Identifying the optimal controller strategy for DC motors

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Abstract: The aim of this study is to design a control strategy for the angular rate (speed) of a DC motor by varying the terminal voltage. This paper describes various designs for the control of direct current (DC) motors. We derive a transfer function for the system and connect it to a controller as feedback, taking the applied voltage as the system input and the angular velocity as the output. Different strategies combining proportional, integral, and derivative controllers along with phase lag compensators and lead integral compensators are investigated alongside the linear quadratic regulator. For each controller transfer function, the step response, root locus, and Bode plot are analysed to ascertain the behaviour of the system, and the results are compared to identify the optimal strategy. It is found that the linear quadratic controller provides the best overall performance in terms of steady-state error, response time, and system stability. The purpose of the study that took place was to design the most appropriate controller for the steadiness of DC motors. Throughout this study, analytical means like tuning methods, loop control, and stability criteria were adopted. The reason for this was to suffice the preconditions and obligations. Furthermore, for the sake of verifying the legitimacy of the controller results, modelling by MATLAB and Simulink was practiced on every controller.

Key words: DC Motor, LQR, PID, PI, controller strategy

1. Introduction

Direct current (DC) motors are an important component in many electrical devices, converting electrical power to mechanical motion. They work by supplying a current through a conductor within a magnetic field; the current is forced by the torque and produces motion. DC motors are ubiquitous, with almost all industries and households using them in various equipment or appliances. These applications require the speed of the motor to be controlled to drive processes such as the arm of a robot. The control can be either manual or automatic. Many studies have investigated the optimal control of DC motors to achieve the desired results.
In this study, we compare several control strategies to evaluate their accuracy and stability. The aim of this study is to design a control strategy for the angular rate (speed) of a DC motor by varying the terminal voltage. This is done by setting the desired angular velocity to unity and examining the best design criteria achieved by different control strategies. The number of techniques and studies reported over recent years shows the importance of this topic.

Here, after presenting a definition of control, we provide an overview of the history of DC motor controlling techniques, and review some of the methods used to implement the work described in this paper. DC motor controllers are an example of controlling devices. They might include a manual or automatic means of starting and stopping the motor, selecting forward or reverse rotation, regulating the speed, limiting the torque, and protecting against overloads and faults [1].

Controlling the motor speed is not a new idea, and is fundamental in the design of feedback control systems. This is done by receiving an input signal from a measured process variable, comparing this value with that of a predetermined control point value (set point), and determining the appropriate output signal required by the final control element to provide corrective action within a control loop. Previous studies and designs have not focused on the real benefit, but have instead applied only slight increments to the control parameters.

Modern controllers use power electronics and microprocessors, and are of varying complexity. The choice of a controller often depends on the control objectives and controller cost. DC motor controllers must be able to handle unknown load characteristics and parameter variations. Proportional-integral-derivative (PID) controllers are most commonly used to control DC motors. These offer several important features and are easy to implement.

The disadvantage of PID controllers is that they often overshoot the desired objective value following sudden changes in load torque. Additionally, PID controller parameters are very difficult to control, making it hard to achieve the optimal state.

To overcome this disadvantage, control methods such as linear quadratic regulators (LQRs) have been developed [2, 13]. LQRs offer robustness in terms of minimizing a given cost function [3, 16]. The simplicity, reliability, and minimal cost of DC motors means that they are often preferred over other motors [4].

The best strategy for controlling the speed of a DC motor is to use a PID controller and LQR, which provides better transient parameters [5]. The robustness of LQR ensures an accurate dynamic response [6]. In addition, the LQR displays better high-range flexibility and control when compared with other controllers [7, 13]. Therefore, the LQR is suitable for robotics applications and process control, because it improves system stability, effective control, and balancing properties [8, 14].

