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The method of learning outcomes assessment based on fuzzy relations

A. MREŁA[1*], O. SOKOLOV[2], and W. URBANIAK[3]

[1] Faculty of Technology, Kujawy and Pomorze University, 55-57 Toruńska St., 85-023 Bydgoszcz, Poland
[2] Faculty of Physics, Astronomy and Informatics, Nicolaus Copernicus University, 5 Grudziądzka St., 87-100 Toruń, Poland
[3] Faculty of Mathematics, Physics & Technical Sciences, Kazimierz Wielki University, 1 Kopernika St., 85-074 Bydgoszcz, Poland

Abstract. The paper presents the method of assessment of learning outcomes acquirement by students. The analysis is based on the results of the final matriculation exam in mathematics. For crisp and both types of fuzzy relations, cut scores (passing scores) can be defined along with the method of preparing rankings of students. The advantage of applying type 2 fuzzy relations is the lack of the necessity for experts to agree to one level (one number) of verification of learning outcomes by items created for the examination. Based on the results of the exam and experts' knowledge, the decision support system for calculating the levels of learning outcomes acquirement, making decisions about passing the examination and preparing rankings of students, can be developed. Additionally, the rank reversal phenomenon does not burden the proposed method.

Key words: mathematical method, fuzzy relations, education, learning outcome, ranking of students.

1. Introduction

In 1999, the Ministers responsible for Education from 29 European countries signed the Bologna declaration and started the Bologna Process, whose main aim was to create the European Higher Education Area [1] and "to ensure more comparable, compatible and coherent systems of higher education in Europe" [1]. Poland took part in the Bologna Process from the very beginning, and nowadays designers of new curricula define learning outcomes first and then describe the training processes needed for students to study them and for teachers to verify their acquirement by students.

For all designed courses, even in the higher level curricula, learning outcomes and processes of studying them are prepared, but there is a problem with assessing their acquirement. There are already some proposals on how to solve this problem [2–6]. For instance, Mreła and Sokolov [7] proposed a method, based on type 1 fuzzy relations, to calculate levels of learning outcomes acquirement, on the example of the "geodesy and cartography" curriculum.

Some of the competences are connected with the studied knowledge and skills, and teachers know how to assess their acquirement. However, the so-called social ones are much more difficult to assess, and teachers have been struggling with evaluating their acquirement. Mitra and Das [3] designed a system where they divided competences into technical and non-technical ones and proposed the methods for evaluating these two types of competences separately and, then explained how to combined the results, and set a grade for students.

Unfortunately, the proposed methods do not satisfy all educators and scientists. For example, Coates [9] states that the means of assessment are not good enough, even on the level of higher education, and that "the quality and productivity of higher education would be improved by reforming almost every facet of assessment". Moreover, he underlines that the situation is difficult to improve because even most academic teachers learn how to assess students achievement by "an informal apprenticeship" and it is important to create "more systematic forms of professional development".

In Poland, the designers of the secondary school curriculum for mathematics have defined five learning outcomes that students should acquire. Teachers verify the acquirement of these learning outcomes during the three years of study, but the final secondary school examination is the most important stage of verification. Written examination in mathematics is compulsory, and tests consist of 34 items, where the first 25 out of them are multiple choice, and 9 are open. For answering the multiple choice items correctly, students earn 1 mark, and for solving the open ones, they can be awarded 2 to 5 points. In this paper, we will present the method of analyzing the verification of learning outcome acquirement taking account of the results of the first part of the test and the first four learning outcomes defined in the Regulation of the Minister of National Education [10]:

- LO1 – The student interprets mathematical texts. After solving the task, the student commentates on the achieved result.
- LO2 – The student uses simple, well-known mathematical objects.
- LO3 – The student chooses a mathematical object for the simple situation and estimates the pertinence of the model critically.
- LO4 – The student applies a strategy that is clear from the content of the task.

To find levels of learning outcome acquirement by students, we will use the theory of fuzzy sets which was introduced by

*e-mail: a.mrela@kpsw.edu.pl

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Zadeh [11] in 1965. Then, in 1975, he generalized the concept of fuzzy sets, which are now called type 1 fuzzy sets, and introduced type 2 fuzzy sets [17].

Recently, the Central Examination Board [13] prepared items which are sent to schools, and the students will solve them during the written examination. Then the examination papers are sent to examination boards where they are marked, and the percentages of marks for all students are calculated. The passing percentage (the cut score, as it was called by Anghoff [14]) is set by the Ministry of National Education on the level of 30% [5]. Thus students who have failed to answer, for instance, 3 items, might pass the examination even if they failed to answer all items verifying one of the learning outcomes.

Since the test in mathematics consists of 25 items and these items verified acquirement of 4 learning outcomes, and the examination was taken by 149 students, there is a large amount of data for analysis. The experts of the Central Examination Board have to analyze the results of all secondary schools in Poland and then prepare reports (for example, [13]) and certificates in six weeks. Because of that, it is very important to prepare an expert system to help experts-examiners decide whether students acquire all required learning outcomes (cut scores can be different for different learning outcomes), and then decide whether the students pass the examination and finally prepare rankings of students. The rankings of students may prove very useful for university recruitment offices.

