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Effect of aiding buoyancy on heat transfer from an isothermal square cylinder in power-law fluids

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The aim of the present study was to explore the influence of aiding buoyancy on mixed convection heat transfer in power-law fluids from an isothermally heated unconfined square cylinder. Extensive numerical results on drag coefficient and surface averaged values of the Nusselt number are reported over a wide range of parameters i.e. Richardson number, $0.1 \le \mathrm{Ri} \le 5$, power-law index, $0.4 \le n \le 1.8$, Reynolds number, $0.1 \le \mathrm{Re} \le 40$, and Prandtl number, $1 \le \mathrm{Pr} \le 100$. Further, streamline profiles and isotherm contours are presented herein to provide an insight view of the detailed flow kinematics.

Keywords: mixed convection, power-law fluids, square cylinder, aiding-buoyancy, Nusselt number

1. INTRODUCTION

As a broad range of industrial fluids exhibit power-law rheology (Bird et al., 1987), considerable research efforts have been directed on studying the flow and heat transfer behaviour of cylinders of various cross-sections (Chhabra, 2011). Several attempts have been made by the researchers to examine the mixed convection heat transfer with aiding buoyancy in unconfined flow past cylinders having various cross-sections such as circular (Srinivas et al., 2009) and square (Sharma and Eswaran, 2005; Sharma et al., 2013; Sharma et al., 2014) in both power-law fluids and Newtonian fluids. Owing to the pragmatic significance of square cylinders in industrial applications and theoretical standpoint, this study aims to bridge the gaps in the state-of-the art for unconfined flows past heated square cylinders immersed in power-law fluids. This work serves as an extension to the study by Sharma et al. (2012) for the effect of aiding buoyancy on an unconfined heated square cylinder. The present study not only extends this work to span the entire power-law fluid behaviour, but the effect of aiding buoyancy is also accounted in more

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detail by taking the value of Ri up to Ri = 5 where natural convection substantially dominates the heat transfer mechanism. Additionally, the effect of Pr has also been incorporated by considering a wide range for Prandtl numbers from Pr = 1 to 100. A detailed analysis is presented for the results in conjunction with that of Sharma et al. (2012).

2. PROBLEM FORMULATION

The schematic configuration of the physical problem considered is presented in Fig. 1. An isothermal unconfined square cylinder is exposed to free stream flow of power-law fluids advancing in upward direction with uniform velocity, V_∞ (aiding-buoyancy) and temperature, T_∞. A constant temperature, T_w, is maintained on the surface of the cylinder. In order to mimic the fictitious unconfined domain, the square cylinder is enveloped in rectangular domain of width, H such that $l/H \leq 0.02$ satisfies the criteria for unconfinement. Since the cylinder is infinitely long in z-direction the flow is assumed to be two-dimensional, i.e., $V_z = 0$ and $\partial V_z/\partial z = 0$. Further, the density of the fluid far away from the cylinder exhibits constant increase eventually approaching the limiting value of ρ_∞ in correspondence to the free stream fluid temperature T_∞. For small temperature variation ($T_w - T_\infty \leq 5$ K), the temperature dependent fluid density is approximated using Boussinesq approximation, i.e., $\rho = \rho_\infty[1 - \beta(T - T_\infty)]$, applied in the buoyancy term of the equation of momentum. Further it is assumed that for the range of parameters considered here, the flow is laminar and steady and as temperature variation is very small, other fluid properties remain constant. Also, the viscous energy dissipation is fairly low to be neglected. Thus, on the