2. Design and implementation

2.1. Mathematical model of a DC motor

A simplified mathematical model of a DC motor can be used to build the motor transfer function. The transfer function is derived from the DC motor equations, which are divided into a mechanical part, electrical part, and the interconnection between them. The equation for the
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The electrical part can be derived as follows [1, 15]:

\[ I_s(s) = \frac{V - K\phi w}{(Ra + SLa)} \]  \hspace{1cm} (1)

where \( E_a = K\phi w \), \( V \) is the motor terminal voltage (V), \( w \) is the motor speed (rad/s), \( Ia \) is the winding current (A), \( K\phi \) is the back electromotive force (EMF) constant (Vs/rad), \( Ra \) is the terminal resistance (\( \Omega \)) and \( La \) is the terminal inductance (H).

The equation for the mechanical part can be derived using Newton’s law, which states that the summation of electrical and load torques is equal to the load and motor inertia multiplied by the derivative of the angular rate.

\[ W(s) = \frac{K\phi Ia - TL}{(Js + b)}, \]  \hspace{1cm} (2)

where \( J \) is the load and motor inertia (kg/m\(^2\)), \( b \) is the damping friction (N.m.s/rad), \( TL \) is the load torque (N.m), \( Te \) is the electrical torque (N.m). As the voltage is the input to the system and the speed is the output, the required transfer function is symbolized by:

\[ \frac{w(s)}{v(s)}. \]  \hspace{1cm} (3)

This form is derived as the flow:

\[ \frac{w(s)}{v(s)} = \frac{K\phi}{(K\phi)^2 + (Ra + SLa)(JS + b)}. \]  \hspace{1cm} (4)

2.2. Dynamic system of a DC motor

Assuming a constant excitation field armature, the voltage can be reformed as:

\[ \frac{d}{dt} Ia = -\frac{Ra}{La} Ia - \frac{K\phi}{La} w + \frac{1}{La} V. \]  \hspace{1cm} (5)

Using Newton’s second law:

\[ J \frac{dw}{dt} = Te - TL - bw, \]  \hspace{1cm} (6)

where \( b \) is the damping friction. Thus,

\[ \left( s + \frac{b}{J} \right) w(s) = \frac{1}{J} K\phi Ia(s) - \frac{1}{J} TL(s). \]  \hspace{1cm} (7)

To use a dynamic system method, we should specify the states, input, and output of the system. In a DC motor system, the current (I) and angular rate (\( dW/dt \)) are the states, the applied voltage (V) is the input, and the angular velocity (w) is the output. As the system is linear, the state space can be written in the form:

\[ \dot{x}(t) = [A] x(t) + [B] u(t), \]  \hspace{1cm} (8)

\[ y(t) = [C] x(t) + [D] u(t), \]  \hspace{1cm} (9)
where $\dot{x}(t)$ denotes the state vectors $i$ and $w$, $y(t)$ is the output $w$, $u(t)$ is the input $V$, 

$$
\dot{x}(t) = \frac{d}{dt} (i \text{ and } w), [A] \text{ is the state matrix } (n \times n), [B] \text{ is the input matrix } (n \times p), [C] \text{ is the output matrix } (q \times n) \text{ and } [D] \text{ is the feed forward (zero) matrix } (q \times p).
$$

Following the method above, and from Equations (4) and (6), the state space becomes:

$$
\frac{d}{dt} \begin{bmatrix} i \\ w \end{bmatrix} = \begin{bmatrix} \frac{Ra}{La} & -\frac{K\phi}{La} \\ \frac{k\phi}{J} & -\frac{b}{J} \end{bmatrix} \begin{bmatrix} i \\ w \end{bmatrix} + \begin{bmatrix} \frac{1}{Ta} \\ 0 \end{bmatrix} V,
$$

(9)

$$
W = [0 \ 1] \begin{bmatrix} i \\ w \end{bmatrix} + [0] V,
$$

(10)

$$
A = \begin{bmatrix} -\frac{Ra}{La} & -\frac{K\phi}{La} \\ \frac{k\phi}{J} & -\frac{b}{J} \end{bmatrix}, \quad B = \begin{bmatrix} \frac{1}{Ta} \\ 0 \end{bmatrix}, \quad C = [0, 1], \quad D = [0].
$$