We have been studying the application of fuzzy relations, especially type 1, to build models of expert systems to solve problems with verification of learning outcomes' acquirement in the case of high school students and university students and graduates. During our studies, we have come to the conclusion that building fuzzy relations (two input and one output) is a sound concept for solving this educational problem.

Therefore, in this article, we are going to develop the principles for applying type 1 and type 2 fuzzy relations to calculate levels of learning outcome acquirement by students, then compare them with the defined cut scores and finally methods for preparing rankings of students. When calculating levels of learning outcomes acquirement with crisp or type 1 fuzzy relations, we obtain values and then it becomes easier to find cut scores and prepare the ranking. When type 2 fuzzy relations are used, there is some proposal on how to calculate one number [15]. The foundations of the methods of preparing rankings of objects which use fuzzy relations are discussed in [16–17].

This paper presents the method for calculating levels of learning outcomes acquirement based on levels of learning outcome verification by items prepared by experts and results of the examination. Moreover, the methods for calculating cut scores and rankings are prepared. The level of interpretability of our methods is high because there are a few steps to understand. At the beginning, the experts prepare two input relations: one based on their subjective knowledge and the other based on the test's results. Then, output relations are developed with the application of S-T-compositions. Finally, the rankings of students are prepared and comparing their results with the passing scores, the decisions of passing individual subjects are made.

2. Crisp relations

Let us consider two crisp relations. The first relation, R_1, between learning outcomes and items, is built based on the information issued by the Central Examination Board [13]. The value $R_1(LO_j, I_k)$ is equal to the level of learning outcome LO_j verification by item I_k, where $j = 1, 2, 3, 4$ and $k = 1, 2, \ldots, 25$ Table 1 presents selected values of relation R_1.

Table 1
Values of crisp relation between learning outcomes and items

Learning outcome	Item						
	I_1	I_2	I_3	I_4	I_5	…	I_{25}
LO_1	0	0	0	0	0	…	0
LO_2	1	1	0	1	1	…	1
LO_3	0	0	1	0	0	…	0
LO_4	0	0	0	0	0	…	0

The second relation, R_2, between items and students is built based on the results of the final secondary school examination in mathematics. The value $R_2(I_k, S_i)$ is equal to 1 if student S_i answered item I_k correctly and is equal to 0 otherwise, where $i = 1, 2, \ldots, 149$ and $k = 1, 2, \ldots, 25$.

To calculate the values of relation , between learning outcomes and students, S-T-composition is used.

Let us recall the definition of S-T composition (see [18]). Let $R = \{(x, y), \mu_R(x, y)\} \subset X \times Y$ and $P = \{(x, y), \mu_P(x, y)\} \subset Y \times Z$ be two relations with their membership functions μ_R and μ_P respectively. Evidently,
in the case of crisp relations, the membership function is the characteristic function of the given set. The S-T composition of relations R and P is a relation $R \circ P \subset X \times Z$ with the membership function defined in the following manner:

$$\mu_{R \circ P}(x, z) = S_{y \in Y}\big(T\big(\mu_R(x, y), \mu_P(y, z)\big)\big). \tag{1}$$

It is shown in [19] that for educational purposes the algebraic T-norm and S-norm are better than the more popular T-norm minimum and S-norm maximum, so we will use the S-T composition with algebraic T-norm and S-norm.

Thus R_3 is a relation which is the S-T composition of relations R_1 and R_2 so $R_3(LO_j, S_i)$ denotes the level of learning outcome LO_j acquirement by student S_i. Table 2 presents some values of relation R_3.

Table 2
Values of crisp relation between learning outcomes and students

Learning outcome	Student						
	S_1	S_2	S_3	S_4	S_5	…	S_{149}
LO_1	1	1	1	1	1	…	1
LO_2	1	1	1	1	1	…	1
LO_3	1	0	1	1	1	…	0
LO_4	1	1	1	1	1	…	1

Since the values of relations R_1 and R_2 are equal to 0 or 1, the values of S-T composition R_3 are also equal to 0 or 1. Student S_2 for example, has acquired all learning outcomes with the exception of LO_3. Because of that, s/he should not pass the examination.

Moreover, the cut score (the passing percentage) can be any number between 0 and 1, and it is difficult to set the ranking of students because many of them have acquired all learning outcomes at level 1 and there is no possibility to distinguish between them.

