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Mathematical modelling of bridge crane dynamics for the time of non-stationary regimes of working hoist mechanism

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Bridge crane is exposed to dynamic loads during its non-stationary operations (acceleration and braking). Analyzing these operations, one can determine unknown impacts on the dynamic behavior of bridge crane. These impacts are taken into consideration using selected coefficients inside the dynamic model. Dynamic modelling of a bridge crane in vertical plane is performed in the operation of the hoist mechanism. The dynamic model is obtained using data from a real bridge crane system. Two cases have been analyzed: acceleration of a load freely suspended on the rope when it is lifted and acceleration of a load during the lowering process. Physical quantities that are most important for this research are the values of stress and deformation of main girders. Size of deformation at the middle point of the main crane girder is monitored and analyzed for the above-mentioned two cases. Using the values of maximum deformation, one also obtains maximum stress values in the supporting construction of the crane.

1. Introduction

Modern cranes need to have very high technical properties and economic parameters. They must be designed to sustain frequent changes of working regimes like any other machines which produce dynamic movement [1]. During this dy-

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namic working regimes, significant dynamic effects appears. Working time of a crane consists of stationary and non-stationary periods of motion. Non-stationary motion regime causes the occurrence of variable loads on the mechanisms and the supporting crane construction. Dynamic analysis needs to be done during crane design process with the aim of analyzing stress and deformation conditions of crane mechanism elements and the supporting structures. Numerous theoretical and experimental researches of dynamics behavior of bridge crane have shown that critical dynamic states occur during load lifting or load lowering [2–4]. Dynamic behavior of large scale machines is in the focus of research in the last decade [5, 6].

Extensive research is carried out to develop strategies for vibration damping [7]. The influence of the restraints type and changing the loading force position on the generated Huber-Misses stress in the gantry crane beam was estimated in paper [8]. In papers [9] and [10], deflection values occurring in the main girder of a portal crane at various payloads were investigated theoretically, numerically and experimentally. The object of the study in paper [11] was the angle of deviation of the crane from the perpendicular to the rails in a horizontal plane. The aim of the work was to develop a mathematical model of the crane's bridge beam misalignment. The model has been created in the Matlab Simulink. Based on the resulting model, one implemented a control system that compensates for the emerging misalignment by speeding up or slowing down the crane drives. The modeling procedure for a flexible knuckle boom crane actuated by hydraulic cylinders and modeled as a planar multibody system, is presented in paper [12]. Paper [13] concerns the analysis of the load motion, taking into account wind pressure and the deformation of the rope system. From the above-mentioned papers one can see that this area of research is in the focus researchers' interest today. The modeling of crane element interconnections in a dynamic calculation is very complex, and it is often not necessary, since not all the factors have the same influence on dynamic loads. Consequently, during mathematical modeling, all the factors that are not essential for the calculations can be ignored [14]. The most important parameters are: the number of concentrated masses, their arrangement along the supporting construction elements, the stiffness of supporting elements and structures and the possibility of their change, the dependence of drive and braking force of the driving mechanisms on time, the speeds and motor revolution frequencies, dimming of oscillation in crane structure and its elements, etc. In each particular case, some of the above-mentioned parameters will have a major impact, and some of them can be ignored.

Bridge cranes are characterized by a special arrangement of individual assemblies (characterized by their respective mass, stiffness and damping). From a dynamic analysis point of view, it can be stated that a crane has an unlimited number of degrees of freedom. The crane mechanisms, as well as the elastic supporting construction, make a complex oscillatory system subjected to oscillatory motion in a vertical plane. Any change in system parameters directly affects dynamic behavior of the system. Most of the research in the last years have been focused on investigating the laws of driving forces change, the elasticity of supporting construction of the crane in a vertical plane and the methods of its discretization [15]. In addition, the problems of damping of elastic oscillation of the rope and supporting structures and elements of the drive mechanism are investigated. Two basic types of mathematical models are used to describe dynamic behavior of bridge cranes: the discrete models, where a continuously distributed mass of load-bearing structure is discretized into a certain number of concentrated masses, and the discrete-continuous ones, where load-bearing structure is represented by its own structural characteristics.