(11)

2.3. Open loop of DC motor angular velocity

To obtain a transient response in the situations studied in this paper (i.e. the open loop condition) we use a simple DC motor model with the parameters listed in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motor terminal voltage</td>
<td>$V$</td>
<td>input</td>
<td>Volt (V)</td>
</tr>
<tr>
<td>Motor speed</td>
<td>$w$</td>
<td>output</td>
<td>rad/s</td>
</tr>
<tr>
<td>Back EMF constant</td>
<td>$K\phi$</td>
<td>0.01</td>
<td>Vs/rad</td>
</tr>
<tr>
<td>Terminal resistance</td>
<td>$Ra$</td>
<td>2</td>
<td>Ohms (Ω)</td>
</tr>
<tr>
<td>Terminal inductance</td>
<td>$La$</td>
<td>0.5</td>
<td>Henrys (H)</td>
</tr>
<tr>
<td>Load and motor inertia</td>
<td>$J$</td>
<td>0.02</td>
<td>Kg/m²</td>
</tr>
<tr>
<td>Damping friction</td>
<td>$b$</td>
<td>0.2</td>
<td>N.m.s/rad</td>
</tr>
</tbody>
</table>

After substituting these parameter values into the variables in Equation (3), the transfer function of the DC motor open loop becomes:

$$
\frac{w(s)}{v(s)} = \frac{0.01}{0.01s^2 + 0.14s + 0.4001} = \frac{1}{(s + 9.998)(s + 4.002)}.
$$

(12)

From question 12 the open loop step response of angular velocity ($w_{ss} = 0.025$ rad/s, $t_s = 1.2$ s) has a large steady-state error (0.975 rad/s).

Even using the dynamic system method, the open loop can be tested by computing the eigenvalues of matrix $A$, which represent the poles of the system: poles = $-4.0017$ and $-9.9983$. From the above result, there are no poles in the right half-plane, which means that the system is unstable in the open loop condition.
2.4. Closed loop of DC motor angular velocity

As the open loop step response has a very large steady-state error and the system is unstable, the closed loop will be designed with different controller strategies to eliminate the error and enhance the system transient response. A feed forward compensator (C) added controllability in sense with the DC Motor with a unity feedback.

2.5. Controlling DC motor angular velocity through different compensation techniques

The main objective of this paper is to design a feed-forward compensator that will drive the DC motor angular velocity to unity. Different control strategies will be applied and compared in terms of the steady-state error in the step response, settling time, and stability. There are three main controllers: a proportional controller, integral controller, and derivative controller.

This paper also examines the LQR controller and three types of compensator: a phase lag compensator, lead integral compensator, and lead lag compensator. Table 2 shows the controller used.

<table>
<thead>
<tr>
<th>Controllers</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportional controller design</td>
<td>$P_{out} = K_p \times e(t)$</td>
</tr>
<tr>
<td>Integral controller design</td>
<td>$P_{out} = K_i \times \int_0^t e(t)$</td>
</tr>
<tr>
<td>Derivative controller design</td>
<td>$P_{out} = K_d \times \frac{de(t)}{dt}$</td>
</tr>
<tr>
<td>Proportional-integral (PI) controller design [9, 12]</td>
<td>$P_{out} = K_p \times e(t) + K_i \times \int_0^t e(t)$</td>
</tr>
<tr>
<td>Proportional-integral-derivative controller design [9, 12]</td>
<td>$P_{out} = K_p \times e(t) + K_i \times \int_0^t e(t) + K_d \times \frac{de(t)}{dt}$</td>
</tr>
<tr>
<td>Phase lag compensator design [10, 11]</td>
<td>$C = \frac{K_c}{a} + \frac{s + 1/T}{s + 1/\alpha T}$</td>
</tr>
<tr>
<td>Lead integral compensator design [3]</td>
<td>$C = \frac{K_c}{a} + \frac{s + 1/T}{s + 1/\alpha T}$</td>
</tr>
<tr>
<td>Lead lag compensator design</td>
<td>$C = K_c \left( \frac{s + \frac{1}{T_{lead}}}{s + \frac{1}{a T_{lead}}} \right) \left( \frac{s + \frac{1}{T_{lag}}}{s + \frac{1}{a T_{lag}}} \right)$</td>
</tr>
</tbody>
</table>

