

Fractal dimensions analysis of branching streamers propagating in mineral oil

VIET-HUNG DANG¹ , ABDERRAHMANE BEROUAL² , PAWEŁ ROZGA³  

¹*Electric Power University
Vietnam*

²*University of Lyon, Ecole Centrale de Lyon
France*

³*Lodz University of Technology
Poland*

e-mail: hungdv79@epu.edu.vn, Abderrahmane.Beroual@ec-lyon.fr, pawel.rozga@p.lodz.pl

(Received: 07.12.2021, revised: 11.04.2022)

Abstract: This article deals with the analysis of the fractal dimension of streamers propagating in mineral oil, under lightning impulse voltage, using the box counting method; the method and technique of calculation are described therein. In the considered experimental conditions, the average velocities of recorded streamers are of 2.4 km/s and 1.8 km/s for positive and negative streamers, respectively; these velocities correspond to the 2nd mode of streamers propagation. It is shown that the streamers present the fractal dimension D ; and the higher D is the bushier are the streamers (i.e. with high branch density). The positive streamers can have higher D than the negative ones, if they are bushier.

Key words: fractal analysis, lightning impulse voltage, mineral oil, streamers

1. Introduction

The characterization of streamers propagating in dielectric liquids is of great interest for predicting the dielectric withstand of insulating systems in HV equipment (particularly HV power transformers). The knowledge of the parameters characterizing these streamers is very helpful in designing and dimensioning insulating systems; this is fundamental for preventing against the breakdown of insulations, in particular that of insulating fluid that could results from switching and lightning over-voltages [1]. The study of streamers in liquid dielectrics is generally performed in point-plane geometry. Depending on the experimental conditions, especially on the



© 2022. The Author(s). This is an open-access article distributed under the terms of the Creative Commons Attribution-NonCommercial-NoDerivatives License (CC BY-NC-ND 4.0, <https://creativecommons.org/licenses/by-nc-nd/4.0/>), which permits use, distribution, and reproduction in any medium, provided that the Article is properly cited, the use is non-commercial, and no modifications or adaptations are made.

voltage (waveform, amplitude, polarity), hydrostatic pressure and temperature, different shapes of streamers can be initiated at the sharp electrode: bushy, hemispherical, umbrella, branched, etc. [2–9]. Those streamers are characterized by their velocity, stopping length, electrical charge, current and light they emit. Generally, the streamers initiated at the negative point electrode are slow and bushy while those initiated at the positive electrode are fast and branchy with the exception of mineral oil where the velocities of both negative and positive streamers are of the same order [2, 6]. Such a classification, based on the shape of the streamers and the polarity of the voltage, remains too general, the speed of the streamers being able to vary from a few tens of m/s to several hundred km/s. A classification in different propagation modes, based mainly on the values of velocity and the inception electric field, has been proposed by Hebner *et al.* [4]. In this classification, streamers are arbitrarily classified into 1st, 2nd and 3rd modes according to the speeds measured; the latter increase by a factor of approximately 10 from one mode to another: 1st mode for velocities of about 0.2 km/s; 2nd mode for velocities ranging 2 to 4 km/s; and 3rd mode for velocities higher than 10 km/s. According to recent work [7–9], the streamers have been classified into four modes: 1st mode corresponding to velocities in the range of 0.1 to 0.5 km/s; 2nd mode corresponding to velocities of about 1 to 4 km/s; 3rd mode corresponding to velocities in the range 10 to 20 km/s; and 4th mode for velocities higher than 10 km/s and that can exceed 100 km/s. This 4th mode is observed in the propagation of positive streamers and it is initiated from a streamer being propagated in another mode. The transition from one mode to another is produced by increasing the applied voltage. Note also that the voltage waveform and its application time have an influence on this transition.

The shape of the streamers presents a stochastic character, the streamers can also be classified according to their fractal dimension. The fractal dimension is also used for the analysis of various forms of objects and phenomena observed in nature [10–14]. The interest of analyzing the fractal dimension of streamers lies in the relationship between the latter and the shape of the streamers and therefore the mechanisms underlying the phenomenon of propagation. Indeed, the denser the streamer and the more branches there will be, the higher its fractal dimension will be. Since streamers are gaseous in nature, the higher the fractal dimension, the more gas there will be in the liquid.

To analyze the fractal dimension of 2D structures and take into account the tendency of discharge branches to propagate “randomly”, Sawada *et al.* [14] introduced a stochastic model. Niemeyer *et al.* [15] elaborated a dielectric breakdown model (DBM) of surface discharges, using the fractal theory; the local electric field was used as a criterion for the growth probability of the discharge patterns. This stochastic model of electrical discharge has been extended to the 3D patterns that were applied to describe electrical trees in solid dielectrics [15–18]. However, even if the developed models introduce the electric field for the propagation of the discharges, they only describe structures generated by simulation and not the real structures of the discharges observed experimentally.

