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ON THE PROBLEM OF UNKNOWN HISTORY IN HEALTH MONITORING

A crucial feature in health monitoring of already existing structures is to be seen particularly in identifying their topical internal structural parameters and controlling their remaining bearing capacity in the course of ageing processes. This is commonly carried out by measuring the deformations/strains caused by test-loading and calculating the parameters on the basis of the metered data.

In the case of elastic response of materials, the information on the parameters is directly related to the time of measurement; in the case of visco-elastic response, however, the history of the time-depending structural response during the period between initial loading and initiating the test-measurements is generally unknown. The problem exists, then, to separate the superimposed strains due to the existing state and to the test-load. For solving the problem, at first the relevant relations between stress/strain and the visco-elastic parameters are considered. Then a procedure will be described how to determine the strain state owing to the test-load only and to calculate the relevant parameters as functions of time. According to the principle of time-shift invariance, the results describe the time-depending response of the visco-elastic material, no matter at which time the loads are applied.

The presented method will be illustrated by two simple but instructive examples.

1. Introduction

A crucial feature in health-monitoring of existing structures that have taken part in an operation for a longer or shorter time is to be seen particularly in identifying the topical state of the characteristic structural parameters, the so-called control-parameters like the material, geometrical, stiffness and compliance-parameters, eigenfrequencies etc. During the time of operation, degradation of the bearing capacity can be observed, indicated by changes of the control-parameters due to different influences like physical and chemical

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deteriorations, changes in loading and environmental conditions, wear, ageing, fatigue etc [1]. The knowledge of the topical parameters is required to assess the remaining bearing capacity and to estimate at least the probability of failure and the rest-service-life [2].

To obtain the information necessary for quite reliable assessment of existing structures, commonly defined test-loads (and/or accelerators) are applied, and the relevant deformations/strains (or frequency-spectra) caused by the test-loading are measured. These metered and eventually statistically processed data are inserted into the respective analytical equations to calculate the control-parameters in an inverse process [3].

Considering a static problem and presupposing linear elastic response of material, the measurements immediately yield the displacements/strains related to the topical values of the parameters at the time of test-loading in view of the state of degradation no matter how long the structure exists already (Fig. 1).

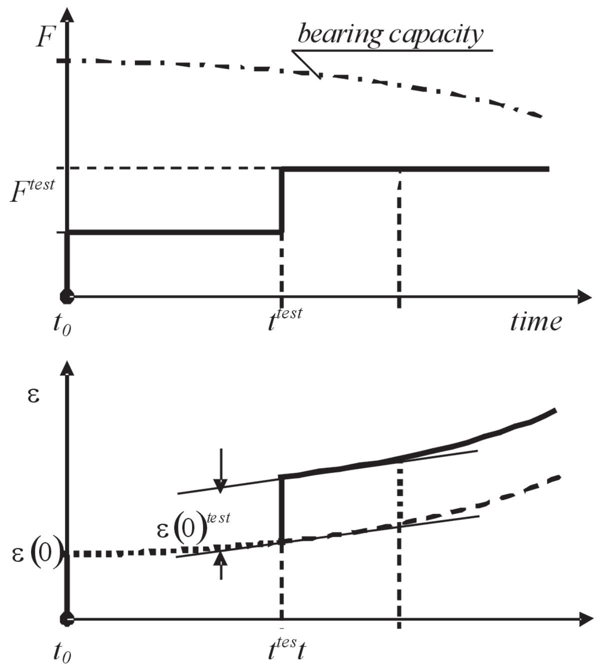


Fig. 1. Evolution of $\varepsilon(t)$ caused by test-load, linear elastic response of material

In the case of visco-elastic material response, however, particular reflections are necessary, if the history of the structure is unknown. The history covers the time-period between the time of initial loading and initiating the monitoring process, includes the changes of loading and environmental conditions, the course of the time-depending visco-elastic parameters and, although of minor influence, the degradation effects. The measurements yield the elastic part of the displacements/strains caused by the test-load at the time of application and the initial visco-elastic parameters but neither the information on the course of the displacements and strains, respectively, owing to the test-load only, nor on the time-functions of the parameters (Fig. 2). The problem must be solved to separate the displacements/strains caused by the test-load from the altogether metered displacements/strains, because this is a presupposition to determine the material time-functions. However, before presenting a solution, we will need to point out some relevant relations concerning the theory of visco-elasticity.

