Central European Journal of Economic Modelling and Econometrics

How Regional Business Cycles Diffuse through Space and Time: Evidence from Spatial Markov Model of Polish NUTS-3 Regions

Agnieszka Rabiej[*], Dominika Sikora-Kruszka[†], Andrzej Torój[‡]

Submitted: 19.07.2023, Accepted: 30.10.2023

Abstract

In a national economy, are individual subnational regions business cycle takers or setters? We address this important regional policy question by investigating regional business cycles at NUTS-3 granularity in Poland ($N = 73$), using two metrics in parallel: GDP dynamics and unemployment. To extract the business cycle, we use a spatial Markov switching model that features both idiosyncratic business cycle fluctuations across regions (as a 2-state chain), as well as spatial interactions with other regions (as spatial autoregression). The posterior distribution of the parameters is simulated with a Metropolis-within-Gibbs procedure. We find a clear division into business cycle setters and takers, the latter being largely (but not only) non-metropolitan regions.

Keywords: business cycle, spatial autoregression, NUTS-3, Markov switching

JEL Classification: C11, C23, C24, R12

[*]Reckitt; e-mail: ga.agnieszka@gmail.com; ORCID: 0000-0003-3240-6120
[†]EY; e-mail: dominikasikora23@gmail.com; ORCID: 0000-0001-7905-1908
[‡]SGH Warsaw School of Economics, Institute of Econometrics; e-mail: atoroj@sgh.waw.pl; ORCID: 0000-0003-3623-168X

1 Introduction

A relatively recent thread of business cycle literature inspects the cycles and their synchronicity at sub-national regional levels. For a number of reasons, sub-national cycles can diverge from the national economy pattern (see Warżała, 2014), posing challenges to both regional and national policymakers, not to mention the supra-national entities pursuing cohesion policies (such as the European Union). These reasons include, broadly speaking, the diverse potential and endowment of regions, resulting in increasing concentration of activity in the most economically attractive geographical areas and their consequential sectoral specialization.

Diversified regional production was found to be a leading source of regional cycle assymetries (Fatas, 1997). A vast literature of economic geography identifies facilitation of trade and economic cooperation as an underlying process. Krugman (1990) claims that the reduction of transaction costs leads to an increased specialization of regions and thus reduces their cycle convergence. Fatas (1997) notes that highly specialized regions experience sectoral shocks as asymmetric local shocks. However, Hamilton and Owyang (2012) point to asymmetric local shocks as another possible source of regional divergence, regardless of the sectoral structure of a region's economy. One example thereof are diversified regional economic policies as such.

Yet another mechanism of business cycle decorrelation across regions are asymmetric reactions to symmetric shocks, as a consequence of structural heterogeneity. For instance, the literature identifies varying reactions to changes in monetary policy (see Owyang and Wall, 2006) or fiscal policy (see Wall, 2007). Warżała (2011) confirms this asymmetry and finds a significant relation between the level of the given region's development and the degree of its sensitivity to the state of the national economy. More economically advanced regions with more diverse production structure exhibit more stability and crisis-resistance.

On the other hand, regional business cycles are exposed to sychronizing forces as well. One of those is the adherence to a common currency area, hypothesized as an endogenously fulfilled optimum currency area criterion (see Frankel and Rose, 1998; Marelli, 2007). On top of that there are country-wide common fiscal policy, uniform legal order, geographical proximity, common language and culture as well as other factors that intensify interregional trade and investment flows and consequently increase the probability of the spatial spillover of a shock, resulting in a synchronization effect.

The extant literature adopts a number of approaches to the analysis of business cycles in general, and to the regional dimension thereof. Time series econometrics dominates the field, in particular the highly popular Markov models which often require the use of Bayesian inference due to their relatively complex specification as confronted with feasible sample sizes. Time series investigations, including the hidden Markov model, look predominantly at the current business cycle position as a function of past observations.

However, a joint investigation of multiple (homogenous) regions opens up new possibilities. Spatial panel models can account for the fact that business cycle position is carried over not only over time, but also through physical space. To accommodate the frequent case of feedback loops between neighbouring regions' cycles, spatial econometric methods need to be introduced.

This paper adopts such a joint, spatio-temporal perspective by looking at regional business cycle positions with their spatial spillovers. To this aim, at the technical level, we combine three tools, i.e. we perform a Bayesian analysis of a hidden panel Markov model with spatial autoregression. We contribute to the literature by inspecting the regional business cycle synchronization and divergences at relatively fine spatial granularity, i.e. for 73 NUTS-3 regions in Poland. The NUTS classification enables the collection and analysis of harmonized statistical data at different levels of geographical aggregation for the EU economic territory, facilitating regional comparisons and formulation of regional policies. The NUTS-3 level represents relatively small territorial units which should fall within the population threshold of between 150 000 and 800 000. The regional breakdown is therefore far more detailed than in both Kondo (2021) and Torój (2020).

