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Reactive power based fair calculation approach for multiobjective load dispatch problem

Harinder Pal Singh[1], **Yadwinder Singh Brar**[1], **D. P. Kothari**[2]

[1] *I.K. Gujral Punjab Technical University*
Jalandhar – Kapurthala Highway, VPO – Ibban, India

[2] *Wainganga College of Engineering and Management*
Wardha Road, Gumgaon, Nagpur, India
e-mails: harinderpal011@yahoo.com, braryadwinder@yahoo.com, dpkvits@gmail.com

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Abstract: This paper proposes a fair calculation approach for the cost and emission of generators. Generators also have reactive power requirements along with the active power demand to meet up the total power demand. In this paper, firstly the reactive power is calculated considering the random active power operating points on the capability curve of a generator then the cost for reactive power generation as well as emission are calculated. In order to develop the mathematical function for the reactive power cost and reactive power emission, a curve-fitting technique is applied, which gives the generalised reactive power cost and reactive power emission functions. At the end, the problem is formulated as a multiobjective problem, considering conflicting objectives such as combined active-reactive economic dispatch and combined active-reactive emission dispatch. The problem is converted from the multiobjective load dispatch problem (MOLDP) into a scalar problem, using the weighting method and the best compromised solution has been calculated using the particle swarm optimization (PSO) technique. A fuzzy cardinal method has been applied to choose the best solution. In order to demonstrate the efficiency of developed functions the proposed method is applied on a 3 generator unit system and a 10 generator unit system, the results obtained show its validity and effectiveness.

Key words: combined active reactive economic dispatch, combined active reactive emission dispatch, economic load dispatch, multiobjective load dispatch

1. Introduction

Under the economic load dispatch (ELD) problem (ELDP) the foremost objective is to minimize the operating cost by scheduling the committed generating unit outputs so as to meet the load demand. The ELDP is defined as the method of decreasing the total generation fuel cost of all committed generators by scheduling them within minimum and maximum limits, thereby satisfying the total load demand and losses [1]. Accumulation this, due to the usage of fossil fuel as a primary energy source of the harmful gasses such as CO_2, SO_2 and NO_x, has been produced. These harmful gases has a major effect on human beings, so emission dispatch (ED) is the another problem which is to be minimized along with the ELDP. Both the ELDP and ED problem (EDP), when solved together, are of conflicting nature, consequently to solve these two conflicting objectives at the same time, the problem is framed as the multiobjective load dispatch problem (MOLDP) [2–3]. Different approaches have been suggested to solve the ELDP, EDP and MOLDP [4–14]. [4] has applied a genetic algorithm (GA), [5] has applied evolutionary programming (EP), [6] has applied a multiobjective evolutionary algorithm (MOEA), in [7], for searching the 'best' weightage pattern with fuzzy set theory, an evolutionary optimization technique was applied. In another research, [8], authors have applied a simplex weighting pattern search technique to solve a multiobjective generation scheduling problem. Differential evolution (DE) is a relatively new member in the family of evolutionary algorithms [9], in [10] authors combine simple arithmetical operators with the classical operators of recombination and mutation to find a final solution. DE is further modified to Multi-objective differential evolution [MODE] [11] to select the best individual by implementing a pareto-based approach. Combined economic emission dispatch using a shuffled frog-leaping algorithm (SFLA) was proposed by [12]. The SFLA is a new addition to the range of intelligent algorithms and a new member to the family of memetic algorithms. The local search is similar in concept to a particle swarm optimization (PSO) [3] algorithm and can search for food based on a colony. Both the PSO and SFLA are meta-heuristic search methods. The PSO is inspired by bird flocking behaviour searching for food while The SFLA is inspired from the memetic evolution of a group of frogs when seeking for food [13, 14]. In the earlier mentioned researches only the active power (AP) cost is considered for the solution of the MOLDP. Apart from this, generators have to supply the reactive power (RP) along with the AP to meet up the total power demand. However, the production of the RP by a generator will diminish its ability to produce the AP, so it becomes important to compare the price of the RP with AP pricing. Many different techniques have been suggested by different researchers for the RP pricing [15–19]. Some of them have focused on formulating the RP pricing [15]. Some have suggested a pricing technique based on minimization of operating cost using decoupled optimal power flow [16], cost allocation of the RP using modified a Y-bus matrix method has been proposed by [17], active and reactive pricing using an interior point method has been suggested by [18]. Cost of production based on the reactive power is highly reliant on the AP output. A fair cost calculation method considering both the AP cost, and RP cost has been suggested by [19], in which author has focused on formulating an objective function of the RP pricing. [20] has deliberated the contingency conditions like going-off that influences the RP price. Authors in [21] have discussed about the wind-diesel isolated hybrid power systems to have cost-effective RP compensation. In [22] the tracing method is integrated with the optimal RP dispatch problem for enhancing the system security. [23] has presented a new approach based on the joint day-ahead active and the RP market.