2.5.1. LQR controller design

State feedback using an LQR follows Equations (7) and (8). The LQR state feedback configuration was found. This design is classified as the optimal control system. However, this will realize practical components that provide the designed operating performance. Therefore, the performance indices can be readily adjusted in the time domain.
As a result, the steady state and the transient performance indices are specified in the time domain. The performance of a control system can be represented by integral performance measures. Therefore, the design of the system must be based on minimizing some performance index, such as the integral of the squared error.

The specific form of the performance index is [13]:

$$ J = \int_{0}^{t_f} (x^T Q x + u^T R u) dt, $$ (13)

where $x$ denotes the state vector, $x^T$ is the transpose of $x$, $t_f$ is the final time, and $R, Q$ denote weighting factors and controller design parameters, respectively, which are selected by trial and error.

The control input $u$ is given by:

$$ u = -Kx = [K_p \ K_i \ K_d \ldots \ Kn]x, $$

where $H$ is known. Its maximum value is given by:

$$ H = x^T Q x + u^T R u + \lambda^T (Ax + Bu), $$

$$ -\dot{\lambda} = \left( \frac{\partial H}{\partial u} \right)^T = Q x + A^T \lambda, $$ (14)

$$ 0 = \frac{\partial H}{\partial a} = -Ru + \lambda^T B, $$ (15)

$$ \lambda(t) = P(t)x(t) \quad \text{or} \quad \lambda = Px, $$

$$ u = -R^{-1} B^T Px, $$ (16)

$$ \dot{P}x + P \left( Ax - BR^{-1}B^T Px \right), $$ (17)

$$ -0 = PA + A^T P - PBR^{-1}B^T P + Q. $$ (18)

3. Results and discussion

The controller designs in the previous section were implemented in a DC motor module to test their capabilities. This section presents the results in detail.

3.1. Proportional controller

The proportional controller is a simple strategy. It works as an amplifier to the input signal and is inversely proportional to the steady-state error. Thus, when the gain of the compensator increases, the steady-state error should decrease. Two proportional controllers were tested. The first has a proportional gain equal to one, which means the input signal is not amplified. This is
identical to the closed loop condition, Figure 1 shows the closed loop step response of the DC motor angular velocity \( w_{ss} = 0.0244 \, \text{rad/s} \), where the steady-state error is no less than its value in the open loop condition \( 0.9756 \, \text{rad/s} \). The settling time is 1.1 s.

In the second proportional controller, the gain was set to 100. Figure 2 shows the step response of the DC motor angular velocity \( w_{ss} = 0.8427 \, \text{rad/s} \), \( t_s = 0.75 \, \text{s} \). The steady-state error is lower \( 0.1573 \, \text{rad/s} \), but has not been eliminated. The system is unstable because there are poles in the right half of the S-plane and the system overshoots the steady state, but it will become stable as the proportional gain is increased.

In conclusion, if the proportional gain is equal to unity, the system will remain unchanged. However, if the gain value is greater than unity, the error signal will be amplified and the steady-state error will decrease, making the system more stable. Unfortunately, using this controller may lead to an increase in the overshoot.

3.2. Integral controller

The integral controller reduces the error by multiplying the transfer function of the system by \( K/s \), where \( K \) is the gain (constant) and \( s \) is the Laplace transform. The gain value is 100.

