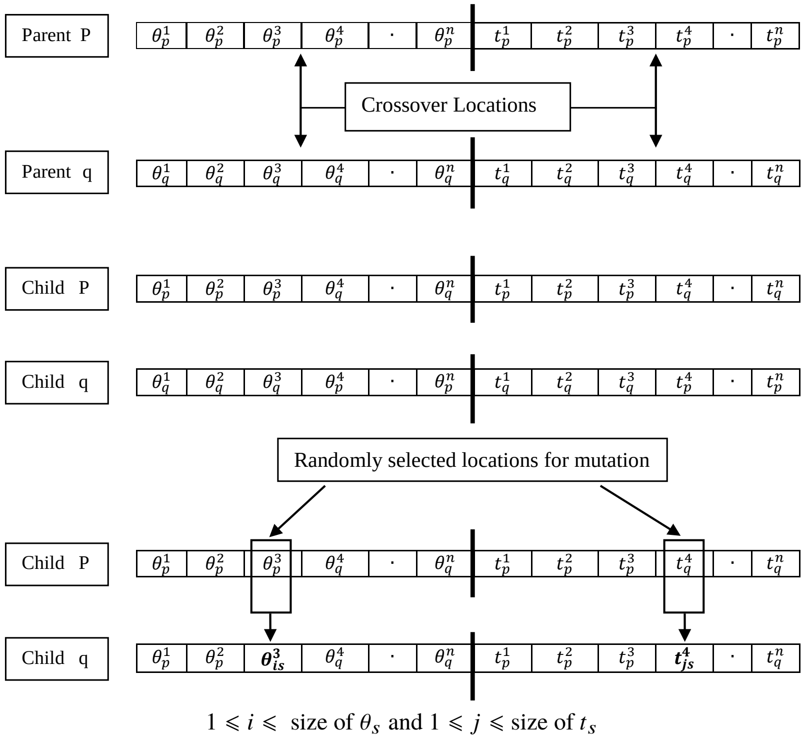

Fig. 4. Chromosome representation, crossover and mutation in OptiComp

Each chromosome will carry n elements only because of symmetric nature of laminate. The crossover and mutation process will be carried out by similar way, as mentioned in Fig. 4.

4. Formulation of optimal design problems

Weight minimization of conventional 0°, ±45° and 90° laminate with uniform ply thickness 0.1 mm has been already carried out in [4] for load cases mentioned in Table 1.

Table 1.

Different load cases used in the study

Load case	Longitudinal force N/mm	Transverse force N/mm	Shear force N/mm
1	3000	3000	0
2	3000	3000	500
3	3000	3000	1000
4	−3000	−3000	0
5	−3000	−3000	500
6	−3000	−3000	1000

These load cases are again considered in this article with an increased search space. As the aim of this article is to demonstrate effectiveness of VTA for finding minimum thickness of laminate, the article is divided in three parts.

i. In the first stage, the minimum laminate thickness solution is obtained using UTA. In this stage, ply angles and number of plies are used as design variables. Each lamina is made of thickness 0.1 mm. The ply angle increment value is reduced to 15° within the range –75° to 90° to obtain optimum weight and effect of ply angle increment value on optimal results is also studied.

ii. In the second stage, the laminate thickness solution is obtained using VTA. Earlier problem is modified by varying the ply thicknesses in discrete form along with ply angles. The discrete values of ply thicknesses used in the VTA analysis are 0.05 mm, 0.075 mm, 0.1 mm, 0.125 mm and 0.15 mm. The number of layers for a particular load case obtained using UTA are kept constant during VTA analysis to study effect of ply thickness variation on minimum laminate thickness.

iii. In the third stage, the effect of different influencing design variables like ply angle increment value $\Delta\theta$, ply thickness increment value Δt and the number of plies ($N_{\max}$) on effectiveness of VTA for finding minimum required laminate thickness to sustain the applied load conditions is investigated. During simulation, values of these influencing variables changed one by one without affecting the remaining two variables used in earlier stage.

The problem statement for first stage can be mathematically expressed as

Problem 4.1 Find $[N_{\max},\ \theta_n]$,

to minimize weight $W = \rho abT, \left(T = \sum_{i=1}^{n} t_i\right)$

Subjected to satisfying

Maximum stress theory/Tsai Wu theory/Tsai Hill theory,

$-75^\circ \leqslant \theta_n \leqslant 90^\circ$ (Ply angle incremental value 15°),

$t_n = 0.1$ mm

$n = 1$ to $N_{\max}$.