Until now authors have focused only on RP pricing strategies, whereas the RP production will also create variation in emission characteristics. Therefore it becomes necessary to formulate an objective function based on the RP emission and this emission should be included with the emission based on the AP for the fair calculations. In this paper, a fair cost and fair emission calculation method is formulated considering the effect of the RP on the AP. The PSO algorithm

[3, 24, 25] has been applied to solve the MOLDP, it consists of conflicting objectives such as combined active-reactive economic dispatch (CAREcD) and combined active-reactive emission dispatch (CAREmD). Unlike the most of the evolutionary algorithm, resolution (individual) in the PSO is related to a randomized velocity and the potential resolutions, called particles, are then "flown" through the problem space. The MOLDP has been transformed into a scalar problem using the weighing method. The best compromising solution has been calculated using a fuzzy cardinal approach.

2. Problem formulation

2.1. Problem objectives

The main objectives of this problem are to minimise fuel cost considering both the active RP generation and emission as well as to take into account both the active RP generation as subject to equality and in equality constraints. The details of objectives are given as below.

2.1.1. Minimization of fuel cost considering AP generation

The fuel cost function considering the AP generation (P_{gi}) can be expressed as [19]:

$$F_1(P_{gi}) = \sum_{i=1}^{NG} \left(a_i P_{gi}^2 + b_i P_{gi} + c_i \right), \tag{1}$$

where a_i, b_i and c_i are the fuel cost coefficients of i-th unit. NG is the number of generators.

2.1.2. Minimization of emission considering AP generation

The amount of emission is given as a function of generator output P_{gi}, such as [26]:

$$F_2(P_{gi}) = \sum_{i=1}^{NG} \left(\alpha_i P_{gi}^2 + \beta_i P_{gi} + \gamma_i + \eta_i \exp(\delta_i P_{gi}) \right), \tag{2}$$

where α_i, β_i, γ_i, η_i and δ_i are the emission coefficients and NG is the number of generators.

2.1.3. Minimization of fuel cost considering RP generation

Production cost considering RP depends on AP output. As seen from Fig. 1 when a generator produces its maximum AP $(P_{g\,\max})$, then there will be no production of RP, subsequently apparent power (S_g) equals $P_{g\,\max}$. Moreover, production of the RP by generators will decrease its ability to produce AP. Hence, the AP production will be reduced due to the production of the RP. Therefore to generate the RP (Q_{gi}) operating at its nominal power $P_{g\,\max}$, it is required to decrease its AP from $P_{g\,\max}$ to P_{gi} [19]

such that $$P_{gi} = \sqrt{P_{g\,\max}^2 - Q_{gi}^2}, \tag{3}$$

therefore, $$Q_{gi} = \sqrt{P_{gi}^2 - P_{g\,\max}^2}, \tag{4}$$

$$\Delta P_g = P_{g\,\max} - P_{gi}\,. \tag{5}$$

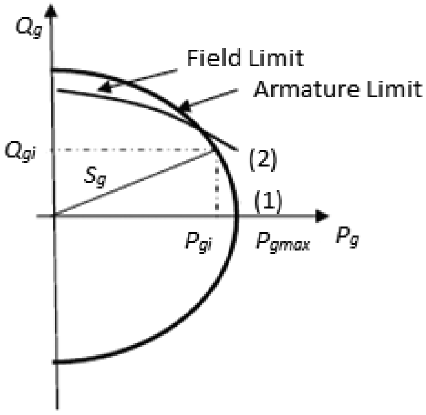

Fig. 1. Capability curve of generator

ΔP_g represents the amount of the AP reduced as a result of the RP generation. To calculate the RP cost accurately it is required to include all the costs imposed on the generator such as Cost ($P_{g\max}$): the cost of generation when producing AP equal to $P_{g\max}$, Cost ($P_{g\max} - P_g$): the cost of the generator when producing both AP and RP equal to P_{gi} and Q_{gi}, (Cost ($P_{g\max}$) – Cost ($P_{g\max} - \Delta P_g$)): decrease in the cost considering AP due to reduction in AP (ΔP_g) as due to generating RP(Q_{gi}), this cost also denotes the cost considering RP production while the operating point shown in Fig. 1 shifts to point (2) starting from point (1) and is given as:

$$\mathrm{Cost}(P_{g\max}) - \mathrm{Cost}(P_{g\max} - \Delta P_g) = \mathrm{Cost}(Q_{gi}) + \frac{\Delta P_g}{P_{g\max}} \mathrm{Cost}(P_{g\max}), \tag{6}$$

where

$$\frac{\Delta P_g}{P_{g\max}} \mathrm{Cost}(P_{g\max})$$

represents the change of the operating point (it is the cost of ΔP_g energy, when the generator is generating its nominal power). From the above equation, the RP cost function based on the AP generation can be written as [19]:

$$\mathrm{Cost}(Q_{gi}) = \frac{P_{g\max} - \Delta P_g}{P_{g\max}} \mathrm{Cost}(P_{g\max}) - \mathrm{Cost}\left(P_{g\max} - \Delta P_g\right), \tag{7}$$

$$F_3(Q_{gi}) = \left(\frac{P_{g\max} - \Delta P_g}{P_{g\max}}\right) F_1(P_{g\max}) - F_1\left(P_{g\max} - \Delta P_g\right). \tag{8}$$

The algorithm steps involved in the calculation of objective function $F_3(Q_{gi})$ are given as below.

