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The Bayesian Methods of Jump Detection: The Example of Gas and EUA Contract Prices

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Abstract

In the paper we present and apply a Bayesian jump-diffusion model and stochastic volatility models with jumps. The problem of how to classify an observation as a result of a jump is addressed, under the Bayesian approach, by introducing latent variables. The empirical study is focused on the time series of gas forward contract prices and EUA futures prices. We analyse the frequency of jumps and relate the moments in which jumps occur to calendar effects or political and economic events and decisions. The calendar effects explain many jumps in gas contract prices. The single jump is identified in the EUA futures prices under the SV-type models. The jump is detected on the day the European Parliament voted against the European Commission's proposal of backloading. The Bayesian results are compared with the outcomes of selected non-Bayesian techniques used for detecting jumps.

Keywords: jump, jump-diffusion model, stochastic volatility, double exponential distribution, commodity markets

JEL Classification: C51, C53, C58, Q41

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1 Introduction

The prices of gas and CO_2 emission allowances depend on political events and decisions as well as the economic situation and other country-specific factors. Modelling and forecasting the prices pose a considerable challenge, which largely results from their time-variable volatility and their sharp changes. The latter are usually referred to as jumps. The very term jump is in common use, although it has not earned a single, widely accepted definition as yet.

The jump-diffusion models are a generalisation of the (pure) diffusion models created by adding a jump component to a 'continuous' part of a stochastic structure. The jump component is used to model sharp movements of time series. The jump-diffusion models are often employed to model prices from financial and commodity markets (e.g. Merton 1976, Kou 2002, Kou and Wang 2004, Weron 2006, Weron *et al.* 2004, Kostrzewski 2014a).

The stochastic volatility processes are frequently applied to model financial time series (e.g. Jacquier *et al.* 1994, Pajor 2003, Jacquier *et al.* 2004, Yu 2005, Omori *et al.* 2007). Extending basic SV structures to the ones with the jump component may handle occasional sharp movements which pop up in prices and the differences in log-prices (returns). The Bayesian SV models with jumps are employed to analyse time series from stock and commodity markets (Chib *et al.* 2002, Li *et al.* 2008, Szerszen 2009, Johannes and Polson 2010, Brooks and Prokopczuk 2013, Kostrzewski and Kostrzewska 2019).

The aim of the paper is to present and apply selected techniques of detecting jumps, to analyse the frequency of jumps, and to relate the times in which jumps occur to calendar effects or political and economic events and decisions. In the study, the time series are modelled by means of three Bayesian models: the Bernoulli jump-diffusion model (the DEJD model) and two discrete-time stochastic volatility structures (SV) with jumps (the SVDEJ and SVNJ models). We choose the best Bayesian specification, i.e. the model with the highest explanatory power.

The models considered in the study have their origin in continuous-time processes, in which 'small' movements of a time series are accounted for by the diffusion or stochastic volatility component, whereas sharp and large values of a time series – by the jump component. In the series of prices or returns we can easily spot 'large' values (in absolute value terms). However, we do not know whether they result from the jump and/or the diffusion component or the stochastic volatility one, since pure jumps are not observed. These issues directly transfer to the discrete-time models which are considered in the paper. In the study we allow for no or a single jump per time interval. More than one jump per time interval is considered by Kostrzewski (2014a). Jumps and stochastic volatility are unobservable. The way of dealing with unobservable quantities consists of introducing them into the model as latent variables. This approach is also adopted in our study, enabling a precise formulation of a jump. It facilitates formal statistical inference about jumps and allows for the detection of jumps and the analysis of the frequency of their occurrences. However, the method leads to a large number of unknown quantities in a model i.e. the number of parameters of the mathematical model as well as latent variables describing jumps and stochastic volatility. In the study we apply the Bayesian statistical framework, which is widely recognized as capable of managing model specifications with latent variables in a formal and internally consistent manner. In the paper, the Bayesian approach for the discrete-time jump-diffusion model and SV models with jumps are applied to analyse contract prices of gas and EUA (CO_2 emission allowances). Moreover, we apply three non-Bayesian techniques to detect jumps: the recursive filter on returns (RFR) technique and two variable return threshold techniques (VRT1 and VRT2) (see e.g. Janczura *et al.* 2013, Kostrzewska and Kostrzewski 2018).

In the paper, the first of the analysed time series comprises one-month gas contract prices (in EUR/MWh). This data set comes from the Dutch Title Transfer Facility (TTF) virtual trading hub. Energy retailers buy gas on volatile spot markets and resell it to consumers at constant prices. The former are at risk of a sharp increase. Predicting prices and periods with a high probability of jumps plays a crucial role in risk management. It might reduce the costs of enterprises which rely on natural gas for their production processes. They might avoid purchasing contracts on the days with higher probability of a jump or hedge their position by buying or signing profitable contracts. On the other hand, identification of periods with a higher probability of steep rises and drops of prices offers an opportunity to speculate on the derivatives market.

The second of the analysed time series comprises the EUA contract prices (EUR/tCO_2). Analyses and forecasts of the prices are relevant to the assessment of the economic viability of investment in technologies used in reducing energy consumption and CO_2 emissions. The detection of jumps contribute to a better understanding of the nature of the prices and the mechanisms of jump formation. It may be useful in obtaining more precise forecasts and effective risk management methods.

In our study we concentrate on the Bayesian methods of jump detection, which are based on the jump-diffusion processes and stochastic volatility models with jumps. We assume the normal or double exponential distribution of jump sizes. The double exponential distribution is not so often considered in the literature as the normal one. However, this distribution allows for modelling negative and positive jumps separately. In the paper we compare the results computed under the formal Bayesian approach with the heuristic approaches. The heuristic techniques hinge on specific statistical principles, and they are often applied to detect jumps (e.g. Janczura *et al.* 2013).

The paper is organised as follows. Section 2 gives a theoretical overview of the Bayesian models used in the study. The models are next applied in Section 3, which presents the empirical analysis concerning gas forward prices and EUA futures contracts. The section discusses the results on jumps detection and their frequency, and compares the explanatory power of the considered models. Conclusions end the paper.

2 Models

We consider three Bayesian models with a discrete time scale. All of them have their counterparts with a continuous time scale. The first one is an example of the Bernoulli jump-diffusion model, and is called the jump-diffusion model. The other two specifications are stochastic volatility models with a leverage effect and jumps but with different distributions of jump values: the normal or double exponential distribution. In the empirical study, the models are fitted to the differences in log-prices.