A dynamic model with three degrees of freedom describing dynamic behavior of a bridge crane first appeared in [16]. After that it has been used by other authors in their research, i.e., [17–21]. All of these researches have been based on a dynamic model whose general form is shown in Fig. 2, which includes all relevant dynamic parameters. The dynamic model in [19] was analyzed in the case of load lifting from the substrate, while in [17] and [18] it was analyzed in the case of lowering a load on an elastic support. However, in these two papers, the damping effects of supporting structure and rope are neglected. The driving forces are expressed in the function of time, according to a linear law. The dynamic model in [18] was analyzed considering damping properties of the substrate. The same dynamic model is used in this paper, however, some modifications have been made. The mentioned modifications are: the stiffness of lifting rope is considered as a constant value, additional crane trolley modes of operation are taken in consideration, most unfavorable cases are selected taking into account the dynamic loads in the supporting structure , and damping effects in the lifting rope and the crane supporting structure are taken in consideration. These modifications make the dynamic model more accurate. The driving force is assumed as a quadratic function and the braking force as a constant value. The mathematical model has been developed using the energy method.

2. Parameters of the dynamic model

The dynamic model is developed according to [19] with the above-mentioned modifications. Dimensions and other parameters for the crane selected as the case study are taken from [22] and [23] with an additional parameter of rope diameter, which is 4 mm. The schematic of bridge crane system is shown in Fig. 1 and the dynamic model in Fig. 2.

The analyzed dynamic model has three degrees of freedom, as follows: q_1 a generalized coordinate that describes oscillation of the reduced mass of the supporting crane construction; q_2 a generalized coordinate that describes oscillation of the load in rope direction; q_3 a generalized coordinate describing displacement of the reduced mass of the lift mechanism. The parameters of dynamic model from Fig. 2 representing the magnitude of concentrated masses are: the discretized mass of supporting structure m_1, the load weight m_2 and the mass of lifting mechanisms reduced to rope direction m_3.

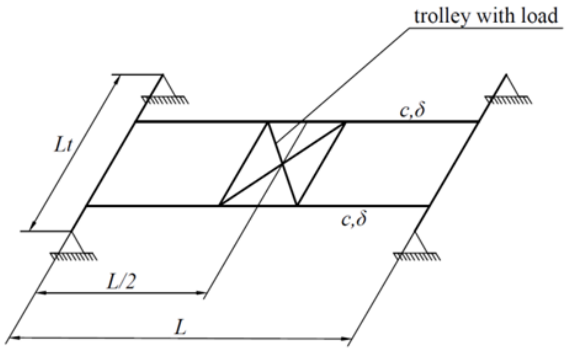

Fig. 1. Schematic representation of crane system

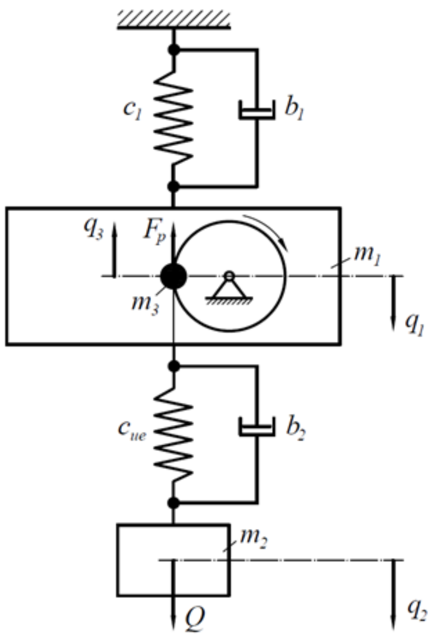

Fig. 2. Dynamic model of bridge crane in acceleration period during load lifting

Consequently, the supporting structure of the bridge crane is represented by a single concentrated mass which is reduced to the point of load suspension. The mass of the trolley is also added to the mass of the supporting structure. The value of the reduced mass of the supporting crane construction is accepted by most authors as being equal to a half of the total mass of the crane carrier. This method of mass reduction is based on experimental studies of dynamic behavior of cranes. For bridge cranes, the first form of oscillation is the most important one. The second form of oscillation is characterized by small amplitudes. The higher forms of oscillation cannot be observed. The coordinate q_1 describes oscillation of the reduced mass of the supporting crane construction. The reduced load mass in papers [22–24] is presented as a single concentrated mass whose size is equal to the rated load capacity. The coordinate q_2 describes oscillation of the load in rope direction. The lift mechanism in papers [22–25] is presented in the form of a single concentrated mass reduced to a translational load displacement. The coordinate q_3 describes displacement of the reduced mass of the lift mechanism. Considering the prevalence of the first oscillation forms for the bridge crane which oscillates in vertical plane, the mass m_1, which is reduced to the hanging point, can be determined from the equation:

$$m_1 = 0.493 m_{\text{nk}} + m_{\text{kol}}, \tag{1}$$

where: m_{nk} is the total mass of the main carrier and the crane cabin, m_{kol} is the mass of the crane trolley.