2.1.3.1. Algorithm: formulation of objective function to calculate the cost of RP generation

1. AP points are varied from $P_{g\max}$ to P_{gi} randomly, i.e. by shifting the operating point to position 2 from position 1 as shown in Fig. 1.

2. Using Step 1, a number of points were marked on the curve corresponding to the values of the AP on the horizontal axis and supposing the use of complete potential of the generator capability and by considering the effect of the operating point as shown in Fig. 1 in such way that its current will be equal to its nominal value, Q will be written as a function of P (Eq. (4)) and the RP points are marked on the vertical axis of Fig. 1.
3. Considering Q_{gi} as a variable (Eq. (4)), the production cost is calculated using Eq. (8).
4. The best curve (regression value 1) is fitted using "the Newton-Gregory interpolation" between the RP as calculated using Eq. (4) and the corresponding RP cost using Eq. (8).

Further, $F_3(Q_{gi})$ is to be expressed as a function of Q_{gi}. Based on the above algorithm the objective function is formulated for the cost calculation considering the RP generation for both the test systems (Test System-I: IEEE 9 bus 3 generator system, Test System-II: New England power system containing 10 generating units). Fig. 2 shows the objective function developed for generator 1 of Test System-I after curve fitting and Fig. 3 shows the objective function developed for generator 1 of Test System-II after curve fitting. Similarly objective functions for the remaining generators of Test System-I and Test System-II are formulated using the above algorithm. The objective function at the degree of polynomial 2 gives a best regression value of 1 for each generator of Test System-I and the objective function at the degree of polynomial 3 gives the best regression value of 1 for each generator of Test System-II. So based on the degree of polynomial

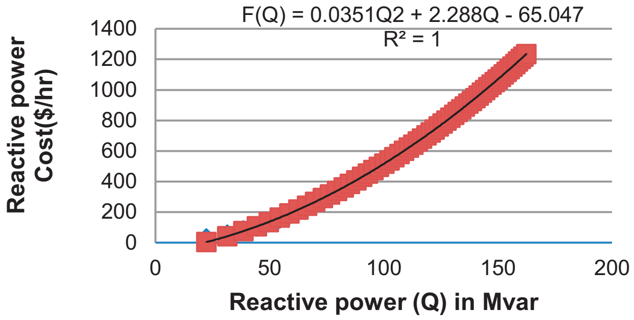

Fig. 2. Cost curve considering RP (Test System-I)

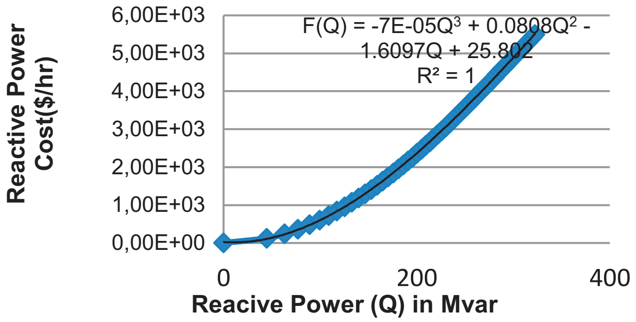

Fig. 3. Cost curve considering RP (Test System-II)

the fuel cost function considering the RP generation can now be expressed in a generalised form as:

$$F_3(Q_{gi}) = \sum_{i=1}^{NG} \left(a_{(n)i} Q_{gi}^{n} + a_{(n-1)i} Q_{gi}^{n-1} + a_{(n-2)i} Q_{gi}^{n-2} + \cdots + a_{0i} Q_{gi}^{0} \right), \tag{9}$$

where, n is the degree of polynomial calculated corresponding to a best regression value of 1, $(a_{(n)i}, a_{(n-1)i}, a_{(n-2)i}, \cdots, a_{0i})$ are the calculated fuel cost coefficients considering the RP generation and NG is the number of generators. The obtained values of these coefficients are shown in Table 1 and Table 3. This method of formulation is very reliable as it is extracted from the power cost function of a generator and provides accurate results in RP pricing [19].

2.1.4. Minimization of emission considering RP generation

Real power loading creates current loading on the generators, so hereby, considering the maximum capability of a generator to supply current, will affect the apparent power due to the requirement of RP from the generator. As the RP requirement increases the supply of AP from the generators also reduces as discussed in section 2.1.3, this also leads to variation in emission from the generators. So it is an important to calculate the emission based on the RP generation, otherwise it may lead to the false calculation. Since an emission function based on the AP generation is available, it is required to formulate the emission function based on the RP generation. In order to calculate the accurate emission of the RP (Q_{gi}), all the emission imposed on generators as given below should be included, such as:

- Emission $(P_{g\,\max})$: emission of the generator when producing the AP equal to $P_{g\,\max}$,
- Emission $(P_{g\,\max} - \Delta P_g)$: emission of the generator when producing both the AP and RP equal to P_{gi} and Q_{gi},
- $\left(\text{Emission}(P_{g\,\max}) - \text{Emission}(P_{g\,\max} - \Delta P_g)\right)$: reduction in the emission of the AP due to reduction in the AP (ΔP_g) as due to generating the RP (Q_{gi}).

