

Wykłady plenarne

P e t e r S i m o n s

Aspects of the Grammar and Logic of Relative Terms

But was not the Boole–Peirce–Schröder line in logic superseded by the Frege–Peano–Russell–Whitehead line? No; it was only eclipsed.

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1. Relations and relatives

One of the principal founding stories of twentieth century analytic philosophy is the triumph of the new logic over the old. The old logic is the traditional term logic originating with Aristotle, sometimes with additional features, but mostly confined to categorical syllogistic. The new logic of Frege, Peano, Whitehead and Russell is marked by three major advances: the provision of a basis in propositional calculus, the introduction of variable-binding quantifiers, and the treatment of relations. These developments come together in modern predicate calculus, which swept the inadequate old logic aside.

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¹ Fisch 1984, xxxi.

This story simplifies by overlooking several complications. One is that the algebra of logic, developed by Boole, Jevons, Schröder and others, though in fact anticipated much earlier by Leibniz, is a more powerful development of term logic. Others are that quantification was invented independently by both Frege and Peirce, that propositional logic was developed by Peirce and MacColl, and that the logic of relations began its development with Boole's contemporary De Morgan and was carried further by Peirce and Schröder. Peirce in particular returned several times to the logic of relations. While his view was that relational predicates are the purest form of expression, his own treatment relies heavily on what he calls *relatives*, which are expressions like *brother*, *enemy*, and so on. His extraordinarily rich and suggestive but difficult 1870 paper "Description of a Notation for the Logic of Relatives, resulting from an Amplification of the Conceptions of Boole's Calculus of Logic",² is the first to introduce relations of arbitrary arity, or number of places, some nine years before Frege's *Begriffsschrift*. However, my purpose here is neither to praise Peirce's many logical achievements, nor to revisit the logic of relations, but to reflect on the way in which Peirce, imitating natural language, generally expresses relational propositions. Let us use the term 'relational predicate' for any predicate (simple or complex) requiring completion by two or more names to give a sentence, for example '— is older than —', '— gives — to —', '— is twice as much heavier than — as — is than —', '— is as far round from — as seen from — as — is from — as seen from —', which are relational predicates of 2, 3, 4 and 6 terms respectively.

When providing examples of relations, Peirce does not use relational predicates, which contain a finite verb (frequently a copula), as in these examples, but rather he uses absolute and relative *terms*, which are common nouns, common noun phrases, or common noun phrase *functors* which require supplementation by one or more other names to form common noun phrases. Here are some of Peirce's examples of absolute terms, binary relative terms, ternary relative terms, and one quaternary relative term:³

Absolute: animal, violinist, Vice-President of the United States

Binary: enemy, benefactor, conqueror, husband, lover, mother

Ternary: giver to — of —, betrayer of — to —

Quaternary: winner over of — to — from —

In 1870 Peirce called binary terms *relative* and ternary or higher *conjunctive* terms. Later in 1880 he called binary terms *dual relatives* and

² Peirce 1984.

³ Peirce 1984, 366.

ternary and higher *plural relatives*. I shall use the expressions ‘relative term’ and (for short) ‘relative’ to cover all, and distinguish by the adjectives ‘binary’, ‘ternary’ etc., as does Schröder.

2. Term logic

The logic of terms was never completely replaced by predicate logic. I do not mean just that traditional logic lingered on in many places — especially in Roman Catholic schools and universities — after predicate logic became the norm. Rather, there was a current of mathematical logic that continued to use terms. Sometimes this was called a calculus of classes, since it was the extensionalist understanding of the semantics of terms that a term stands for a class. We find this in Boole, Jevons and Schröder, for example. It makes a late guest appearance in the one worked-out example Tarski gives of a theory of truth in his seminal work “The Concept of Truth in Formalized Languages”.⁴ But in the twentieth century, term logic was retained as a serious enterprise in modernized form in Poland, by Tarski’s two logic teachers, Jan Łukasiewicz and Stanisław Leśniewski. In Łukasiewicz’s case this was a deliberate anachronism: he was intent on showing how Aristotle’s syllogistic could be put into a modern guise, as based on propositional calculus.⁵ Łukasiewicz also, in his 1929 Warsaw University textbook *Elements of Mathematical Logic*, probably for pedagogical reasons, supplemented propositional logic with term logic,⁶ rather than predicate logic as is standard today. Leśniewski’s use of terms is however not anachronistic, but is an integral part of his mathematical logic. Leśniewski’s system is hierarchically organised. Its basis is a propositional calculus augmented by quantifiers and functors, which he calls Protothetic. Then comes, and here I quote him,