The fractal dimension of actual shapes (i.e., observed experimentally) of surface discharges propagating over solid insulating immersed in dielectric liquids has been analyzed in an original work by Kebbab and Beroual [19]; the shape of these discharges recalls Lichtenberg figures [20]. The fractal dimension of this type of discharge has been conducted subsequently by other researchers using the same technique/software as these authors, considering different solid insulating materials and different liquids [21, 22]. All these works showed that these surface discharges have

a fractal dimension which depends on the physicochemical properties of the solid/liquid dielectric structure, the applied voltage and hydrostatic pressure.

The analysis of the fractal dimension of the streamers propagating within insulating liquids, in a divergent electric field, is difficult because they develop in the three-dimensional space. To get around this difficulty, it is possible to consider different sections of the streamer (that is to say in the two-dimensional space) resulting in a plane rotating around the head of the sharp electrode (electrode point) so as to describe the whole streamer.

This paper deals with the fractal analysis of branching streamer shapes propagating in mineral oil under lightning impulse voltage.

2. Experiment

The diagram of the experimental system, presented in Fig. 1, consists of a Marx generator (500 kV – 2.2 kJ – 1.2/50 μ s), a visualization/photography system and a test cell containing a point-plane electrode arrangement. The point HV electrode was made of tungsten, the radius of curvature r_p of which was 250 μ m while the grounded electrode was made of an aluminum plate of 150 mm in diameter. A flat pressboard of 5 mm thickness and the same diameter as the grounded electrode was deposited on the plane electrode to prevent breakdown, so the effective electrode gap was 20 mm. The investigated liquid is mineral oil (MO) of the naphthenic type; the used volume of oil was 26 L. The fundamental parameters of the tested mineral oil are included in Table 1.

The test cell was equipped with four symmetrically placed glass windows. Two of these were used for the shadowgraph photography registration system, while the other two ones for the registration system of light emitted by streamers thanks to a photomultiplier. The shadowgraph

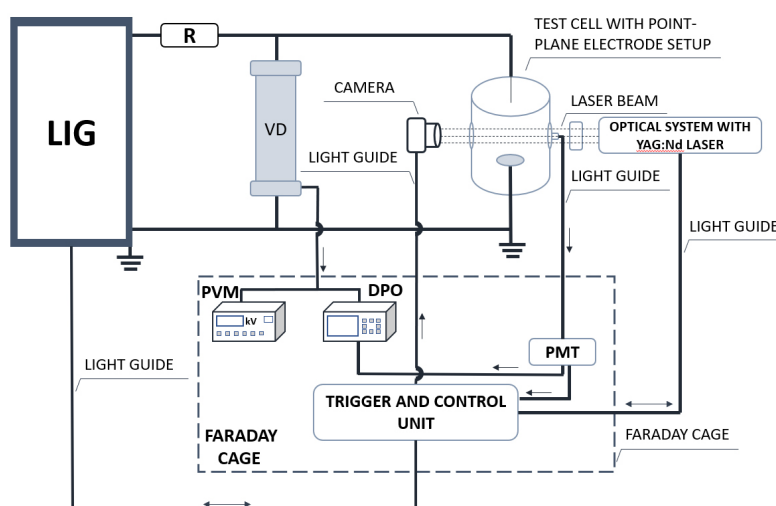


Fig. 1. Laboratory experimental system: LIG – lightning impulse generator, VD – voltage divider, R – limiting resistor, PMT – photomultiplier, PVM – peak value meter, DPO – digital phosphore oscilloscope

Table 1. Parameters of the mineral oil under investigation

Parameter	Unit	Value
Density at 20°C	kg/dm ³	0.87
Viscosity at 40°C	mm ² /s	9.1
Flash point	°C	146
Dielectric dissipation factor at 90°C	–	0.001
AC breakdown voltage as per IEC	kV	66
Moisture content	ppm	12
Electrical permittivity at 90°C	–	2.2

photographs were taken using the single-shot mode by applying the Q-switched YAG neodymium laser. This means that during one discharge, one streamer photography could be taken out. The laser generates a short, 10 ns in width, light pulse, which is controlled such that it allows one to take a photography at any time from the moment of streamer initiation. The time of taking the photography could be changed with 0.1 μ s resolution. The triggering of the system of streamer photography registration was synchronized with the lightning wave generated by a Marx generator and the signal from the photomultiplier informing about the discharge initiation.