This emission also represents the emission of the RP production while the operating point shown in Fig. 1 shifts to point (2) starting from point (1) and it can be written as:

$$\begin{aligned} &\text{Emission}(P_{g\,\max}) - \text{Emission}\left(P_{g\,\max} - \Delta P_g\right) = \\ &= \text{Emission}(Q_{gi}) + \frac{\Delta P_g}{P_{g\,\max}} \text{Emission}(P_{g\,\max}), \end{aligned} \tag{10}$$

where

$$\frac{\Delta P_g}{P_{g\,\max}} \text{Emission}(P_{g\,\max})$$

is related to the change in the operating point.

The above equation can also be written as:

$$\text{Emission}(Q_{gi}) = \frac{P_{g\,\max} - \Delta P_g}{P_{g\,\max}} \text{Emission}(P_{g\,\max}) - \text{Emission}\left(P_{g\,\max} - \Delta P_g\right), \tag{11}$$

$$F_4(Q_{gi}) = \left(\frac{P_{g\,\max} - \Delta P_g}{P_{g\,\max}} \right) F_2(P_{g\,\max}) - F_2\left(P_{g\,\max} - \Delta P_g\right). \tag{12}$$

The algorithm steps involved in the calculation of objective function $F_4(Q_{gi})$ are given as below.

2.1.4.1. Algorithm: formulation of objective function to calculate the emission considering RP generation

1. Value of active and RP is noted down using step 1 and 2 of article 2.1.3.1.
2. For each operating point, emission for RP generation is calculated Eq. (12).
3. The best curve (regression value 1) is fitted between the RP as calculated using Eq. (4), and RP emission as calculated using Eq. (12).

$F_4(Q_{gi})$ is to be expressed as a function of Q_{gi}. Based on the above algorithm the objective function is formulated for the RP emission calculation. Fig. 4 shows the objective function developed for generator 1 of Test System-II after curve fitting based on a best regression value of 1 and similar curve fitting is done for the remaining generators. The function at the degree of polynomial 4 gives a best regression value of 1 for the formulation of objective functions for each generator. The emission function considering the RP generation for all the test systems can now be expressed in generalised form as:

$$F_4(Q_{gi}) = \sum_{i=1}^{NG} \left(\alpha_{(n)i} Q_{gi}^{n} + \alpha_{(n-1)i} Q_{gi}^{n-1} + \alpha_{(n-2)i} Q_{gi}^{n-2} + \cdots + \alpha_{0i} Q_{gi}^{0} \right), \tag{13}$$

where n is the degree of polynomial calculated corresponding to best regression value of 1, $(\alpha_{(n)i}, \alpha_{(n-1)i}, \alpha_{(n-2)i}, \cdots, \alpha_{0i})$ are the emission coefficients considering RP which are calculated using curve fitting and NG is the number of generators. The obtained values of these coefficients are shown in Table 3. This is an accurate emission function for RP calculation as all the variation in emission imposed on generator due to RP requirements have been included during formulation and best curve is fitted based on regression of 1 using Newton-Gregory interpolation.

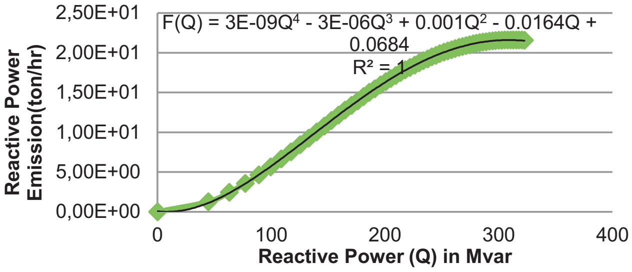

Fig. 4. Emission curve considering RP (Test System-II)

2.2. Constraints

2.2.1. Active and RP balance constraints

The total generation considering AP must balance the demand plus the losses [26].

$$\sum_{i=1}^{NG} P_{gi} - (P_D + P_L) = 0, \tag{14}$$

where P_D is the AP demand and P_L is the AP losses.

The total generation considering RP must equal to demand plus the losses.

$$\sum_{i=1}^{NG} Q_{gi} - (Q_D + Q_L) = 0, \tag{15}$$

where Q_D is the RP demand and Q_L is the RP losses.