For the second stage, the problem statement can be mathematically expressed as

Problem 4.2 Find $[\theta_n,\ t_n]$,

to minimize weight $W = \rho abT, \left(T = \sum_{i=1}^{n} t_i\right)$

Subjected to satisfying

Maximum stress theory/Tsai Wu theory/Tsai Hill theory,

$-75^\circ \leqslant \theta_n \leqslant 90^\circ$ (Ply angle incremental value 15°),

$0.05 \leqslant t_n \leqslant 0.15$ (Ply thickness incremental value 0.025 mm),

$n = 1, \ldots, N_{\max}$ ($N_{\max}$ as obtained in earlier stage).

The problem statement for third stage can be mathematically expressed as

Problem 4.3 Find $[\theta_n, t_n]$,

to minimize weight $W = \rho abT, \left(T = \sum_{i=1}^{n} t_i\right)$

Subjected to satisfying

Maximum stress theory/Tsai Wu theory/Tsai Hill theory,

$-75^\circ \leqslant \theta_n \leqslant 90^\circ$ (Ply angle incremental value $\Delta\theta$),

$0.05 \leqslant t_n \leqslant 0.15$ (Ply thickness incremental value Δt mm),

$n = 1, \ldots, N_{\max}$ ($N_{\max}$ considered as one of the variables).

5. Results

The simulation studies have been carried out for the problem statements mentioned in section 4 on specimen carbon/epoxy balanced symmetric laminate with in plane dimensions $a = 1000$ mm, $b = 1000$ mm. The population size of 200 is used for all simulation results presented in the paper. The physical properties of the unidirectional carbon/epoxy laminate material are listed in Table 2 [4].

Table 2.

Material properties for carbon/epoxy

Property	E_{11} [GPa]	E_{22} [GPa]	G_{12} [GPa]	γ_{12}	S_{Lc} [MPa]	S_{Lt} [MPa]	S_{Tc} [MPa]	S_{Tt} [MPa]	S_{Lts} [MPa]	ρ kg/m^3
Value	116.6	7.673	4.173	0.27	1701	2062	240	70	105	1605

5.1. Results of problem 4.1 for different load cases

Direct value-coded genetic algorithm selected from OptiComp with UTA option is applied for finding optimum weight of laminate for all the load cases mentioned in Table 1. The obtained results in terms of number of plies, ply angle stacking sequence and optimum weight/ laminate thickness are provided in Table 3 to Table 8 for load cases 1 to 6, respectively.

Ply angles are the dominating design variables when UTA is applied for design optimization of composite laminates. In order to investigate the effect of ply angle increment value on composite laminate design, minimum laminate thicknesses obtained using 15° ply angle increment value and minimum laminate thicknesses obtained using 45° ply angle increment value [4] are compared in Table 9.

The difference between reference results and the newly obtained results for 15° ply angle increment value goes on increasing with the increase in shear force value. The maximum stress theory, as well as Tsai Wu theory, are susceptible to ply angle increment value. It is observed that ply angle increment value has negligible or

Table 3.

Optimum results for load case 1 by UTA

Load case	Failure criteria (N_{max})	Stacking sequence	Laminate thickness [mm]	Weight [kg] by OptiComp
1	MS (66)	[-45/75/30/15/90/-30/60_2/-75/0/-45/-30/45/-30/-45/ -30/45/60/-60/-15/75/90/30/45/-15/75/-15/30/60/ -60/45/-45/30]s	6.6	10.593
	TW (72)	[-75/60/30/60/30/-30/75/30/15/0/45/-30/90/-15/-30/ -15/15/90/-30/75/-60/-30/-75/60/75/60/-15/45/-60/ -15/-30/-75/90/30/-75/-15]s	7.2	11.556
	TH (72)	[-60/60/75/0/-75/-30/-60/30/-45/-15/75/-15/-60/-15/ 45/75/15/0/-45/90/-30/60/-75/45/60/0/60/-60/-15/ -60/45/15/30/60/0/-75]s	7.2	11.556

Table 4.