Figures 2 to 3 show the selected representative photographs of negative and positive streamers propagating under lightning impulse voltage, the peak value of which being 104 kV. The distinctive photographs are identified by the time they were taken versus the time the voltage was applied. This time roughly corresponds to the propagation time of streamers; it includes the initiation time. This time is used to evaluate the average propagation velocity of streamers. In details, the propagation velocity is assumed to be the slope of the regression line in the $l_s = f(t)$ co-ordinate system. The l_s was the length of the longest streamer channel read from the photography with an accuracy of 0.1 mm, while t is the time of the streamer propagation above mentioned. It is important to point out herein that all the considered streamers are initiated after the crest of the lightning impulse waveform [23]. Only 8 photographs per polarity are chosen for further analysis due to the fact that they do not differ from each other significantly when comparing the same polarity and similar propagation time. This is done not to repeat the calculation for the same or very similar data (photographs).

The average velocities of streamers are of 2.4 km/s and 1.8 km/s for positive and negative streamers, respectively. These values are of the same range as those reported elsewhere [6, 9, 24–26].

At the first glance, if we limit ourselves to the classification of streamers according to their form, this result is surprising regarding the effect of polarity. Indeed, it was commonly accepted that the positive streamers are faster and more branchy than the negative ones, whereas we observe the opposite: the positive streamers are a little bit faster (i.e. a 1.3 times) than the negative ones but they are bushier (i.e. with a higher branch density). However, this result is not as surprising as it sounds when one considers the different modes of propagation of streamers [7, 9]. Indeed, according to [9], the positive streamer in the third mode of propagation (10 km/s) in mineral oil is bushier than the positive streamer propagating in the second mode (2 km/s).