2.1 The DEJD model

The Merton model (Merton 1976) is presumably the most famous jump-diffusion structure. It assumes the normal distribution of jump values. The Bayesian version of this model (in a discrete time scale) is considered by Kostrzewski (2014a). The double exponential jump-diffusion (DEJD) model in a continuous-time version is similar to the Merton's specification. The only difference resides in different distributions of jump values. The double exponential distribution is assumed in the DEJD model. Hence, negative and positive jumps follow exponential distributions with different values of their parameters. The main motivation behind the structure is the assumption of the asymmetry of values of negative and positive jumps and their frequency. The DEJD model defines the dynamics of an asset under the Kou model (Kou 2002, Kou and Wang 2004), which originates in pricing derivative securities. The specification is a particular case of the Pareto-Beta jump-diffusion model proposed by Ramezani and Zeng (1998), where two Poisson processes govern the arrival rate of 'bad' and 'good' news.
In the following study, the discrete version of the double exponential jump-diffusion model is considered and it is also called the DEJD model. The model is specified as:

$$y_{t+1} = y_t + \left(\mu - \frac{1}{2}\sigma^2\right)\Delta + \sigma\varepsilon_{t+1}\sqrt{\Delta} + J_{t+1}, \tag{1}$$

where $y_t = \ln(S_t)$ and S_t is a price at time t, $\{\varepsilon_t\} \sim iid\ N(0,1)$, $\Delta > 0$, and $\{J_t\}$ is a sequence of independent and identically distributed variables with the probability density:

$$f_{J_t}(x) = p_D \frac{1}{\eta_D}\exp\left(\frac{1}{\eta_D}x\right)\mathbb{I}_{(-\infty,0)}(x) + p_U\frac{1}{\eta_U}\exp\left(-\frac{1}{\eta_U}x\right)\mathbb{I}_{(0,\infty)}(x), \tag{2}$$

where $p_D \geq 0$, $p_U \geq 0$, $p_D + p_U = 1$, $\eta_D > 0$, $\eta_U > 0$. Parameter $\mu \in \mathbb{R}$ represents the drift of S, while σ is the volatility parameter. Parameters p_D and

p_U are interpreted as the probabilities of a negative and a positive jump, respectively, if a jump occurs. The probability of no jump is assumed to equal $\frac{1}{1+\lambda\Delta}$, where λ is the intensity of jumps. Moreover, $-\eta_D$ and η_U are means of negative and positive jump sizes, correspondingly. The specification is an example of the so-called Bernoulli jump-diffusion model. Additional properties of the DEJD structure are presented by Kostrzewski (2015).

2.2 The SVDEJ model

The jump-diffusion structures are criticized for the assumption of a constant value of the volatility parameter σ. In an attempt to address this disadvantage, the next model is formulated so as to incorporate, additionally, a stochastic volatility component. Therefore, in what follows, we consider the following (discrete-time) model with the double exponential distribution of jump values (SVDEJ):

$$\begin{aligned} y_{t+1} &= y_t + \mu + \sqrt{\exp(h_t)}\varepsilon_{t+1}^{(1)} + J_{t+1}, \\ h_{t+1} &= h_t + \kappa_h (\theta_h - h_t) + \sigma_h \left(\rho\varepsilon_{t+1}^{(1)} + \sqrt{1-\rho^2}\varepsilon_{t+1}^{(2)}\right), \end{aligned} \tag{3}$$

where $y_t = \ln(S_t)$, $\left\{\varepsilon_t^{(1)}\right\} \sim iid\ N(0,1)$, $\left\{\varepsilon_t^{(2)}\right\} \sim iid\ N(0,1)$, $\varepsilon^{(1)}$ and $\varepsilon^{(2)}$ are independent, and $\{J_t\}$ is a sequence of independent and identically distributed variables with the probability density:

$$\begin{aligned} f_{J_t}(x) = p_D \frac{1}{\eta_D} \exp\left(\frac{1}{\eta_D}x\right) \mathbb{I}_{(-\infty,0)}(x) + p_0 \delta_{(0)}(x) \\ + p_U \frac{1}{\eta_U} \exp\left(-\frac{1}{\eta_U}x\right) \mathbb{I}_{(0,\infty)}(x), \end{aligned} \tag{4}$$

where $\delta_{(0)}$ is the Kronecker delta, $p_D \geq 0$, $p_0 \geq 0$, $p_U \geq 0$, $p_D + p_0 + p_U = 1$. The parameters p_D, p_0, and p_U are interpreted as the probabilities of a negative jump, no jump and a positive jump occurrence, respectively.
An additional restriction of $0 < \kappa_h < 2$ is assumed in order to ensure the stationarity of the log-volatility process h_t. The value of $1 - \kappa_h$ for $\kappa_h \in (0, 1)$ is interpreted as a speed of mean reversion of the log-volatility process h_t towards its mean level θ_h, and σ_h is the volatility parameter of h_t. Parameter $\rho \in (-1, 1)$ is the correlation coefficient between log-price shocks $\varepsilon_t^{(1)}$ and log-volatility shocks $\rho\varepsilon_t^{(1)} + \sqrt{1-\rho^2}\varepsilon_t^{(2)}$. If $\rho < 0$, the leverage effect exists. If $\rho > 0$, the inverse leverage effect exists. The specification incorporates the stochastic volatility structure and jumps.

2.3 The SVNJ model

The stochastic volatility model with normal jumps (SVNJ) is similar to the previous one. The only difference is the choice of the normal distribution of jumps $N\left(\mu_J, \sigma_J^2\right)$, instead of the double exponential one. The model is defined as:

$$\begin{aligned} y_{t+1} &= y_t + \mu + \sqrt{\exp\left(h_t\right)}\varepsilon_{t+1}^{(1)} + J_{t+1}, \\ h_{t+1} &= h_t + \kappa_h\left(\theta_h - h_t\right) + \sigma_h\left(\rho\varepsilon_{t+1}^{(1)} + \sqrt{1-\rho^2}\varepsilon_{t+1}^{(2)}\right), \end{aligned} \tag{5}$$

where $y_t = \ln\left(S_t\right)$, $0 < \kappa_h < 2$, $\left\{\varepsilon_t^{(1)}\right\} \sim iid\ N\left(0,1\right)$, $\left\{\varepsilon_t^{(2)}\right\} \sim iid\ N\left(0,1\right)$, $\varepsilon^{(1)}$ and $\varepsilon^{(2)}$ are independent, and $\{J_t\}$ is a sequence of independent and identically distributed variables with the probability density:

$$f_{J_t}\left(x\right) = p_J\frac{1}{\sqrt{2\pi\sigma_J^2}}\exp\left(-\frac{1}{2}\frac{\left(x-\mu_J\right)^2}{\sigma_J^2}\right) + \left(1-p_J\right)\delta_{(0)}\left(x\right), \tag{6}$$

where $p_J \geq 0$ is interpreted as a probability of a jump and $\delta_{(0)}$ is the Kronecker delta. Parameter $\mu_J \in \mathbb{R}$ represents the mean of a jump value and $\sigma_J^2 \in (0,\infty)$ denotes the variance of a jump. The normal distribution is most often assumed for the value of a jump.

The above structure is further referred to as the stochastic volatility model with normal jumps and denoted here by the acronym SVNJ. Note that the log-volatility h_t (and not h_{t+1} as in (Jacquier *et al.* 2004) appears in the formula for y_{t+1}, which is in line with the idea presented by Yu (2005), Omori *et al.* (2007), Li *et al.* (2008), and Johannes and Polson (2010).

Similarly to the SVDEJ model, the SVNJ structure incorporates the stochastic volatility structure, jumps and the conditional correlation ρ between log-prices and log-volatility. The SVNJ specification, in Bayesian framework, is proposed by Szerszen (2009).

2.4 What is a jump? The Bayesian perspective on a jump

The Bayesian statistical model is defined by the joint density:

$$p\left(y,\theta\right) = p\left(y\left|\theta\right.\right)p\left(\theta\right), \tag{7}$$

where $y = (y_1, ..., y_n)$ is the observed data, θ is a vector of unknown parameters, $p\left(y\left|\theta\right.\right)$ is a sampling density and $p\left(\theta\right)$ is a prior density. Given y, $p\left(y\left|\theta\right.\right)$ – as a function of θ – is the likelihood function. The Bayesian inference rests upon the posterior density $p\left(\theta\left|y\right.\right) = p\left(y\left|\theta\right.\right)p\left(\theta\right)/p\left(y\right)$ of θ given data y (Bernardo and Smith 2009).