the theory I have designated Ontology, which forms a modernized traditional logic of a certain kind, and which – in its content and power – most approaches Schröder’s ‘class calculus’ when this is considered to include the theory of ‘individuals’.⁷

The final part of his system, built on Ontology, is his Mereology, or theory of part, whole and collective class. We discuss the use of relative terms in mereology below.

⁴ Tarski 1983, 152–278; *vide* § 2, 165–185.

⁵ Łukasiewicz 1957.

⁶ Łukasiewicz 1958, 1963, Chapter V.

⁷ Leśniewski 1992, 176 f., 412; 2015, 297, 493.

Jerzy Słupecki called Leśniewski's Ontology his 'calculus of names',⁸ but it is much more than that, subsuming all of syllogistic, the Boolean algebra of logic, predicate calculus, and, *in potentia*, simple type theory. The principal feature of the names (terms) of Ontology, which are the new basic syntactic category introduced there, is that, unlike the singular names of Frege and Russell, they may be empty (denoting nothing), as in latter-day free logic, but also plural (denoting several things), as in traditional logic. This not only confers considerable expressive power on Leśniewski's system, but renders it closer in "feel" to natural language than predicate logic.

Leśniewski standardly based his Ontology on a single logical constant, the functor of singular inclusion ' ε '. A sentence ' $a \varepsilon b$ ' is true if and only if the subject term ' a ' denotes a single individual, and this is one of the one or more individuals denoted by the predicate term ' b '. We can best read ' ε ' as "is one of". That something is an individual may be expressed by the sentence ' $a \varepsilon a$ ': " a is one of a ". This is what Leśniewski means by saying that his theory incorporates Schröder's account of individuals.

3. Relative terms

What Peirce calls relative terms or relatives are those which are *functors*; they are not themselves terms but yield terms when one or more terms are inserted in the relevant argument slots. So for example 'husband' yields a term when augmented by a further term, as 'husband of Queen Elizabeth II'. We may then use this term, a subject term and a copula to give a sentence, as 'Prince Philip is husband of Queen Elizabeth II'. Completed term expressions like 'husband of Queen Elizabeth II' I shall call *relational terms*. A relative may be used on its own, as in 'Philip is a husband', but it is then *derelativized*, and means 'Philip is a husband of *someone*'. Relative terms are extremely common, and form a major resource in natural languages for expressing relational propositions, notably in such areas as kinship and social relations as well as official roles, correlatives, ranks, order, and spatial and temporal relations. Relative terms are found not just for binary relations, but also for those of higher arity. Take for instance the ternary relation of giving, ' — gives — to — ' being the form of the predicate. For each of the three argument places or slots there is a relative term, and since the relation has three places, the relative terms require two completions to yield a complex absolute term. Here are suitable relative terms for those three places in order:

⁸ Słupecki 1955.

donor – gift – recipient.

Taking the form

a gives *b* to *c*

and completing each relative term by its two arguments we get the following three absolute terms:

donor of *b* to *c*

recipient of *b* from *a*

gift of *a* to *c*

and taking in each case the omitted term and making it the subject of a copulative sentence, we get three predications analytically equivalent to the original

a gives *b* to *c*,

namely

a is donor of *b* to *c*

c is recipient of *b* from *a*

b is gift of *a* to *c*

Notice that I am being slavonically sparing with the indefinite article in the copula: in each of these cases one could with equal or greater felicity have ‘is a’ instead of ‘is’. In some cases, the naked English copula connotes uniqueness, as in

Mary is (the) mother of Jesus

Philip is (the) husband of Queen Elizabeth

but for present purposes I am leaving the complications of definiteness or indefiniteness out of consideration.