2.2.2. Active and RP operating limits

The AP and RP generation by each unit must lie between minimum and maximum limits.

$$P_{gi}^{\min} \le P_{gi} \le P_{gi}^{\max}, \tag{16}$$

$$P_{gi}^{\min} \le P_{gi} \le P_{gi}^{\max}, \tag{17}$$

where $P_{gi}^{\min}$ and $P_{gi}^{\max}$ are the minimum and maximum limits for the AP generation. $Q_{gi}^{\min}$ and $Q_{gi}^{\max}$ are the minimum and maximum limits for the RP generation by i-th unit.

2.3. Combined active and RP cost

To obtain an accurate cost function, the RP cost is to be counted in the AP cost function. The total cost is given by combining the cost considering AP generation as given in Eq. (1) and the cost considering the RP generation as given in Eq. (9). The objective function becomes as:

$$\text{Minimize } F_1^{\text{total}} = \sum_{i=1}^{NG} F_1(P_{gi}) + F_3(Q_{gi}), \tag{18}$$

$$\text{Subjected to } \sum_{i=1}^{NG} P_{gi} - (P_D + P_L) = 0, \tag{19}$$

$$\sum_{i=1}^{NG} Q_{gi} - (Q_D + Q_L) = 0, \tag{20}$$

$$P_{gi}^{\min} \le P_{gi} \le P_{gi}^{\max} \quad (i = 1, 2, \ldots, NG), \tag{21}$$

$$Q_{gi}^{\min} \le Q_{gi} \le Q_{gi}^{\max} \quad (i = 1, 2, \ldots, NG), \tag{22}$$

2.4. Combined active and RP emission

In order to obtain an accurate emission function, the RP emission is to be counted in with the AP emission function. The total emission is given by combining the emission considering the AP generation, as given in Eq. (2), and emission considering the RP generation, as given in Eq. (13). The objective function becomes as given below.

$$\text{Minimize } F_2^{\text{total}} = \sum_{i=1}^{NG} F_2(P_{gi}) + F_4(Q_{gi}), \tag{23}$$

$$\text{Subjected to } \sum_{i=1}^{NG} P_{gi} - (P_D + P_L) = 0, \tag{24}$$

$$\sum_{i=1}^{NG} Q_{gi} - (Q_D + Q_L) = 0, \tag{25}$$

$$P_{gi}^{\min} \le P_{gi} \le P_{gi}^{\max} \quad (i = 1, 2, \ldots, NG), \tag{26}$$

$$Q_{gi}^{\min} \le Q_{gi} \le Q_{gi}^{\max} \quad (i = 1, 2, \ldots, NG). \tag{27}$$

2.5. Weight method

The objectives, as mentioned in Eq. (18) and Eq. (23), are of conflicting nature. Therefore, to minimize these conflicting objectives all together and to produce the non-inferior solution for the MOLDP, the weighting approach has been applied. Aggregation Eq. (18) and Eq. (23), using the weight method, can be written as given below [2].

$$\text{Minimize } \; F = \sum_{k=1}^{M} w_k F_k^{\text{total}}, \tag{28}$$

$$\text{Subjected to } \; \sum_{k=1}^{M} w_k = 1 w_k \ge 0, \tag{29}$$

where M denotes the number of objectives, w_k represents the levels of normalized weights in the range of 0 to 1.

3. Solution approach

3.1. Evaluation of objective function

Power balance constraints are satisfied by calculating the errors, which are given as:

$$E_1 = \sum_{i=1}^{NG} P_{gi} - (P_D + P_L), \tag{30}$$

$$E_2 = \sum_{i=1}^{NG} Q_{gi} - (Q_D + Q_L), \tag{31}$$

where P_D is the AP demand, P_L represents the AP losses, similarly Q_D is the RP demand, Q_L represents the RP losses. Errors as calculated in Eq. (30) and Eq. (31) are then added in Eq. (1), Eq. (2), Eq. (9) and Eq. (13) to penalize their fitness value and now changed to the following generalized forms:

$$F_1 = F_1(P_{gi}) + r \times (E_1)^2 \quad (i = 1, 2, \ldots, NG), \tag{32}$$

$$F_2 = F_2(P_{gi}) + r \times (E_1)^2 \quad (i = 1, 2, \ldots, NG), \tag{33}$$

$$F_3 = F_3(Q_{gi}) + r \times (E_2)^2 \quad (i = 1, 2, \ldots, NG), \tag{34}$$

$$F_4 = F_4(Q_{gi}) + r \times (E_2)^2 \quad (i = 1, 2, \ldots, NG), \tag{35}$$

where r is the penalty value taken as 10 000 in this problem.