Optimum results for load case 2 by UTA

Load case	Failure criteria (N_{max})	Stacking sequence	Laminate thickness [mm]	Weight [kg] by OptiComp
2	MS (66)	[75_3/60/75/0/45/60/75/0/-15/15/75/-15/-30/60/0/-15/ -30_2/75_2/90_2/75/60/-30/15/-15/0/75/15/75]s	6.6	10.593
	TW (72)	[60/75/0_2/-15/0_2/90_2/0_2/60_3/75/90/75_2/0/90/45/-15/ 60/90/-15/60/ 15/0_3/-75/15/-45_2/90_2]s	7.2	11.556
	TH (72)	[75_2/60/-15/75/15/0_2/15/-15/-30/-15/15/90/0/75/0/ 90_2/0_2/60/75/90/90/75/-30/75/30/75/60/90/0_2/75/ -15]s	7.2	11.556

Table 5.

Optimum results for load case 3 by UTA

Load case	Failure criteria (N_{max})	Stacking sequence	Laminate thickness [mm]	Weight [kg] by OptiComp
3	MS (68)	[75/15/-15/0/60/75/-15/15/75/45/75_2/-30/75/-15/90/ -15/60/60/-30/90/30/75/90/-15/60/-15/15/-15/90/0/ 45/75/-30]s	6.8	10.918
	TW (72)	[-15_2/75_2/15_2/75/90/30/0/75/15/90/75/60/75_2/15_2/ 75/15/-15/75/-75/75_2/0/15/30/-60/30/15/90/60/0/ -15]s	7.2	11.556
	TH (72)	[-30/-45/60/0/60/75/45/-30/0/60_2/-30/60/-15/60/ -30_2/75/-15/-30_2/-15/15/75/-45/90/60/75/90/60_2/ 30/-15/75/-15/60]s	7.2	11.556

Table 6.

Optimum results for load case 4 by UTA

Load case	Failure criteria (N_{max})	Stacking sequence	Laminate thickness [mm]	Weight [kg] by OptiComp
4	MS (34)	[-15/-45/0_2/15/75/90/45/-45/90/-15/0/60/90/75/-30/75]s	3.4	5.457
	TW (32)	[15/-75/-30/30/-60_2/75/15/60_2/45/-45/75/-15/0/-45]s	3.2	5.136
	TH (38)	[-30/45/-15/45/-30/-60/-30/0/75/-45/60_2/75/-45/45/-45/-15/60_2]s	3.8	6.099

Table 7.

Optimum results for load case 5 by UTA

Load case	Failure criteria (N_{max})	Stacking sequence	Laminate thickness [mm]	Weight [kg] by OptiComp
5	MS (34)	[-30/-15/-30/75/-30_2/75/-15/75/-30/30/75/90/0/75/-30/90]s	3.4	5.457
	TW (32)	[-60/60/-45/30/-75/60/75/-15/-452/30/-75/-15/15/-45/-30]s	3.2	5.136
	TH (38)	[0_2/-75/0/90/75/-75/-45/75/90/-60/0/15/-30/-75/-30/60/-30/0]s	3.8	6.099

Table 8.

Optimum results for load case 6 by UTA

Load case	Failure criteria (N_{max})	Stacking sequence	Laminate thickness [mm]	Weight [kg] by OptiComp
6	MS (34)	[-30/15/-30_2/ 75_2/-30/60_2/-30_2/-15/75_2/-30_2/75]s	3.4	5.457
	TW (32)	[75/-45/15/-15/75/60/-75/-60/-30_2/-45_2/-15/0/-60_2]s	3.2	5.136
	TH (38)	[30/-15/90/60/0_2/-15/-75/-45/-30/-45/-30/-60/60/90/-45_2/-60/75]s	3.8	6.099

very small effect on laminate thickness when laminate is subjected to pure biaxial force condition only. Ply angle increment value has major impact on design of laminates subjected to a significant amount of shear forces along with in plane biaxial loads.

The laminate thickness obtained using UTA does not represent true lowest possible laminate thickness for that load case, as UTA constraints all the plies to

Table 9.

Optimum laminate thicknesses for different ply angle increment values

Load case	Failure criteria	Laminate thickness in mm for ply angle increment value 45° [4]	Laminate thickness in mm for ply angle increment value 15°
1	MS	6.8	6.6
	TW	7.2	7.2
	TH*	–	7.2
2	MS	7.2	6.6
	TW	7.6	7.2
	TH*	–	7.2
3	MS	8	6.8
	TW	8	7.2
	TH*	–	7.2
4	MS	3.6	3.4
	TW	3.2	3.2
	TH*	–	3.8
5	MS	4	3.4
	TW	3.6	3.2
	TH*	–	3.8
6	MS	4.4	3.4
	TW	3.6	3.2
	TH*	–	3.8

* reference results are not available

have same thickness. To get the real representation of minimum laminate thickness required for a particular case, it is necessary to vary ply thickness along with ply angle.