Let us assume that a time series of prices or returns is a trajectory of a stochastic process. In practice, we do not actually know if a given observation has been generated by pure diffusion, the SV process, or (co-)generated by the jump component. In order to address the problem, latent variables $q = (q_t)$, $\xi = (\xi_t)$ are introduced. Moreover, the sequence of latent variables $h = (h_t)$ represents the unobserved log-volatilities under the SV-type models. In the SVNJ model the variable q_t takes two values: 0 or 1. The first one corresponds to no jump, whereas the second one with a jump occurrence. If a jump occurs, a variable ξ_t represents a size of a jump. In the models with the double exponential distribution (the DEJD and SVDEJ models) q_t takes three values: -1, 0 or 1, corresponding to a negative jump, no jump and a positive jump, respectively. If a negative jump occurs at time t (i.e. $q_t = -1$), variable $-\xi_t^D$ represents a size of a jump. If a positive jump occurs at time t (i.e. $q_t = 1$), variable ξ_t^U stands for a value of the jump. Formally, an occurrence of a jump at time t is equivalent to an event $q_t \neq 0$. Unfortunately, the values of q_t are not observed; however, the posterior probability of a jump $P\left(q_t \neq 0 \middle| y\right)$ can be assessed.

Finally, we define that a jump occurs at time t if the posterior probability of a jump exceeds an arbitrarily chosen value (here 0.5, i.e. $P\left(q_t \neq 0 \middle| y\right) > 0.5$). Additionally, under the double exponential distribution, the magnitude of a negative jump at time t equals $-\xi_t^D$, and a positive one ξ_t^U. Similarly, we define a size of a jump under the normal distribution of jumps as a value of ξ_t.

The Bayesian models with extended parameter space including also the latent variables are defined as

$$p\left(y, \theta, h, q, \xi\right) = p\left(y \left| \theta, h, q, \xi \right.\right) p\left(\theta, h, q, \xi\right). \tag{8}$$

It leads to the following Bayesian specifications with a discrete time scale denoted using the same symbols as in the above mentioned DEJD, SVDEJ and SVNJ models. For the unknown quantities of the models, standard proper prior distributions reflecting our prior uncertainty are assumed. In what follows, the Bayesian models are presented. The details of Bayesian specification of DEJD, SVDEJ and SVNJ can be found in, respectively, Kostrzewski (2015), Kostrzewski (2016) and Szerszen (2009).

- The **DEJD** model (Kostrzewski 2015):

$$\begin{aligned} y_{t+1} &= y_t + \left(\mu - \tfrac{1}{2}\sigma^2\right)\Delta + \sigma\varepsilon_{t+1}\sqrt{\Delta} + J_{t+1}, \\ J_{t+1} &= -\xi_{t+1}^D \cdot \mathbb{I}\left(q_{t+1} = -1\right) + 0 \cdot \mathbb{I}\left(q_{t+1} = 0\right) + \xi_{t+1}^U \cdot \mathbb{I}\left(q_{t+1} = 1\right), \end{aligned} \tag{9}$$

  where $y_t = \ln\left(S_t\right)$, $\left\{\varepsilon_t\right\} \sim iid\ N\left(0,1\right)$, $\left\{\xi_t^D\right\} \sim iid\ Exp\left(\eta_D\right)$, $\left\{\xi_t^U\right\} \sim iid\ Exp\left(\eta_U\right)$.

  In order to define the Bayesian model we employ the following reparametrisations: $\mu^{'} = \mu - \frac{1}{2}\sigma^2$, $h = \frac{1}{\sigma^2}$, $L = \lambda\Delta$, where $\Delta = \frac{1}{252}$. The prior structure is given as follows: $h \sim G\left(\nu_h, A_h\right)$ (a gamma distribution with mean $\frac{\nu_h}{A_h}$ and variance $\frac{\nu_h}{A_h^2}$), $\mu^{'} | h \sim N\left(\mu_0, \left(hA_\mu\right)^{-1}\right)$ (a normal distribution with mean μ_0 and variance $\left(hA_\mu\right)^{-1}$), $L \sim \chi^2\left(\nu\right)$ (a χ^2 distribution with ν degrees of freedom and mean ν), $\eta_U \sim IG\left(\nu_{U,\eta}, A_{U,\eta}\right)$ (an inverse gamma distribution with mean $A_{U,\eta}/(\nu_{U,\eta}-1)$ for $\nu_{U,\eta} > 1$), $\eta_D \sim IG\left(\nu_{D,\eta}, A_{D,\eta}\right)$, $p_U \sim B\left(a_U, b_U\right)$

(a beta distribution with mean $a_U/(a_U + b_U)$). The values of the hyperparameters of the prior structure are given in Table 1. The values are the same as in (Kostrzewski 2015).

Table 1: The values of the hyperparameters of the prior distribution for the DEJD model

ν_h	A_h	μ_0	A_μ	ν	$\nu_{D,\eta}$	$A_{D,\eta}$	$\nu_{U,\eta}$	$A_{U,\eta}$	a_U	b_U
5	1	0	1	$\frac{10}{252}$	1.86	0.43	1.86	0.43	1	1

- The **SVDEJ** model (Kostrzewski 2016):

$$\begin{aligned} y_{t+1} &= y_t + \mu + \sqrt{\exp(h_t)}\varepsilon_{t+1}^{(1)} + J_{t+1}, \\ h_{t+1} &= h_t + \kappa_h(\theta_h - h_t) + \sigma_h\left(\rho\varepsilon_{t+1}^{(1)} + \sqrt{1-\rho^2}\varepsilon_{t+1}^{(2)}\right), \\ J_{t+1} &= -\xi_{t+1}^D \cdot \mathbb{I}(q_{t+1} = -1) + 0 \cdot \mathbb{I}(q_{t+1} = 0) + \xi_{t+1}^U \cdot \mathbb{I}(q_{t+1} = 1), \end{aligned} \tag{10}$$

where $\left\{\varepsilon_t^{(i)}\right\} \sim iid\ N(0,1)$, $\left\{\xi_t^D\right\} \sim iid\ Exp(\eta_D)$ and $\left\{\xi_t^U\right\} \sim iid\ Exp(\eta_U)$.
The Bayesian version of the model is considered in (Kostrzewski 2016) where it is denoted by a longer acronym SVLEDEJ.
In order to define the Bayesian model we apply the reparametrisation $(\sigma_h, \rho) \to (\phi_h, \omega_h)$, where $\phi_h = \sigma_h\rho$ and $\omega_h = \sigma_h^2(1-\rho^2)$. The prior structure is given as follows: $\mu \sim N(m_\mu, w_\mu)$, $\kappa_h \sim N(m_{\kappa_h}, w_{\kappa_h})\mathbb{I}_{(0,2)}$ (a truncated normal distribution), $\theta_h \sim N(m_{\theta_h}, w_{\theta_h})$, $\omega_h \sim IG(a_{\omega_h}, b_{\omega_h})$, $\phi_h|\omega_h \sim N\left(0, \frac{1}{2}\omega_h\right)$, $\eta_D \sim IG(a_{\eta_D}, b_{\eta_D})$, $\eta_U \sim IG(a_{\eta_U}, b_{\eta_U})$, $(p_D, p_0, p_U) \sim Dirichlet(d_D, d_0, d_U)$.
The values of the hyperparameters are given in Table 2. The values are the same as in (Kostrzewski 2016).