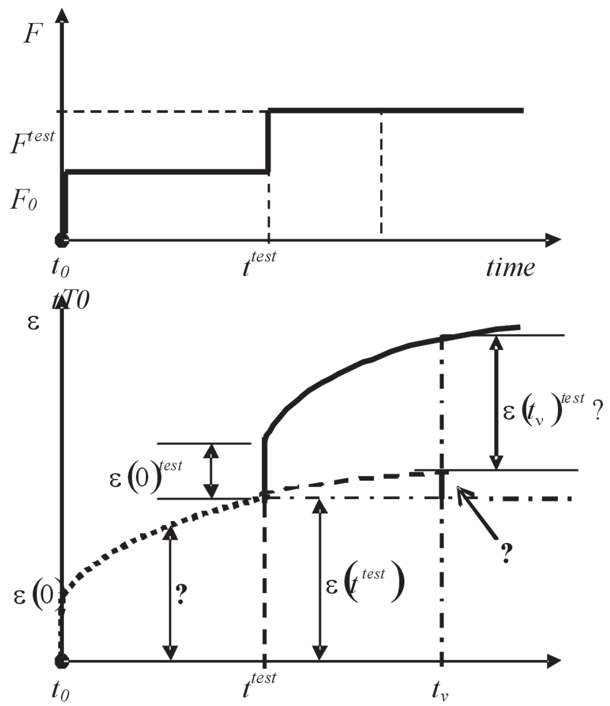


Fig. 2. Evolution of $\varepsilon(t)$ caused by test-load, linear visco-elastic response of material

2. Relations concerning the theory of visco-elasticity

According to the theory of linear visco-elasticity [4], [5], the strain state $\varepsilon(t)$ in a one-dimensional stress-strain state is always proportional to the stress state, for given σ_0 = const yielding the function of the creep compliance

$$J(t) = \varepsilon(t)/\sigma_0 \tag{1}$$

If, however, the stress state depends on the strain state, as for instance because of the effects of lateral contraction,

$$\sigma(t) = \sigma_0 \cdot f[\varepsilon(t), \mu] \tag{2}$$

the stress-strain relations are described by the hereditary integral

$$\sigma(t) = K(0^+) \cdot \varepsilon(t) - \int_{0^+}^{t} \frac{d}{d\tau} K(t-\tau) \cdot \varepsilon(\tau) \cdot d\tau \tag{3}$$

where $K(t)$ denotes the relaxation modulus. This equation belongs to the class of VOLTERRA's integral equations of the 2^{nd} kind. Such equations are well-posed and have a unique and stable solution always in an appropriate setting, which means that for $0 \le \tau \le t$ the functions are continuous in the time space [6]. As the strain data are taken in discrete intervals Δt, it is advisable to transform eq. (3) into a discrete formulation [7]:

$$\sigma_0 \cdot f[\varepsilon(t), \mu] = \frac{1}{2} \left\{ K(0) \cdot [\varepsilon(t_n) - \varepsilon(t_{n-1})] - \sum_{i=1}^{n-1} K(t_n - t_i)[\varepsilon(t_{i-1}) - \varepsilon(t_{i+1})] + \right.$$
$$\left. + K(t_n) \cdot [\varepsilon(t_i) + \varepsilon(0^+)] \right\} \tag{4}$$