Nowadays, to prepare the ranking of students, the sum or average can be calculated. After calculating the values of relation R_3 the ranking of students can be prepared. For example, let us consider all students who earned the total number of points equal to 14 points $(S_{26}, S_{36}, S_{42}, S_{49}, S_{78}, S_{91}, S_{96}, S_{112}, S_{117}, S_{129}, S_{149})$. When applying the average as the basis for the ranking all these students hold the same position. However, after calculating the values of relation R_3, the experts can use the lexicographical order or the weighted average. If they assume, for example, that learning outcome LO_1 is the most important one, and then LO_2, next LO_3, and the least important is LO_4, then the experts can rank these 14 students as follows:

1. $S_{26}, S_{49}, S_{78}, S_{112}, S_{117}, S_{129}, S_{149}$,
2. S_{36},
3. S_{42}, S_{91}, S_{117}.

Assume now that the weighted average is applied. If the experts decide, for example, that LO_4 is the most important learning outcome (40%), then LO_3 (30%), next LO_2 (20%) and the least important learning outcome is LO_1 (10%), thus they arrive at the following ranking:

1. $S_{26}, S_{49}, S_{78}, S_{112}, S_{129}, S_{149}$,
2. S_{42}, S_{91}, S_{117},
3. S_{36}, S_{96}.

Thus before preparing the ranking of students, the experts must decide on the importance of learning outcomes. The ranking based on the importance of learning outcomes can be used, for example, by recruitment offices at universities.

3. Type 1 fuzzy relations

Let X be the space. According to Zadeh (1965), the fuzzy set $A \subset X$ is a set of pairs $A = \{(x, \mu_A(x)), x \in X\}$, where $\mu_A\colon X \to [0, 1]$ is a membership function. Value $\mu_A(x)$ describes the level of membership of element x in set A.

The designers of the written examination have prepared 3 items to verify the acquirement of learning outcome LO_1, 17 items for LO_2, 3 items for LO_3 and 2 items for LO_4. If for at least one item defined to verify the acquirement of learning outcome LO_j student S_i answered correctly, then the level of acquirement of this learning outcome by this student is equal to 1. Thus the probability of acquirement of learning outcome LO_2 is much higher than that of LO_4. Hence, instead of calculating S-T composition with values 0, 1 (Table 2), shares of each item defined for verification of the learning outcome by individual items are used to calculate (with the use of formula (1)) the new fuzzy relation R_3 whose values are presented in Table 3.

Table 3
Values of type 1 fuzzy relation between learning outcomes and students

Learning outcome	Student						
	S_1	S_2	S_3	S_4	S_5	…	S_{149}
LO_1	0.70	0.56	0.70	0.70	0.56	…	0.70
LO_2	0.62	0.42	0.60	0.57	0.52	…	0.57
LO_3	0.56	0.00	0.70	0.70	0.56	…	0.00
LO_4	0.75	0.50	0.75	0.75	0.50	…	0.75

If the cut score is equal to, for instance, 0.5, then students S_2 and S_{149} will not pass the examination because they have not acquired learning outcomes LO_2 and LO_3, respectively.

To prepare the ranking of students who earned 14 points, the lexicographic order $(S_8, S_{29}, S_{30}, S_{65}, S_{84}$ and so on$)$ can be used. Moreover, the examiners can use the weighted average to prepare the ranking of students, and they can decide that learning outcome LO_1 is the most important one and set the weight equal to 0.4 for it, followed by: LO_2 – 0.3, LO_3 – 0.2 and LO_1 – 0.1. Table 4 presents the weighted averages.

Table 4
Values of weighted averages of levels of learning outcomes acquirement

Student	S_1	S_2	S_3	S_4	S_5	…	S_{149}
Weighted average	0.65	0.40	0.68	0.67	0.54	…	0.17

Using the weighted average, the ranking of students $(S_8, S_{29}, S_{30}, S_{65}, S_{84}$ and so on$)$ can now be compiled.

Assume now that the designers of fuzzy relation R_1, between learning outcomes and items, do not need to use values 0 or 1, and they can decide that the level of verifying learning outcomes by items can belong to the interval of [0, 1]. Assume that the experts described the values of this relation, which are presented in Table 5.

Table 5
Values of the fuzzy relation between learning outcomes and items

Learning outcome	Student						
	I_1	I_2	I_3	I_4	I_5	…	I_{25}
LO_1	0.1	0.3	0.1	0.3	0.2	…	0.0
LO_2	0.9	0.8	0.3	0.9	1,0	…	1.0
LO_3	0.3	0.2	0.7	0.2	0.3	…	0.3
LO_4	0.2	0.2	0.2	0.3	0.2	…	0.2

After calculating S-T composition (1) of relations R_1 and R_2, the values of fuzzy relation R_3 are presented in Table 6.

Similarly to before, let the cut score for the acquirement of each learning outcome be defined on the level of e.g. 0.5, so the experts can decide whether students have acquired these learning outcomes on the required levels and then applying,

Table 6
Values of type 1 fuzzy relation between learning outcomes and students

Learning outcome	Student						
	S_1	S_2	S_3	S_4	S_5	…	S_{149}
LO_1	0.99	0.75	0.99	0.99	0.83	…	0.93
LO_2	0.96	0.66	0.98	0.96	0.81	…	0.89
LO_3	0.92	0.40	1.00	0.99	0.86	…	0.60
LO_4	0.98	0.72	1.00	1.00	0.75	…	0.95

for example, the weighted average can decide about passing the examination by students. Furthermore, as in the previous section, the experts can prepare the rankings of students applying the lexicographic order or weighted average.