- The **SVNJ** model (Szerszen 2009):

$$\begin{aligned} y_{t+1} &= y_t + \mu + \sqrt{\exp(h_t)}\varepsilon_{t+1}^{(1)} + J_{t+1}, \\ h_{t+1} &= h_t + \kappa_h(\theta_h - h_t) + \sigma_h\left(\rho\varepsilon_{t+1}^{(1)} + \sqrt{1-\rho^2}\varepsilon_{t+1}^{(2)}\right), \\ J_{t+1} &= 0 \cdot \mathbb{I}(q_{t+1} = 0) + \xi_{t+1} \cdot \mathbb{I}(q_{t+1} = 1), \end{aligned} \tag{11}$$

where $\left\{\varepsilon_t^{(i)}\right\} \sim iid\ N(0,1)$ and $\{\xi_t\} \sim iid\ N\left(\mu_J, \sigma_J^2\right)$.
In order to define the Bayesian model we apply the reparametrisation $(\sigma_h, \rho) \to (\phi_h, \omega_h)$, where $\phi_h = \sigma_h\rho$ and $\omega_h = \sigma_h^2(1-\rho^2)$. Then, the prior structure is given as follows: $\mu \sim N(m_\mu, w_\mu)$, $\kappa_h \sim N(m_{\kappa_h}, w_{\kappa_h})\mathbb{I}_{(0,2)}$, $\theta_h \sim N(m_{\theta_h}, w_{\theta_h})$, $\omega_h \sim IG(a_{\omega_h}, b_{\omega_h})$, $\phi_h|\omega_h \sim N\left(0, \frac{1}{2}\omega_h\right)$, $p_J \sim B(a_J, b_J)$,

$\mu_J \sim N(m_{\mu_J}, w_{\mu_J})$, $\sigma_J^2 \sim IG\left(a_{\sigma_J^2}, b_{\sigma_J^2}\right)$. The values of the hyperparameters are given in Table 3. The values are the same as in (Szerszen 2009) except a_J and b_J. Szerszen assumes the Jeffreys' prior distribution of p_J (i.e. $a_J = b_J = 0.5$). In our study, we assume $a_J = 2$ and $b_J = 15$ therefore $P(p_J < 0.5) \approx 1$. Moreover, the mode, mean and standard deviation of the prior distribution of p_J equal 0.067, 0.118 and 0.076, respectively. The distribution refers to prior expectation of 'sporadic' jumps.

Table 2: The values of the hyperparameters of the prior distribution for the SVDEJ model

m_μ	w_μ	m_{θ_h}	w_{θ_h}	m_{κ_h}	w_{κ_h}	a_{ω_h}	b_{ω_h}
0	10	0	10	1	6	3	$\frac{1}{20}$
a_{η_D}	a_{η_U}	b_{η_D}	b_{η_U}	d_D	d_0	d_U	
1.86	1.86	0.43	0.43	1	1	1	

Table 3: The values of hyperparameters of the prior distribution for the SVNJ model

m_μ	w_μ	m_{θ_h}	w_{θ_h}	m_{κ_h}	w_{κ_h}	a_{ω_h}
0	10	0	10	1	6	3
b_{ω_h}	m_{μ_J}	w_{μ_J}	$a_{\sigma_J^2}$	$b_{\sigma_J^2}$	a_J	b_J
$\frac{1}{20}$	0	10	3	$\frac{1}{20}$	2	15

The Bayesian inference about unknown parameters and latent variables is based on posterior distributions. In the study posterior characteristics of the unknown quantities are calculated by means of the Markov Chain Monte Carlo (MCMC) methods (Gamerman and Lopes 2006), combining the Gibbs sampler and the Metropolis-Hastings algorithm, as well as the acceptance-rejection sampling method (Chib and Greenberg 1995). A convergence of the MCMC algorithms is verified by the visual inspection of CUMSUM, ergodic means and standard deviations plots (Yu and Mykland 1998).

3 Empirical results

In order to evaluate the Bayesian methods of jumps detection, we employ them for gas and CO_2 allowances contract prices.

3.1 Gas forward contracts

Data presentation

Energy is an important factor exerting an impact on economy, politics, stability and people's everyday lives. Natural gas is one of the most important sources of energy and often the basic ingredient of the energy mix. Moreover, natural gas is crucial for both electricity production and heating. The development of gas markets, observed in last decades, is a result of increasing worldwide deregulations and liberalisations of the gas markets.

Gas injected into the transmission network is traded at a virtual trading hub. It is not important, at the hub, which entry or exit is chosen for the natural gas injection or withdrawal. The first of the analysed time series comprises one-month gas contract prices (in EUR/MWh). This data set comes from the Dutch Title Transfer Facility (TTF) virtual trading hub. The data span the period from January 21, 2008 to April 22, 2015. The number of observations equals 1840. Kostrzewski (2016) considers the same time series and presents the results of parameters estimation and the estimation of probabilities of jumps calculated only under the SVDEJ model. However, the analysis of the frequency of jumps and the moments in which they occur is not conducted in (Kostrzewski 2016).

According to Hagfors *et al.* (2016), most stock and commodity prices are not stationary. However, in our study, the results of the Augmented Dickey–Fuller test, Phillips-Perron Unit Root Test (the p-values are less than 0.01) and the Kwiatkowski-Phillips-Schmidt-Shin test (the p-value higher than 0.1) suggest stationarity of the differences in gas forward contract log-prices. Moreover, the Łomnicki-Jarque-Bera test and Shapiro-Wilk test reject normality of the time series of differenced log-prices.

Frequency of jumps

We analyse the number of jumps detected under the Bayesian models. Additionally, we apply three non-Bayesian techniques to detect jumps (Janczura *et al.* 2013):

1. The RFR technique, where returns (the differences in log-prices) exceeding the mean of returns by three sample standard deviations are treated as jumps, with the jump values removed one by one ('recursive filter').

2. The VRT1 technique, where 2.5% of the highest and 2.5% of the lowest returns are treated as jumps.

3. The VRT2 technique, where 10% of the highest and 10% of the lowest returns are treated as jumps.

Table 4 shows the number of negative and positive jumps detected for each day of a week in the differences in log-prices of gas forward contracts. The number of discovered jumps depends on a detection method. In the Bayesian models, the highest

number of jumps is detected under the jump-diffusion structure. The results for the SVDEJ and SVNJ models are similar to each other (30 jumps), and the number of jumps detected under the models is much lower than the number of jumps detected under the DEJD model (265 jumps). We note that the stochastic volatility strongly reduces the number of observations classified as a result of the jump component under the SV-type models (see also Figs. 1–3).
The VRT2 technique detects the greatest number of jumps (368 jumps), whereas the fewest number of jumps is detected under the SV-type models (30 jumps). The methods applied in the study detect more positive jumps than negative ones, except for the VRT1 and VRT2 techniques where the numbers are equal. It is in line with our expectations, because higher gas prices are generally less desirable. It justifies our approach to treat negative and positive jumps separately. Moreover, jumps most frequently appear on Mondays. In the DEJD model and the VRT2 technique the numbers of jumps are high and 73% of these jumps occur on the same days. About 94% of the observations classified as a result of a jump component under the SVDEJ model coincide with the jumps identified also under the DEJD structure. Moreover, all jumps detected under the SVDEJ model are also classified as such by the VRT2 technique.