The solution of this equation yields the relaxation modulus $K(t)$, and with that the creep compliance $J(t)$ is given also, because both these parameters are depending on each other; in the LAPLACE-transform the relation holds with p the LAPLACE variable

$$J(p) \cdot K(p) = p^{-2} \tag{5}$$

To complete the picture, some remarks are added concerning general stress states. The components of the stress tensor are to be expressed as functions of the strain tensor, the bulk modulus $K(t)$, the relaxation shear modulus $G(t)$ and POISSON's ratio $\mu(t)$.

$$\sigma_{ij}(t) = \Phi\left[K(t), G(t), \mu(t); \varepsilon_{ij}(t),\right]; i, j \in [1/3] \tag{6}$$

POISSON's ratio is related to both the other parameters; in the LAPLACE-transform the relation reads

$$\mu(p) = \frac{1}{3}(3K(p) - 2G(p)) \cdot (6K(p) + 2G(p))^{-1} \tag{7}$$

With eq. 7, transformed into the time-space, eq. 6 becomes nonlinear. However, it turns out that POISSON's ratio is almost constant over time, which has been proved by many researchers as well as by the author himself [8].

The functions $K(t)$ and $G(t)$ are to be calculated on the basis of equilibrium conditions like for instance considering

a) a plane plate: $\sigma_{\alpha\beta,\beta} = 0, \quad \alpha,\beta = 1,2$

b) a plate in bending [8]: $m_{\alpha\beta,\beta} - q_\alpha = o, \quad \alpha,\beta = 1,2$

3. Method for strain-separation

As already described above, the metered strain data consist of two parts, the strain increasing owing to the initial state and loading of the structure on the one hand and the effects caused by the test-loading on the other hand. To determine the material time-functions, both these parts are to be separated from each other, as will be described in the following.

Beginning at an arbitrary reference time t_A before applying the test load, increments $\Delta\hat{\varepsilon}(t); t_A < t_\nu \leq t_N$ are measured in time intervals Δt (Fig. 3). The symbol "roof" denotes measured quantities.

The data are then to expand in a polynomial series

$$\Delta\tilde{\varepsilon}(t) = \sum_{\kappa=0}^{k} a_\kappa \cdot t^\kappa, \tag{8}$$

where a_k denote the coefficients, which are to be calculated according to the GAUSSIAN least-square method on the basis of $\Delta\hat{\varepsilon}(t_\nu)$, carrying out the calculations proceeding stepwise from $t_\nu$to $(t_\nu + \Delta t_\nu)$, beginning at t_N^+

After applying the test-load at time t_N, the measurement yields $\hat{\varepsilon}\left(t_N^+\right)$ and immediately the elastic strain caused by the test-load

$$\varepsilon(0^+) = \hat{\varepsilon}\left(t_N^+\right) - \Delta\hat{\varepsilon}(t_N) \tag{9}$$

For $t_\nu, \nu > N$ the increments $\Delta\hat{\varepsilon}(t_\nu)$ can not be measured separately; they are to extrapolate stepwise by means of the balancing polynomial, eq. (8).

$$\Delta\tilde{\varepsilon}(t_\nu) = \Delta\tilde{\varepsilon}(t_{\nu-2}) + 2\sum_{\kappa=0}^{k} \frac{d}{dt}(a_\kappa \cdot t^\kappa)\,\Delta t \tag{10}$$

Finally, the temporal course of strains related to the test-load only is determined in discrete intervals Δt

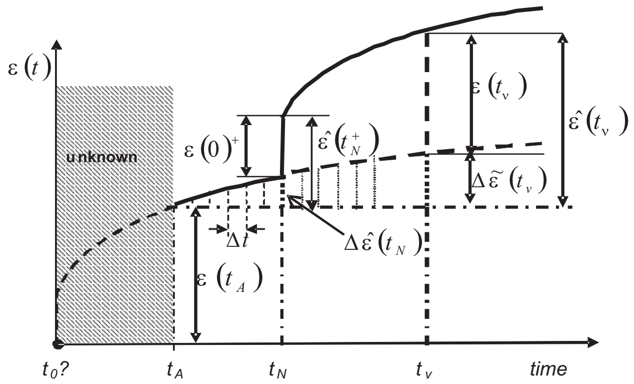


Fig. 3. Principle of the procedure to determine $\varepsilon\,(t_\nu)$ referring to the test-load in case of unknown history.

$$\varepsilon\,(t_\nu) = \hat{\varepsilon}\,(t_\nu) - \Delta\tilde{\varepsilon}\,(t_\nu)\,;\, t_\nu > t_N \tag{11}$$

The course of the described procedure is presented in Fig. 4.