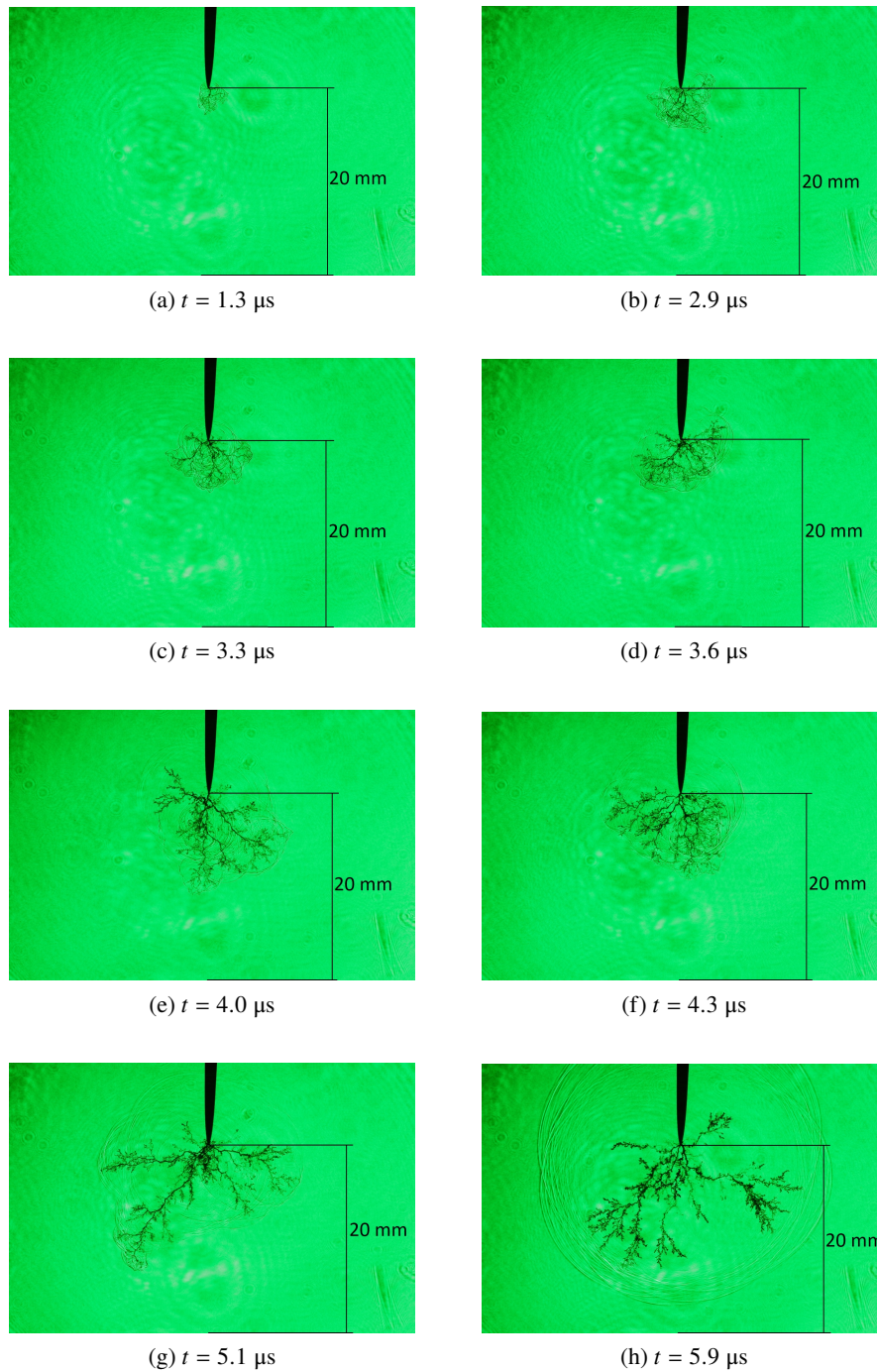


Fig. 2. Photographs of streamers propagating in mineral oil, under negative lightning impulse voltage the peak value of which is 104 kV

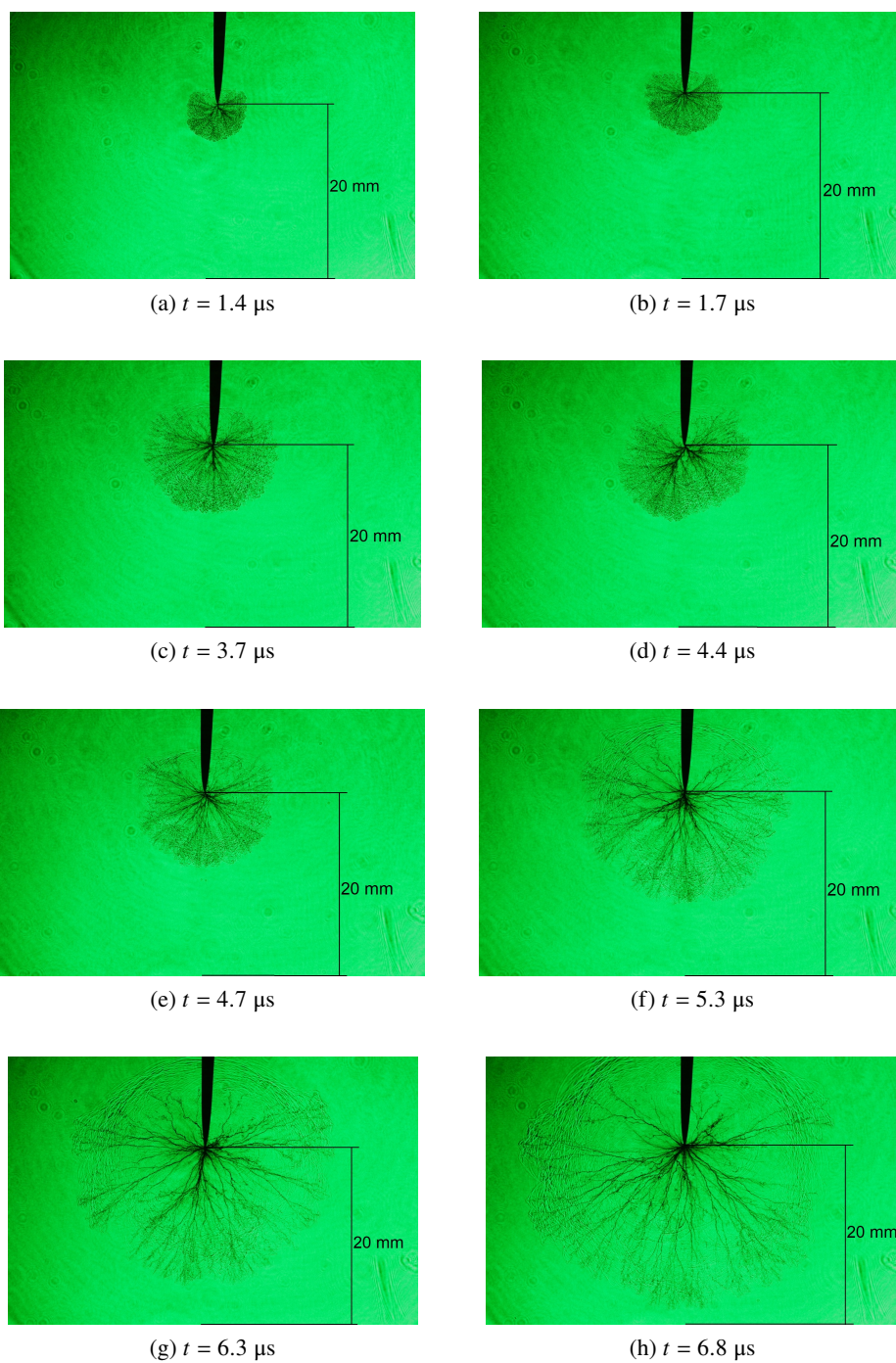


Fig. 3. Photographs of streamers propagating in mineral oil, under positive lightning impulse voltage the peak value of which is 104 kV