4. Type 2 fuzzy relations

Sometimes, instead of using type 1 fuzzy relations, it is more convenient to apply type 2 fuzzy relations. Zadeh [7] proposed interval-valued fuzzy sets, whose membership functions are interval-valued. They are easier to work with than type 2 fuzzy sets [15], and they are generalizations of fuzzy sets (type 1). These sets were also applied in preparing assessment of students' achievements. For example, Hameed [21] presented the simplified application of interval type-2 fuzzy system, to reduce uncertainties and try to develop a fairer and more transparent system.

Assume now that four experts $E_1 - E_4$describe the values of the relation between learning outcomes and items separately. In previous sections, the experts had to agree on a single value, but now it is not necessary. Let Table 7 present the levels of verification of learning outcomes by items estimated by these experts.

Table 7
Values of fuzzy relation between two learning outcomes and the items described by 4 experts

Item	LO_1				LO_2			
	E_1	E_2	E_3	E_4	E_1	E_2	E_3	E_4
I_1	0.9	0.7	0.8	0.9	0.9	0.8	0.7	1.0
I_2	0.9	0.8	0.4	0.2	0.7	0.8	1.0	0.9
I_3	0.8	0.4	0.2	0.3	0.2	0.1	0.3	0.4
I_4	0.4	0.3	0.4	0.3	0.8	1.0	1.0	0.6
I_5	0.4	0.8	0.9	0.3	1.0	1.0	1.0	0.9
I_6	0.6	0.7	0.8	0.9	0.2	0.3	0.4	0.3
I_7	0.9	0.7	0.8	0.9	0.9	0.8	0.7	1.0

In this situation, we calculate the averages and standard deviations of levels of learning outcomes' verification, and use type 2 fuzzy relations [7]. According to Rutkowski [18], the type 2 fuzzy set $\tilde{A}$, defined on the intercourse space X, is the set of pairs $\{x, \mu_{\tilde{A}}(x)\}$ where $x \in X$ and its level of membership in fuzzy set $\tilde{A}$ is the type 1 fuzzy set, defined on interval $J_x \subset [0, 1]$ so $\mu_{\tilde{A}}(x) = \int_{u \in J_x} f(u)/u$ Function f_x: $[0, 1] \rightarrow [0, 1]$ is called the secondary membership function and its values $f_x(u)$ are levels of secondary membership. Interval J_x is called the basic membership of x.

Let $A = \{-0.6 + s \cdot 0.4, \text{ where } s = 0, 1, \ldots, 56\}$ be the basic membership set for all the secondary membership functions discussed below.

Let $m_{j,k}$ and $s_{j,k}$ denote the average and standard deviation of values set by the experts for learning outcome LO_j and item I_k, where $j = 1, 2, 3, 4$ and $k = 1, 2, \ldots, 25$. Let each secondary membership function of the relation between learning outcome LO_j and item I_k be defined as a Gaussian function $\mu_1(x, LO_j, I_k) = exp\left(-(x - m_{j,k})^2/s_{j,k}\right)$ for each j, k and $x \in A$. For example, type 2 fuzzy relation R_1 between learning outcome LO_1 and items $I_1 - I_4$ is presented in Fig. 1.

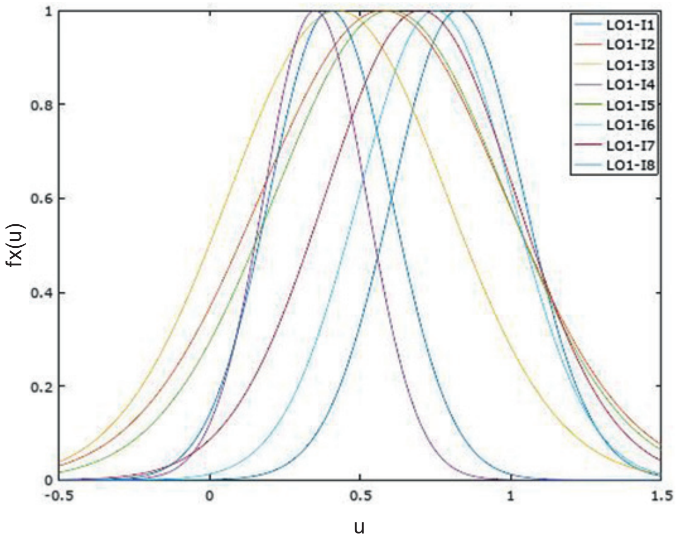

Fig. 1. Type 2 fuzzy relation between learning outcome and 8 items

During the examination, each student earned 1 point if they answered correctly and 0 points if incorrectly. If student S_i answered item I_k correctly, then the secondary membership function of type 2 fuzzy relation between items and students be the Gaussian functions defined in the following way: $\mu_2(x, I_k, S_i) = exp\left(-(x - 1)^2/0.1\right)$ for each $x \in A$. If student S_i answered item I_k incorrectly, then the secondary membership function is equal to for each $x \in A$. We do not fuzzify this value because if the student answered the item incorrectly, there is no increase of the levels of acquirement of all learning outcomes.