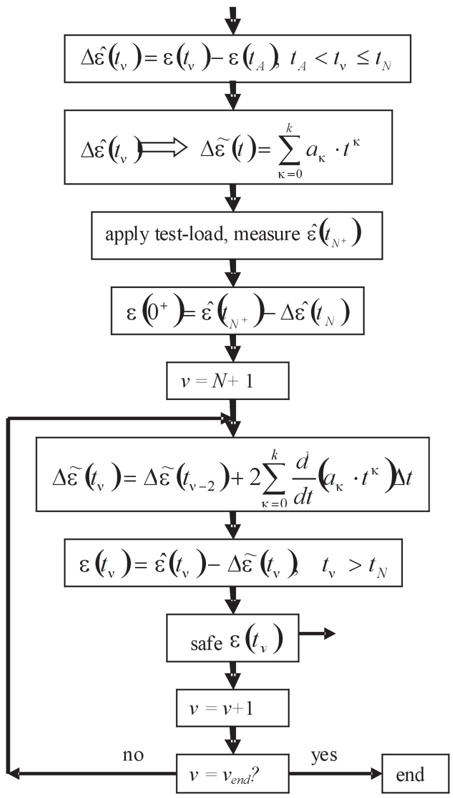


Fig. 4. Flow-diagram to calculate $\varepsilon\,(t_\nu)$ referring for the test-load

It must be mentioned that depending on the response of the respective materials and on the time the considered structure exists already, the creep-effects can fade away more or less. The gradients of the characteristic functions may approach asymptotically a constant or even approach zero, which can be proved by the metered results in the period t_A to t_N. In such cases, the evaluations can be done without a balancing polynomial (Fig. 5).

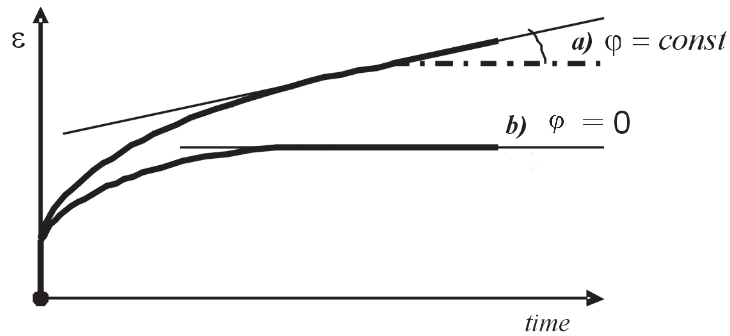


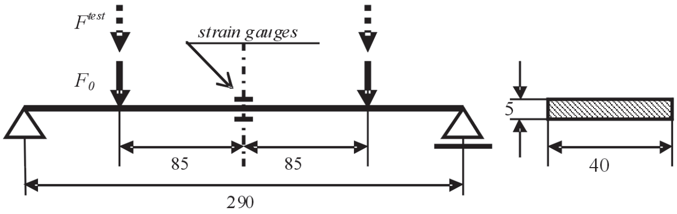
Fig. 5. Upper and lower bound creep curves

Having determined the course of strains related to the test-load in discrete time-intervals, these values are inserted into the relevant visco-elastic relations the solution of which then yields the characteristic parameters $K(t)$, $J(t)$, and $G(t)$, respectively. According to the principle of time-shift invariance in linear visco-elasticity, these functions describe completely the time-depending response of the material, no matter at which time loads are applied to the structure, although the history prior to the test is unknown. The method described above will be illustrated by two simple but instructive examples.