3. Calculation method of fractal dimension

There are many methods to estimate the fractal dimension D of a self-similar structure [10]. For its ease of use, the box-counting method is the one used in this work. This has been described in detail in previous work [19, 21, 22]. For the sake of continuity, let us recall here the main lines of this method. This consists, in the first step, in converting the original images of the discharges into binary (black and white), as illustrated by Figs. 4(a) and 4(b). The information contained in

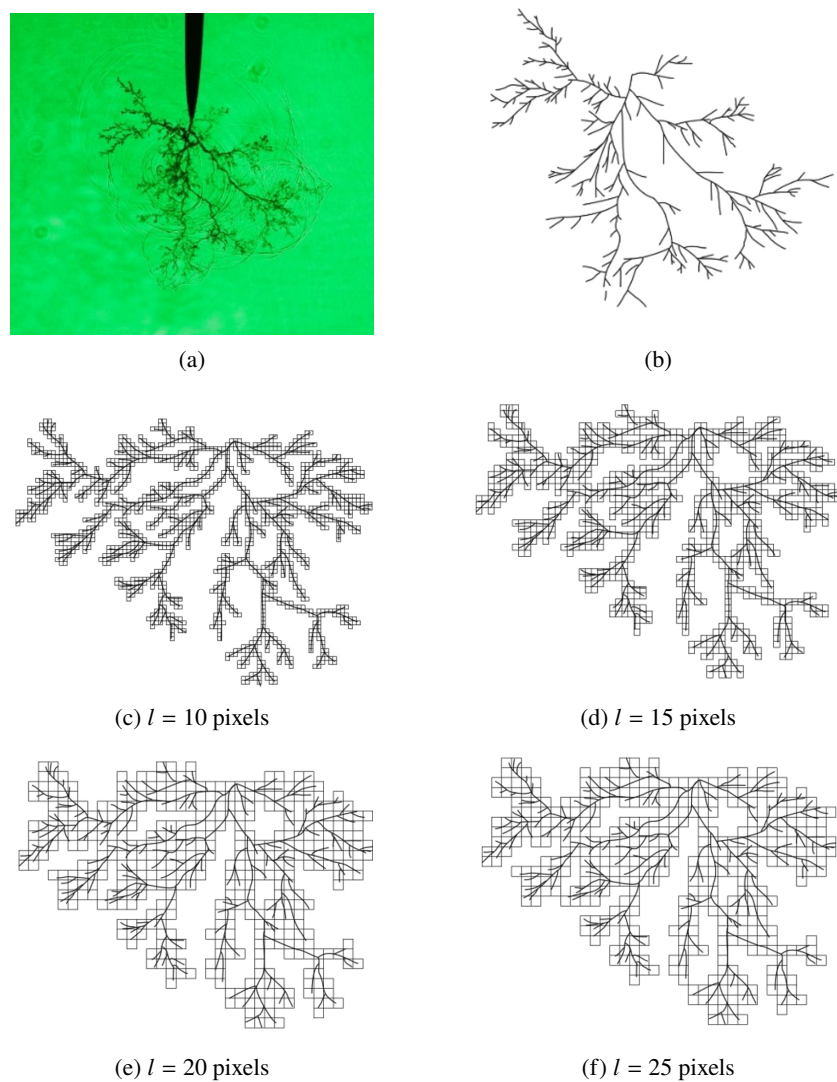


Fig. 4. Application of box counting method: (a) the original image, (b) the extracted binary image, (c) and (d) the binary image covered by squares of side $l = 10$ pixels and $l = 25$ pixels, respectively. The treated photo is that of negative streamer of Fig. 2(e)

the streamer image is transformed into a 580×780 element matrix. The elements of the matrix are integers with a value between 1 (represents black) and 255 (represents white) representing the data relating to the color of each pixel in the image. Values between 1 and 255 represent shades of gray. These images are then processed by a computer code designed for this purpose in Matlab, described in early work [21]. Its principle consists in generating a square mesh of the side l completely covering the discharge. The magnitude of the square side l is changed at each step. The number of squares (or boxes) of the mesh containing a branch, however small it may be, of the discharge $N(l)$ is counted for each value of l . An illustration of the grid method is given in Figs. 4(c) to 4(f). After this operation, the characteristic $N = f(l)$ is plotted in a logarithmic coordinate system. The slope of the linear part of each plotted characteristic makes it possible to deduce the value of the corresponding fractal dimension D of the discharge according to the following relationships

$$N(l) \sim l^{-D}. \quad (1)$$

So,

$$D = \lim_{l \rightarrow 0} \left(\frac{\log N(l)}{\log l} \right)^n. \quad (2)$$

4. Results and discussion

After treating Figs. 2 and 3, we get a total number of boxes N versus the side of boxes l for both positive and negative streamers, respectively. Figures 5 and 6 present the variations of N versus l , deduced from this processing; and Tables 2 and 3 summarize the fractal dimension values of the images of negative and positive streamers depicted in Figs. 2 and 3, respectively.