5.2. Results of problem 4.2 for different load cases

The VTA strategy is applied for all the load cases mentioned in Table 1 and the obtained results are provided in Table 10 to Table 15. In all the results, T denotes the obtained minimum laminate thickness required to sustain the applied load for given number of plies. The meaning of the obtained stacking sequences in terms of ply angles and ply thicknesses is explained in Fig. 5.

Majority of researchers are using UTA for weight minimization of composite laminate. The laminate thicknesses obtained using UTA with ply angle increment value 15° and VTA are compared with the reference laminate thicknesses [4] in

Table 10.

Optimum results for load case 1 using VTA

Load case	Failure criteria (N_{max})	Stacking sequence	Average thickness T [mm]	Weight [kg] by OptiComp
1	MS (66)	θ_n = [75/0/-60/0_2/15_2/-45/30/-60/-45/75/45/-60/90/75/45/-30/45/-45/60/-60/60/-60/30/15/60/-45/15/-30/0/30/90]$_s$ t_n = [0.1/0.075/0.1/0.075/0.1_3/0.075_2/0.1/0.075/0.125_2/0.15/0.1_2/0.125/0.15/0.05/0.1/0.075/0.1_3/0.05/0.075/0.1/0.075/0.15/0.075/0.1/0.15/0.125]$_s$	0.0963 (6.55)	10.512
	TW (72)	θ_n = [-15/-75/90/30/-60/60/45/-60/75/0_2/-30/-45/75/-15/60/75/-60/90/0/75_2/60/-15/45/-45_2/30/0_2/-30/-15/-45/0/75/60]$_s$ t_n = [0.075/0.1/0.075/0.125/0.05/0.15/0.075/0.1/0.125/0.075/0.125/0.05_3/0.15/0.1/0.05/0.01_2/0.075/0.125/0.15/0.10_2/0.125/0.1_2/0.125_3/0.05/0.1/0.125/0.075/0.1_2]$_s$	0.0979 (7.05)	11.315
	TH (72)	θ_n = [-30/-60/60/15/-60/45/-60/-45/15_2/90/0/-60_2/0/-15/15/0/75_2/30/75/-75/90/-15/30/-15/-75/60/0/75/15/-45/60/45/75]$_s$ t_n = [0.125/0.1_2/0.05/0.15/0.1/0.125_2/0.075/0.1/0.075/0.125/0.075/0.05_2/0.15/0.1/0.075/0.05/0.1_2/0.125/0.075/0.1_3/0.125_2/0.075/0.1/0.125/0.075/0.1/0.075/0.1/0.125]$_s$	0.0979 (7.05)	11.315

Table 11.

Optimum results for load case 2 using VTA

Load case	Failure criteria (N_{max})	Stacking sequence	Average thickness T [mm]	Weight [kg] by OptiComp
2	MS (66)	θ_n = [75/60/15/60/-60/90/-60/15/45/75/-60/75/15/30/60/-75/45/75/45/0/-15/30/15/90/-30_2/0/15/-60/60/75/0_2]$_s$ t_n = [0.1_2/0.125/0.1/0.075_2/0.1/0.075/0.15/0.075/0.1_2/0.05/0.075_2/0.15/0.075/0.1/0.15/0.1/0.075/0.15/0.05/0.15/0.125_2/ 0.1/0.075/0.15/0.05/ 0.1/ 0.125/0.05]$_s$	0.0963 (6.55)	10.512
	TW (72)	θ_n = [30/75/-45/-15/45/60/30/75/-15/60/-15/75/45/45/-75/-15/-60/60/90/45/0/75/0/45_2/30/75/30_2/-45/75/-30/60/-60/15/-45]$_s$ t_n = 0.125/0.075/0.125/0.15/0.05_2/0.1/0.15/0.1/0.075/0.1/0.075/0.1/0.125/0.1/0.125/0.05/0.1/0.125/0.05/0.125/0.1/0.075/0.1/0.125/0.05_2/0.15/0.05/0.125/0.15/0.075_2/0.1/0.075/0.150]$_s$	0.0979 (7.05)	11.315
	TH (72)	θ_n = [-45/0_2/15/-15/75_2/-30/-60/45/15_2/-60/75/-60/90/-75/15/45/75/15/45/-45/15_2/90/75/60/-45/75/0_2/30/45/75/30]$_s$ t_n = [0.075/0.125/ 0.075/ 0.125/0.075/ 0.1/0.15/0.1_2/0.125/0.075/0.125/0.075/0.15/0.1/0.075/0.125/0.075/0.125/0.075_2/0.1/0.075/0.05/0.1/0.125/0.1/0.05/0.125/0.1_2/0.075/0.1/0.075/0.125/0.1]$_s$	0.0979 (7.05)	11.315

Table 12.

