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EXPERIMENTAL APPROACH TO THE DESIGN OF PIEZO-ACTIVE STRUCTURE

The examination of a smart beam is presented in the paper. Experimental investigations were carried out for flexible beam with one fixed end and free opposite end. Piezoelectric strips were glued on both sides of the beam. One strip works as a sensor, and the second one as an actuator. It is a single input and single output system. The study focuses on the analysis of natural frequencies and modes of the beam in the relation to the position of the piezo-elements. The natural frequencies, mode shapes, generated control forces, and levels of the measured signals are considered and calculated as a functions of the piezo-element locations. We have found correlations between mode shapes, changes of natural frequencies, control forces and measured signals for the lowest four modes. In this way, we can find the optimal localization of the distributed sensors and actuator on the mechanical structure directly by the using of the finite elements method (FEM).

1. Introduction

In the design of vibration control of the mechanical structure we should solve the following problems.

1. The choice of the sensors and actuators and their localization along the structure.

2. Mathematical modeling of the structure as a control plant.

3. The choice of the control method and the design of the control law.

4. Implementation of the control system and its validation.

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In the paper, first of all we consider the first problem since it is also important for proper analysis and finding solutions to other problems.

The sensors and actuators can act at specific points on the structure, or can be distributed along the structure [1], [2]. The localization of the actuators influences the level of the exerted forces [3], while localization of the sensors has the influence on the power of the measurement signals [3]. The vibration process is sufficiently fast to reduce the number of actuators we can use to generate the control forces. Electromagnetic suspensions, magnetic bearings, piezo-strips, or piezo-stocks can be such actuators. Among these actuators, the piezo-strips can be considered as the distributed actuators.

Piezo-strips can be also used as distributed sensors. Distributed actuators and sensors act on the vibration control system in an unpredictable manner. This means that as long as optimal distribution of the piezo-stripes is not known, we do not know how these piezos act on the structure and how they influence the vibrations. Therefore, there is a need to find any simple tool to estimate this influence. The best solution would be to express this influence by an indicator which can be obtained by means of the finite element method (FEM) analysis of the structure. Presently such analysis is carried out during the design of each more loaded structure.

In the paper, we investigate the dynamics of the uniform beam without piezo-strips, and with two such strips fixed on it. One of them works as an actuator while the other one is used as a sensor. The dynamics is investigated for the different locations of the strips along the beam using both an analytical solution and the finite element solution. We find the natural frequencies, first four mode shapes, the generated control forces, and levels of the measured signals. We also calculate the correlations between mode shapes, changes of natural frequencies, control forces and measured signals for the four lowest vibration modes.

2. Finite element model of the smart beam

The laboratory model of the smart beam consists of a steel beam in cantilever configuration with two piezoelectric stripes bonded onto its surface. It is shown in Figure 1. The steel beam has the dimensions of 25×280×1 [mm], while the sensor and the actuator are single piezoelectric stripes of 25×56×0.38 [mm] and 25×56×0.75 [mm], respectively.

In the first step, the passive beam without piezoelectric elements was investigated to determine natural frequencies. For this purpose we used the software *Ansys*, and a proper finite element method was generated.

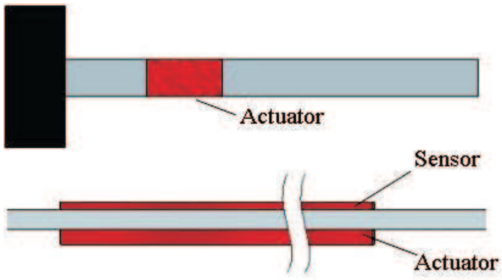

Fig. 1. The experimental smart beam

In the next step, in the same procedure we examined the smart beam. In this case, the finite element model (FEM) was rebuilt taking into account the piezoelectric parameters given in Table 1.

Table 1.