The mass (load) is reduced to rope direction and is represented as a concentrated mass m_2 whose value is equal to the nominal load capacity. The load lifting mechanism is presented with a concentrated mass m_3 reduced to the translational displacement of load. Determination of the stiffness coefficient of the supporting construction is directly related to the number and position of concentrated masses for which the elastic structure is discretized. For this dynamic model, the stiffness coefficient of the supporting construction can be determined according to the equation:

$$c_1 = \frac{48EI}{L^3}, \tag{2}$$

where: EI is the bending stiffness of crane girder in vertical plane, L is the span of bridge crane.

The value of load rope stiffness is one of fundamental dynamics parameters when analyzing dynamics behavior of cranes, and according to the recommendations of the literature [16, 19] the stiffness of ropes can be considered as a constant value, especially for the case of determining the maximum dynamics of crane. The value of rope stiffness for lifting can be determined according to the equation [22, 23]:

$$c_{\text{ue}} = \frac{E_u A_u}{L_u}, \tag{3}$$

where: E_u is the elasticity modulus of the rope, it is considered as a constant, A_u is the cross-sectional area of the rope and L_u is the length of rope at the beginning of lifting.

The damping value is also a significant dynamic parameter for dynamic analysis of cranes. The intensity of attenuation can be determined from logarithmic decimation of low-frequency oscillations of crane load due to the fact that oscillations at the second frequency are damped considerably faster than the oscillation at the first frequency. The oscillation coefficient of the oscillating supporting construction can be determined according to the following equation:

$$b_1 = \frac{m_1 \delta \sqrt{c_1/m_1}}{\pi}, \tag{4}$$

where: m_1, c_1 is the reduced mass and stiffness of the main girder, respectively, δ is the logarithmic oscillation decay of crane bridge without load at the position of trolley. The winch is placed in the middle of the bridge span.

The dynamics behavior of the crane is influenced by the dynamic behavior of its driving mechanisms, which depends on the characteristics of engine and brakes which are usually given in the form of moments or forces described as aperiodic functions of time.

The driving force can be determined according to equation (5), and the braking force according to equation (6) [22]]:

$$F_p = Q + F_{\text{din}} \left(1 - \frac{t^2}{T^2}\right), \tag{5}$$

$$F_k = \frac{2M_k}{D_d}, \tag{6}$$

where:

$$F_{\text{din}} = \frac{2\Delta M'_{\text{sr}} i_m \eta_m}{D_d}, \tag{7}$$

$$\Delta M'_{\text{sr}} = \frac{m_q v_{\text{diz}} D_d}{2T i_k i_m \eta_m} + 1.1 \frac{J_1 n_{\text{em}}}{9.55T}, \tag{8}$$

$$M_k = \frac{Q D_d}{2t} + \frac{m_q v_{\text{diz}} D_d}{2 i_k t} + 1.1 \frac{J_1 n_{\text{em}} i_m \eta_m}{9.55t}, \tag{9}$$

T, t is the acceleration and breaking time, respectively, D_d is the rope winding drum diameter, Q is the load capacity of crane, m_q is the load weight, v_{diz} is the lifting speed, i_k is the elevators' transmission ratio for load lifting, n_{em} is the frequency of rotation of the electric motor for lifting, J_1 is the moment of inertia of rotating masses of the lifting mechanism, i_m is the gear ratio of the lift mechanism gearbox, η_m is the degree of utilization of the lift mechanism gearbox.

3. Mathematical model for acceleration in the case of load lifting

In the analyzed dynamic model from Fig. 2, the load at the start of lifting mechanism is at a certain height and is on standby. The supporting crane construction is deformed and is in a position around which it will oscillate in the non-stationary work regime.

Also, the load lifting rope, at that initial moment, is deformed, or elongated by the length of static deformation f_{st}.