From Figure 3, it is clear that the integral controller can fully eliminate the steady-state error \( w_{ss} = 1.2202 \, \text{rad/s} \). However, it also has the disadvantage of producing a closed loop system
with a slower response time \((t_{ss} = 2.95\, \text{s})\), large overshoot value \((22.0151)\), and the potential for system instability as the gain increases.

It can be concluded that the benefit of this type of controller is its ability to reduce the error to zero. However, stability is not guaranteed, as the system may oscillate randomly. This controller also has a slower response time.

3.3. Proportional-integral controller

In the PI controller described by Equation (9), \(K_p\) and \(K_i\) were each set to 100. Thus, the equation becomes:

\[
c = 100 \cdot \left(1 + \frac{1}{s}\right) / s.
\]

Figure 4 shows that the PI controller eliminates the steady-state error \((w_{ss} = 0.9998\, \text{rad/s})\). The system remains stable as the controller gain increases, and the overshoot is reduced to zero, but the system settling time is still large \((t_{ss} = 3.25\, \text{s})\).
3.4. Derivative controller

This type of the system applies feed-forward control equal to the derivative of the error. Thus, from Figure 5, the derivative controller drives the motor speed to zero, so the steady-state error will be unacceptable.

\[ c = K_p + \frac{K_i}{s} + K_d \cdot s. \]

In addition, the noise signal produced in the system is amplified, so the derivative controller cannot be used alone. However, this type of controller has the advantage of an improved transient response. Hence, there is no need to use a derivative controller if the control objective has a slow response, but it could be beneficial if the control objective responds quickly.

3.5. Proportional-integral-derivative controller

Transferring Equation (18) from the time domain to the S-domain, we obtain:

In Figure 6, we see that the PID controller eliminates the steady-state error \( w_{ss} = 1.0122 \text{ rad/s} \), but the settling time
remains large ($t_{ss} = 3.4355$ s). The root locus plot indicates that the system will have poles on the imaginary axis as the gain increases.

$$u = -Kx = [K_p \ K_i \ K_d \ldots \ Kn]x.$$ 

3.6. Phase lag compensator

Setting $kc$, $\alpha$, and $T$ to 1000, 10, and 10, respectively, in Equation (21), the feed-forward phase lag compensator controlling the motor can be written as:

$$c = 100 \cdot \frac{s + 0.1}{s + 0.01}.$$

From Figure 7, we can see that the steady-state error is practically eliminated ($w_{ss} = 0.9611$ rad/s, $e_{ss} = 0.0385$). This technique produces a slow response (settling time is very large, $t_{ss} = 34$ s) and large rise time. The system is not subject to instability as the gain increases.

![Fig. 7. Step response, root locus, and Bode plot of DC motor system angular velocity using a phase lag compensator](image)

3.7. Lead integral compensator

This controller combines the integral compensator to reduce the steady-state error with the lead compensator to improve the settling time. To achieve this, we multiply Equation (22) by $1/s$ to give:

$$c = Kc \cdot \frac{s + 1}{s \left(s + \frac{1}{\alpha T}\right)}.$$

Setting $Kc$, $\alpha$, and $T$ to 100, 10, and 10, respectively, we have:

$$c = 100 \cdot \frac{s + 0.1}{s + 0.01}.$$

From Figure 8, it can be noted that the lead integral compensator eliminates the steady state error ($w_{ss} = 0.9999$ rad/s), but has a large settling time ($t_{ss} = 14.75$ s) and results in an unstable system as the gain increases. Thus, using this technique gives a slow response, but will improve the steady-state error.
3.8. Lead lag compensator

The lead compensator provides a fast response but results in an unstable system, whereas the lag compensator gives a stable system with a slow response time. Therefore, using a lead lag compensator may be more accurate. The system is controlled by a compensator equal to:

\[ c = \frac{100(s + 10)(s + 0.1)}{(s + 100)(s + 0.01)} \]

Figure 9 shows that the steady-state error is eliminated \( \omega_{ss} = 0.7136 \text{ rad/s} \), but the settling time is very large \( t_{ss} = 127.65 \text{ s} \). The system is not subject to instability as the gain increases.