4. Examples

As example no.1, a one-dimensional four-point bending experiment was chosen, the specimen was made of polymer material FRP 2X. The dimensions, the loading conditions and the principle of the experimental set-up are given in Fig. 6.

The deformation of the cross-section due to lateral contraction has been proved to be negligibly small, [9]. Therefore, the impressed stress $\sigma_1 1$ remains constant throughout the time. The strains $\varepsilon_{11}(t), \varepsilon_{22}(t)$ were measured by electrical strain gauges, 90°-rosettes, applied at the centre of the beam on the upper and lower surface. POISSON's ratio was found equal to $\mu = 0.42 \approx$ const. For the reason of comparison, the strains were measured in



material: *FRP 2X;*
initial load $F_0 = 2\times1N$; test load $F^{test} = 2\times1N$;
initial bending moment $M_0 = 60\ Nmm$, stress $\sigma_{11}(0) = \pm0.36 N/mm^2$

Fig. 6. Example no. 1. Beam under 4-point loading

two epochs. In the 1st epoch, the specimen was loaded by $F_1 = 2\times1$ N. After 20 min the test-load $F^{test} = 2\times1$ N was applied. The evolution of metered strain $\varepsilon_{11}(t)$ over the two epochs is shown in Fig. 7.

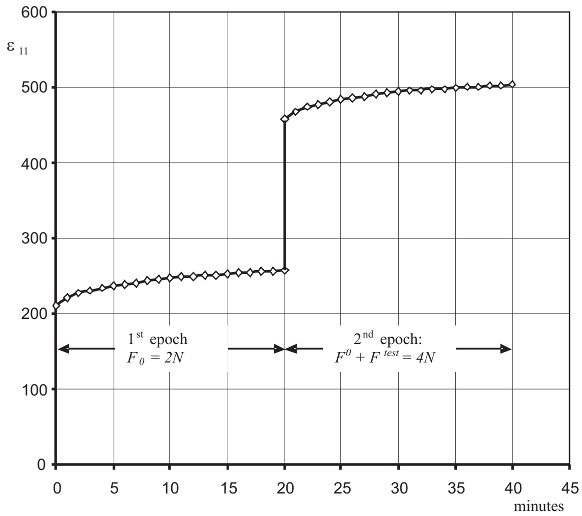


Fig. 7. Evolution of the metered strain over two epochs

The experiment was repeated, the increments $\Delta\hat{\varepsilon}_{11}(t)$ related to the reference time $t_A = 14$ min. were taken as input data to calculate the coefficients of the balancing polynomial, eq. (11), extrapolating $\Delta\tilde{\varepsilon}(t_v), v \geq N$ step by step in intervals Δt.

The separation process has yielded the evolution of the strain $\varepsilon_{11}(t_v)$; $t_v > t_N$, caused by the test-load only (Fig. 8). With these values the relaxation modulus $K(t)$ was calculated according to eq. (4) (Fig. 9). For comparison the function of the relaxation modulus was determined also on the basis of the metered course of strain in the 1st epoch. The correspondence can be accepted as quite sufficient. The deviations might be caused by truncation errors in the extrapolation process. Following the creep compliance $J(t)$ has been calculated also (Fig. 10).