Parameters of structure

Parameters	steel beam	actuator	Sensor
dimensions [m]	0.28×0.025×0.001	0.056×0.025×0.00075	0.056×0.025×0.00038
mass [kg]	0.0546	0.00675	0.00342
Material	hard steel	Piezo-cristal	Piezo-cristal
volume [m^3]	$7^{*}10^{-6}$	$0.9375^{*}10^{-6}$	$0.475^{*}10^{-6}$
Young's modulus (E) [GPa]	200	0.18	0.18
Poisson's coefficient (ν)	0.3	0.28	0.28
density (ρ) [kg/m^3]	7800	7200	7200

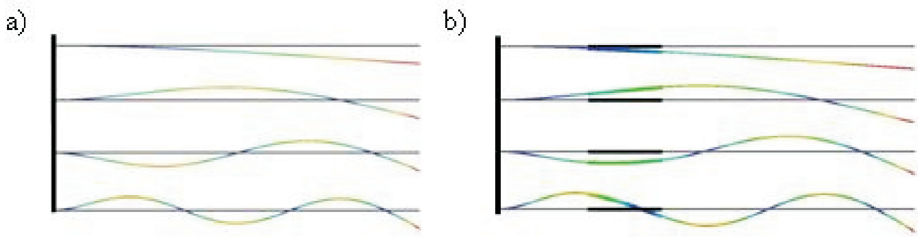

Fig. 2. Natural frequencies of examined beam: a) steel beam, b) beam with piezo-elements

The natural frequencies and modal shapes of the beam were calculated. The obtained results are shown in Fig. 2. Fig. 2a shows the mode shapes of the steel beam without piezo-elements. Fig. 2b shows the mode shapes for the same beam with piezo-elements located at a distance of 0.056 [m] from the fixed end (control force and strain for the fourth mode has a maximal value).

The first four natural frequencies of the steel beam and the smart beam are given in Table 2. We can notice that adding the piezo-elements to the structure results in its greater stiffness. This causes a slight increase of the natural frequencies. The reason for this is that the length of the piezo-elements is relatively small in the comparison to the length of the whole beam (the ratio is 0.17).

Table 2.

First four natural frequencies of the beam

freqencies [Hz]	passive beam	smart beam	Increase [%]
f1	10.51	10.82	2.94
f2	64.14	65.83	2.63
f3	184.42	191.58	3.88
f4	361.82	372.95	3.08

For further calculations, the smart beam was divided into ten equal segments. The first segment, located on the free end of the beam had the smallest stiffness. The tenth segment, located on the clamped end of beam had the greatest stiffness. We divided the beam into 10 segments, because the piezo-elements were moved with steps equals to half the length of the piezo-stripes (0.028 [m]). The divided beam is shown in Fig. 3.

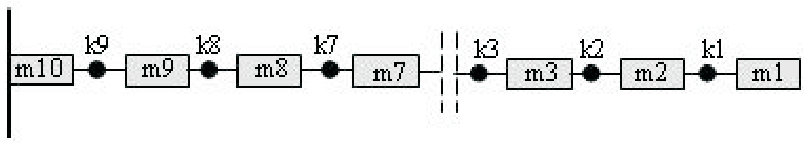

Fig. 3. Divided beam

Table 3.

Calculated coordinates of elements

	Location of piezo-stripes (center point)								
	0.028	0.056	0.084	0.112	0.140	0.168	0.196	0.224	0.252
amplitude for first mode	0.133	0.53	1.17	1.96	2.88	3.91	5.00	6.25	7.44
amplitude for second mode	−0.75	−2.56	−4.53	−5.81	−6.09	−5.05	−2.81	0.68	4.61
amplitude for third mode	1.93	5.13	6.22	4.06	−0.078	−4.00	−5.44	−3.14	2.19
amplitude for fourth mode	−3.2	−5.67	−3.17	3.57	5.82	2.22	−3.4	−5.24	0.018

The total length was measured from the fixed end. In the next step, another simulation was performed. The displacement coordinates were measured for the four lowest natural frequencies. The obtained results are collected in Table 3 and the modes are plotted in Fig. 4. The results are the same as in the case of FEM. The horizontal axis shows the next center points location of piezo-stripes on the beam, and the vertical axis shows the deflection amplitude. In accordance to the vibration theory of beam the values in points describe the shapes of the vibration modes.