The availability of relative terms corresponding to relational predicates is common, and there are regular ways to form them if they are not already lexically present: expressions like ‘employer’ and ‘employee’ are already available for the subject and object of the verb ‘employs’, and while novel coinages may be awkward — ‘kisser’ and ‘kissee’ for ‘kisses’, for example — there seems to be no principled objection to having distinct relative terms for each of the distinct slots in a relational predication. Where a relation is symmetric, the same term may be used for the two or more related items: for ‘— is married to —’ we have ‘spouse’; for ‘— has the same parents as —’ we have ‘sibling’.

The objects which are denoted by relative terms in expressing such relational propositions satisfy or fill what I call *case roles*.⁹ By ‘case’ I do not mean here the sort of thing expressed by the various grammatical cases in inflected languages, such as agent, direct object, instrument, location, etc., though there are connections, but the specific role for the relation in question. Thus in the three predications

a kisses *b*

a kicks *b*

a kills *b*

the item *a* is always the agent and the item *b* is the patient (and the patient may be made grammatical subject by passivization), but there are *six* case roles, not two, determined by both the nature of the relation and the role of the item in the relationship: kisser, kicker, killer; kissee, kickee, killee. Case roles are not linguistic features but relative features of the items involved in the relationship. They will prove their usefulness below. Our theoretical conjecture is that a relational predicate will always correspond or can be made to correspond to a suitable number of case roles, terms for which may be coined *ad hoc* if need be.

It might be objected that because often a relative term is morphologically derived from a verb, as ‘lover’ and ‘beloved’ are derived from the verb ‘love’, this shows that the relation is conceptually prior to the case roles, the terms for which derive from the verb. I do not dispute the order of morphological dependence in such cases, but the thesis that a predicate is equivalent to a suitable group of case role terms is not one about priority, but about equivalence of lexical means of expression. The idea is that one could replace predicates by groups of case role terms, joined together by a verb component which we might call a “copula”, which could carry tense, number and mood as required, and which will be not a single binary functor as in natural languages but one of a family depending on how many case roles it links.

4. The grammar of relative terms

In term logic, as in traditional logic, there is no syntactic divide between singular terms and non-singular terms. ‘Socrates’, ‘Pegasus’, ‘philosopher’, ‘horse’ and ‘unicorn’ are all terms out of the same syntactic bag. This is a moderate simplification by comparison with such languages as English, where terms are subdivided into common terms and definite terms. For logical

⁹ In database terminology these are sometimes called *slots*.

purposes however, it can be helpful to not insist on a syntactic divide, allowing the differences between definite, indefinite and indeterminate to be carried by specific logical particles.

Using the letters ‘S’ and ‘N’ for the basic syntactic categories of sentence and term (name) respectively, and denoting functors which take expressions of categories β_1, \dots, β_n as arguments to yield an expression of category α by $\alpha(\beta_1 \dots \beta_n)$, an n -placed predicate in predicate logic has category $S(N_1 \dots N_n)$, while terms and term functors have categories $N, N(N), N(NN)$ etc., and the copula is a logical predicate of category $S(NN)$. A more complete linguistic description of how relative terms work in English would need to bring in the role of prepositions such as ‘of’, ‘by’, ‘from’ and ‘to’, which perform analogous jobs in relatively uninflected languages such as English to case endings in more highly inflected languages. However, as this paper is about logic and not about linguistics, the occasionally messy and logically obfuscatory details are here foregone.

5. Relatives and cardinality

In one of the 59 definitions of his 1929 lectures “Elementary Outline of Ontology”, Leśniewski defines a binary predicate as *relational* if it holds only among individuals:¹⁰

$$D35 \forall \varphi \ulcorner \text{rel}(\varphi) \leftrightarrow \forall ab \ulcorner \varphi(ab) \rightarrow . a \varepsilon a \wedge b \varepsilon b \urcorner \urcorner$$

and he subsequently lists a number of definitions and theorems mimicking the theory of relations in *Principia Mathematica*. A better name for such predicates is *bisingular*, and predicates which require singularity in one place may be called respectively left-singular and right-singular. But this prompts the general question as to what constraints a given relational predicate or a relative may place on the cardinalities of objects satisfying it. When, as in Leśniewski, terms need not be singular, such predicates need not be bisingular, for example the predicates ‘outnumber’ and ‘are twice as numerous as’, but also such non-arithmetical predicates as ‘host’, ‘defeat’, ‘hate’ and ‘intermarry with’. In the predicate ‘are twice as numerous as’, the first argument cannot be singular if the predicate is used to express a truth, as in for example

The holes in a round of golf are twice as numerous as the provinces of
Austria
and

¹⁰ Leśniewski 1988, 53.