Now the combined total cost and emission are given by

$$f_1^{\text{total}} = F_1 + F_3, \tag{36}$$

$$f_2^{\text{total}} = F_2 + F_4. \tag{37}$$

Now the objective of the problem is to

$$\text{Minimize } f = \sum_{k=1}^{M} w_k \left[f_k^{\text{total}} \right], \tag{38}$$

subjected to equality and inequality constraints, Eq. (24) to Eq. (27) and

$$\sum_{k=1}^{M} w_k = 1. \tag{39}$$

3.2. Decision making

The degree of a membership function is set between 0 and 1. The 0 value indicates inconsistency with sets, while 1 indicates full consistency. The fuzzy sets are represented by the equation called membership function $\mu(f_i)$, expressed as [2]:

$$\mu(f_i) = \begin{cases} 1; & f_i \le f_i^{\min} \\ \dfrac{f_i^{\max} - f_i}{f_i^{\max} - f_i^{\min}}; & f_i^{\min} < f_i < f_i^{\max} \\ 0; & f_i \ge f_i^{\max} \end{cases}. \tag{40}$$

In order to decide the best solution, K non-dominated values of membership values are calculated as:

$$\mu_D^K = \frac{\left[\sum_{i=1}^{M} \mu(F_i^K) \right]}{\left[\sum_{k=1}^{K} \sum_{i=1}^{M} \mu(F_i^k) \right]}. \tag{41}$$

The maximum value of membership μ_D^K, among all the fuzzy set is the 'best' solution

$$\text{Max} \left[\mu_D^K : k = 1, 2, \ldots, K \right]. \tag{42}$$

3.3. Algorithm for solution technique

As per the above discussion, the following practice can be used for executing the PSO algorithm.

```
– For each one particle $P_i$
    • Initially calculate the particle's position randomly in the lower and upper limits using the equations:
      $P_{ij} = P_j^{\min} + r_{ij}\left(P_j^{\max} - P_j^{\min}\right)$ and $Q_{ij} = Q_j^{\min} + r_{ij}\left(Q_j^{\max} - Q_j^{\min}\right)$.
DO
Assign the weights $W_1$ and $W_2$
($W_1 = 1$, $W_2 = 1 - W_1$)
    • Estimate the fitness of each particle using Eq. (38) and then find the minimum fitness out of each,
    • Assign all the initial positions as the particle's best known position (local),
    • Assign the global best position according to the minimum value to the local best fitness,
    • Initially calculate the velocity of particles within min-max boundaries.
DO
Increment iteration counter, IT = IT + 1, until a termination criterion is met, repeat.
    • Generate random vectors $R_1$ and $R_2$, modify the velocity using the equation:
      $v_{ij}^{\text{new}} = W \times v_{ij} + C_1 \times R_1 \times (X_{ij}^{\text{best}} - X_{ij}) + \left(C_2 \times R_2 \times (G_j^{\text{best}} - X_{ij})\right)$,
    • Modify the position using the equation: $X_{ij}^{\text{new}} = X_{ij} + v_{ij}^{\text{new}}$,
    • Evaluate the fitness using Eq. (38) based on new positions.
    • IF the new calculated fitness is less than the preceding calculated fitness
      THEN
    • Update the new positions as the local best position and the new fitness as a local fitness,
    • Find the minimum fitness from the local best fitness,
    • Modify the global best position according to minimum fitness value.
While (IT < $\text{IT}_{\max}$)
At the end, the best new position gives the global best solution.
– As per the global best values, compute $f_1^{\text{total}}$, $f_2^{\text{total}}$ using Eq. (36) and Eq. (37), respectively.
  $W_1 = W_1 - 0.1$, while ($W_1 < 0$)
– Compute the membership function from Eq. (40),
– Compute the fuzzy cardinal priority of the non-dominated solutions from Eq. (41),
– Choose the solution that achieves the maximum membership in the fuzzy set so obtained.
STOP
```

4. Results and discussion

The proposed algorithm discussed in section 3.3 has been tested on two test systems.

- Test System-I consist of 3 generating units whose input data is obtained from ref. [19].
- Test System-II consist of 10 generating units whose input data is obtained from ref. [26].

4.1. Results of Test System-I

Table 1 shows the derived values of RP cost coefficients, these values are derived using a curve-fitting technique as discussed in article 2.1.3.

As only economic objectives are considered in this test system, therefore the problem is solved only for ELD. Using input data from ref. [19] and Table 1, the proposed algorithm is applied on Test System-I. Table 2 shows the obtained value of the AP, RP, cost considering AP, cost considering RP and combined (active and reactive) operating cost of Test System-I.

Table 1. Derived fuel cost coefficients considering RP (Test System-I)

Gen. no.	a_{2i}^2	a_{1i}^1	a_{0i}^0	$Q_{\min}$	$Q_{\max}$
1	0.035	2.29	−65.04	−300	300
2	0.025	1.55	−48.44	300	300
3	0.038	2.40	−70.99	300	300

Table 2. AP Generation (P_G), RP Generation (Q_G), combined cost in $/h

Gen. no.	P_G**(MW)**	Q_G**(Mvar)**
1	112.824700	21.288760
2	128.743800	82.631210
3	73.431460	11.080020
Cost ($/h)	5250.3430	210.17720
Combined cost ($/h)	5460.5205	