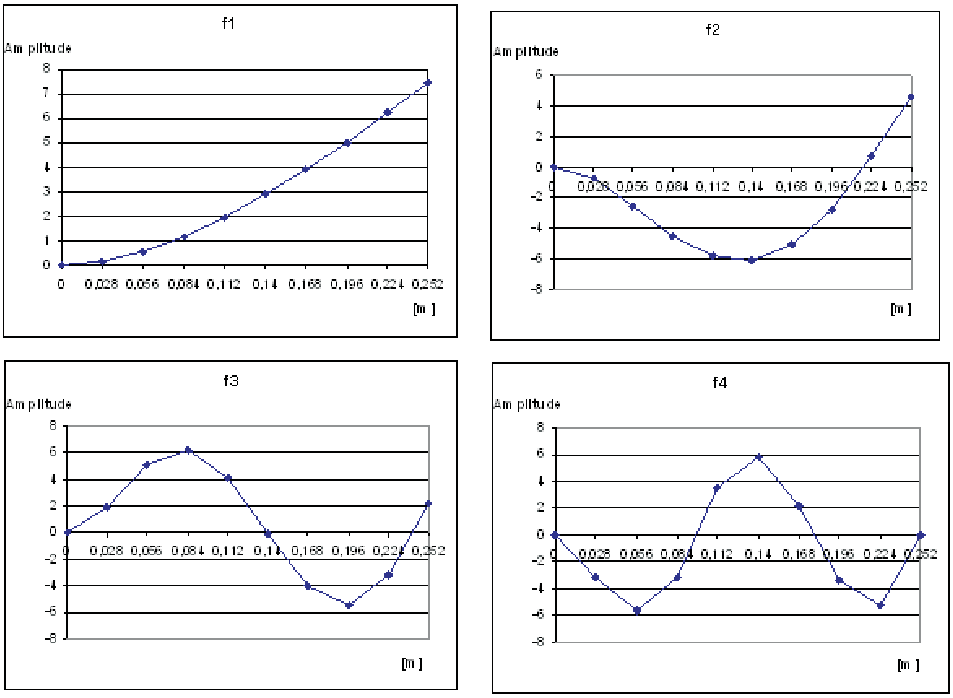

Fig. 4. Calculated coordinates of divided beam

3. The influence of piezoelement position on the beam natural frequencies

It is well known that location of actuators and sensors has a significant influence on vibration control system of mechanical structures. It is much easier to determine such influence in the case of point actuators and sensors. Piezoelectric actuator and sensor are distributed-parameter elements. In this section, we try to find a simple method, which would allows us to find the best location for such elements. First we will check how location of the elements influences natural frequencies of a smart beam. In order to do so, both sensor and actuator are shifted from the fixed end to the opposite end in 28 mm steps. Fig. 5 shows the start and end position of piezo-elements. In every step (repeated 9 times), the first four natural frequencies are calculated.