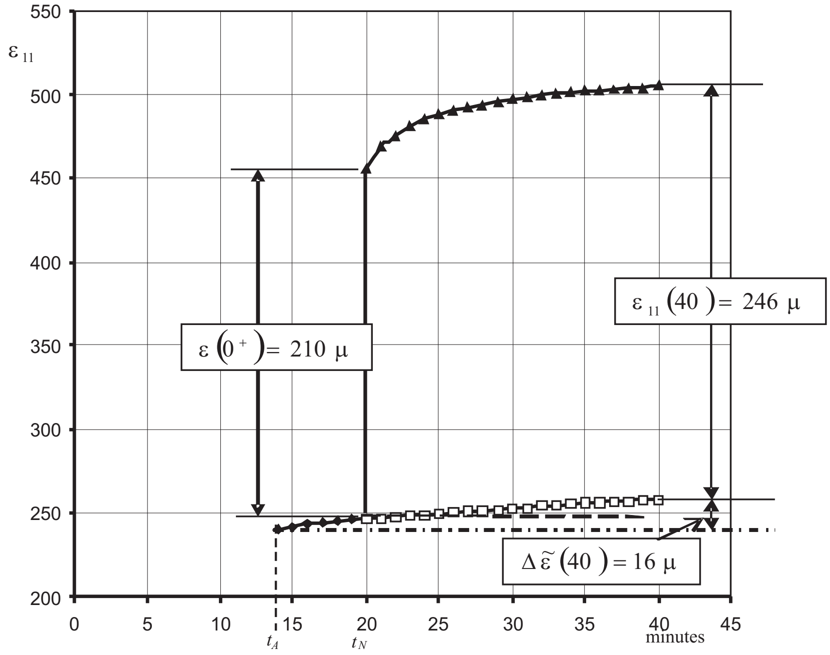


Fig. 8. Determination of the course of strain due to the test-load

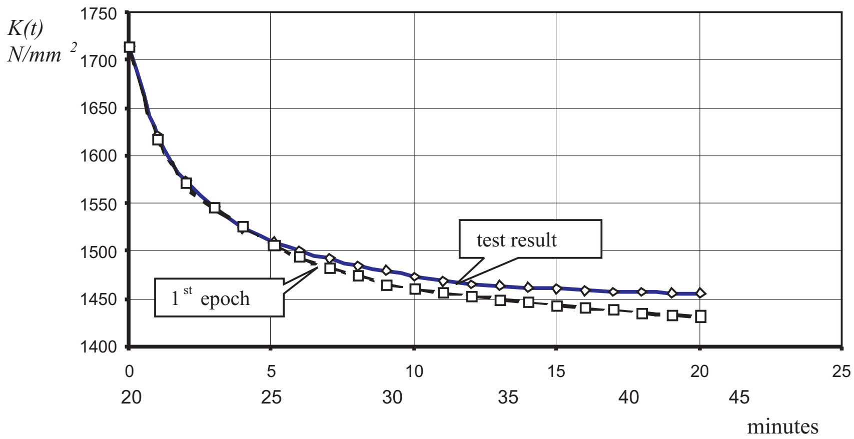


Fig. 9. Evolution of $K(t)$ due to the test-loading; for comparison: $K(t)$ referring to the 1st epoch

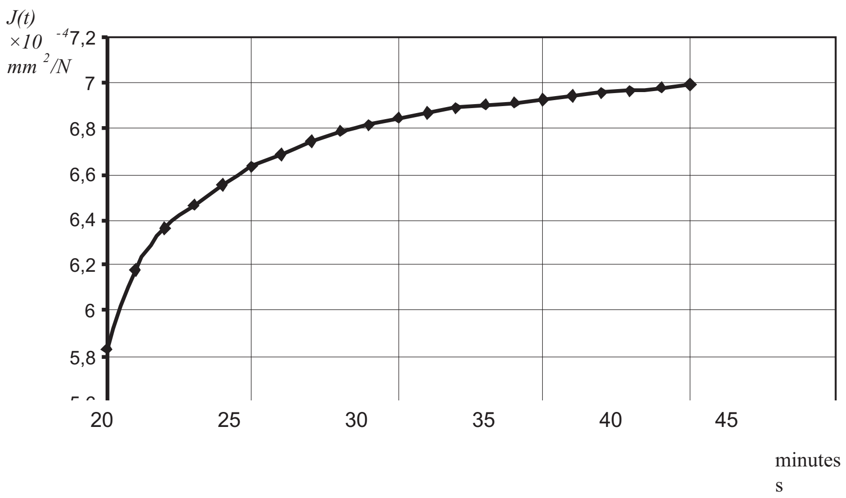


Fig. 10. Evolution of the creep compliance $J(t)$

The example no. 2 refers to tensile testing of thin polymer film material NYLON, part of a Chinese/German cooperation project. The dimensions of the specimen and the loading conditions are given in Fig. 11.