Optimum results for load case 3 using VTA

Load case	Failure criteria (N_{max})	Stacking sequence	Average thickness T [mm]	Weight [kg] by OptiComp
3	MS (68)	θ_n = [45$_2$/15/-60/45/30/0/45/60/75/-30/45/-75/90/45/60/-30$_2$/45/60$_2$/30/60/75/-30/15/-45/30/15/45/15/60/45/90]$_s$ t_n = [0.15/0.125/0.075/0.15/0.1/0.125/0.05/0.125$_2$/0.05/0.125$_2$/0.1/0.05$_2$/0.075$_2$/0.125/0.1/0.125/0.05/0.125/0.075/0.1/0.075$_3$/0.15/0.1/0.125/0.075/0.05$_2$/0.15]$_s$	0.097 (6.6)	10.593
	TW (72)	θ_n = [45$_2$/90/0/75/45/15/60/-45/30/45/15/75/-60/0/30/-15/60/0/30/-60/30/75/-15/60/45/-30/75/-15/30/75/60/75$_2$/-60/-30]$_s$ t_n = 0.075$_3$/0.125/0.075/0.15/0.125/0.1/0.05$_2$/0.075/0.15/0.125/0.05/0.1/0.05/0.125/0.15/0.05/0.125/0.075/0.125/0.075/ 0.125/0.075$_3$/0.1$_2$/0.125/0.15/0.1/0.125/0.125/0.075/0.1]$_s$	0.098 (7.05)	11.315
	TH (72)	θ_n = [15/45/90/75/60/30/-30/45/-60/0/60/15/0/60/-75/-30/15/-75/60/75/30/90/45/60/45/15/-15/0/-45/30/45/30/60$_2$/-30/45]$_s$ t_n = [0.1$_3$/0.15/0.125/0.05/0.075/0.1/0.125/0.075/0.1/0.125/0.15/ 0.125/0.05$_3$/0.125/0.1/0.05/0.075$_2$/0.1/0.125$_2$/0.05/0.125/0.1$_2$/0.05/0.125/0.1/0.15/0.075/0.15/0.075]$_s$	0.098 (7.05)	11.315

Table 13.

Optimum results for load case 4 using VTA

Load case	Failure criteria (N_{max})	Stacking sequence	Average thickness T [mm]	Weight [kg] by OptiComp
4	MS (34)	θ_n = [-45/0/15/-30/-60/45/60/30/75/-60/0/90/-60/60/15/-45/75]$_s$ t_n = [0.1/0.075/0.075/0.1$_2$/0.15/0.05/0.1$_2$/0.075/0.125/0.075/0.1$_3$/0.125/0.1]$_s$	0.097 (3.3)	5.296
	TW (32)	θ_n = [60/15/75/60/-60/45/-30/ 90/-45$_3$/30/15/-30$_2$/30]$_s$ t_n = [0.1$_3$/0.15/0.05$_2$/0.125$_2$/0.1/0.15/0.075$_2$/0.05$_2$/0.075/0.15]$_s$	0.0953 (3.05)	4.895
	TH (38)	θ_n = [15/45/0/-30/-60$_3$/75$_2$/-30/-15$_2$/75/90/15/75/15/45/90]$_s$ t_n = [0.075/0.15/0.125/0.075$_2$/0.075/0.15/0.1$_2$/0.125/0.1/0.125/0.1/0.05 /0.075/0.1$_2$/0.075$_2$]$_s$	0.097 (3.7)	5.938

Table 14.