Within this framework, our main goal is to decompose each regional business cycle into three sources: own dynamics (region's Markov switching process), other region's values (spatial spillovers) and innovations. Regions with a high share of the first component can be interpreted as business cycle setters, while the second component points to business cycle takers.

Fine spatial granularity calls for methods that do not assume observational independence, but also poses some challenges: the need for both more computing power and room for potential considerable reinterpretation of previous results as the spatial spillovers can potentially be stronger with smaller regions under consideration. This is of particular importance as the Polish NUTS-3 regional division entails mixed urban-rural areas to a little extent. Instead, it splits metropolies and peripheral areas as separate regions. The practical importance of regarding individual regions as business cycle takers or setters is non-negligible. Regional development policies may react differently to region-specific shocks provided that it generates a large degree of externality. Consequently, the business cycle setting regions are of greater interest to country-wide monetary policy.

The rest of the paper is organized as follows. Section 2 reviews the tools widely used in the (regional) business cycle literature, with focus on hidden Markov models and regional applications. Section 3 introduces the data and methodology for the empirical analysis that follows in Section 4 where detailed results can be found. Finally, Section 5 concludes the paper and discusses potential extensions of the study.

2 Literature review: from country-level Markov switching to regional spatial models

First introduced by Hamilton (1989), the hidden Markov model quickly spread through the field of business cycle research. It differentiates between two states: the state of fast economic growth (expansion) and slow economic growth (recession). Basic Hamiltonian model measures the growth rate of an economic activity indicator as a sum of a state-dependent average growth rate and random disturbance. Expansion and recession are assumed to be a stochastic, 2-state homogenous Markov chain process. Therefore, the probability of switching between the states depends only on the previous period state. The surge in popularity of the hidden Markov models in business cycle analyses, against the alternative of factor models (Stock and Watson, 1991), resulted i.a. from their ability to detect the turning points. Chauvet and Piger (2003) show that they can detect regime change more quickly than the Business Cycle Dating Committee NBER.

The growing body of Markov-switching model applications inspired a number of researchers to propose case-specific extensions. To obtain a more comprehensive and robust picture, Chauvet and Piger (2003) investigate two economic indicators instead of one: the quarterly growth rate of real GDP and the monthly growth rate of employment in the US. Furthermore, the frequency of publication favours the labour market data as a source of information about the business cycle because, according to Chauvet and Hamilton (2005), it can lead to more precise conclusions. Lawrence et al. (2021) investigated the relationship between unemployment and the business cycle using a dynamic stochastic general equilibrium (DSGE) model. They found that changes in the unemployment rate were closely linked to fluctuations in economic activity, suggesting that the unemployment rate serves as a reliable indicator of business cycle movements.

On the specification level, more profound extensions were proposed by i.a. Hansen (1992), who considered the identification of regime change by switching parameters other than the average growth rates, in particular the variance of the error term and the autoregression parameter. Krolzig (1997), in turn, modified the model by using a multi-dimensional density function that was not represented by a Gaussian process. The density function does not necessarily need to have a normal distribution but e.g. a t-student distribution with the number of degrees of freedom changing accordingly to the regime change as in the case of Dueker (1997). In a number of studies, a 3-regime specification has been established by dividing the expansion phase into normal and rapid growth phase (see Bernardelli and Dędys, 2015; Kim and Murray, 2002). Sims and Zha (2006) showed that the Bayesian approach enables the estimation of a model with multiple regimes. In their case, models with 7-10 states exhibit the best fit.

Another possible alteration of the model includes the regime change endogeneity assumption. Kim et al. (2004) claim that macroeconomic shocks can be correlated with business cycle fluctuations and that measurement errors of an unobservable variable can also lead to endogeneity. One can also reject the fixed transition probabilities assumption by allowing them to change according to the fluctuations of economic activity indicators. Diebold et al. (1994) decided to model the transition probabilities as parametric logistic functions of exogenous variables. New form of the model incorporates time-varying transition probabilities, thus making the anticipation of changes in the economy possible. Filardo and Gordon (1998) argue that the decreasing expected duration of the current cycle phase can be a forecast of an upcoming regime switch.