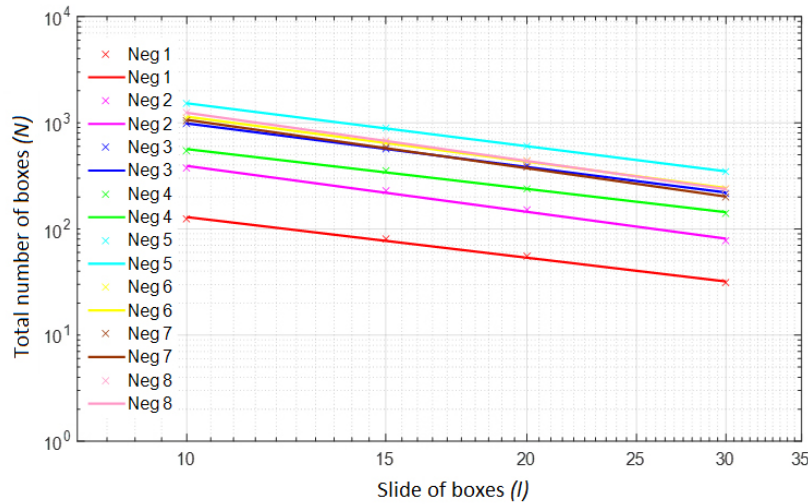


Fig. 5. Total number of boxes N versus the side of boxes l obtained from the analysis of negative streamers in Fig. 2, using the box counting method

It is observed that at each moment of propagation, the fractal dimension of positive streamers is higher than that of negative ones. This makes sense given the fact that the positive streamers appear bushier (i.e. with higher branch density) than the negative ones (see Figs. 2(d) and 3(c) that are taken at a close time namely $3.6 \mu\text{s}$ and $3.7 \mu\text{s}$, respectively).

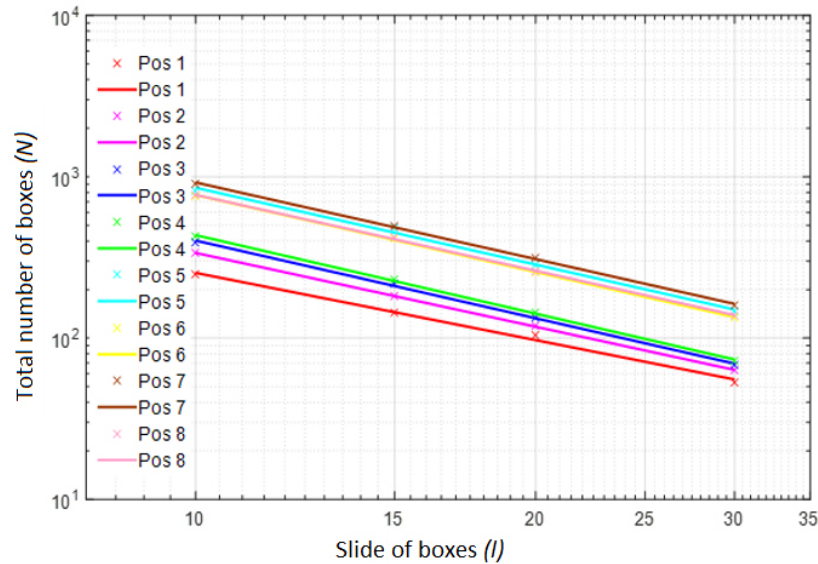


Fig. 6. Total number of boxes N versus the side of boxes l obtained from the analysis of positive streamers in Fig. 3, using the box counting method

Table 2. Fractal dimension of the images of negative streamers depicted in Fig. 2

Photo	2(a)	2(b)	2(c)	2(d)	2(e)	2(f)	2(g)	2(h)
D	1.271	1.435	1.363	1.245	1.343	1.407	1.514	1.507

Table 3. Fractal dimension of the images of positive streamers depicted in Fig. 3

Photo	3(a)	3(b)	3(c)	3(d)	3(e)	3(f)	3(g)	3(h)
D	1.385	1.521	1.601	1.617	1.583	1.578	1.578	1.564