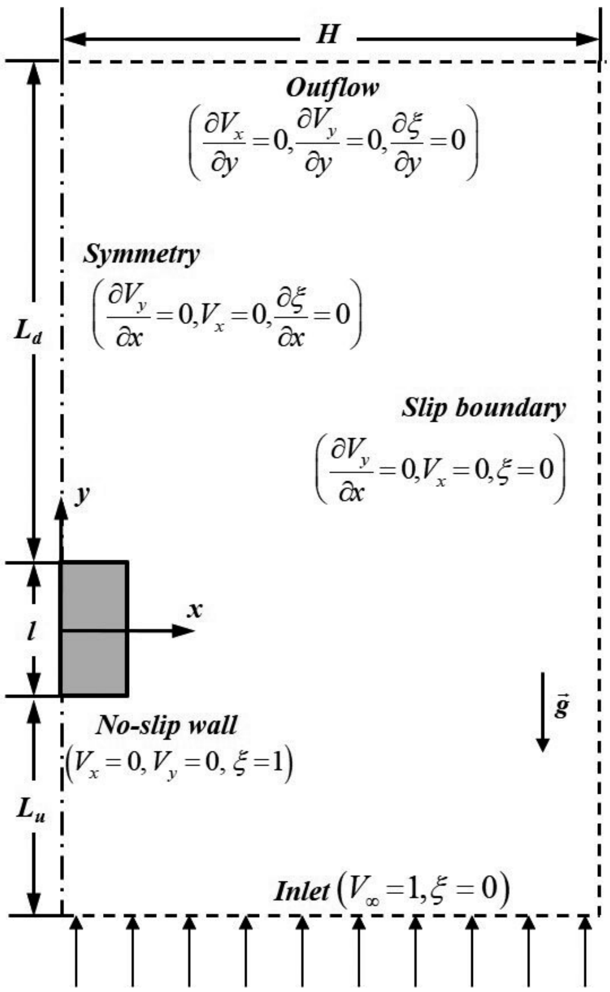

Fig. 1. Schematic representation of the flow and computational domain

basis of the aforementioned assumptions the dimensionless forms of equations for conservation of mass, momentum and energy coupled with the Boussinesq approximation are mentioned below.

$$\nabla V = 0 \tag{1}$$

$$(V \cdot \nabla)V = -\nabla p + \nabla\tau/\mathrm{Re} + \mathrm{Ri}\xi\delta_{12} \tag{2}$$

$$(V \cdot \nabla)\xi = \nabla^2\xi/\mathrm{Re}\,\mathrm{Pr} \tag{3}$$

Since, the constitutive equations for the power-law fluids have been reported by Srinivas et al. (2009), these have not been included here for brevity. To capture the physics of the problem, appropriate boundary conditions are prescribed on all surfaces and boundaries. The boundary conditions and their mathematical definition both are shown in Fig. 1.

2.1. Numerical solution scheme

To ensure the accuracy and reliability of the present numerical scheme, the domain and grid independent test were performed. It was found that a domain with upstream length, $L_u = 25\,\mathrm{l}$ and $L_d = 30\,\mathrm{l}$ and a grid with 240 000 elements were optimum for the present study. Next, the accuracy of the numerical methodology was confirmed by performing validation of present results with relevant literature. The present numerical results are compared with those of Bird (1987) for local Nusselt number for mixed convection from an isothermal circular cylinder in air. The results are seen to be within ±1% with the literature values. The numerical values of gross parameters C_D, C_{DP} and Nu are compared with those of Srinivas et al. (2009) and Sharma et al. (2012) are found to be within ±3% of literature values.

3. RESULTS AND DISCUSSION

In the present study, the new numerical results delineate the influence of aiding buoyancy mixed convection on the flow and heat transfer from an isothermally heated square cylinder in power-law fluids over wide ranges of parameters, such as: $0.1 \le \mathrm{Ri} \le 5$, $0.1 \le \mathrm{Re} \le 40$, $1 \le \mathrm{Pr} \le 100$ and $0.4 \le n \le 1.8$.

3.1. Streamlines and isotherm contours

The representative streamlines and isotherm contours are shown at Re = 40 in Figs. 2(a) and 2(b). It is clear that for all values of Pr and Ri the wake size decreases with increase in either Pr or Ri. The momentum and energy equations are coupled via body force term. At Pr = 10, wakes are observed at the back of the square cylinder for all values of Ri. Also, at a constant value of Pr it can be clearly observed that as Ri increases from Ri = 0.1 to Ri = 5, the wake length is seen to gradually reduce. Thus, it could be reasoned that Ri causes decrease in wake length and at higher values of Ri the flow field gets completely suppressed which could be ascribed to the fact that as the increase in plume strength does not permit the formation of adverse pressure gradient at the rear of the object even at higher values of Re, Pr and Ri used here. In other words, increasing buoyancy tends to stabilize the flow field.