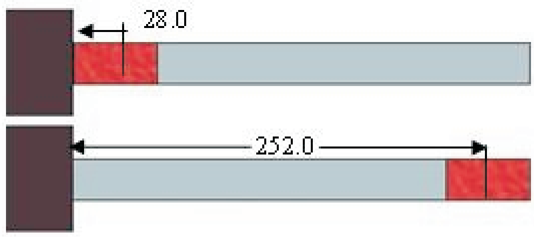

Fig. 5. Examination of piezo-element position influence

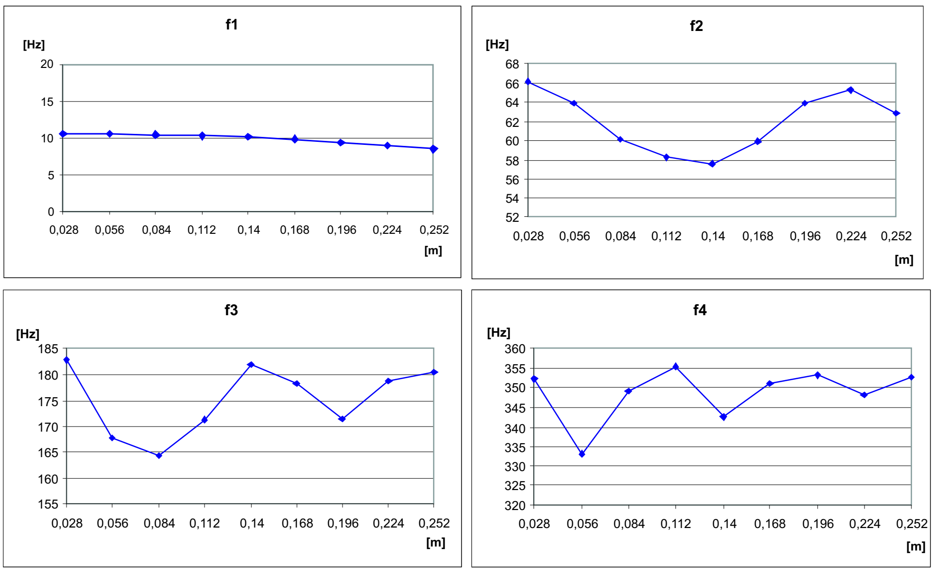

Fig. 6. Natural frequencies of the smart beam versus the piezo-element position

The obtained results are presented in Fig. 6. The first mode is shown in the top left corner and the fourth mode is shown in the bottom right corner. Only the first natural frequency monotonically falls when the sensor and the actuator depart from the fixed end of the beam. Other three frequencies oscillate around average values. It appears that all the natural frequencies of the smart beam depend on the piezo-elements position. On the one hand, these piezo-elements act as an additional masses, and on the other hand, they change the mechanical parameters of the beam.

4. Control forces

A question arises whether the above result can give us the information about the best position of sensors and actuators in the control system. We consider the smart beam where the bending moment of piezoelement is represented by three forces as it is shown in Fig. 7.

The model of the piezoelements was considered as a "static coupled model" [7] with the difference that the whole piezoelement was divided into two equal segments. In each segment the bending moment was represented by a couple of opposite forces concentrated at the segment's edge. Such a modeling of an actuator is allowable in the case when the distances between nodal points of controlled modes are greater than a length of the segment. Therefore, a force equal to Q is introduced at position x_{p1}, at x_{p2} – force 2Q and at x_{p3} – force Q.

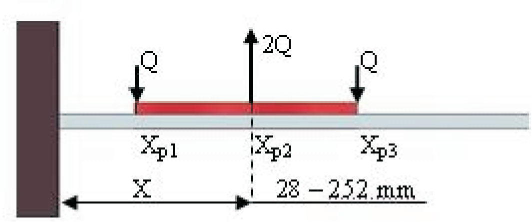

Fig. 7. Forces generated by the piezoelement

The equation of forced vibrations of the beam has the well-known form [4]:

$$\frac{\partial^4 y(x,t)}{\partial x^4} + a_z^2 \frac{\partial^2 y(x,t)}{\partial t^2} = \frac{1}{EI} Q_A \left(-\delta\left(x - x_{p1}\right) + 2\delta\left(x - x_{p2}\right) - \delta\left(x - x_{p3}\right)\right) \tag{1}$$

where δ is the Dirac function. After modal transformation

$$Q_A \left(-\delta\left(x - x_{p1}\right) + 2\delta\left(x - x_{p2}\right) - \delta\left(x - x_{p3}\right)\right) = \sum_{n=1}^{\infty} Q_{An} U_n(x) \tag{2}$$

the modal transversal force of piezo-actuator is represented by the equation

$$Q_{An} = Q_A \left(-U_n\left(x_{p1}\right) + 2U_n\left(x_{p2}\right) - U_n\left(x_{p3}\right)\right) \tag{3}$$

The modal force in function of the piezo-actuator location is shown in Fig. 8.