4.2. Results of Test System-II

Table 3 shows the derived values of RP cost coefficients, emission coefficients, minimum and maximum limit of the RP, and these values are derived using a curve-fitting technique as discussed in article 2.1.4. Both economic and emission objectives are considered in this test system, therefore the problem is solved for the MOLDP. Using input data from ref. [26] and Table 3, the proposed algorithm is applied on Test System-II. To find the best solution in the MOLDP, the programme has run at different value of w_1 and w_2, the combined cost(active and reactive) and combined emission (active and reactive) are calculated corresponding to these weights. After calculating the combined cost and combined emission, the membership functions (μ_1, μ_2) and then membership function for non-dominated solutions (μ_D) are calculated. The maximum value of μ_D gives the best solution. When $w_1 = 1$ and $w_2 = 0$, the cost considering AP generation, cost considering RP generation and cost considering combined (active and reactive) generation comes out to be as minimum as 349867.900 $/h, 3645.783 $/h and 353513.683 $/h at the expense of increase in emission considering the AP generation, increase in emission considering the RP generation and increase in emission considering combined (active and reactive) generation as given by 109112.100 ton/h, 78597.310 ton/h and 187709.41 ton/h. The cost increases and the emission decreases when w_1 approaches between 1 and 0 and w_2 approaches between 0 and 1, at the end when $w_1 = 0$ and $w_2 = 1$, the AP emission, RP emission and combined (active and reactive) emission comes out to be as minimum (ED) as 295.747 ton/h, 709.770 ton/h and 1005.518 ton/h at the expense of increase in cost, considering an AP of 405270.1 $/h, cost considering an RP of 16498.020 $/h and a combined (active and reactive) cost of 421768.1 $/h. To find the best solution for multiobjective generation, scheduling the fuzzy cardinal ranking method has been applied,

Table 3. Derived fuel cost, emission coefficients considering RP (Test System-II)

Gen. no.	a_{3i}^{3}	a_{2i}^{2}	a_{1i}^{1}	a_{0i}^{0}	α_{4i}^{4}	α_{3i}^{3}	α_{2i}^{2}	α_{1i}^{1}	α_{0i}^{0}	$Q_{\min}$	$Q_{\max}$
1	−7.00E-05	8.00E-02	−1.61	25.8	3.00E-09	−3.00E-06	0.001	−0.016	0.068	0	300
2	−3.00E-05	6.10E-02	−9.18E-01	17.45	1.00E-08	−2.00E-05	0.009	−0.178	0.958	0	300
3	−7.00E-06	1.10E-02	−1.97E-01	3.751	9.00E-09	−2.00E-05	0.006	−0.119	0.646	0	300
4	−1.00E-05	1.90E-02	−3.26E-01	6.203	9.00E-09	−2.00E-05	0.006	−0.119	0.646	0	300
5	−4.00E-06	1.10E-02	−0.16	3.463	3.00E-07	0	0.237	−5.021	31.37	0	300
6	−4.00E-06	9.00E-03	−0.135	2.927	3.00E-07	0	0.237	−5.021	31.37	0	300
7	−3.00E-06	6.00E-03	−0.096	2.079	1.00E-06	−0.002	0.952	−20.43	123.6	0	300
8	−3.00E-06	6.00E-03	−0.096	2.079	1.00E-06	−0.002	0.952	−20.43	123.6	0	300
9	−3.00E-05	6.20E-02	−0.0963	18.3	2.00E-07	0	0.08	−1.547	7.799	0	300
10	−4.00E-05	8.80E-02	−1.32	25.18	1.00E-08	−2.00E-05	0.009	−0.178	0.958	0	300

Table 4. Power dispatch for ELD problem, ED problem and MOLDP (Test System-II)

Gen. no.	**ELD**		**ED**		**MOLDP**	
	AP(P_{gi}) MW	RP(Q_{gi}) Mvar	AP(P_{gi}) MW	RP(Q_{gi}) Mvar	AP(P_{gi}) MW	RP(Q_{g}) Mvar
1	104.431600	52.18288	486.6306	300	122.8789	300
2	300	0	560.4518	246.0391	300	188.075
3	653.225000	300	575.2646	300	687.8835	300
4	412.885300	203.5398	575.1877	300	478.822	300
5	424.346500	300	582.1768	13.15452	721.6416	68.1843
6	805.111300	300	582.1901	0.00E+00	737.7504	69.0105
7	900.000000	0	538.8544	11.79944	725.393	28.5934
8	900.000000	300	538.8039	11.79243	725.6301	28.7791
9	500	0	500	17.19974	500	91.8363
10	500	44.27737	560.4392	300	500	125.491
Cost ($/h)	349867.900	3645.783	405270.100	16498.020	352079.30	10511.19
Emission (ton/h)	109112.100	78597.310	295.747	709.770	3866.178	3074.07
Total cost ($/h)	353513.683		421768.1		362590.500	
Total emission (ton/h)	187709.41		1005.518		6940.249	

membership functions μ_1 and μ_2 are calculated then a membership function for non-dominated solutions (μ_D) is calculated. The maximum value of μ_D gives the best solution. For this problem at $w_1 = 0.6$ and $w_2 = 0.4$, it gives the best solution at a combined (active and reactive) cost of 362590.500 $/h and corresponds to a combined (active and reactive) emission of 6940.249 ton/h. The power generation dispatch that corresponds to economic load dispatch (for both AP and RP), emission dispatch (for both AP and RP) and multiobjective economic emission dispatch (for both AP and RP) is shown in the Table 4.