Optimum results for load case 5 using VTA

Load case	Failure criteria (N_{max})	Stacking sequence	Average thickness T [mm]	Weight [kg] by OptiComp
5	MS (34)	θ_n = [0/-15/75/30/0$_3$/-60/-75/-45/45/-60/75/-45/ -60/75/-75]$_s$ t_n = [0.125/0.15/0.125/0.1/0.075/0.05/0.1/0.075$_2$/ 0.15/0.075$_2$/0.125/0.1/0.125/0.05/0.075]$_s$	0.097 (3.3)	5.296
	TW (32)	θ_n = [45/-15/-60/0/-45/-30/30/-75/-60/30/60$_2$/-45/ -60/30/-30]$_s$ t_n = [0.05$_2$/0.125$_2$/0.05/0.1/0.125/0.15/0.125/0.1$_2$/ 0.075/0.125/0.1/0.05/0.075]$_s$	0.0953 (3.05)	4.895
	TH (38)	θ_n = [-45/75/-60/-45/-30$_2$/60/0/15/-45/45/-30/75/- 45/30/-15/75/60/90]$_s$ t_n = [0.125/0.075/0.125/0.15/0.1/0.075/0.15/0.125/ 0.075$_3$/0.125/ 0.05/0.1/0.125/0.075/0.05/0.075 /0.125]$_s$	0.098 (3.75)	6.018

Table 15.

Optimum results for load case 6 using VTA

Load case	Failure criteria (N_{max})	Stacking sequence	Average thickness T [mm]	Weight [kg] by OptiComp
6	MS (34)	θ_n = [-60/-45-45-45/-30/-15/15/-15/-75/-60/30/-60/ 75/-60/45/-45/-75]$_s$ t_n = [0.1/0.05/0.125$_4$/0.15/0.1/0.05/0.075/0.1/ 0.075/0.15$_2$/0.075/ 0.05/ 0.075]$_s$	0.1 (3.4)	5.457
	TW (32)	θ_n = [-45/-60/-45/-75/0/-45$_3$/60/45/-75/60/-30/ 30/0/-45]$_s$ t_n = [0.075$_2$/0.1/0.125$_2$/0.1/0.125$_2$/0.1/0.05/0.1/ 0.075/0.125/0.075/0.1/ 0.05]$_s$	0.095 (3.05)	4.895
	TH (38)	θ_n = [-30$_2$/75/60/75/15/-45/-30/60$_2$/-30/0/-75/-45/ 0/-60/-15/45/-60]$_s$ t_n = [0.05/0.1/0.075$_2$/0.05/0.075/0.1/0.125$_2$/0.1/ 0.075/0.1/0.125/0.1$_2$/0.15$_2$/0.075/0.15]$_s$	0.098 (3.75)	6.018

Table 16 for all the load cases. Comparison of these results exposes limitations of UTA and effectiveness of VTA while calculating minimum laminate thickness for the applied load condition.

The reduction in laminate weight/thickness obtained using VTA over the reference results for all the load cases is the combined outcome of reduced ply angle increment value and ply thicknesses in discrete form. So, drastic reduction in laminate weight/ thickness is observed using VTA for all the load cases. In zero shear

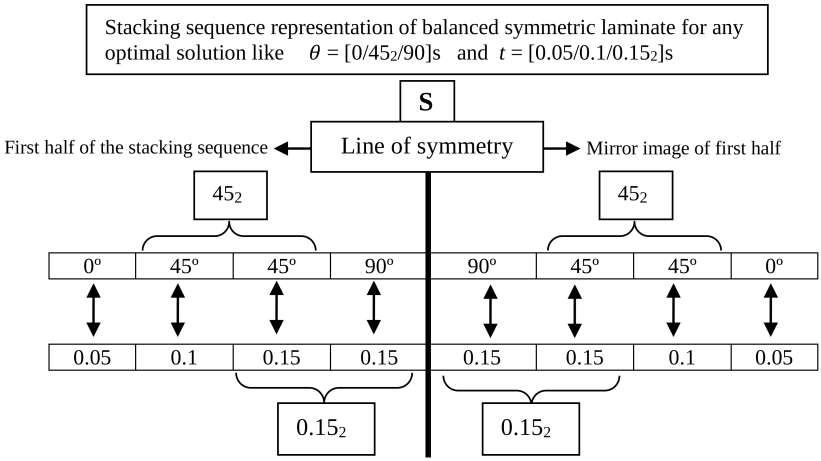

Fig. 5. Explanation of the optimum stacking sequences obtained in terms of ply angles and ply thicknesses

Table 16.