The generalized coordinates q_i ($i = 1, 2, 3$) describe possible displacements of the concentrated mass. The concentrated mass system acts as a force F_p in the direction of rope to lift the load and represents the driving force of the lifting mechanism electric motor. Using the energy method (Lagrange equation) one can describe dynamic behavior of a bridge crane in non-stationary work regime of lifting mechanism. In a general form, it has the form of equation [22]:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial E_k}{\partial \dot{q}_i}\right) - \frac{\partial E_k}{\partial q_i} + \frac{\partial \varphi}{\partial \dot{q}_i} + \frac{\partial E_p}{\partial q_i} = Q_i\,, \tag{10}$$

where: E_k is the kinetic energy of the system as a function of generalized coordinates and velocities, E_p is the potential energy of the system as a function of generalized coordinates, Φ is the function of dissipation of the system as a function of generalized velocities, q_i, $\dot{q}_i$ are generalized coordinates and speed, respectively, and Q_i is the non-potential force.

It is important to note that oscillations in this system have small amplitudes, and can be analyzed by applying the theory of small oscillations. Therefore, kinetic and potential energy of the system should be calculated with an accuracy to a small second order [26]. Kinetic energy of the system can be determined according to equation [22]:

$$E_k = \frac{1}{2}\sum_{i=1}^{3} m_i v_i^2\,. \tag{11}$$

Velocities of the concentrated mass of the system are represented by the equation:

$$\begin{aligned} v_1 &= \dot{q}_1\,, \\ v_2 &= \dot{q}_1 + \dot{q}_2 - \dot{q}_3\,, \\ v_3 &= \dot{q}_3 - \dot{q}_1\,. \end{aligned} \tag{12}$$

Adding the value v_i from equation (12) to equation (11) and performing elemental transformations, one obtains final equation for kinetic energy of the given system as:

$$E_k = \frac{1}{2}\begin{bmatrix} (m_1 + m_2 + m_3)\,\dot{q}_1^2 + m_2\dot{q}_2^2 + \\ (m_2 + m_3)\,\dot{q}_3^2 + 2m_1\dot{q}_1\dot{q}_2 - \\ 2\,(m_2 + m_3)\,\dot{q}_1\dot{q}_3 - 2m_2\dot{q}_2\dot{q}_3 \end{bmatrix}. \tag{13}$$

Potential energy of the system (14) consists of three components, potential energy of the supporting structure (15), potential energy of the lifting rope (16) and potential energy of the load (17) [22].

$$E_p = E_p^{\text{str}} + E_p^{u} + E_p^{\text{ter}}, \tag{14}$$

$$E_p^{\text{str}} = \frac{1}{2}\{q\}^T[c]\{q\} = \frac{1}{2}c_1\,q_1^2, \tag{15}$$

$$E_p^{u} = \frac{1}{2}c_{\text{ue}}\,(q_2 + f_{\text{st}})^2, \tag{16}$$

$$E_p^{\text{ter}} = m_2 g\,(y_2 - H + l), \tag{17}$$

where: $[c]$ is the stiffness matrix of the supporting structure, $\{q\}$ is the vector of generalized coordinates, c_{ue} is the equivalent stiffness of ropes for lifting loads, H is the lifting height, and l is the initial rope length.

In order to determine potential load energy (17), we use the vertical crane planar diagram shown in Fig. 3.

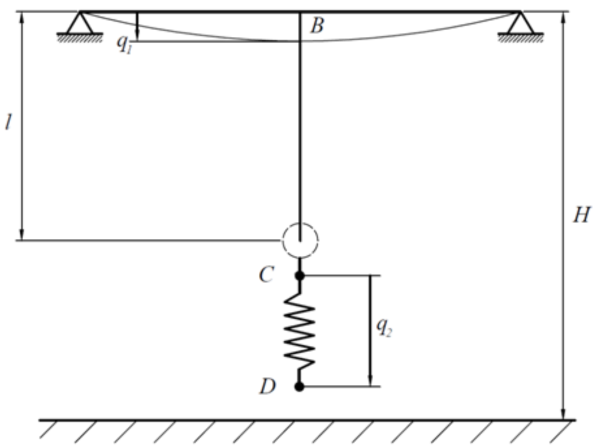

Fig. 3. Scheme of bridge crane in vertical plane

The size y_2 is determined by equation (18), and the size $\overline{BC}$ with equation (19) [22]:

$$y_2 = H - q_1 - \overline{BC} - q_2, \tag{18}$$

$$\overline{BC} = l - q_3\,. \tag{19}$$

When the values from equation (18) and (19) are substituted into equation (17), the final equation for potential energy of load (20) is obtained. Using equation (20), (15) and (16) one gets the final equation for potential energy of the whole system (21).