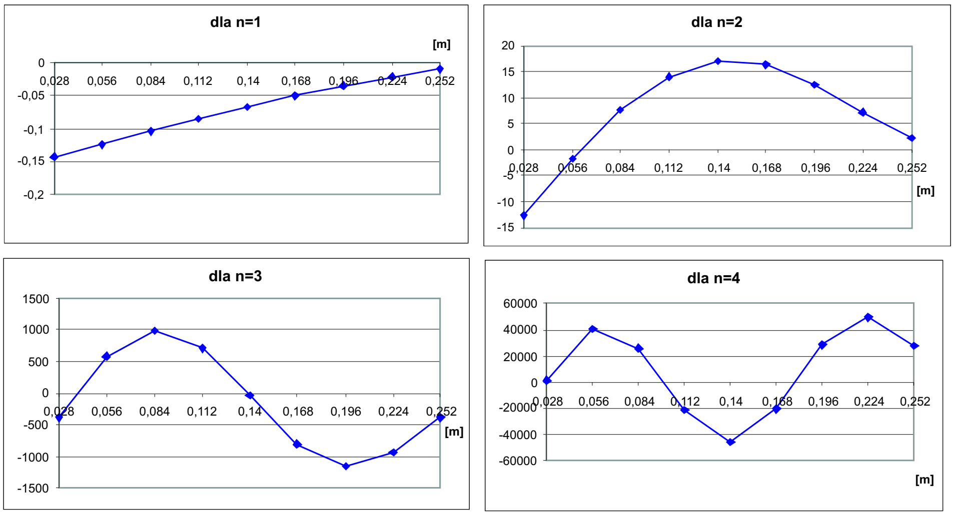

Fig. 8. Forces generated by piezo-actuator

5. Measurement signals

The extension of piezo-sensors results in voltage signal generated on its walls. Let us assume that piezo-element in Fig. 7 is now a piezo-sensor. The extension is the result of angular deformation of the beam, which produces strain in the piezo-sensor. The angular deformation of the piezo-sensor will be considered separately for both of its parts indicated in Fig. 7. We express the unit elongation of each part in terms of angular deformations in the following form:

$$\varepsilon_1(t) = -\frac{\Delta r T(t)}{L}\left(\left.\frac{\partial U(x)}{\partial x}\right|_{xp2} - \left.\frac{\partial U(x)}{\partial x}\right|_{xp1}\right) \tag{4}$$

$$\varepsilon_2(t) = -\frac{\Delta r T(t)}{L}\left(\left.\frac{\partial U(x)}{\partial x}\right|_{xp3} - \left.\frac{\partial U(x)}{\partial x}\right|_{xp2}\right) \tag{5}$$

The total unit elongation is a sum of unit elongations of both parts of the piezo-sensor:

$$\varepsilon(t) = \varepsilon_1(t) + \varepsilon_2(t) \tag{6}$$

where $U(x)$ is the function of mode shape, Δr is radius of the beam curvature, $T(t)$ is the time function of mode vibrations.

All unit elongations are presented in Fig. 9 for four modes. The elongations for the first mode are shown in the top left corner and for the fourth mode the elongations are shown in the bottom right corner. In proper easier interpret the graphs, the variable in horizontal axis shows the next positions piezoelectric's centre, and that in vertical axis is corresponding value of deformation. Obtained results for measurement signals are very similar to the control forces but they have the opposite phase.

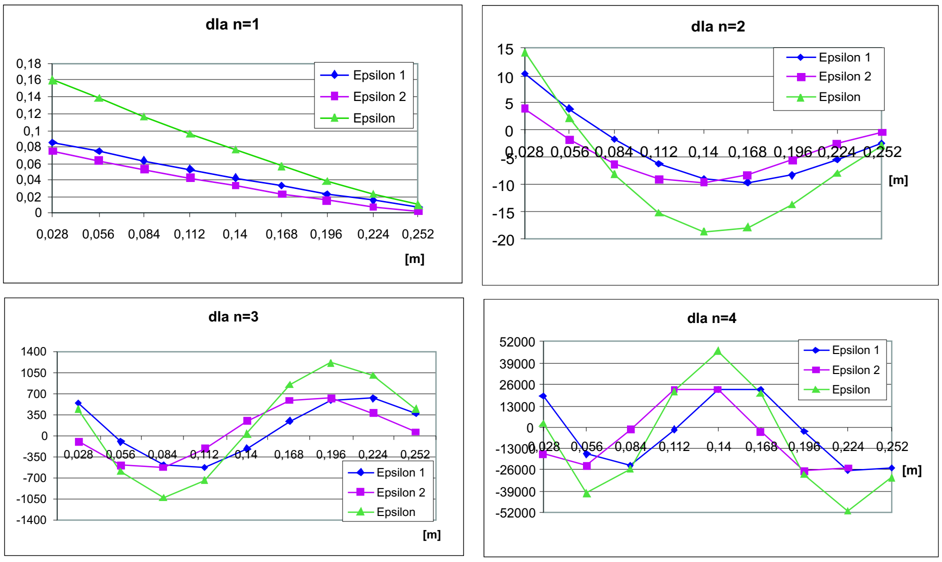

Fig. 9. Unit elongations (strains) of piezo-sensor versus its location. The maximal point of the green line indicates the best position of the piezo-element from measurement side (for a given mode)

6. Corelation between obtained results

In this section, we collect and compare the above results. As a tool for the comparison we used correlation matrices. The Matlab software was used to design four matrixes. Individual matrices are associated with the four lowest vibration modes. Every matrix has nine rows which represent the centre position of the piezoelectric element along the beam and four columns which contain values of: control forces, mode shapes, natural frequency changes, and unit elongations. Using these matrices, we obtained correlation coefficients between control force, mode shape, changes of natural frequencies and unit elongation for each vibration mode. The correlation coefficients are presented in Tables 4–7.