3.9. Linear quadratic regulator

The LQR function is used to determine the state feedback control gain as \( k = 41.0100 \), and from Figure 10 shows the step response, root locus, and Bode plot of the DC motor system using an LQR controller. In the steady state, the error has been fully eliminated \( \omega_{ss} = 11 \text{ rad/s} \), and the settling time is small \( t_{ss} = 1.0488 \approx 1 \text{ s} \). There is no overshoot and the system is completely stable.
3.10. Comparison between all strategies

The results from all strategies are presented in Table 3, and the step responses of all controllers considered in this study are shown in Figure 11. From these results, it is clear that the LQR controller is the most stable, as the rise time and settling time are very small and the steady-state error is zero. The phase lag controller exhibits good stability, but suffers from a high rise time and settling time.

Table 3. Comparison of study results

<table>
<thead>
<tr>
<th>Controller</th>
<th>Velocity</th>
<th>Steady-state error</th>
<th>Settling time</th>
<th>Rise time</th>
<th>Overshoot</th>
<th>Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportional $c = 1$</td>
<td>0.0244</td>
<td>0.9756 (large)</td>
<td>1.1</td>
<td>0.6</td>
<td>0</td>
<td>unstable</td>
</tr>
<tr>
<td>Proportional $c = 100$</td>
<td>0.8427</td>
<td>0.2858 (quite large)</td>
<td>0.75</td>
<td>0.15</td>
<td>17.9888</td>
<td>unstable</td>
</tr>
<tr>
<td>Integral</td>
<td>1.2202</td>
<td>1.5987e–014 (eliminated)</td>
<td>2.95</td>
<td>0.55</td>
<td>22.0151</td>
<td>unstable</td>
</tr>
<tr>
<td>Proportional integral</td>
<td>0.9998</td>
<td>6.1062e–015 (eliminated)</td>
<td>3.25</td>
<td>0.25</td>
<td>0</td>
<td>unstable</td>
</tr>
<tr>
<td>Derivative</td>
<td>0</td>
<td>Not acceptable</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>unstable</td>
</tr>
<tr>
<td>Proportional integral derivative</td>
<td>1.0122</td>
<td>0 (fully eliminated)</td>
<td>3.4355</td>
<td>2.2122</td>
<td>1.2232</td>
<td>unstable</td>
</tr>
<tr>
<td>Phase lag</td>
<td>0.9611</td>
<td>0.0385 (eliminated)</td>
<td>34</td>
<td>12.4000</td>
<td>0</td>
<td>stable</td>
</tr>
<tr>
<td>Lead integral</td>
<td>0.9999</td>
<td>7.8160e–014 (eliminated)</td>
<td>14.75</td>
<td>8.1</td>
<td>0</td>
<td>unstable</td>
</tr>
<tr>
<td>Lead lag</td>
<td>0.7136</td>
<td>0.2858 (quite large)</td>
<td>127.65</td>
<td>70.3</td>
<td>0</td>
<td>unstable</td>
</tr>
<tr>
<td>Linear quadratic regulator</td>
<td>1</td>
<td>0 (fully eliminated)</td>
<td>1.0488</td>
<td>0.5617</td>
<td>0</td>
<td>stable</td>
</tr>
</tbody>
</table>
4. Conclusion

Many applications require the speed of a DC motor to be accurately controlled. Therefore, a control system for a DC motor was designed with the objective of controlling the angular speed to be unity with the best steady state and transient performance. Several types of controllers were applied to the problem, and the results given by the different controller strategies were compared.

Based on the step response, root locus, and Bode plot results for each controller considered in this study, it is clear that the LQR controller achieves the best steady state and transient response performance. This controller can fully eliminate the steady-state error with a very small transient settling time. There is no overshoot and the system is completely stable.

References