Table 4: The number of jumps identified under the Bayesian models and non-Bayesian techniques in gas contract prices in the period January 21, 2008 – April 22, 2015

		Mon.	Tues.	Wed.	Thurs.	Fri.	Total
DEJD:	negative	36	26	23	18	28	131
	positive	42	28	24	21	19	134
	all	78	54	47	39	47	265
SVDEJ:	negative	1	1	0	0	0	2
	positive	15	7	2	2	2	28
	all	16	8	2	2	2	30
SVNJ:	negative	2	1	0	1	1	5
	positive	12	7	2	2	2	25
	all	14	8	2	3	3	30
RFR:	negative	12	10	6	4	6	38
	positive	22	15	11	6	6	60
	all	34	25	17	10	12	98
VRT1:	negative	16	11	7	4	8	46
	positive	16	13	10	5	2	46
	all	32	24	17	9	10	92
VRT2:	negative	51	34	34	29	36	184
	positive	53	37	31	34	29	184
	all	104	71	65	63	65	368

Table 5 indicates the number of jumps detected on a set of special dates. We also report the percentage of jumps out of the number of all jumps, months, weekends

Table 5: The number of jumps detected on special dates. The percentage of jumps out of the number of all jumps, months, weekends or breaks in trade in gas contract prices in the period January 21, 2008 – April 22, 2015

	The first trading day of a month	The last trading day of a month	After a break
DEJD	39 jumps 14.7% of jumps 45.3% of months	17 jumps 6.4% of jumps 19.8% of months	90 jumps 34% of jumps 23.8% of weekends 23.4% of breaks
SVDEJ	23 jumps 76.7% of jumps 26.7% of months	0 jumps 0% of jumps 0% of months	18 jumps 60% of jumps 4.8% of weekends 4.7% of breaks
SVNJ	23 jumps 76.7% of jumps 26.7% of months	1 jump 3.3% of jumps 1.2% of months	16 jumps 53.3% of jumps 4.2% of weekends 4.2% of breaks
RFR	26 jumps 26.5% of jumps 30.2% of months	2 jumps 2% of jumps 2.3% of months	39 jumps 39.8% of jumps 10.3% of weekends 10.2% of breaks
VRT1	26 jumps 28.3% of jumps 30.2% of months	4 jumps 4.3% of jumps 4.7% of months	37 jumps 40.2% of jumps 9.8% of weekends 9.6% of breaks
VRT2	42 jumps 11.4% of jumps 48.8% of months	20 jumps 5.4% of jumps 23.3% of months	117 jumps 31.8% of jumps 31% of weekends 30.5% of breaks

or breaks in trade. For example, in the DEJD model: 90 jumps are identified after a break (that is after a day without trade, e.g. just after a weekend or a holiday), and they account for 34% of all jumps identified by this model. Under the SVDEJ model, we observe a jump in every 3 to 4 beginnings of a month (26.7%), moreover, we observe jumps less often than every 21^{st} weekend (4.8%). Under SVDEJ and SVNJ (76.7%) most jumps are identified on the first day of a month. On the other hand, the highest number of jumps under the DEJD model and non-Bayesian techniques is identified after breaks in trade. About 50% of jumps detected under the DEJD model and the non-Bayesian techniques and about 90% of jumps detected under the SV-type models are explained by the calendar effect (cf. Table 6). There are still jumps which occur after no break and neither on the first nor the last day of a month.

Table 6: The number of jumps explained by the calendar effect and the percentage out of all jumps identified in gas contract prices in the period January 21, 2008 – April 22, 2015

Method	No. of jumps	(%)
DEJD	121	(45.7%)
SVDEJ	27	(90.0%)
SVNJ	26	(86.7%)
RFR	53	(54.1%)
VRT1	51	(55.4%)
VRT2	153	(41.6%)

The results obtained by both SV-type models are similar. In both cases only few jumps cannot be explained by the calendar effects. Under the SVDEJ model there are three such jumps, identified on 26 March 2013, 20 June 2014 and 10 February 2015, with the posterior probabilities equal to 0.7747, 0.6842 and 0.7287, respectively. We try to link the dates of jump occurrences with some news or events which might have been the reason of sharp price movements. The positive jump observed on 26 March 2013 might be explained by a supply shortage of gas in the UK caused by the coldest March over the last 50 years, which led to the need of an unplanned purchase of an extra amount of gas. Next, the steep rise on 20 June 2014 might have resulted from the Russia-Ukraine gas war. On 19 June 2014 Russia terminated the talks with Ukraine about gas supplies. The Dutch government announced a reduction of Groningen gas output, which might have spurred the positive jump on 10 February 2015.

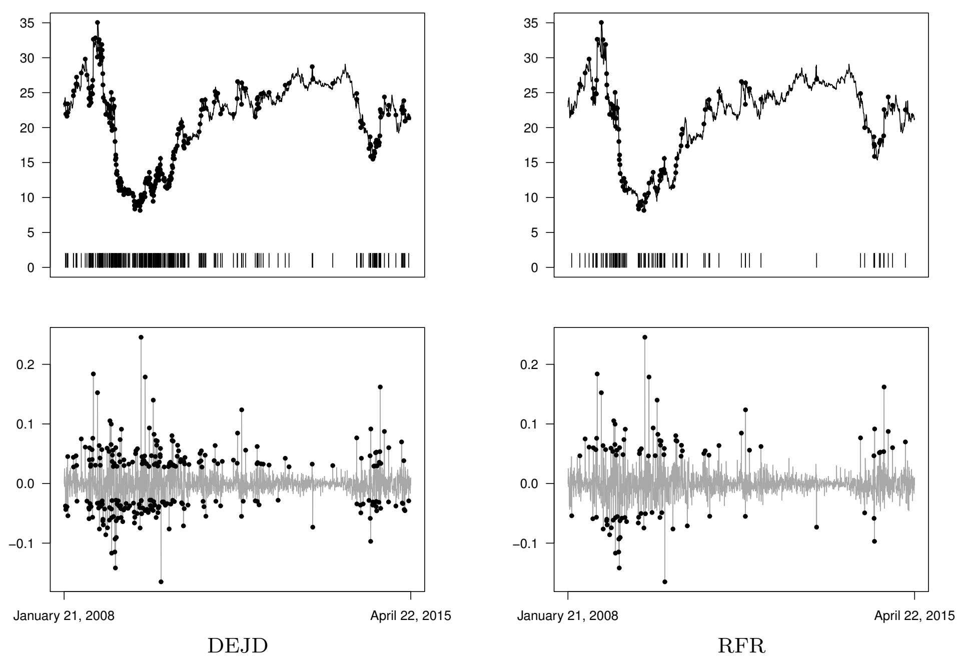
Under the SVNJ model, there are four jumps which are not explained by the calendar effect: on 26 March 2013, 19 June 2014, 20 June 2014 and 10 February 2015, with the corresponding probabilities: 0.5448, 0.7523, 0.7849 and 0.6897. Hence, about 86.7% of all jumps are explained by the calendar effects in that case.

Taking the results into account, we recommend traders not to buy contracts on the first and last trading day of a month and a day just after a break in trade because of a higher probability of a jump occurrence on these days.