Now let the S-T composition between relations R_1 and R_2 be defined as follows. If $x \in A$, then for each $j = 1, 2, 3, 4$ and $i = 1, 2, \ldots, 149$,

$$\mu_3(x, LO_j, S_i) = 1 - \left(1 - \mu_1(x, LO_j, I_1) \cdot \mu_1(x, I_1, S_i)\right) \cdot \\ \cdot \ldots \cdot \left(1 - \mu_1(x, LO_j, I_{25}) \cdot \mu_1(x, I_{25}, S_i)\right).$$

After applying the S-T composition, the secondary membership functions for learning outcome LO_1 and first 9 students $(S_1 - S_9)$ are presented in Fig. 2.

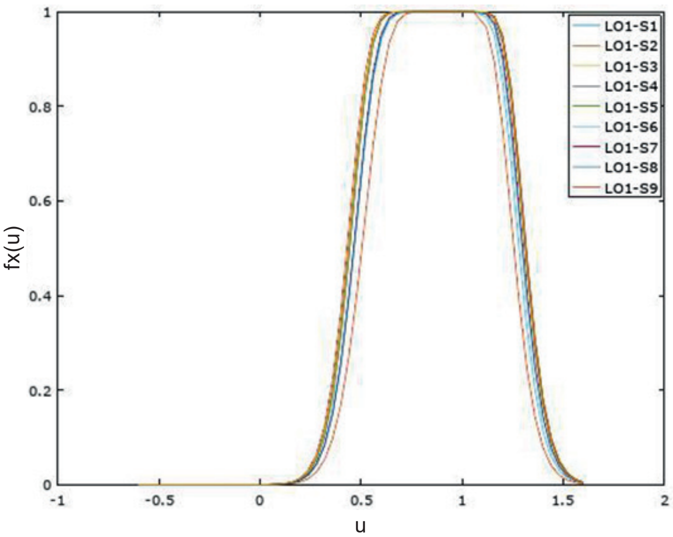

Fig. 2 Secondary membership functions of the relation between learning outcome LO_1 and students $S_1 - S_9$

For each student S_i, we can build the type 2 fuzzy relation whose secondary membership functions show the relation between this student and the learning outcomes. Type 2 fuzzy relations between students for students S_1, S_2, S_5 and S_{149} are presented in Fig. 3.

Now we define the level of the acquirement of learning outcome LO_j by student S_i in the following manner. Student S_i acquires learning outcome LO_j at level a if $\mu_3(a, LO_j, S_i) = max_{x \in A}\,\mu_3(x, LO_j, S_i)$ and in the case when there is more than one such value a, then the level of the given learning outcome is equal to the smallest one. The levels of acquirement of learning outcomes $LO_1 - LO_4$ by a number of students are presented in Table 8.

As previously, the cut score should be defined by experts. If it is equal to, for example, 0.5, then students who would acquire this learning outcome on a level lower than 0.5 would not pass the test. Moreover, based on levels of learning outcomes acquirement, the ranking can be prepared with the use of the lexicographical order or the weighted average.