The ears of a cat are twice as numerous as its tail.

There are many definitions that can be made around such numerical constraints. Left-singular predicates for example tend to go easily with associated relatives, a device much used in traditional logic to trade in verbal predicates for terms, as in

Juliet loves Romeo
Juliet is a lover of Romeo.

We can always define a related but not synonymous left-singular predicate. Take for instance the true predication

The England team defeated the West German team in the 1966 World Cup Final

This is neither left- nor right-singular — the rules of football constrain it to be *biplural* — but there are related singular predicates, as in the predications

Gordon Banks was one of the England team that defeated the West German team

and, conversely

Franz Beckenbauer was one of the West German team defeated by the England team

where notice that in the latter case the verb is passivized in order to bring the otherwise second argument into the left-singular subject position.

There appear in fact to be no universal constraints regarding the number of objects falling under relative terms: it depends on the predicates and terms in question. For example, ‘parent of X ’, for suitable X , denotes two objects in sexually reproducing species; in asexually reproducing species it denotes one, in modern fertility medicine it may denote three. ‘wife of X ’ denotes one female in monogamous societies, but may denote several in polygamous ones, or (as in ‘wife of Henry VIII’) may denote several serially.

6. Relatives and functions

In mathematics, since Peano, functions have often been regarded as derived from right-unique relations, that is, ones such that

for all x , y and z , if $R(x,y)$ and $R(x,z)$, then $y = z$

In those circumstances, mathematicians employ a function expression $f(x)$ to stand for the unique y such that $R(x,y)$. This was how Whitehead and Russell treated functions in *Principia Mathematica*, as definite descriptions.¹¹ But in natural language as well as in mathematical language, functional expressions are relatives. Consider relational terms like

sum of 2 and 3
 cube of 12
 quotient of 3 by 2
 limit of $(f(a + h) - f(a))/h$ as $h \rightarrow 0$

and those in the definitional equation

$$\mathbf{x} \cdot \mathbf{y} = |\mathbf{x}||\mathbf{y}|\cos \theta$$

Underneath the neat mathematical notation, relatives are ubiquitous. One need only listen to mathematicians speaking their formulas out loud. The last equation, defining the vector dot or scalar product, can be read as

The dot product of the vectors \mathbf{x} and \mathbf{y} is the product of the absolute values of \mathbf{x} and \mathbf{y} and the cosine of the angle between them.

In this mathematicians' patter there are no fewer than five relatives:

dot product of — and —
 product of — and — and —
 absolute value of —
 cosine of —
 angle between — and —

At this point it may appear that I am cavalierly omitting the definite article, which occurs six times in that English sentence. While not wanting to discount the importance of the definite article, this is not my topic here, so let me just translate that English sentence into another non-mathematical language:

Iloczyn skalarny wektorów \mathbf{x} i \mathbf{y} jest iloczynem wartości bezwzględnych \mathbf{x} i \mathbf{y} i cosinusa kąta między nimi.

This language is that of many great mathematicians, and they did and do manage perfectly well without definite articles and the vagaries of their usage in English and other articulated languages. There are exactly as many relatives in the Polish as in the English, namely five:

iloczyn skalarny

¹¹ Whitehead and Russell 1910, *30.

iloczyn
wartość bezwzględna
cosinus
ką

and just as many as there are in the mathematical notation (it being understood that ‘ θ ’ stands for the angle between \mathbf{x} and \mathbf{y}), and the number of places they have are in each case the same as with the English. Relatives are a better guide to the content than the superficialities of articles, prepositions and inflexions. Now consider expressions such as

square root of 2
arccos(0.5)
root of the equation $x^n + 1 = 0$
logarithm of z (where z is a complex number)

In all of these cases, post-Fregean logical orthodoxy would insist that because these do not denote single values, they cannot contain functional expressions. But these come out of exactly the same logico-grammatical bag as those that are single valued. Mathematicians and computer scientists have long put up with partial functions, those not “defined” for every argument, such as the division function, which is “undefined” for divisor 0, or the expression

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

which stands for a divergent series, and so is likewise “undefined” (better: does not denote).