The isotherm contours are observed to exhibit qualitatively close resemblance with those of streamline profiles. It can be observed from Figs. 2(a) and 2(b) that the clustering of isotherms in the upstream direction increases with the increasing Re, notwithstanding the fact that influence of Ri is relatively stronger as compared to Pr. A close inspection of Fig. 2 reveals that isotherm are closely packed in shear thinning fluids relative to shear thickening fluids. This is obviously due to the fact that thermal boundary layers are relatively thinner in shear thinning fluids as compared to shear thickening fluids.

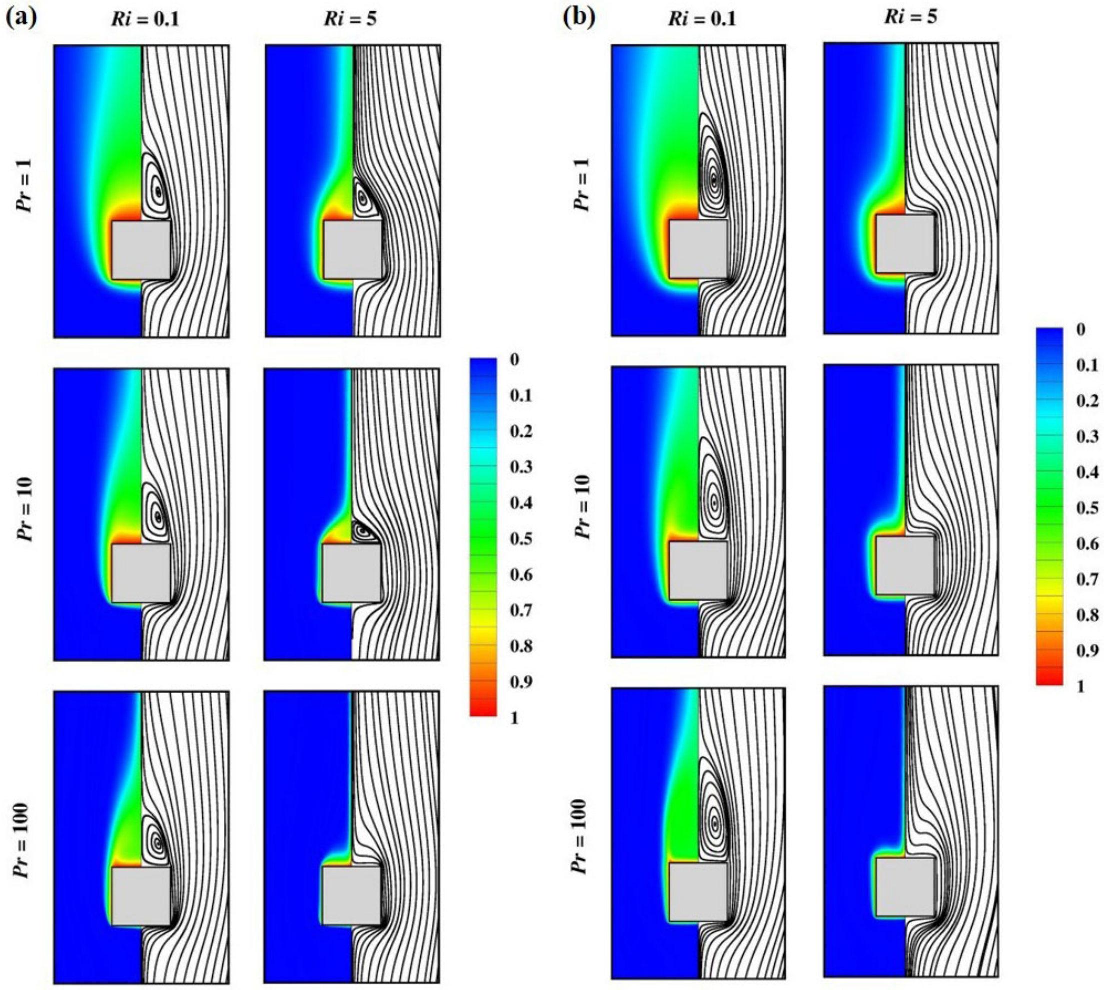

Fig. 2. Representative isothermal contours (left) and streamlines (right) for extreme values of Ri and Pr at Re = 40 for (a) $n = 0.4$ and (b) $n = 1.8$

3.2. Drag coefficient and average Nusselt number

Figure 3 shows the combined effect of n, Ri, Re and Pr on drag coefficient. As seen here, the role of Ri is very complex, it not only causes an increase in mean flow due to increasing velocity gradients but also, causes suppression of wake phenomenon leading to an increase in total drag. Also, it can be clearly observed that even in the mixed convection regime drag coefficient tends to exhibit inverse variation with Re. Also, it was found that ratio C_{DP}/C_D decreases on incrementing the value of Ri which further supports our assertion that increase in buoyancy causes lowering of surface pressure. However, for smaller values of Re and Pr the effects of the thermal buoyancy are superseded by viscous forces.