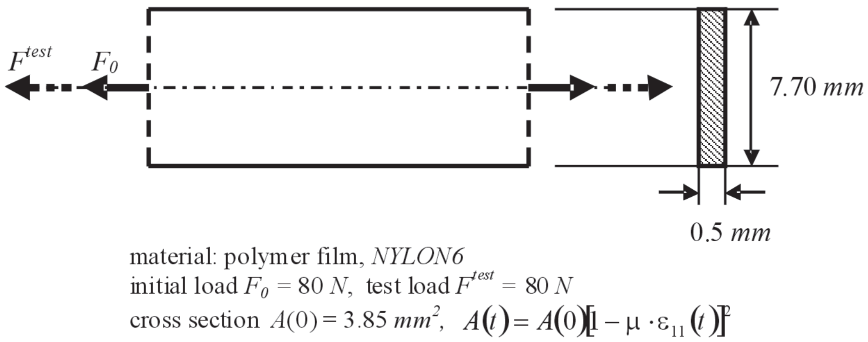


Fig. 11. Example no.2: Tensile test

The strains $\varepsilon_{11}(t), \varepsilon_{22}(t)$ were measured by means of the *marker-identification method*, POISSON's ratio was found to be $\mu \approx 0.40 \approx$ const. The lateral contraction was taken into account.

$$\sigma_{11}(t) = 20.89\,[1 - \mu \cdot \varepsilon_{11}(t)]^{-2}\ \mathrm{N/mm^2}$$

Similarly as in the first example, the measurements were carried out in two epochs (Fig.12).

However, it turns out that the gradient of the creep curve at the time t_N of test-loading tends to be constant already. Determining the strain $\varepsilon_{11}(t_v)$;

$t_v > t_N$, caused by the test-load only, one could abstain from applying a balancing polynomial (see Fig. 5). The further evaluation has yielded the relaxation modulus $K(t)$ related to the test-loading. The evolution (Fig. 13) proves the almost perfect correspondence to the modulus related to the 1st epoch.

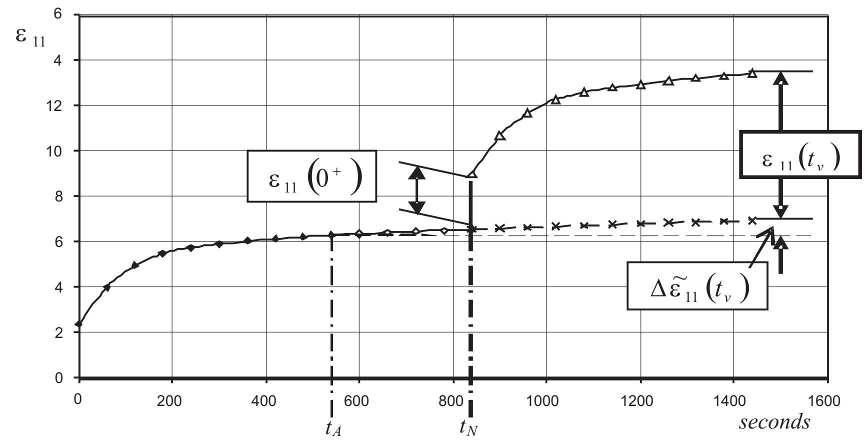


Fig. 12. Evolution of the metered strain due to two epochs and of the strain due to test-loading