Model comparison

The comparison of the models is conducted by means of the decimal logarithm of Bayes factors: $\log_{10}\left(\frac{p(DEJD|y)}{p\,(SVNJ|y)}\right)$ and $\log_{10}\left(\frac{p(SVNJ|y)}{p\,(SVDEJ|y)}\right)$, where $p\,(M_i|y) = \frac{p\,(M_i)\,p\,(y|M_i)}{\sum_j p\,(M_j)\,p\,(y|M_j)}$, with M_i standing for the i-th model. We do not prefer any model, thereby, we assume $p\,(M_i) = p\,(M_j)$. Hence $p\,(M_i|y) \propto p\,(y|M_i)$. In order to estimate the marginal data density $p(y|M_i)$, we apply the Newton-Raftery (N-R) estimator (Newton and Raftery 1994), also called the harmonic mean estimator. It is consistent, however, without finite asymptotic variance. Moreover, it can be unstable (Raftery *et al.* 2007, Pajor and Osiewalski 2013, 2014, Pajor 2017) and 'simulation pseudo-biased' (Lenk 2009). In order to avoid such 'pseudo-bias', Pajor and Osiewalski (2013, 2014) and Osiewalski and Osiewalski (2013) recommended working with appropriately chosen subsets of the parameter space (extended to cover latent variables as well). Estimating $p(y|M_i)$ on the basis of the MCMC sample from the subset of the parameter space requires correcting the original N-R estimator by the prior probability of the chosen subset; Osiewalski and Osiewalski (2013, 2016) propose using multivariate cubes. This, however, can be criticised as an arbitrary solution. Ultimately, our examples are based on N-R estimator and very long realizations of the Markov chains.

Figure 1: *Upper panels*: Gas forward contract prices in the period January 21, 2008 – April 22, 2015 (solid line) and the observations identified as jumps (dots) marked also by vertical line segments. *Lower panels*: The series of the differences in log-prices of gas forward contracts and the observations identified as jumps (dots). The results based on DEJD (*left panels*) and RFR (*right panels*)



The calculations of the harmonic mean estimators are based on 6,000,000 iterations after rejecting burn-in iterations. The values of the logarithmic Bayes factors are as follows: $\log_{10}(p(DEJD|y)/p(SVDEJ|y)) = 63$, $\log_{10}(p(SVDEJ|y)/p(SVNJ|y)) = 9.53$.
In both cases the numbers are higher than 2. According to the interpretation of Kass and Raftery (1995), it means that the models from the numerators very strongly outperform the models from the denominators. The simplest one, DEJD, gains the highest posterior probability. Moreover, the SV-type model with the double exponential distribution of jumps (SVDEJ) is more probable *a posteriori* than its counterpart with normal sizes of jumps. To sum up, the results of the Bayesian comparison unambiguously point to the simplest model DEJD as the best one, and the model with a normal distribution of jumps as the worst one. It again justifies the choice of the double exponential distribution of jump sizes.

Figure 2: *Upper panels*: Gas forward contract prices in the period January 21, 2008 – April 22, 2015 (solid line) and the observations identified as jumps (dots) marked also by vertical line segments. *Lower panels*: The series of the differences in log-prices of gas forward contracts, the observations identified as a result of the jump component (dots) and stochastic volatility with mirror reflection along the horizontal axis (solid line). The results based on SVDEJ (*left panels*) and SVNJ (*right panels*)

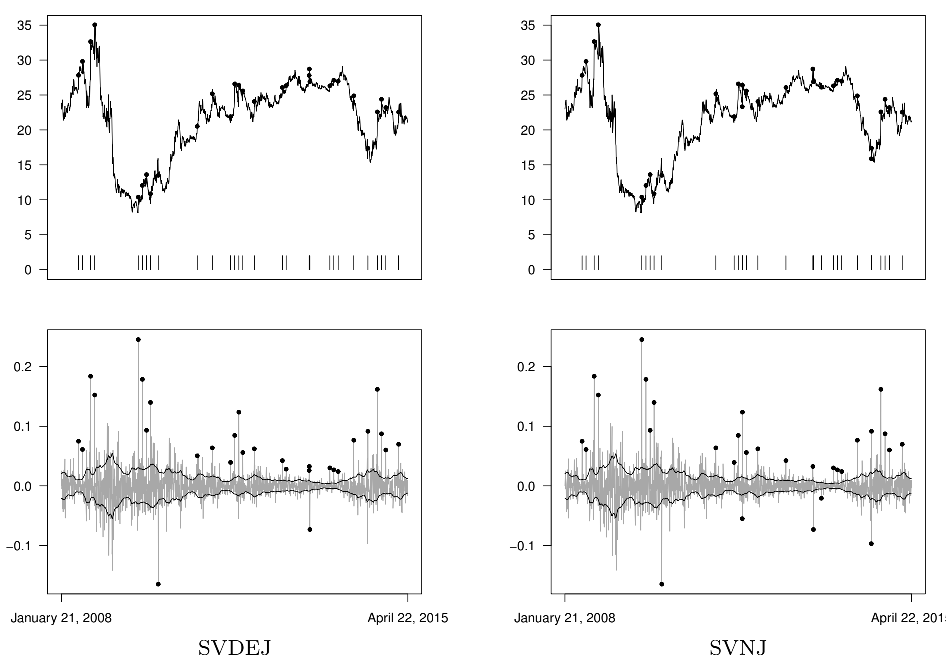
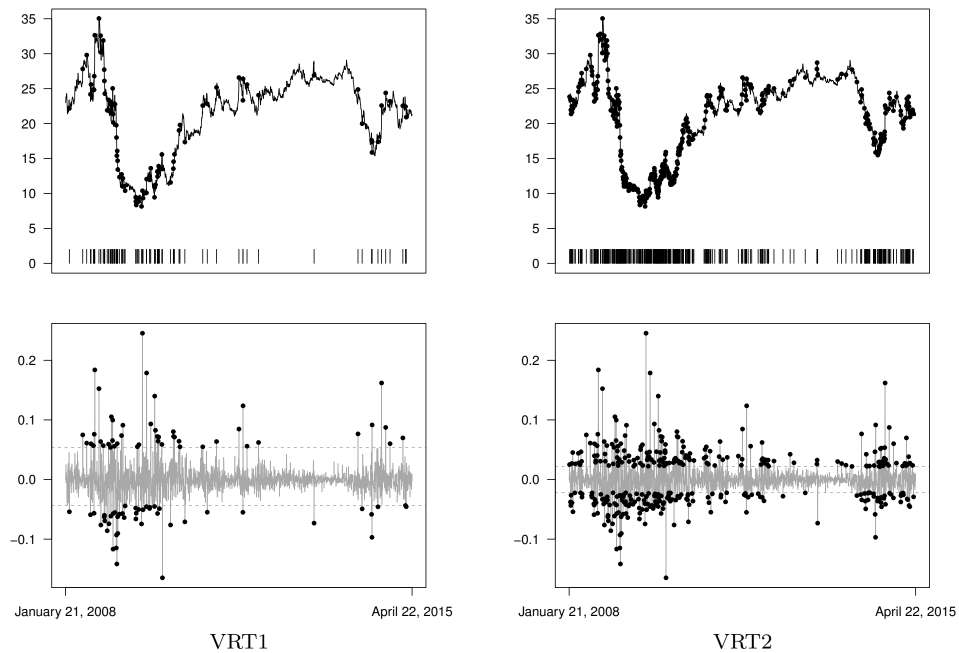


Figure 3: *Upper panels*: Gas forward contract prices in the period January 21, 2008 – April 22, 2015 (solid line) and the observations identified as jumps (dots) marked also by vertical line segments. *Lower panels*: The series of the differences in log-prices of gas forward contracts and the observations identified as jumps (dots). The results based on VRT1 (*left panels*) and VRT2 (*right panels*)



Figs. 1–3 present gas forward contract prices and the differences in log-prices in the period January 21, 2008 – April 22, 2015. The dots represent the values which are found to result from the jump component under the Bayesian models and the values which are identified as jumps under the non-Bayesian techniques.