$$E_p^{\text{ter}} = m_2 g\,(-q_1 - q_2 + q_3), \tag{20}$$

$$E_p = \frac{1}{2}c_1 q_1^2 + \frac{1}{2}c_{\text{ue}}\left(q_2^2 + 2f_{\text{st}}q_2 + f_{\text{st}}^2\right) + m_2 g\,(-q_1 - q_2 + q_3), \tag{21}$$

The dissipation function can be determined according to equation (22), in which b_2 is defined by the rope damping coefficient.

$$\begin{aligned}\varphi &= \frac{1}{2}\{\dot{q}\}^T[B]\{\dot{q}\} = \frac{1}{2}\{\dot{q}_1 \;\; \dot{q}_1\}\begin{bmatrix} b_1 & 0 \\ 0 & b_2 \end{bmatrix}\begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{Bmatrix} \\ &= \frac{1}{2}\left(b_1 q_1^2 + b_2 q_2^2\right). \end{aligned} \tag{22}$$

The generalized non-potential force $Q_3 = F_p$ also acts on the system, and its intensity is defined by equation (5).

Finally, in accordance with equation (10), one can formulate the differential motion equation for the whole system of the dynamic model shown in Fig. 2 as (23). This differential equation describes dynamic behavior of the bridge crane in vertical plane during non-stationary work regime of the lifting mechanism.

$$\begin{aligned} (m_1 + m_2 + m_3)\,\ddot{q}_1 + m_2\ddot{q}_2 - (m_2 + m_3)\,\ddot{q}_3 + b_1\dot{q}_1 + c_1 q_1 - m_2 g &= 0, \\ m_2\ddot{q}_1 + m_2\ddot{q}_2 - m_2\ddot{q}_3 + b_2\dot{q}_2 + c_{\mathrm{ue}} q_2 &= 0, \\ -(m_2 + m_3)\,\ddot{q}_1 - m_2\ddot{q}_2 + (m_2 + m_3)\,\ddot{q}_3 + m_2 g = Q + F_{\mathrm{din}}&\left(1 - \frac{t^2}{T^2}\right) \end{aligned} \tag{23}$$

The equation system (23) can be solved by numerical methods taking into account the initial conditions, which are, for the case analyzed, represented by equation (24):

$$\begin{aligned} q_1(0) &= q_{1\mathrm{st}}\,; \quad q_2(0) = 0; \\ q_3(0) &= 0\,; \qquad \dot{q}_i(0) = 0, \end{aligned} \tag{24}$$

The system of differential equations (23) was solved using the software specially developed by the authors of this paper. To solve the system of differential equations, one used the Runge–Kutta fourth-order method (RK4). The Fortran programming language was used. The obtained results have also been confirmed using the Wolfram Mathematica software package.

4. Mathematical model of bridge crane for braking mode during lowering the load

The dynamic model which is considered for the case of non-stationary work regime of lifting mechanism is shown in Fig. 4. Using the same methodology as described in Section 3, the following equations are formed here: kinetic energy of the system (25), potential energy of the system (26), as well as the function of dispersion (22). The external force of the stimulus acting on the system is a generalized non-potential force $Q_3 = -F_k$. Its value is determined by equation (6).

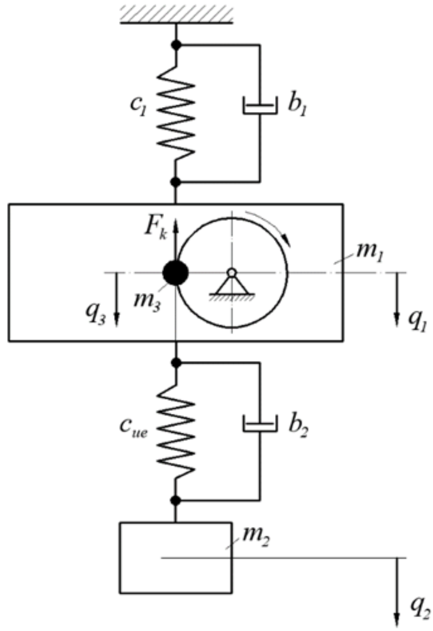

Fig. 4. Dynamic model of a bridge crane for load lowering process

$$E_k = \frac{1}{2}\begin{bmatrix} (m_1 + m_2 + m_3)\,\dot{q}_1^2 + m_2\dot{q}_2^2 \\ +\,(m_2 + m_3)\,\dot{q}_3^2 + 2m_2\dot{q}_1\dot{q}_2 \\ -2\,(m_2 + m_3)\,\dot{q}_1\dot{q}_3 + 2m_2\dot{q}_2\dot{q}_3 \end{bmatrix}, \tag{25}$$

$$E_p = \frac{1}{2}c_1 q_1^2 + \frac{1}{2}c_{\text{ue}}\,(q_2 + f_{\text{st}})^2 + m_2 g\,(-q_1 - q_2 - q_3)\,, \tag{26}$$