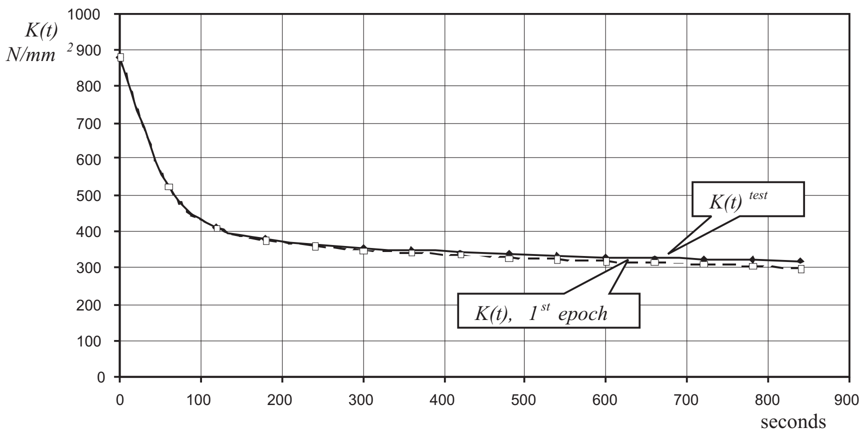


Fig. 13. Evolution of $K(t)$ due to test-loading, for comparison $K(t)$ due to the 1st epoch

5. Summary and conclusion

In health monitoring of ageing structures, which consist of time-dependent material and which are in service already for a shorter or longer time, the non-destructive testing method, i.e. applying test-loads and measuring the structural response, presupposes the knowledge of the entire history, which means information on the time of initial loading and/or putting into service, changes in loading- and environmental conditions, the time-functions of the visco-elastic material parameters and probably the state of degradation. But in practice the history is mostly unknown. Especially in the case of visco-elastic response of material, it is substantial to know about the course over time of the creep-function and the relaxation-function respectively. To overcome the lack of information, a method has been described, which makes it possible to determine these time-depending functions. The method has been illustrated by two adequate examples with quite satisfying results, which verify the time-shift invariance of linear visco-elastic materials.

The above considerations were restricted to one-dimensional problems; however, it must be pointed out - with reference to eq. (6) – that the method can obviously be applied to two- and three-dimensional problems, as well. The method provides a useful tool for reliable supervising ageing structures. In this concern, it must be pointed out that the previous considerations were focussed only on the influence and effects of visco-elastic material response in ageing processes of structures. Of course, other effects like those mentioned above must be taken into account in a comprehensive analysis of structural bearing capacity [1].

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Problem nieznanej historii w monitorowaniu stanu struktur

S t r e s z c z e n i e

Sprawą o zasadniczym znaczeniu w monitorowaniu stanu istniejących konstrukcji jest identyfikacja aktualnych parametrów strukturalnych i kontrola nośności, jaką struktura zachowała w przebiegu procesu starzenia. Wykonuje się to zwykle przez pomiary deformacji/odkształceń wywołanych przez obciążenia testujące i obliczanie parametrów na podstawie danych pomiarowych.

W przypadku materiałów, których odpowiedź ma charakter sprężysty, informacja o parametrach jest bezpośrednio związana z czasem pomiaru. Jednak, w przypadku odpowiedzi lepkosprężystej, historia czasowo-zależnej odpowiedzi strukturalnej w okresie pomiędzy obciążeniem wstępnym a inicjalizacją pomiarów testujących jest na ogół nieznana. Powstaje więc problem jak odseparować odkształcenia wynikające z istniejącego stanu od powodowanych obciążeniem testowym. Dla rozwiązania tego problemu, rozpatrzone będą na wstępie relacje między naprężeniami i odkształceniami a parametrami lepkosprężystości. Następnie przedstawiony będzie opis procedury pozwalającej wyznaczyć stan odkształceń powstały tylko w wyniku obciążenia testowego i obliczyć właściwe parametry jako funkcje czasu. Zgodnie z zasadą niezmienniczości względem przesunięcia w czasie, otrzymany wynik opisuje czasowo-zależną odpowiedź materiału lepkosprężystego, niezależnie od momentu, w którym zastały przyłożone obciążenia.

Zaprezentowana metoda będzie zilustrowana dwoma prostymi, lecz pouczającymi przykładami.