We observe the jump clustering phenomenon – periods of no jumps alternate with the ones of frequent jumps – for all models, apart from the SVDEJ and SVNJ models. The SV component explains many sharp movements of the series and may be a reason why the jump clustering phenomenon does not manifest itself in SV-type models (see Fig. 2). Moreover, the lower panels of Figure 2 present the posterior means of stochastic volatility estimated under the SVDEJ and SVNJ models. We observe the periods of higher volatility alternating with those of lower volatility. The results are indicative of the stochastic volatility clustering.

The jumps found under the Bayesian models do not correspond to the values below or above some fixed thresholds – there exist values classified as a result of the jump component whose sizes are lower (or higher) than observations not classified as such. Moreover, the results for the SVNJ model are similar to the ones obtained for the SVDEJ specification.

3.2 EUA futures contracts

Data presentation

Emissions trading is used as an instrument of international ecological politics in the fight with global warming. It was established to prompt, in a financial way, with the use of the cap and trade principle, factories, power stations, civil aviation industry etc. to reduce the amounts of industrial greenhouse CO_2 emissions. Trading carbon dioxide allowances is one of the ways in which CO_2 emission can be reduced. The European Union Emissions Trading Scheme was launched on January 1, 2005, and was preceded by the international agreement in Kyoto in 2002.

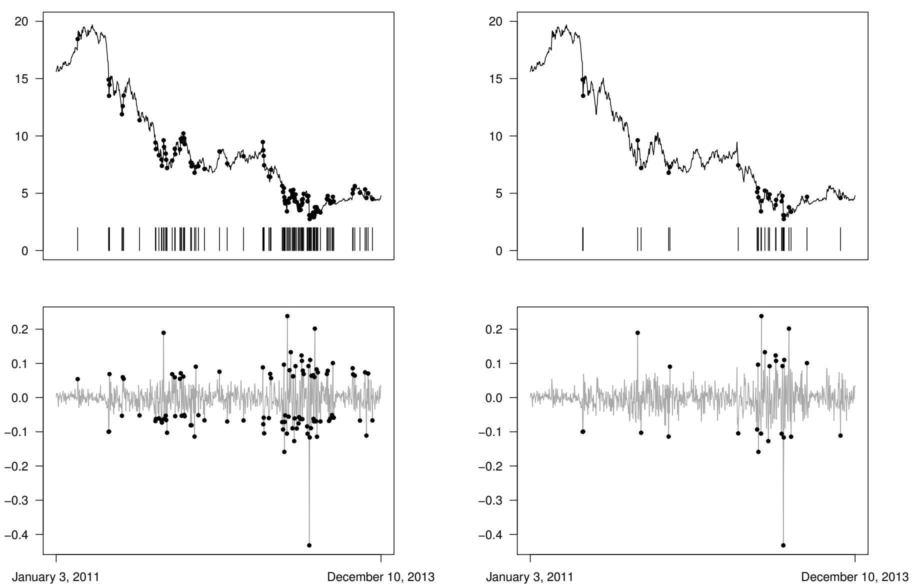
Table 7: The number of jumps identified under the Bayesian models (DEJD, SVDEJ, SVNJ) and non-Bayesian techniques (RFR, VRT1, VRT2) in the series of EUA contract prices in the period January 3, 2011 – December 6, 2013

		Mon.	Tues.	Wed.	Thurs.	Fri.	Total
DEJD:	negative	14	12	17	7	11	61
	positive	5	6	8	12	11	42
	all	19	18	25	19	22	103
SVDEJ:	negative	0	1	0	0	0	1
	positive	0	0	0	0	0	0
	all	0	1	0	0	0	1
SVNJ:	negative	0	1	0	0	0	1
	positive	0	0	0	0	0	0
	all	0	1	0	0	0	1
RFR:	negative	3	2	3	3	3	14
	positive	0	2	3	4	3	12
	all	3	4	6	7	6	26
VRT1:	negative	4	5	3	3	3	18
	positive	2	2	3	6	5	18
	all	6	7	6	9	8	36
VRT2:	negative	17	13	21	8	14	73
	positive	11	20	13	16	15	75
	all	28	33	34	24	29	148

European Union CO_2 emission allowances (EUA) are traded on the ICE Futures Europe electronic platform which is a part of the Intercontinental Exchange (ICE). Each of EU allowances entitles one to emit one tonne of carbon dioxide equivalent. EUA futures contracts are standardised products. Future prices are quoted in euro

per metric tonne. The contract unit amounts to $1,000$ CO_2 EU allowances. The trading model is based on continuous trading.
The second dataset considered in this study is the series of daily differences in the logarithm of closing prices of EUA futures. The empirical study covers the period of surpluses of allowances in a market, which resulted in low contract prices. Unfortunately, EUA low prices did not result in the industry's investments in limiting emissions, as it was cheaper to buy allowances.
The following analysis is focused on the closing prices of EUA futures contracts expiring on December 16, 2013. The data span the period from January 3, 2011 to December 6, 2013. The number of observations equals 757. The period under study concerns Phase II (2008–2012) and Phase III (2013–2020) of the European Union Emissions Trading Scheme (EU ETS).

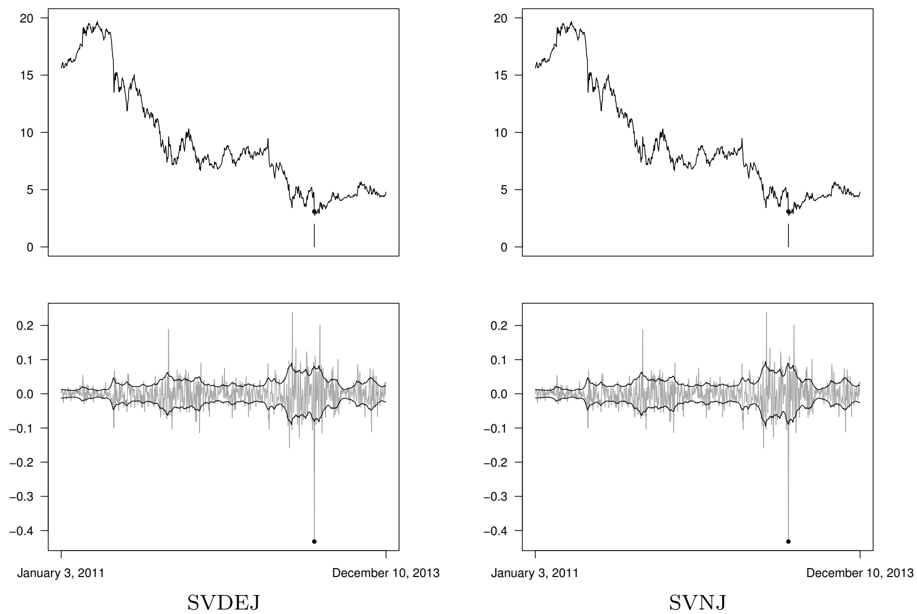
Figure 4: *Upper panels*: Prices of EUA futures contracts in the period January 3, 2011 – December 6, 2013 (solid line) and the observations identified as jumps (dots) marked also by vertical line segments. *Lower panels*: The series of the differences in log-prices of EUA futures contracts and the observations identified as jumps (dots). The results based on DEJD (*left panels*) and RFR (*right panels*)



DEJD RFR

Similarly to gas contracts, the results of the Augmented Dickey–Fuller test, Phillips-Perron Unit Root Test (the p-values are less than 0.01) and the Kwiatkowski-Phillips-Schmidt-Shin test (the p-value are higher than 0.1) suggest stationarity of the differences in EUA futures log-prices. Moreover, the Łomnicki-Jarque-Bera test and Shapiro-Wilk test reject the normality of the time series.