5. Comparison of results

To show the effectiveness of the proposed algorithm, two test systems are investigated. Firstly, the developed algorithm is tested on Test System-I and the results obtained are compared with the results obtained by ref. [19]. As seen in Table 5, it is found from the result that the combined (active and reactive) fuel cost (5460.5205 $/h) obtained from the proposed approach comes out to be less as compared to the combined (active and reactive) fuel cost (5690.612 $/h) calculated from the approach discussed by ref [19]. Then the developed algorithm is tested on Test System-II for an ELD problem, considering AP generation by keeping $W_1 = 1$, $W_2 = 0$. As seen from the results in Table 6, the cost of generation comes out to be minimum, which is 3.498×10^5 $/h, as compared

Table 5. Comparison of combined (active and reactive) cost obtained (Test System-I)

Gen. no.	**Proposed approach**		**Ref. [19]**	
	P_G(MW)	Q_G(Mvar)	P_G(MW)	Q_G(Mvar)
1	112.824700	21.288760	86.5714	34.3719
2	128.743800	82.631210	134.3834	47.4364
3	73.431460	11.080020	94.0452	33.1917
Combined cost ($/h)	5460.5205		5690.612	

Table 6. Comparison of cost and emission considering AP (Test System-II)

	ELD proposed approach		**ELD ref. [26]**		**ED proposed approach**		**ED ref. [26]**	
	P_G	Q_G	P_G	Q_G	P_G	Q_G	P_G	Q_G
Cost ($/hr)	3.498×10^5	3645.783	3.508×10^5	–	4.0527×10^5	16498.02	3.966×10^5	–
Total cost ($/h)	353513.683		–		421768.1		–	
Emission (ton/h)	1.091×10^5	78597.31	7.681×10^4	–	295.747	709.77	318.08	–
Total emission (ton/h)	187709.41		–		1005.518		–	

to ref. [26] cost – 3.508×10^5 \$/h, and corresponding emission, which is 1.091×10^5 ton/h, also found to be comparable with ref [26] emission – 7.681×10^4 ton/h. This shows that the proposed algorithm is effective to handle the ELD problem. After the ELD, the developed algorithm is tested for a ED problem by keeping $W_1 = 0$ and $W_2 = 1$, as seen from the results the emission comes out to be minimum, which is 295.747 ton/h, as compared to emission of ref [26] – 318.083 ton/h, and corresponding cost, which is 4.0527×10^5 \$/h, also found to be comparable with ref [26] cost – 3.966×10^5 \$/h. Since, as discussed in this paper, RP cost and emission is also important for fair calculation, so cost and emission based on the RP is also calculated. So, from the results it is clear that when problem is solved for the ELDP the actual cost (total cost for both active and RP) comes out to be 353513.683 \$/h at the expense of emission (total emission for both active and RP), 187709.41 ton/h. When the problem is solved for the ED the actual emission (total emission for both active and RP) comes out to be 1005.518 ton/h at the expense of cost (total cost for both active and RP), 421768.1 \$/h. The cost and emission calculated using a fair calculation approach is greater because of the addition of the RP cost and emission, which in turn may give a positive signal for stakeholders to think about investment in the RP supplies. This will result in a more safe operation of the system in the future, especially in restructured power systems.

6. Conclusion

In order to solve ELDP, the authors mainly focused on cost calculation based on AP generation since the generators also have RP requirement to meet the total power demand, so the generators have to supply the RP. The generation of the RP affects the real power output, therefore, for accurate calculations, it is an important to consider the RP cost along with the real power cost. Based on this cost function, considering the RP is formulated. A PSO algorithm is applied on Test System-I to solve the ELDP based on active and RP cost functions. The results obtained are compared with ref. [19] and are found better. In thermal power plants ED is the second main objective to be considered along with the ELD and the problem is formulated as an MOLDP. The PSO algorithm is applied on Test System-II and the obtained results are compared with ref. [26] and are found better. The authors till now have focused on formulating the RP cost function. In the MOLDP, emission is the second main objective to be considered. Just as reactive cost is important for fair cost calculation, similarly the RP contributes in emission therefore, it is important to consider the emission based on the RP along with the emission based on the AP, otherwise it may lead to the false calculation of emission. Therefore, the emission function based on the RP is formulated in this paper. MOLDP based conflicting objectives such as CAREcD and CAREmD are solved using the PSO algorithm. The weight method is applied to convert the MOLDP into a scalar problem and the best compromise solution is calculated using a fuzzy cardinal approach. The results obtained show there validity and effectiveness. In future, large power system networks considering more objectives such as voltage profile improvement, minimization of losses and voltage stability improvement (L-index) can be solved using different techniques such as the SLFA, bacteria foraging optimization algorithm (BFOA), hybrid PSO-SFLA, etc.

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