Comparison of optimum thicknesses obtained at different stages

Load case	Failure criteria	Laminate thickness by [4] [mm]	Laminate thickness using UTA (15°) [mm]	Laminate thickness by VTA strategy [mm]	% reduction in thickness using UTA- over reference thickness	% reduction in thickness using VTA- over reference thickness
1	MS	6.8	6.6	6.55	2.94	3.68
	TW	7.2	7.2	7.05	0	2
	TH*	–	7.2	7.05	–	2**
2	MS	7.2	6.6	6.55	2.94	3.68
	TW	7.6	7.2	7.05	2.69	4.67
	TH*	–	7.2	7.05	–	2**
3	MS	8	6.8	6.6	14.96	17.5
	TW	8	7.2	7.05	10	11.87
	TH*	–	7.2	7.05	–	2**
4	MS	3.6	3.4	3.3	5.55	8.34
	TW	3.2	3.2	3.05	0	4.7
	TH*	–	3.8	3.7	–	2.63**
5	MS	4	3.4	3.3	10.32	17.5
	TW	3.6	3.2	3.05	11.11	15.28
	TH*	–	3.8	3.75	–	1.31**
6	MS	4.4	3.4	3.4	22	22
	TW	3.6	3.2	3.05	20	23.75
	TH*	–	3.8	3.75	–	1.31**

* Reference results not available;
** Percentage difference against UTA results for 15° ply angle increment value.

force cases (1 and 4) also, VTA can reduce the laminate thickness, which is the effect of discrete values of ply thicknesses. The comparison of the ply thicknesses obtained at different levels of analysis for all load cases under tensile loading and compressive loadings is provided in Fig. 6 and Fig. 7, respectively. Subscripts 1 to 6 denotes different load cases, while UTA 45 and UTA 15 denote ply angle increment values used for UTA in Fig. 6 and Fig. 7. It is observed that VTA produced maximum laminate thickness reduction of 23.75% for load case 6 over reference results.

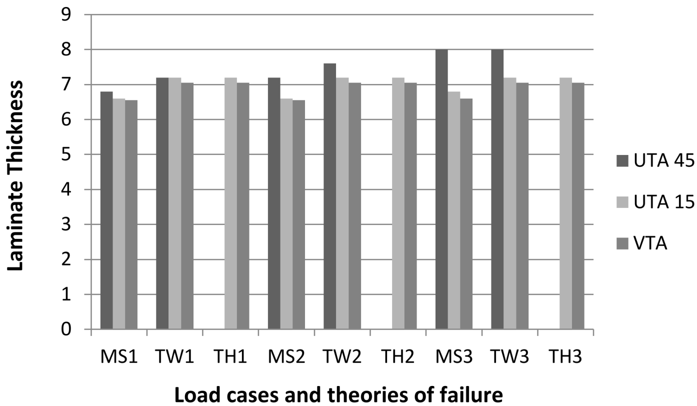

Fig. 6. Comparison of laminate thicknesses for biaxial tensile load cases

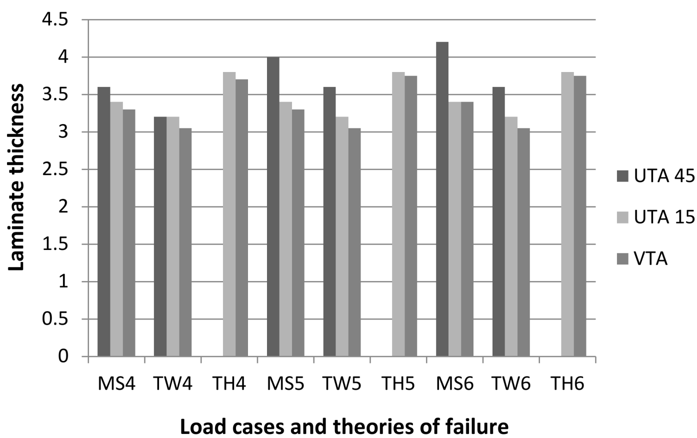

Fig. 7. Comparison of laminate thicknesses for biaxial compressive load cases

5.3. Results of problem 4.3 for different load cases

Even though, VTA results in reduced optimum weight for all the load cases, it also carries manufacturing complexity. Variable thickness composite structures can be fabricated by ply drops and splicing, which are not necessarily cost-efficient due to their high manufacturing cost. Though, it can be preferred for designing highly critical aerospace components [16]. Unlike the two stage methodologies defined by previous researchers [14, 16], VTA can be efficiently used for calculating minimum laminate thickness required for sustaining the applied load in one stage only. In this section, load case 2 is considered for demonstrative purpose and performance of VTA while calculating minimum laminate thickness is tested against variation in different design variables like the number of layers and increment value of ply angles and ply thicknesses within given limit bounds.