Numerous authors point to business cycle asymmetry as a non-negligible feature. The cycle phases can vary in the steepness, deepness, sharpness and persistence to shocks (see Clements and Krolzig, 1998). Regime-switching models proved to be able to encapsulate the sharpness asymmetry (i.e. asymmetry of business cycle turning points) because they allow for the introduction of different parameters depending on the current regime. Diebold and Rudebusch (1996) combined two alternative approaches to business cycle research: the Hamiltonian Markov-switching model and the dynamic factor model (see Stock and Watson, 1989). They introduced a model with factors subject to regime changes which enabled them to include the two crucial features of the business cycle: its non-linearity and the coincidence of many macroeconomic indicators. They also included the dynamics in the form of autoregression.

Multiple estimation approaches can be found in the literature for models of this nature. Kim and Nelson (1998) proposed the multi-move Gibbs sampling in a Bayesian framework for a dynamic factor model with regime switching. Gibbs sampler draws from the conditional posterior density for each parameter. The authors proved that the sampling results converge to the joint posterior distribution of parameters for a sufficiently large number of iterations.

A relatively rich literature investigating the business cycle synchronization and divergence at the sub-national level exists for the United States of America. For instance, Harding and Pagan (2006) measured the strength of two business cycles synchronization through the percentage of time when the two economies found themselves in the same regime. The results showed that various regions can be synchronized with the national economy to a different extent.

Owyang et al. (2005) confirmed the existence of significant differences between American states in both regimes by applying a regime-switching model to economic time series for each state. There were, however, some similarities found between regions in a given territory and with a common specialization. Another observation was that transition probabilities were usually close to 0 or 1 which implied that the identification of the current business cycle phase was relatively easy. The authors also proved the occurrence of distinct differences in the timing and length between the national and regional business cycles. First of all, each region could switch regimes at a different point in time than the whole country. Second of all, a recession in the national economy did not determine the state of all regional economies.

The rising interest in sub-national analyses of business cycles has led to methodological questions regarding the joint analysis of multiple geographic areas. The key disadvantage of the original Hamiltonian model was its one-dimensionality, i.e. it was only applicable to national and regional cycles separately. This problem was addressed by Hamilton and Owyang (2012) who converted the model into a panel which enabled them to analyze the fluctuations in many regions simultaneously. The authors used a multi-dimensional approach by grouping regions with similar characteristics into clusters, which then helped them identify 3 such clusters in the US. Econometrics of panel data can help capture both dynamic and geographical relations.

Multiple researchers confirmed the significance of the spatial element in the business cycle analysis. Artis et al. (2011) constructed a spatial ARMA model where the spatial dependence between regions was represented by a spatial lag of the dependent variable or the error term. This specification also enables further model development through a spatial weights matrix. To address the problem of numerous regions and a large transition matrix, Frühwirth-Schnatter and Kaufmann (2008) proposed the idea of aggregating the regions into clusters.

Shibaev (2016) further emphasized that geographical units are rarely independent. The author inspected spatial relations in a Markov-switching model with the use of spatial autoregressive error specification which allows for a spatial autocorrelation of shocks in different regions. This application confirmed the existence of the spatial spillover effect in the US which was also proved by Artis et al. (2011), whereas the phenomenon of regional business cycle synchronization was confirmed by Beraja et al. (2016) and Hamilton and Owyang (2012).

A panel Markov-switching model was also implemented in the works of Kondo (2021) and Torój (2020). Both authors introduced the spatial lag into a panel hidden Markov model in order to investigate the spatial spillover effect. Estimation methods included the multi-move Gibbs sampler and the Metropolis-Hastings algorithm. In terms of methodology, this paper continues that strand of literature by combining Markov chains, panel and spatial econometrics with the Bayesian inference. It also fills the gap for regional business cycle analysis in Poland.

The work of Torój (2020) does not fit with the rest of the literature insofar as it investigates the profitability of enterprises, rather than standard business cycle measures, for NUTS-2 regions in Poland. There are a few other, more typical applications of the Markov-switching model for Poland at the national level, including Podgórska and Decewicz (2001), where the authors decided to apply the model in the analysis of the Polish industry and found that the assumption of two regimes is sufficient to examine periodical occurrences of both downturns and upturns in the economy. Other Polish researchers also point to Markov-switching models as tools useful for the purpose of business cycle inspection. Bernardelli and Dędys (2015) prove that they are successful in the description of variables which observe significant

fluctuations, while Kośko et al. (2016) agree that Markov-switching models can be used for the effective identification of the current state of the economy and the turning points of the business cycle.
Nevertheless, Polish literature on regional business cycles remains scarce, particularly in the case of the two-dimensional approach. Warżała (2015) indicated that the Polish regions can differ in their sensitivity to economic shocks and pointed to the correlation between this sensitivity and the level of economic development of a region. Differences in the economic potential, labour force availability, management quality and innovativeness can influence the fluctuations and lead to regional business cycle divergence (see Drozdowicz-Bieć, 2008). In this context, an appropriately detailed level of data granularity is crucial for the quality of the regional business cycle analysis, particularly in the case of Polish regions.