Reflecting on the fact that relative terms need not be left- or right-singular, we should simply accept that terms like ‘square root of 2’ are just as good as others, merely differing in the cardinality of the values they denote. Multi-valued functions have indeed long been accepted among mathematicians (as distinct from logicians), as Oliver and Smiley stress in their *Plural Logic*, citing as authorities just three: Euler, Hardy, and Penrose.¹² Those are good enough authorities for me.

¹² Oliver and Smiley 2013, 143 f.

7. Relatives and abstraction

Another natural place to find relatives is where abstraction occurs. In abstractive equivalences of the form

$$a E b \Leftrightarrow \S(a) = \S(b)$$

where E is an equivalence relation, the expression taking the place of ‘ \S ’ is naturally a relative term. Examples are expressions like

weight of —
 height of —
 cost of —
 age of —
 number of —

and to take a previous example (from a four-place equivalence relation)

angle between — and —.

This is understandable, because abstraction is *away from* something in a base, the a and b and their kind, so the abstractum is *of* this, and therefore naturally expressed by a relative term with the “this” as argument. So relatives are a highly natural way to express the products of abstraction.

However, such equivalences do not always mark abstraction and do not always indicate any kind of ontological priority of the supposed base items which are terms of the relational predication. A similar equivalence is

$$a \text{ is a sibling of } b \Leftrightarrow \text{the parents of } a = \text{the parents of } b$$

but parents, unlike heights and numbers, are hardly abstract objects. Here ‘sibling’ feels like the derived term. So the mere availability of relative terms equivalent to relational ones tells us nothing about metaphysical priority: it varies with the case.

8. Relations and relatives in mereology

Mereology, the formal theory of part and whole, can be formulated in two ways. The way which aligns with predicate logic is to use unanalysed binary mereological predicates meaning e.g. ‘is a part of’, ‘overlaps with’, and ‘is disjoint from’. Mereology so formulated was originally called the *theory of extension* by Whitehead and somewhat later the *calculus of individuals* by Leonard and Goodman. But in the original formulation by the inventor of the

term ‘mereology’, Leśniewski, the mereological constants are not predicates but relative terms such as ‘part (of)’, ‘ingredient (of)’, ‘overlapper (of)’, ‘external (to)’, ‘class (of)’ and ‘collection (of)’. This has one straightforward, overt technical advantage, and one covert metaphysical advantage. The overt technical advantage is that because Leśniewski’s terms are not restricted to the singular, all the standard mereological functors have the same grammatical form, namely, that of relative terms. While the terms corresponding to binary predicates, such as ‘part of’, are bisingular, that is, if A is part of B , then both A and B are individuals, the collective terms ‘class of’ (*klasa*) and ‘collection of’ (*mnogość, zbiór*) are also relative terms, but their arguments need not be (and typically are not) singular, for example as in ‘ A is a collection of boats’, ‘ B is the class of (all) snails’. In predicate-logical formulations however, the constants corresponding to ‘class’ (i.e. complete collection) has to be variable-binding operators, as in ‘the class of x such that $F(x)$ ’, and there is simply no easy way to speak of incomplete collections. The Leśniewskian version, using relative terms, is thus smoother and more elegant.