Figure 4 delineates the distribution of average Nusselt number over Ri at $n = 0.4$ and $n = 1.8$. It is noted that heat transfer increases on incrementing the magnitudes of Re, Pr and Ri which could be due to the gradual thinning of thermal boundary layers. Moreover, increasing trends of average Nusselt number with Ri, show that heat transfer enhances with increasing buoyancy forces. At higher value of Pr the temperature gradients are high which further enhances the effect due to thermal buoyancy forces. Additionally, it can be observed here that heat transfer is higher in shear thining fluids (see Fig. 3, $n = 0.4$) as compared to shear thickening fluids (see Fig. 3, $n = 1.8$). This is eventually due to rising levels of fluid viscosity which hamper the rate of heat transfer from the square cylinder to the ambient flowing fluid past it. Also, at Ri = 0.25, there is a sudden drop in Nu for $n = 1.8$ which could be due to the complex behaviour of non-linear terms Re, Pr and Ri which are described as functions of power-law index (n).

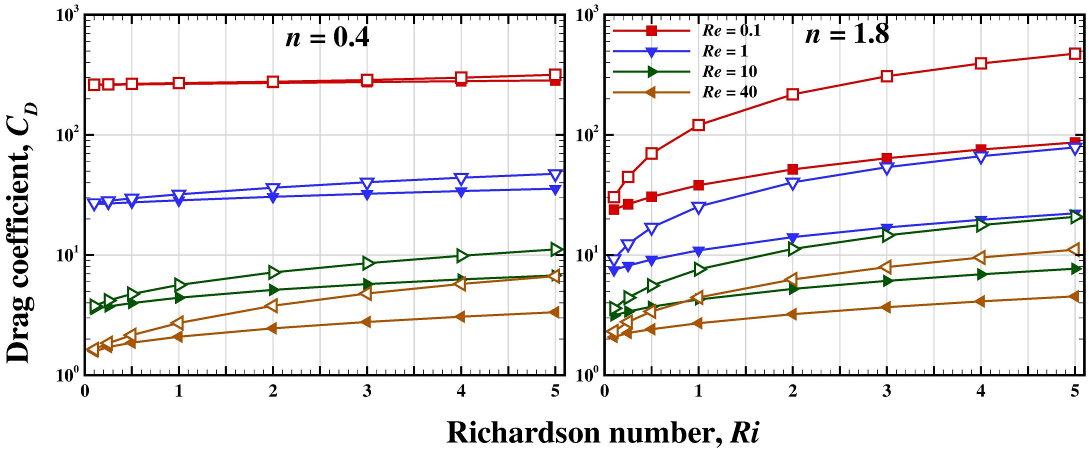

Fig. 3. Variation of total drag coefficient C_D for Pr = 1 (unfilled symbols) and Pr = 100 (filled symbols)