We observe that the fractal dimension of positive streamers is higher than that of the negative ones. If we limit ourselves only to the classification of streamers according to the polarity of the sharp electrode, namely that the positive streamers are faster and branchy (less bushy) than the negative ones which are slow and bushy [6, 19, 21], this result seems surprising, even if in mineral oil the positive streamers are only slightly faster than the negative ones (1.3 times faster). However, in the classification of streamers according to their modes of propagation, and therefore, according

to their velocities and the electric field of their initiation, negative streamers can also be branchy and fast [7, 9]. In the case of Figs. 2 and 3, propagations for both positive and negative cases are according to the 2nd mode. Note that in mineral oil, the streamers can propagate according to the 4th mode (typical velocity of 100 km/s) [9]. Indeed, from a certain threshold voltage U_a , called acceleration voltage, one channel (or two) can propagate from the initial discharge with an ultra-fast speed which exceeds the 100 km/s and fills most of the interelectrode space [7, 9].

So, according to the observations reported in [9], fast streamers can be bushy as it is the case of the 3rd mode in mineral oil. Thus, one cannot relate the fractal dimension to the modes of propagation of streamers as for the high mode, the streamers can be bushy and fast.

5. Conclusions

By using the box counting method to calculate the fractal dimension D , it is shown that the branching streamers propagating in mineral oil present a fractal dimension that depends on the polarity of the voltage (electric field). The velocities of streamers we considered in this work are in the same range and correspond to the 2nd propagation mode. The more branched the streamer (i.e. the lower the number of streamer branches), the smaller its fractal dimension. As fast streamers can be bushy (i.e. with high density of branches), one cannot relate the fractal dimension to the modes of propagation of streamers.

References

- [1] Abu Shehab W.F., Ali S.A., Alsharari M.I., *Lightning protection for power transformers of Aqaba Thermal Power Station*, Archives of Electrical Engineering, vol. 69, no. 3, pp. 645–660 (2020), DOI: [10.24425/ae.2020.133923](https://doi.org/10.24425/ae.2020.133923).
- [2] Devins J.C., Rzađ S.J., Schwabe R.J., *Breakdown and pre-breakdown phenomena in liquids*, Journal of Applied Physics, vol. 52, pp. 4531–4545 (1981), DOI: [10.1063/1.329327](https://doi.org/10.1063/1.329327).
- [3] Beroual A., Tobazeon R., *Prebreakdown phenomena in liquid dielectrics*, IEEE Transactions on Electrical Insulation, vol. 21, no. 4, pp. 613–627 (1986), DOI: [10.1109/TEI.1986.348967](https://doi.org/10.1109/TEI.1986.348967).
- [4] Hebner R.E., *Measurements of Electrical Breakdown in Liquids*, in *The Liquid State and its Electrical Properties*, vol. B193, Plenum Press (1988).
- [5] Badent A., Kist K., Schwabe R.J., *Voltage Dependence of Prebreakdown Phenomena in Insulating Oil*, Conference Record of the IEEE International Symposium on Electrical Insulation, Pittsburg, PA, USA, pp. 414–417 (1994).
- [6] Beroual A., Zahn M., Badent A., Kist K., Schwabe A.J., Yamashita H., Yamazawa K., Danikas M., Chadband W.G., Torshin Y., *Propagation and Structure of Streamers in Liquid Dielectrics*, IEEE Electrical Insulation Magazine, vol. 14, no. 2, pp. 6–17 (1998), DOI: [10.1109/57.662781](https://doi.org/10.1109/57.662781).
- [7] Lesaint O., *Prebreakdown phenomena in liquids: propagation “modes” and basic physical properties*, Journal of Physics D-Applied Physics, vol. 49, no. 14, 22 (2016), DOI: [10.1088/0022-3727/49/14/144001](https://doi.org/10.1088/0022-3727/49/14/144001).
- [8] Rozga P., Beroual A., Przybylek P., Jaroszewski M., Strzelecki K., *A Review on Synthetic Ester Liquids for Transformer Applications*, Energies, vol. 13, 6429 (2020), DOI: [10.3390/en13236429](https://doi.org/10.3390/en13236429).
- [9] CIGRE Group TB 856, *Dielectric performance on insulating liquids for transformers*, WG D1.70 TF3 (2021).