The metaphysical advantage is one which may or may not have been apparent to Leśniewski. It is this. In a relational predication such as ‘Juliet loves Romeo’ it is natural to look for some material item which in some way corresponds to the predicate, e.g. a universal of loving or a relational mental trope of loving. But in the case of mereology, it is extremely implausible to hold that there is any kind of item corresponding to the part-predicate or other mereological predicates. On the other hand, the existence of the various parts of a given object is not much in doubt¹³ (though there may be metaphysical disputes as to *which* parts an object has). So, the relational term ‘part of X ’ and its plural ‘parts of X ’ correspond without doubt to some things in the world, namely those parts themselves. This is not only congenial to Leśniewski’s own nominalism, but perhaps coincidentally reflects the fact that there is no universal of parthood or particular relational parthood tropes. I previously always assumed Leśniewski’s formulation of mereology was simply a reflection of his preferred choice of idiom, namely Ontology, but maybe it was more than that. As a matter of historical fact, Leśniewski formulated mereology first (1916), in a regimented Polish, before the formal language Ontology was developed (1920) so his choice of idiom was probably steered by natural language. Since he does not express any opinion on the metaphysics as distinct from the logic of parthood, it is hard to say whether this played any part in his choice of idiom, but from my own metaphysical viewpoint his formulation is preferable to the predicate-logical one.

¹³ One must add, except among philosophers, who are often hell-bent on contradicting common understanding.

9. The order and direction problems

There has for some time been intensive discussion on what may be called the *order problem* for relations. The two propositions

Jan loves Maria
Maria loves Jan

appear to involve the same constituents, but the order of their terms matters, since they are logically independent of one another. The question is whether the order comes with the relation itself or is somehow imposed from without, with the relation itself being neutral as to order. A related *direction problem* concerns such pairs as

Jan loves Maria — Maria is loved by Jan
Jan is taller than Maria — Maria is shorter than Jan
Edinburgh is north of London — London is south of Edinburgh

where the question is whether there are two necessarily mutually converse relations corresponding to the two different predicates, or just one relation, in which the order matters (as in the first problem). Various positions and solutions have been proposed to resolve these interlinked problems, but no clear consensus has emerged.¹⁴

By allowing every relation to be represented by its case roles, both problems simply disappear. The case roles and their number completely determine the relation, and all we need to do is to assign arguments to each case role, fill the slots, to complete the predication. So the examples we have given can be represented respectively by

Jan : lover — beloved : Maria
Maria : lover — beloved : Jan
Jan : taller — shorter : Maria
Edinburgh : further north — further south : London

It does not matter in which order we mention the case roles: all that matters is which objects fill which roles. The order problem simply disappears, and there is no unnecessary multiplication of relations as in some responses to the direction problem. It seems not unlikely that the order and direction problems arose because predicate logic, having been developed for mathematics, is highly schematic, whereas the examples I have given are from natural language, which provides the solution through its relative terms. In these

¹⁴ For a survey of the order and direction problems and responses thereto, see MacBride 2016.

examples note the central complex, consisting of the two case roles and the link (copula) between them. This will prove useful in the next section.

10. Case roles and Russell's multiple relation theory

Between 1906 and 1919 Bertrand Russell upheld, in one form or another, what is called his *multiple relation theory* of belief, judgment, and other so-called propositional attitudes.¹⁵ For brevity I shall use the word 'thinking' for all such attitudes, whether they be occurrent or dispositional. Russell's motivations for this at first sight unlikely theory are twofold. He wishes to avoid on the one hand *unfacts* – objective falsehoods or non-obtaining states of affairs – such as are proposed by Meinong, and on the other hand *propositions*, in the sense of Frege's *Gedanken*, that is, abstract propositional senses. His reasons for the latter denial are those he sets out in his famous paper "On Denoting",¹⁶ namely that our supposed cognitive access to propositions, as to other senses, leaves them wholly mysterious. Both motivations are praiseworthy, so it is worth considering whether Russell's third way offers a better account of cognitive attitudes.