In the considered case of non-stationary work regime of the load lifting mechanism, the differential equations are represented by equation (27):

$$\begin{aligned} (m_1 + m_2 + m_3)\,\ddot{q}_1 + m_2\ddot{q}_2 + (m_2 + m_3)\,\ddot{q}_3 + b_1\dot{q}_1 + c_1 q_1 - m_2 g &= 0, \\ m_2\ddot{q}_1 + m_2\ddot{q}_2 + m_2\ddot{q}_3 + b_2\dot{q}_2 + c_{\text{ue}} q_2 &= 0, \\ (m_2 + m_3)\,\ddot{q}_1 + m_2\ddot{q}_2 + (m_2 + m_3)\,\ddot{q}_3 &= m_2 g - F_k\,. \end{aligned} \tag{27}$$

The initial conditions necessary for solving the differential equation systems (14) are given by equation (15):

$$\begin{aligned} &q_1(0) = q_{1\text{st}}\,; \quad q_2(0) = 0; \quad q_3(0) = 0; \\ &\dot{q}_1(0) = 0; \quad\;\; \dot{q}_2(0) = 0; \quad \dot{q}_3(0) = v_{\text{diz}}\,. \end{aligned} \tag{28}$$

Also, the system of differential equations (10), and the system (14) were solved using the software specially developed by the authors of this paper. The initial values of generalized coordinates $q_{1\text{st}}$ for acceleration and braking periods represent the size of deformation of the supporting construction of the bridge crane due to its own weight and nominal load.

5. Results

The bridge crane parameters shown in Fig. 1a, which are necessary to solve the differential equations (23) and (27), are listed in Table 1. The change in deformation size of the main thrust girder center, and the change of generalized coordinates q_1, in the acceleration case for load lifting process, are shown in Fig. 5, while the change in the size q_1 for braking during the load lowering process is shown in Fig. 6.

Table 1. Bridge crane parameters

m_1 [kg]	m_2 [kg]	m_3 [kg]	
75	250	Acceleration	Braking
		4150	2980
H [m]	l [m]	Q [kN]	F_{din} [kN]
2	1.5	2.5	1
c_1 [kN/cm]	c_{ue} [kN/cm]	b_1 [kNs/cm]	b_2 [kNs/cm]
18.90	6.25	0.12	0.0375
F_k [kN]	T [s]	t_k [s]	v_{diz} [m/s]
2.53	1.5	1	0.14

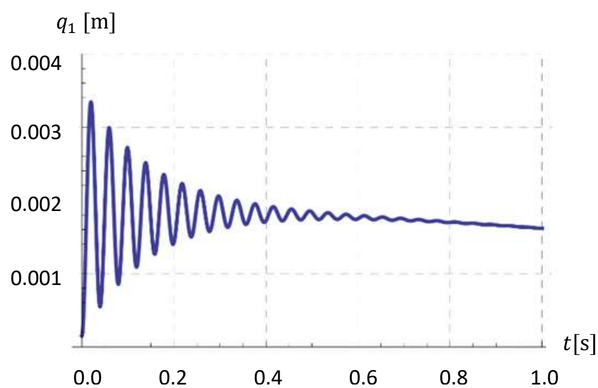

Fig. 5. Size of q_1 in the case of acceleration for load lifting

Based on the known maximum deformation values of the support structure (Figs. 5 and 6), and using software packages for static and dynamic structural analysis (the CATIA V5 software is used in this case), one can calculate maximum stresses in the bearing structure of the crane can. In the case of acceleration during load lifting the maximal stress is $\sigma_u = 80.64$ MPa. For braking, during load lowering, the stress value is $\sigma_k = 60.48$ MPa.