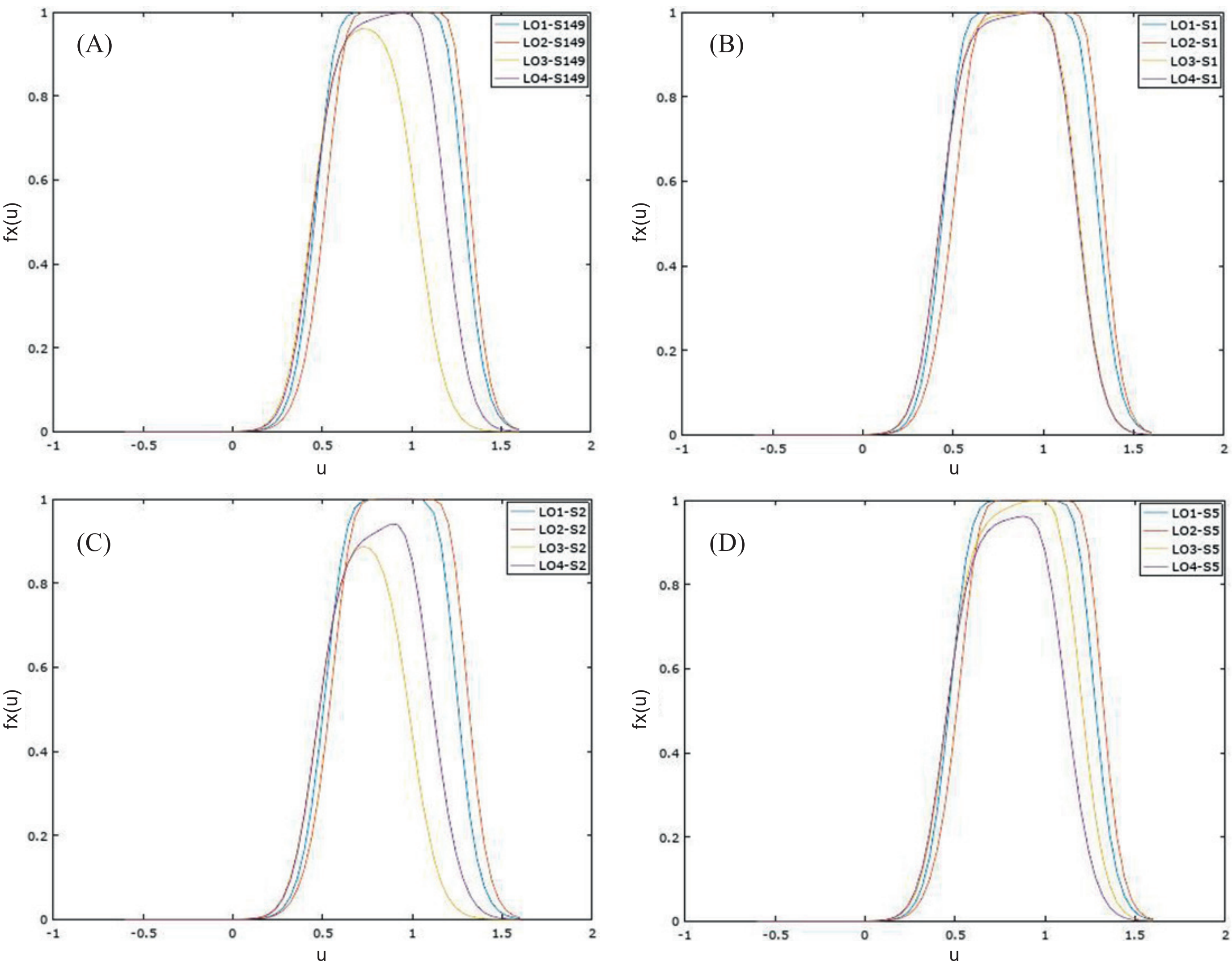

Fig. 3. A) Type 2 fuzzy relation for student S_1, B) Type 2 fuzzy relation for student S_2, C) Type 2 fuzzy relation for student S_5, D) Type 2 fuzzy relation for student S_{149}

Table 8
Levels of acquirement of learning outcomes by students

Learning outcome	Student						
	S_1	S_2	S_3	S_4	S_5	…	S_{149}
LO_1	0.92	0.92	0.92	0.92	0.92	…	0.92
LO_2	0.92	0.96	0.96	0.96	0.96	…	0.96
LO_3	0.92	0.72	0.96	0.96	0.96	…	0.72
LO_4	0.96	0.88	0.96	0.96	0.88	…	0.96

Let us compare the results of students S_1, S_2, S_5, and S_{149} (see Fig. 3). It can be noticed that some secondary membership functions are "slim" and some of them are "wide". When the secondary membership function is "slim", the teachers can be more certain that the level of acquirement of the given learning outcome is right and if the function is "wide", the teachers are less certain. Thus we assume that in the first case the likelihood of acquirement of this learning outcome is higher than in the second case. For calculating the likelihood of describing the level of acquirement of learning outcome LO_j by student S_i, the range will be defined as follows:

$$range(LO_j, S_i) = A(a_1) - A(a_2),$$

where $a_1 = min_{x \in A}\, \mu_3(x, LO_j, S_i) > 0.5$ and $a_2 = max\, \mu_3(x, LO_j, S_i) > 0.5$ Levels of acquirement of learning outcomes with the ranges of these learning outcomes acquirement by some students are presented in Table 9.

Table 9
Ranges of levels of learning outcomes acquirement by students

Learning outcome	Student						
	S_1		S_2		…	S_{149}	
	value	range	value	range	…	value	range
LO_1	0.9	0.7	0.8	0.9	…	0.92	0.8
LO_2	0.9	0.8	0.4	0.2	…	0.96	0.8
LO_3	0.8	0.4	0.2	0.3	…	0.72	0.56
LO_4	0.4	0.3	0.4	0.3	…	0.96	1.12

Thus, since in the case of student S_1 the level of acquirement of learning outcome LO_1 is equal to 0.92 and the range of its acquirement is 0.84, the likelihood of this learning outcome acquirement is higher than in the case of learning outcome LO_4, where the range is equal to 1.16. If we assume that the learning outcome LO_1 is the most important one, then LO_2, followed by LO_3 and LO_4 is the least important learning outcome, and the ranking of students can be prepared by the lexicographical order.