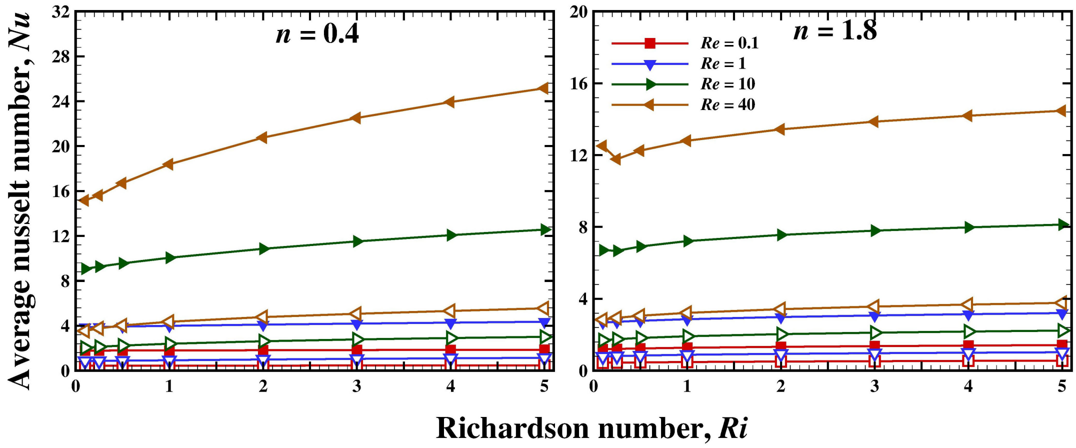

Fig. 4. Variation of the average Nusselt number, Nu for Pr = 1 (unfilled symbols) and Pr = 100 (filled symbols)

4. CONCLUSIONS

The influence of aiding buoyancy on flow and heat transfer from an isothermally heated unconfined square cylinder in power-law fluids for mixed convection regime has been investigated numerically for very broad ranges of conditions ($0.1 \leq \text{Re} \leq 40$, $0.4 \leq n \leq 1.8$, $1 \leq \text{Pr} \leq 100$ and $0.1 \leq \text{Ri} \leq 5$). While extensive results on flow and temperature fields have been detailed in terms of streamlines and isotherms, results for global parameters which includes drag coefficient and average Nusselt number are also reported. Both of these global parameters are positively influenced by Ri.

SYMBOLS

C_D	drag coefficient, $C_D = 2F_D/\rho_\infty V_\infty^2 l$, –
C	thermal heat capacity of fluid, J/(kg·K)
g	acceleration due to gravity, m/s^2
Gr	Grashof number, $\text{Gr} = g\beta(T_w - T_\infty)l^3 \left(\rho_\infty/m\,(V_\infty/l)^{1-n}\right)^2$, –
h	heat transfer coefficient, W/(m·K)
k	thermal conductivity of fluid, W/(m·K)

l	side length of square cylinder, m
L_d, L_u	downstream and upstream length, m
m	power-law consistency index, Pa·s^n
n	power-law index, –
Nu	average Nusselt number, –
Pr	Prandtl number, $\mathrm{Pr} = Cm/k(V_\infty/l)^{n-1}$, –
Re	Reynolds number, $\mathrm{Re} = \left(\rho_\infty V_\infty^{2-n} l^n\right)/m$, –
Ri	Richardson number, $\mathrm{Ri} = \mathrm{Gr}/\mathrm{Re}^2$, –
T	temperature of fluid, K
V	velocity vector, –
x, y	Cartesian coordinates, –

Greek symbols

β	coefficient of volumetric expansion, 1/K
ρ	density of the fluid, kg/m^3
ξ	non-dimensional temperature, $\xi = (T - T_\infty)/(T_w - T_\infty)$, –

Subscripts

∞	free stream

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