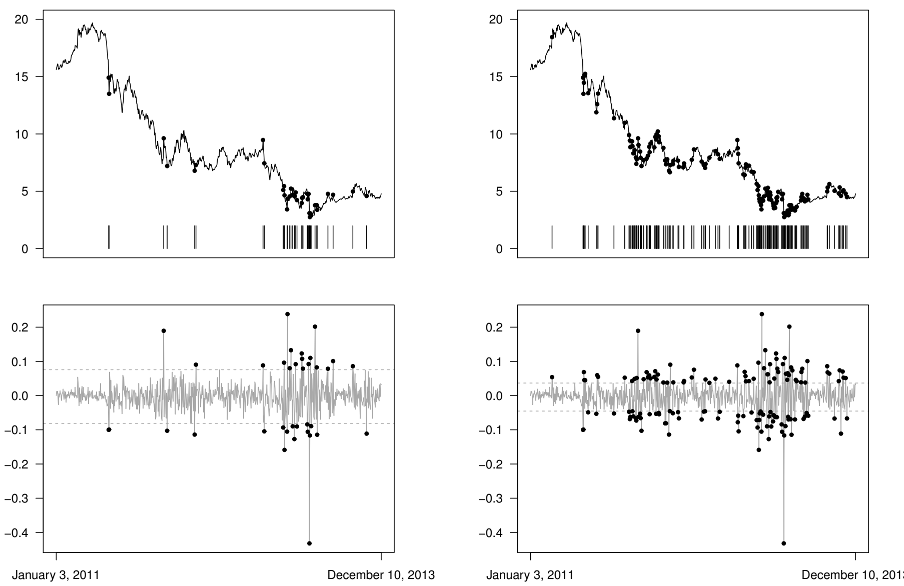
Figure 5: *Upper panels*: Prices of EUA futures contracts in the period January 3, 2011 – December 6, 2013 (solid line) and the observations identified as jumps (dots) marked also by vertical line segments. *Lower panels*: The series of the differences in log-prices of EUA futures contracts, the observations identified as a result of the jump component (dots) and stochastic volatility with mirror reflection along the horizontal axis (solid line). The results based on SVDEJ (*left panels*) and SVNJ (*right panels*)



Frequency of jumps

We analyse the number of jumps detected under three Bayesian models: DEJD, SVDEJ, SVNJ, and by means of the non-Bayesian techniques: VRT1, VRT2, RFR. Table 7 presents the number of negative and positive jumps. The number of negative jumps in the series of differences in log-prices of EUA futures contracts is equal or higher than the number of positive jumps for each method of jump detection apart from VRT2 (see also Figs. 4–6). Similarly to the prices of gas contracts, the greatest number of jumps is detected under VRT2 (148 jumps) and DEJD (103 jumps). Under the SV-type models the stochastic volatility component explains nearly all variability and there is only a single data point with a high probability of a jump occurrence under the SVDEJ and SVNJ models. Moreover, jumps detected after a break in trade amount to about 20% of all jumps under the DEJD, RFR, VRT1 and VRT2 methods.

Figure 6: *Upper panels*: Prices of EUA futures contracts in the period January 3, 2011 – December 6, 2013 (solid line) and the observations identified as jumps (dots) marked also by vertical line segments. *Lower panels*: The series of the differences in log-prices of EUA forward contracts and the observations identified as jumps (dots). The results based on VRT1 (*left panels*) and VRT2 (*right panels*)



Model comparison

In the paper (Kostrzewski 2014b), a shorter time series of the EUA futures contract prices is analysed by means of the DEJD model. In order to compare the Bayesian models, the calculations of the harmonic mean estimator are based on $3,000,000$ iterations after rejecting the burn-in iterations. The values of the logarithmic Bayes factors are as follows: $\log_{10}(p(SVNJ|y)/p(DEJD|y)) = 33.24$, $\log_{10}(p(SVNJ|y)/p(SVDEJ|y)) = 0.269$.
The SV-type models gain the highest posterior probabilities. However, the difference between the posterior probabilities of SVNJ and SVDEJ is very small and negligible. According to Kass and Raftery (1995), if the logarithmic Bayes factor is lower than 0.43, the difference between the models is 'not worth more than a bare mention'. The results indicate that the jump value distribution does not play a key role in the

EUA future contract prices. The result is not surprising because only one jump is detected under the SV-type models, and almost all variability is explained by stochastic volatility.
Figs. 4–6 show the prices of EUA futures contracts and the differences in log-prices in the period January 3, 2011 – December 6, 2013. The dots represent the values which are found to result from the jump component under the Bayesian models and the values which are identified as jumps under the non-Bayesian techniques. The only jump identified under the SV-type models occurs on April 16, 2013. On this day the European Parliament voted against the European Commission's proposal of delaying the auction of 900 million allowances from the first three years (2013–2015) of the 3rd ETS trading period (2013–2020) (so-called backloading), which caused a sharp decline in prices.

4 Conclusions and remarks

The study employs Bayesian models (DEJD, SVDEJ, SVNJ) and three non-Bayesian techniques (RFR, VRT1, VRT2) to detect jumps.
In gas forward contract prices the number of positive jumps is higher or equal than the number of negative jumps. We might expect that 'bad' news has greater impact on prices than 'good' news, which may be justified by the phenomenon of 'crashophobia'. Generally, in gas contracts 'bad' news or a 'bad' event implies a rise of the price. The results justify our approach to treat negative and positive jumps separately by employing the double exponential distribution of jumps.
Many jumps in gas contract prices are explained by the calendar effects. The results reveal the relation between jumps and the scheme of trading. Many jumps appear just after a break in trade, so traders should be very cautious buying contracts on such days.
The results of the application of the jump-diffusion model and non-Bayesian techniques indicate not only a high frequency of jumps, but also that the jump activity varies over time. Introducing the stochastic volatility component into the model structure strongly reduces the number of identified jumps. Under the SV-type models periods of higher volatility alternating with periods of lower volatility can be observed. The SV part explains many sharp movements of the series and models time series in periods with high variations. In the case of gas contract prices, the simplest model, DEJD, gains the highest posterior probability.
In the case of the EUA futures contracts, the number of negative jumps in prices is higher than or equal to the number of positive jumps in all methods but VRT2. The models with stochastic volatility have significantly higher posterior probabilities than the DEJD specification. The SV structure explains nearly all variation of the time series, the only jump detected is linked with the backloading.
Generally, the jumps which are found under the Bayesian models do not correspond to the values below or above fixed thresholds. In our opinion, jumps might be treated

as residuals of the ‘continuous’ part of the model rather than (global) outliers. In other words, a jump is a value which is ‘considerably’ higher or lower than the values in its neighbourhood, rather than in comparison with the values across the whole data set. These results are different than the ones obtained using non-Bayesian techniques (VRT1, VRT2, RFR).

Our empirical results might be useful on at least three counts. First, they show the impact of calendar effects and political and economic decisions on jump appearances, which might be valuable for traders. Second, the results indicate the models which best fit the data in question. Last but not least, the results of the study provide valuable tools for risk management connected with the identification of periods with a higher probability of steep rises and drops of prices.

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