5. Discussion of methods

Nowadays, educators and teachers use information technologies and multimedia to help students study more and more frequently. Educational systems, the dynamics ones in particular, allow to assess and self-assess every part of a teaching material and to prepare individual paths for every single student. Many new methods and algorithms are developed as foundations of systems designed to monitor the process of learning and to help assess students' achievements. Barón at al. [22] present a learning assessment system based on fuzzy cognitive maps to facilitate the evaluation of the learning process in interactive environments.

Hameed and Sorensen [23] have compared two three-nodes fuzzy evaluation systems designed for evaluating students' achievements and preparing the ranking. As a result of the comparison, they showed, in the example, that the system based on Gaussian membership functions is more robust than the one based on triangular membership functions. These systems took account of the difficulty, importance and complexity of items.

Fuzzy relations type 1 and type 2 can be used to find out about the levels of learning outcome acquirement by students. The teachers-experts have to set the values of the relation between learning outcomes and items whose values show the level of verification of the given learning outcome by the specific item. Moreover, the teachers have to decide whether values of the second relation, namely the relation between students and items, should be transformed or whether it can be filled in with the simple results of the examination.

During the process of setting values of the relation between learning outcomes and items, the experts can be asked, like in this paper, about the value of the relation (the number). However, the experts can use linguistic terms describing the level of verification of the given learning outcome by the given item, such as e.g. "very high", "medium" or "low" [18]. The application of linguistic terms brings the experts closer to natural language and generates the necessity of deeper understanding of the ideas underlying the models (…) for the analysis of natural-language documents' [24].It is necessary to study the different methods of measuring similarities of data given in the form of natural-language documents [25].

6. Conclusions

The application of type 1 and type 2 fuzzy relations lets teachers define one cut score for all learning outcomes or different cut scores for different learning outcomes, or a cut score for the weighted average. They can then decide whether students have acquired the learning outcomes on the required levels and finally whether they have passed the examination.

The application of type 2 fuzzy relations does not force experts to agree on a single level of learning outcomes verification by items, and it gives the experts information on the likelihood of acquiring them.

Using these results, an expert system for calculating levels of learning outcomes acquirement, making decisions on passing the examination and preparing rankings can be developed.

We have also tested the common problem of many analytic hierarchy processes [26, 27], namely the rank reversal phenomenon. Since all levels of learning outcomes' acquirement are calculated for each learning outcome and student individually, the problem of preparing rankings of students is reduced to setting ranks of real numbers, so that adding or subtracting a few students does not cause the rank reversal phenomenon. Moreover, we are going to conduct more research on preparing rankings of students when all or parts of learning outcomes are equally important, so some aggregation functions of the levels of LOs' acquirement are to be defined.

Furthermore, the presented method can be used to develop expert systems for calculating the levels of some social and vocational traits and skills required to apply for certain professional positions or jobs. This can be done based on the knowledge of experts and some tests. After the calculation of levels of the traits and skills discussed, statistical analysis can be carried out, and decisions about recruitment can be made.

References

[1] EHEA, European Higher Education Area and Bologna Process, www.ehea.info, (access date: 12.07.2016).

[2] A.Varghese, J. P. Sreedhar, S. Kolamban, and S. Nayaki, "Outcome based Assessment using Fuzzy Logic", International Journal of Advanced Computer Science and Applications, Vol. 8, No. 1, 2017.

[3] M. Lambovska, "A Fuzzy Model for Team Control and its Application", *Management and Production Engineering Review* 9 (3), (2018), 108–119 DOI: 10.24425/119540.

[4] D.H. Rao, S.R. Mangalwede, and V. Deshmukh. "Student performance evaluation model based on scoring rubric tool for network analysis subject using fuzzy logic", 1–5, 2017. 10.1109/ICEECCOT.2017.8284623.

[5] R. Ilahi, I. Widiaty, and A.G. Abdullah, "Fuzzy system application in education", IOP Conference Series: Materials Science and Engineering. 2018, 434. 012308. 10.1088/1757–899X/434/1/012308.

[6] M.G. Voskoglou and A. Papadopoulou, "A Fuzzy Logic Evaluation of Beliefs and Attitudes of Greek Secondary Teachers about Teaching the Euclidian GeometryGeometry", *American Journal of Computational and Applied Mathematics*, 2017; 7(3): 80–85, doi:10.5923/j.ajcam.20170703.03.

[7] A. Mreła and O. Sokolov, "Compositions of fuzzy relations applied to verification learning outcomes on the example of the major Geodesy and Cartography", *Journal of Education, Health and Sport*, 5(5), 2015.

[8] M. Mitra and A. Das, "A Fuzzy Logic Approach to Assess Web Learner's Joint Skills", IJMECS, vol.7, no.9, pp.14–21, 2015. DOI: 10.5815/ijmecs.2015.09.02

[9] H. Coates, "Assessment of Learning Outcomes". In: A. Curaj, I. Matei, T. Pricopie, J. Salmi, P. Scott (eds) *The European Higher Education Area*. Springer, Cham, 2015, DOI https://doi.org/10.1007/978–3-319–20877–0_26.