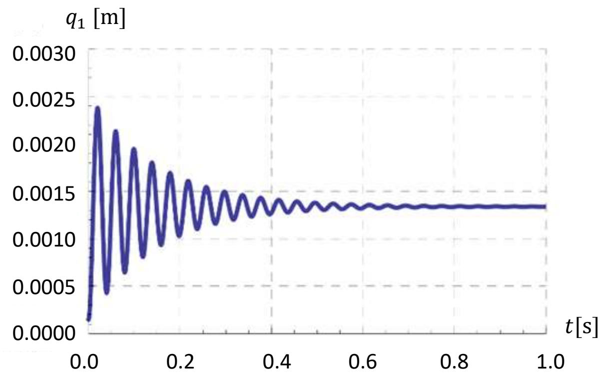

Fig. 6. Size of q_1 in the case of braking for load lowering

6. Conclusion

In this research, we have applied a well-known dynamic model with some additional corrections, in order to develop a more accurate model. The following corrections are introduced: the stiffness of rope is assumed constant, additional working conditions are taken into consideration, working conditions most problematic from the aspect of dynamic load are chosen, damping effects in the crane construction and in the rope are taken in consideration, the load force is considered in a form of quadratic equation and the braking force is assumed as a constant value.

When designing a bridge crane, the most significant parameters which must be analyzed are the value of deflection of support construction and the stress value in the middle of the main bridge span. The maximum value of deflection for a certain construction crane is defined and limited by both national and international standards. It is therefore necessary to consider changing the value of q_1 during the non-stationary work regime of the lifting mechanism. Figs. 5 and 6 show that for the considered bridge crane, at the point of maximum deflection of the bridge crane, more critical is the acceleration period during load lifting then the braking period during load lowering. The maximum size of deformation of the supporting construction (Figs. 5 and 6) was used to obtain maximum values of stress in the supporting structure. This was done using software packages for static and dynamic structural analysis. It can be noticed that the highest stress value in the crane's main girder appears in its middle point, which could be expected. The stress value in the load acceleration period is $\sigma_u = 80.64$ MPa which is greater than the stress value for the braking time during load lowering $\sigma_k = 60.48$ MPa. The higher stress value in the acceleration period is explained by the fact that deformation of the supporting structure in the braking period is smaller in comparison to that in the acceleration period. According to the calculations carried out using regulations and standards BAS ISO 9374-5 and EN1993-1-1: 2005, the stress value in the analyzed section of the main crane is $\sigma = 83.19$ MPa. It is important to note that this stress value is obtained by a calculation that takes into account the prescribed value of dynamic coefficient $\vartheta = 1.15$. By comparing the results obtained from this research and the results obtained on the basis of the calculation using the dynamic coefficient, one can observe that the stress value in the acceleration period during load lifting is lower by 3,07% than the value obtained by the application of BAS ISO 9374-5 and EN1993-1-1: 2005 regulations. To generalize this notion it is necessary to carry out a wider research on the family of bridge cranes, which can provide data for further development in this area.

References

[1] Q. Yang, X. Li, H. Cai, Y-M. Hsu, J. Lee, C. Hung Yang, Z. Li Li, and M. Yi Li. Fault prognosis of industrial robots in dynamic working regimes: Find degradation in variations. *Measurement*, 173:108545, 2021. doi: 10.1016/j.measurement.2020.108545.

[2] S. Wang, Z. Ren, G. Jin, and H. Chen. Modeling and analysis of offshore crane retrofitted with cable-driven inverted tetrahedron mechanism. *IEEE Access*, 9:86132–86143, 2021. doi: 10.1109/access.2021.3063792.

[3] Q. Jiao, B. Li, Y. Qin, F. Wang, J. Gu, J. Wang, and C. Mi, Research on dynamic characteristics of lifting rope-breaking for the nuclear power crane. *Journal of Failure Analysis and Prevention*, 21:1220–1230, 2021. doi: 10.1007/s11668-021-01154-2.

[4] D. Cekus, P. Kwiatoń, and T. Geisler. The dynamic analysis of load motion during the interaction of wind pressure. *Meccanica*, 56:785–796, 2021. doi: 10.1007/s11012-020-01234-x.

[5] J. Yuan, C. Schwingshackl, C. Wong, and L. Salles. On an improved adaptive reduced-order model for the computation of steady-state vibrations in large-scale non-conservative systems with friction joints. *Nonlinear Dynamics*, 103:3283–3300, 2021. doi: 10.1007/s11071-020-05890-2.

[6] H. Zhu, J. Li, W. Tian, S. Weng, Y. Peng, Z. Zhang, and Z. Chen. An enhanced substructure-based response sensitivity method for finite element model updating of large-scale structures. *Mechanical Systems and Signal Processing*, 154:107359, 2021. doi: 10.1016/j.ymssp.2020.107359.

[7] I. Golvin and S. Palis. Robust control for active damping of elastic gantry crane vibrations. *Mechanical Systems and Signal Processing*, 121:264–278, 2019. doi: 10.1016/j.ymssp.2018.11.005.