We can notice that all variables are strongly correlated, except of the variable describing the changes of natural frequency. The frequency changes have twice more extreme points than other variables. So, after some modifications (for example the correlation with half of the values of the natural frequency changes) we could also obtain high correlation for this variable. We can ten find the best localization of the sensors and actuators by the analysis of the structure with the help of FEM.

Table 4.

Correlation coefficients for the first natural frequency

	Control force	Mode shape	Freq. change	Elongation
Control force	1.0000	0.9614	−0.9016	−0.9927
Mode shape	0.9614	1.0000	−0.9843	−0.9783
Freq. change	−0.9016	−0.9843	1.0000	0.9287
Elongation	−0.9927	−0.9783	0.9287	1.0000

Table 5.

Correlation coefficients for the second natural frequency

	Control force	Mode shape	Freq. change	Elongation
Control force	1.0000	−0.5596	−0.7579	−0.9999
Mode shape	−0.5596	1.0000	0.6926	0.5464
Freq. change	−0.7579	0.6926	1.0000	0.7525
Elongation	−0.9999	0.5464	0.7525	1.0000

Table 6.

Correlation coefficients for the third natural frequency

	Control force	Mode shape	Freq. change	Elongation
Control force	1.0000	0.9279	−0.5959	−0.6301
Mode shape	0.9279	1.0000	−0.4444	−0.6765
Freq. change	−0.5959	−0.4444	1.0000	−0.0705
Elongation	−0.6301	−0.6765	−0.0705	1.0000

Table 7.

Correlation coefficients for the fourth natural frequency

	Control force	Mode shape	Freq. change	Elongation
Control force	1.0000	−0.9128	−0.1654	−0.9989
Mode shape	−0.9128	1.0000	0.2387	0.8934
Freq. change	−0.1654	0.2387	1.0000	0.1541
Elongation	−0.9989	0.8934	0.1541	1.0000

7. Conclusions

In this paper, we have shown strong correlation between the optimal location of distributed sensors and actuators and the results of the finite element analysis (FEM). By using FEM analysis, we obtained changes of natural frequencies and vibration mode shapes for different localization of piezo-element strips. In the future work we will find the following:

1. The optimal localization of a single strip used for control/measurement a few modes (SISO).
2. The optimal localization of several strips used for control/measurement of many modes (MIMO).
3. Identification method for obtaining a control plant model without spillover effects.
4. Design method of control law.

Such approach do the vibration control system design can be called the fast prototyping of the smart mechanical systems.

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Eksperymentalne podejście przy projektowaniu układów aktywnego sterowania drganiami konstrukcji z wykorzystaniem elementów piezoelektrycznych

S t r e s z c z e n i e

Szybkie prototypowanie układu aktywnego sterowania drganiami wybranych konstrukcji z wykorzystaniem elementów piezoelektrycznych realizowane jest w czterech etapach:

1. Określenie optymalnego położenia piezo-elementów pomiarowych i wykonawczych na konstrukcji dla przyjętych kryteriów.
2. Przyklejenie piezoelektryków i eksperymentalna identyfikacja modelu układu otwartego.
3. Zaprojektowanie praw sterowania i ich implementacja w wybranych sterownikach.
4. Weryfikacja eksperymentalna działania układu zamkniętego.

W artykule skoncentrowano się na rozwiązaniu problemów związanych z pierwszym etapem. Badania przeprowadzono dla stalowej belki wraz przyklejonymi do niej obustronnie piezo-elementami. Jeden z piezo-elementów pracuje jako aktuator drugi zaś jako sensor. Przesuwając paski piezoelektryczne wzdłuż belki wyliczono zmiany wartości naturalnych częstotliwości własnych drgań belki,

postaci drgań, sił sterujących oraz sygnałów pomiarowych. Na podstawie zebranych danych symulacyjnych zostały utworzone macierze korelacyjne dla pierwszych czterech postaci drgań. Wszystkie prezentowane badania zostały wykonane z wykorzystaniem metody elementów skończonych. Wyznaczono położenia piezoelektryków dla których uzyskuje się największe wartości modalnych sił sterujących i modalnych sygnałów pomiarowych.