[10] Regulation of the Minister of National Education of 25 April 2013 amending the regulation on the conditions and manner of assessing, classifying and promoting pupils and students as well as conducting tests and public examinations, 2013, (access date: 14.07.2016), [in Polish].

[11] L.A. Zadeh, "Fuzzy sets", *Information and Control* 8, pp. 338–353, 1965.

[12] L. A. Zadeh, "The Concept of a Linguistic Variable and Its Application to Approximate Reasoning–1", *Information Sciences*, vol. 8, pp. 199–249, 1975.

[13] Rules for assessing task solutions, matriculation examination in 2014/2015, Central Examination Board, 2015, www.cke.edu.pl, (access date: 12.07.2016), [in Polish].

[14] W.H. Angoff, "Norms, and equivalent scores", in R.L. Thorndike (ed.), "Educational Measurement", *American Council on Education*, Washington D.C., pp.514–515, 1971.

[15] W.T. Dobrosielski, J. Szczepański, and H. Zarzycki, "A Proposal for a Method of Defuzzification based on the Golden Ratio – GR", conference IWIFSGN, Kraków 2015, ISBN 978–3-319–26211–6, ISSN 2194–5357, Novel Developments in Uncertainty Representation and Processing, pp 75–84, Springer International Publishing, 2016.

[16] W. Duch, T. Winiarski, J. Biesiada, and A. Kachel, "Feature Ranking, Selection, and Discretization", *Proc. Joint Int. Conf. on Artificial Neural Networks (ICANN) and Int. Conf. on Neural Information* Processing (ICONIP), Istanbul, pp. 251–254, 2003.

[17] W. Duch, T. Wieczorek, J. Biesiada, and M. Blachnik, "Comparison of feature ranking methods based on information entropy", *Proc. of Int. Joint Conf. on Neural Networks* (IJCNN), Budapest 2004, IEEE Press, pp. 1415–1420, 2004.

[18] L. Rutkowski, "Metody i techniki sztucznej inteligencji", PWN, Warszawa 2009, [in Polish].

[19] A. Mreła, O. Sokołov and T. Katafiasz, "Types of fuzzy relations' composition applied to validation of learning outcomes at mathematics during final high school examination", in: A. Mreła and P. Wilkoszewski (ed.), "Nauka i technika u progu III tysiąclecia", Wydawnictwo Kujawsko-Pomorskiej Szkoły Wyższej w Bydgoszczy, Bydgoszcz, pp.119–132, 2015.

[20] C. Cornelis, G. Deschrijver, and E.E. Kerre, "Advances and challenges in interval-valued fuzzy logic", *Fuzzy Sets and Systems*, 157, 2006, pp. 622–627.

[21] I.A. Hameed, A Simplified Implementation of Interval Type-2 Fuzzy System and its Application in Students' Academic Evaluation, 2016 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), 2016, DOI: 10.1109/FUZZ-IEEE.2016.7737748.

[22] H.B. Barón, R.G. Crespo, J.P. Espada, and O.S. Martínez, "Assessment of learning in environments interactive through fuzzy cognitive maps", *Methodologies and Applications*, Soft Comput 19:1037–1050, 2015, DOI 10.1007/s00500–014–1313-x.

[23] I.A. Hameed and C.G. Sorensen, Fuzzy Systems in Education: A More Reliable System for Student Evaluation, in: A.T. Azar (ed.), *Fuzzy Systems*, IntechOpen, Rijeka 2010, DOI 10.5772/7216, https://doi.org/10.5772/7216.

[24] A. Bolshoy, Z. Volkovich, V. Kirzhner, and Z. Barzily, "Mathematical Models for the Analysis of Natural-Language Documents", in: "Genome Clustering". *Studies in Computational Intelligence*, vol 286. Springer, Berlin, Heidelberg, 2010.

[25] Z. Volkovich, O. Granichinc, O. Redkinc, and O. Bernikovac, "Modeling and visualization of media in Arabic", *Journal of Informetrics*, Elsevier, Vol. 10, Issus 2, May 2016, pp. 439–453, 2016.

[26] A. Darko, A.P.C. Chan, E.E. Ameyaw, E.K. Owusu, E. Pärn, and D. J. Edwards, "Review of application of analytic hierarchy process (AHP) in construction", *International Journal of Construction Management*, 2018, DOI: 10.1080/15623599.2018.1452098.

[27] M.S.D. Putra, S. Andyana, Fauziah, and A. Gunaryati, "Fuzzy Analytical Hierarchy Process Method to Determine the Quality of Gemstones", *Advances in Fuzzy Systems*, Vol. 2018, Article ID 9094380, https://doi.org/10.1155/2018/9094380.