[8] L. Sowa, W. Piekarska, T. Skrzypczak, and P. Kwiatoń. The effect of restraints type on the generated stresses in gantry crane beam. *MATEC Web Conferences*, 157:02046, 2018. doi: 10.1051/matecconf/201815702046.

[9] Y.A. Onur and H. Gelen. Design and deflection evaluation of a portal crane subjected to traction load. *Materials Testing*, 62(11):1131–1137, 2020. doi: 10.3139/120.111597.

[10] Y.A. Onur and H. Gelen. Investigation on endurance evaluation of a portal crane: experimental, theoretical and finite element analysis. *Materials Testing*, 62(4):357–364. 2020. doi: 10.3139/120.111491.

[11] A. Komarov, A. Grachev, A. Gabriel, and N. Mokhova. Simulation of the misalignment process of an overhead crane in Matlab/Simulink. *E3S Web Conferences*, 304:02008, 2021. doi: 10.1051/e3sconf/202130402008.

[12] A. Cibicik, E. Pedersen, and O. Egeland. Dynamics of luffing motion of a flexible knuckle boom crane actuated by hydraulic cylinders. *Mechanism and Machine Theory*, 143:103616, 2020. doi: 10.1016/j.mechmachtheory.2019.103616.

[13] D. Cekus and P. Kwiatoń. Effect of the rope system deformation on the working cycle of the mobile crane during interaction of wind pressure. *Mechanism and Machine Theory*, 153:104011, 2020. doi: 10.1016/j.mechmachtheory.2020.104011.

[14] D. Ostric, N. Zrnic, and A. Brkic. A modeling of bridge cranes for research of dynamic phenomena during their movement. *Tehnika – Mašinstvo*, 51(3-4):1–6, 1996.

[15] T. Wang, N. Tan, X. Zhang, G. Li, S. Su, J. Zhou, J. Qiu, Z, Wu, Y. Zhai, and R. Donida Labati. A time-varying sliding mode control method for distributed-mass double pendulum bridge crane with variable parameters. *IEEE Access*, 9:75981–75992, 2021. doi: 10.1109/access.2021.3079303.

[16] M.S. Komarov. Dynamics of load-carrying machines. *Madagiz*, Moscow, 1962. (in Russian).

[17] S. Dedijer. *Dynamic coefficients in operation of bridge cranes of small and medium load capacity*. D.Sc. Thesis, Faculty of Mechanical Engineering, Belgrade, Jugoslavia, 1970.

[18] D. Scap. Dynamic loads of the bridge crane when lifting loads. *Tehnika - Strojarstvo*, 24(6):307–315, 1982.

[19] H.A. Lobov. Dynamics of load-carrying cranes. *Mechanical Engineering*, Moscow, Russia, 1987. (in Russian).

[20] D. Ostric, A. Brkic, and N. Zrnic. The analysis of influence of swing of the cargo and rigidity of driving shafts of mechanism for moving to the dynamic behaviour of the bridge crane. *Proceedings of IX IFToMM Congress*, Milano, 1995.

[21] D. Ostric, A. Brkic, and N. Zrnic. The analysis of bridge cranes dynamic behaviour during the work of hoisting mechanism. *Proceedings of XIV IcoMHaW*, Faculty of Mechanical Engineering, Belgrade, 1996.

[22] M. Delić, M. Čolić, E. Mešić, and N. Pervan. Analytical calculation and FEM analysis main girder double girder bridge crane. *TEM Journal*, 6(1):48–52, 2017. doi: 10.18421/TEM61-07.

[23] M. Delić, N. Pervan, M. Čolić, and E. Mešić. Theoretical and experimental analysis of the main girder double girder bridge cranes. *International Journal of Advanced and Applied Sciences*, 6(4):75–80, 2019. doi: 10.21833/ijaas.2019.04.009.

[24] H. A. Hobov. Calculation of dynamic loads of bridge cranes when lifting a load. *Bulletin of Mechanical Engineering*, 5:37–41, 1977. (in Russian).

[25] D. Ostric, A. Brkic, and N. Zrnic. Influence of driving-shaf to dynamic behavior of the bridge crane in horizontal plane, modeled with several concentrated masses during the acceleration. *FME Transactions*, 2: 25–30, 1993.

[26] S.G. Kelly. *Mechanical Vibrations – Theory and Applications*, Global Engineering, Stamford, USA, 2012.