5.3.1. Effect of number of plies on VTA

In section 5.1, it is observed that the number of plies required to sustain load case 2 are 66, 72 and 72 for maximum stress theory, Tsai Wu theory and Tsai Hill theory, respectively. These values are in near vicinity to 70. Investigation of the effect of number plies on performance of VTA for load case 2 can be done by selecting number of plies other than 70. With this consideration, the minimum laminate thickness is obtained for an arbitrary selected number of layers 50, 60 and 80 using VTA without changing other design variables mentioned in 4.2. The obtained results are provided in Table 17.

Table 17.

Minimum laminate thickness for different number of layers using VTA

Load case	Number of layers	Failure theory	Laminate thickness	Laminate thickness already obtained (number of layers)
2	50	MS	6.65	6.55 (66)
		TW	7.1	7.05 (72)
		TH	7.05	7.05 (72)
	60	MS	6.55	6.55 (66)
		TW	7.1	7.05 (72)
		TH	7.05	7.05 (72)
	80	MS	6.6	6.55 (66)
		TW	7.1	7.05 (72)
		TH	7.05	7.05 (72)

The number of layers affects the performance of VTA in this regards to a certain extent. Excessively small number of layers or very high number of layers may affect the performance of VTA. In such cases, performance of VTA can be improved by selecting wide range between limit bounds of ply thicknesses or by reducing ply thickness increment value within limit bounds.

5.3.2. Effect of ply thickness increment value on VTA

Now, problem 4.3 is solved for obtaining minimum laminate thickness by using different ply thickness increment values within the limits 0.05 mm to 0.15 mm keeping ply angles and number of layers unaffected. The obtained results are provided in Table 18.

Table 18.
Minimum laminate thickness for load case 2 with reduced ply thickness increment value

Load case	Number of layers	Failure theory	Laminate thickness with 0.01 mm increment	Laminate thickness with 0.025 mm increment	Laminate thickness with 0.05 mm increment
2	66	MS	6.52	6.55	6.55
	72	TW	7.04	7.05	7.1
	72	TH	7.02	7.05	7.1

The results provided in Table 18 show that ply thickness increment value 0.01 mm gives more accurate minimum laminate thickness for load case 2 compared to higher ply thickness incremental values for all the failure theories.

5.3.3. Effect of ply angle increment value on VTA

The performance of VTA in terms of laminate thickness calculations is tested against ply angle increment value for load case 2 keeping ply thicknesses and number of layers unaffected, and the obtained results are provided in Table 19.

Table 19.
Minimum laminate thickness for load case 2 against ply angle increment value

Load case	Number of layers	Failure theory	Laminate thickness with 5° angle increment	Laminate thickness with 15° angle increment	Laminate thickness with 45° angle increment
2	66	MS	6.55	6.55	6.55
	72	TW	7.05	7.05	7.05
	72	TH	7.05	7.05	7.05

The obtained results show that the ply angle increment value does not affect performance of VTA while calculating minimum laminate thickness.

6. Conclusions

The current article deals with demonstration of effectiveness of Variable Thickness Approach (VTA) for finding minimum required laminate thickness to sustain the applied load condition. Initially, Uniform Thickness Approach (UTA) is used for minimizing weight/thickness of laminate with reduced ply angle increment value 15°. It is observed that reduction in ply angle increment value is not that much effective in weight reduction when shear force is not acting along with biaxial forces. At the same time, because of constraint of working with uniform thickness, UTA cannot reach to minimum required laminate thickness for any load case. VTA can be used to overcome this limitation of UTA and has the ability to find minimum required laminate thickness with substantial accuracy in one stage only. The results provided in Graph 1 and Graph 2 prove capability of VTA in obtaining minimum laminate thickness for any load case as well as for any failure theory. Because of the use of all the design variables in discrete form and handling of different design variables simultaneously, VTA is computationally efficient. The obtained results show that number of plies and ply thickness increment value significantly affect performance of VTA, while ply angle increment value hardly affects performance of VTA in this regards. VTA can be coupled with any other optimization algorithm for this purpose, which can handle multiple design variables of different nature in discrete form simultaneously.

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