In the initial version of Russell's theory, he simply said that a person's thinking consists not in their having a binary relation to a unitary something, which he variously called a proposition, complex, objective, or fact, but in having multiple relations to the components of what would be such a unitary something were it to exist. When Ottoline believes that Alys loves Bertie, Ottoline stands in several relations, to Alys, Bertie and loving. While this allows for false as well as true thinking, and avoids a unitary object of thought, it faces two problems. One is that it does not distinguish Alys loving Bertie from Bertie loving Alys, which is an instance of the order problem; the other is that the relation is not distinguished in any way from the other components, so can be replaced by something else, such as this table, resulting in nonsense. Wittgenstein pointed this out to Russell,¹⁷ which caused Russell to revise his theory by introducing an additional component, the logical form of the thought relation.¹⁸ But this did not solve the order problem, and as formulated by Russell it reintroduced the unhappy feature that every thinking must be true, which Russell desperately wished to avoid. So, after its final desultory appearance in the *Philosophy of Logical Atomism* lectures,¹⁹ where Russell

¹⁵ For an insightful overview of Russell's shifting views on judgement, see Candlish 1996.

¹⁶ Russell 1905.

¹⁷ Wittgenstein 1979, 103.

¹⁸ Russell 1984, 117 f.

¹⁹ Russell 1986, 118 f.

barely attempts to defend the view, he dropped it and — regrettably — moved over to neutral monism and behaviourism.

But Russell's theory can be made to work by exploiting relatives expressing case roles. As noted above, these solve the order and direction problems. The multiple relation theory comes in as follows. If that which is thought has a relation with n terms, the thinking relation has $n + 2$ terms. It has two new case roles, one for the thinker (the subject), and one for the subordinate, thought relation, with its case roles, but now shorn of the fillers for these. The remaining n case roles are derived from those of the thought relation. In the Ottoline example, the believed relation has the two case roles *lover* and *beloved*. The believing relation has this and Ottoline filling the two case roles *believed relation* and *believer* respectively. The other two case roles are *believed-as-lover* and *believed-as-beloved*, and they are filled by Alys and Bertie respectively. This fulfils Russell's desire to let the belief have the objects themselves as arguments, but the modification means that false thought is just as possible as true, and what makes the difference between truth and falsity is whether the objects now lifted to the *believed-as-lover* and *believed-as-beloved* roles also in fact respectively fill the *lover* and *beloved* roles of the love relation, and this is nothing other than a correspondence theory of truth for thinking, something after which Russell persistently strove.

The thus modified theory does everything Russell wanted for it. There are a couple of residual problems: one is that thought "about" non-existent objects needs to be explained in a non-Meinongian way. Russell himself can work around this with his descriptive theory of names, but it is not a perfect solution. Another problem is that it does not work for non-atomic thoughts, although Russell probably did not intend it to. However, if names are replaced by descriptions, as Russell thought they should be, and descriptions are taken as incomplete symbols and analysed away as in "On Denoting", then what look like atomic thoughts turn out to be non-atomic, and need to be dealt with as such. Also, a trope nominalist like myself would need to eliminate the apparent reference to universals that infest the theory as described. I am moderately confident that this last problem can be overcome by noting that thinkings are particular occurrent events or dispositional states, but more needs to be said, and the other problems would need to be addressed in some way before we could claim that the multiple relation theory offers an overall satisfactory account of thinking. However, it is much less hopeless a theory than posterity (Russell himself included) has generally thought.

11. Conclusion

What I hope to have made clear is that relative terms are neither a whimsical oddity nor a mere historical relic, but are an important class of expressions found all over natural and mathematical languages, that merit further investigation for their own sake, and that can also be philosophically fruitful.

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P e t e r S i m o n s

Aspects of the Grammar and Logic of Relative Terms

Keywords: *term, relative, C.S. Peirce, B. Russell, function, multiple relation theory of belief*

We are familiar with the grammar and logic of relational predicates in predicate calculus, chiefly as transmitted through Whitehead and Russell. In natural languages however, relations are frequently expressed using what Peirce called *relatives*, that is, expressions like *brother, gift, head, effect, successor*, which require completion by one or more definite terms to yield general names or terms. Peirce developed a logic of such relatives which influenced Schröder and Tarski. Later, Leśniewski used relative terms such as *part, overlapper, class* etc. to formulate his mereology, rather than the predicates and operators subsequently and more standardly used. In this paper I consider aspects of the grammar and logic of such relative terms, particularly in regard to several areas of general logico-philosophical interest: cardinality; functions; abstraction; the order problem of relations; and Russell's multiple relation theory of belief and judgment.