- [10] Mandelbrot B.B., *Fractals, Form, Chance and Dimension*, Freeman, San Francisco, USA (1977), DOI: [10.1016/0012-8252\(79\)90075-8](https://doi.org/10.1016/0012-8252(79)90075-8).
- [11] Djemai Z., Beroual A., *Fractal Dimension of Discharges Propagation on Insulating Interfaces*, Archives of Electrical Engineering, vol. 3, pp. 249–254 (1998).
- [12] Boroujeni F.M., Maleki A., *Fractal Analysis of Noise Signals of Sampo and John Deere Combine Harvesters in Operational Conditions*, Archives of Acoustics, vol. 44, no. 1, pp. 89–98 (2019), DOI: [10.24425/aoa.2019.126355](https://doi.org/10.24425/aoa.2019.126355).
- [13] Ficker T., *Electrostatic discharges and multi-fractal analysis of their Lichtenberg figures*, Journal of Physics D: Applied Physics, vol. 32, pp. 219–226 (1999).
- [14] Sawada Y., Ohta S., Yamazaki M.Y., Honjo H., *Self-similarity and a phase transition-like behaviour of a random growing structure governed by a non-equilibrium parameter*, Physics Review A, vol. 26, 3557 (1982), DOI: [10.1103/PhysRevA.26.3557](https://doi.org/10.1103/PhysRevA.26.3557).
- [15] Niemeyer L., Pietronero L., Wiesmann H.J., *Fractal dimension of dielectric breakdown*, Physical Review Letters, vol. 33, pp. 1033–1036 (1984), DOI: [10.1103/PhysRevLett.52.1033](https://doi.org/10.1103/PhysRevLett.52.1033).
- [16] Wiesmann H.J., Zeller H.R.A., *A fractal model of dielectric breakdown and prebreakdown in solid dielectrics*, Journal of Applied Physics, vol. 60, pp. 1770–1773 (1986), DOI: [10.1063/1.337219](https://doi.org/10.1063/1.337219).
- [17] Fujimori S., *Electric Discharge and Fractals*, Japan Journal of Applied Physics, vol. 24, no. 9, pp. 1198–1203 (1985).
- [18] Kudo K., *Fractal analysis of electrical trees*, IEEE Transactions on Dielectrics and Electrical Insulation, vol. 5, no. 5, pp. 713–727 (1998), DOI: [10.1109/94.729694](https://doi.org/10.1109/94.729694).
- [19] Kebbabi L., Beroual A., *Fractal analysis of creeping discharge patterns propagating at solid/liquid interfaces: Influence of the nature and geometry of solid insulators*, Journal of Physics D: Applied Physics, vol. 39, pp. 177–183 (2006), DOI: [10.1088/0022-3727/39/1/026](https://doi.org/10.1088/0022-3727/39/1/026).
- [20] Lichtenberg G.C., *Nova methodo naturam ac motum fluidi electrici investigandi*, Commentatio Prior, Novi Commentarti Soc. Reg. Sc. Gottingensis, vol. 8, pp. 168–180 (1778).
- [21] Beroual A., Dang V-H., *Fractal analysis of lightning impulse surface discharges propagating over pressboard immersed in mineral and vegetable oils*, IEEE Transactions on Dielectrics and Electrical Insulation, vol. 20, pp. 1402–1408 (2013), DOI: [10.1109/TDEL.2013.6571462](https://doi.org/10.1109/TDEL.2013.6571462).
- [22] Beroual A., Coulibaly M.-L., *Relationship between the Fractal Dimension of Creeping Discharges Propagating at Solid/Gas Interfaces and the Characteristics Parameters of Interfaces*, International Review on Electrical Engineering, vol. 9, no. 2, pp. 460–465 (2014).
- [23] Rozga P., *Influence of paper insulation on the prebreakdown phenomena in mineral oil under lightning impulse*, IEEE Transactions on Dielectrics and Electrical Insulation, vol. 18, no. 3, pp. 720–727 (2011), DOI: [10.1109/TDEL.2011.5931058](https://doi.org/10.1109/TDEL.2011.5931058).
- [24] Rozga P., Jayasree T., Mohan Rao U., Fofana I., Picher P., *Prebreakdown and Breakdown Phenomena in Ester Dielectric Liquids*, in *Alternative Liquids Dielectrics for High Voltage Transformer Insulation Systems: Performance Analysis and Applications*, Wiley-IEEE Press, pp. 147–183 (2021), DOI: [10.1002/9781119800194.ch6](https://doi.org/10.1002/9781119800194.ch6).
- [25] Rozga P., Rapp K.J., Stanek M., *Lightning Properties of Selected Insulating Synthetic Esters and Mineral Oil in Point-to-Sphere Electrode System*, IEEE Transactions on Dielectrics and Electrical Insulation, vol. 25, pp. 1699–1705 (2018), DOI: [10.1109/TDEL.2018.007069](https://doi.org/10.1109/TDEL.2018.007069).
- [26] Lundgaard L.E., Linhjell D., Berg G., *Streamer/leaders from a metallic particle between parallel plane electrodes in transformer oil*, IEEE Transactions on Dielectrics and Electrical Insulation, vol. 8, pp. 1054–1063 (2001), DOI: [10.1109/94.971465](https://doi.org/10.1109/94.971465).