3 Data and methodology

3.1 Data

This work utilizes publicly available data from reputable sources. The GDP values for Poland are obtained from Eurostat, covering the years 2000-2018. The yearly data is measured in billions of euro and represents GDP at current market prices by NUTS-3 regions. The unemployment rate monthly data for Poland is sourced from the Local Data Bank (BDL, Statistics Poland), covering the years 2011-2021. The data represents the share of registered unemployed individuals in the working-age population in 73 NUTS-3 regions. Table 1 contains the summary of basic data characteristics for both datasets. Prior to using the data in the model and conducting simulations, visualization techniques were employed to draw meaningful conclusions.

Table 1: Datasets summary

Parameters	Dataset I	Dataset II
Country	Poland	Poland
Explained variable	GDP	unemployment rate
Data source	Eurostat	BDL, Statistics Poland
Data frequency	annual	monthly
Timeframe	2000-2018	2011-2021
Spatial granularity	NUTS-3	NUTS-3
Number of regions	73	73
Units of measurement	billion EUR	%
Average value	13 bln EUR	10.7%

Regarding GDP in Poland, an overall upward trend is observed with breaks in 2003 and during the financial crisis in 2009 (see Figure 1). Despite the downturns, Polish NUTS-3 regions recovered quickly and resumed their economic growth. The national average GDP within the sample amounted to 13 billion euros, with only Miasto Warszawa subregion exceeding the national average at 43 billion euros. Thus, the presence of high GDP values in the capital city significantly influences the overall average. Most subregions recorded their maximum GDP values in the last period of the sample, with Miasto Warszawa subregion reaching the highest value of 68 billion euros. Figure 2 indicates that high GDP levels are observed in subregions within large urban agglomerations, particularly in the southwestern part of Poland.

The unemployment rate in Polish NUTS-3 regions displays a downward trend for the majority of cases, reaching the maximum around 2013-2014 (see Figure 4). The highest recorded value was 27.3% for Ełcki subregion in February 2013. After reaching their respective peaks, most subregions experienced a negative trend until reaching a minimum in 2019. Then the trend was reversed, and unemployment rates started to rise again due to factors such as the pandemic and the imposition of sanitary restrictions affecting market operations, including the labor market. The average unemployment rate for Poland during the analyzed period was approximately 10.7%. High values were observed in the northern and southeastern parts of the country, while subregions in the central areas generally exhibit lower values (see Figure 3). Large cities and their surrounding areas tend to have both lower unemployment rates and higher GDP values.

3.2 Hidden panel Markov model with spatial autoregression

Our model, strongly inspired by the works of Kondo (2021) and Torój (2020), builds on the following equation:

$$\mathbf{y_t} = \rho \mathbf{W} \mathbf{y_t} + \mathbf{m_0} \odot (\mathbf{1_N} - \mathbf{s_t}) + \mathbf{m_1} \odot \mathbf{s_t} + \boldsymbol{\varepsilon_t}, \tag{1}$$

where $\mathbf{y_t}$ denotes a vertical vector of the analyzed variable (either GDP dynamics or unemployment change) in $N = 73$ Polish NUTS-3 regions in period t, $\mathbf{s_t}$ – unobservable N-element vector of binary states (1 – expansion, 0 – recession) in period t, $\mathbf{m_0}$ and $\mathbf{m_1}$ are vertical, N-element vectors containing the constant for each region during, respectively, recession and expansion, $\mathbf{W} = [w_{n,n}]$ $(n = 1, ..., N)$ is a strictly exogenous, row-normalized spatial weight matrix based on the inverted distance between regions' centroids, $\boldsymbol{\varepsilon_t}$ is an N-dimensional vector of errors, independent for individual $t = 1, ..., T$ periods, with identical multivariate normal distributions of zero means and variance-covariance matrix comprising the elements of vector σ^2 on the diagonal and zero elsewhere. Due to limited data availability for Poland, we have $T = 18$ for GDP dynamics (yearly data with a time range of 2000-2018, % changes calculated vs previous year) and $T = 110$ for unemployment (monthly data for the

Figure 1: GDP in Polish subregions in years 2000-2018 (billion EUR)

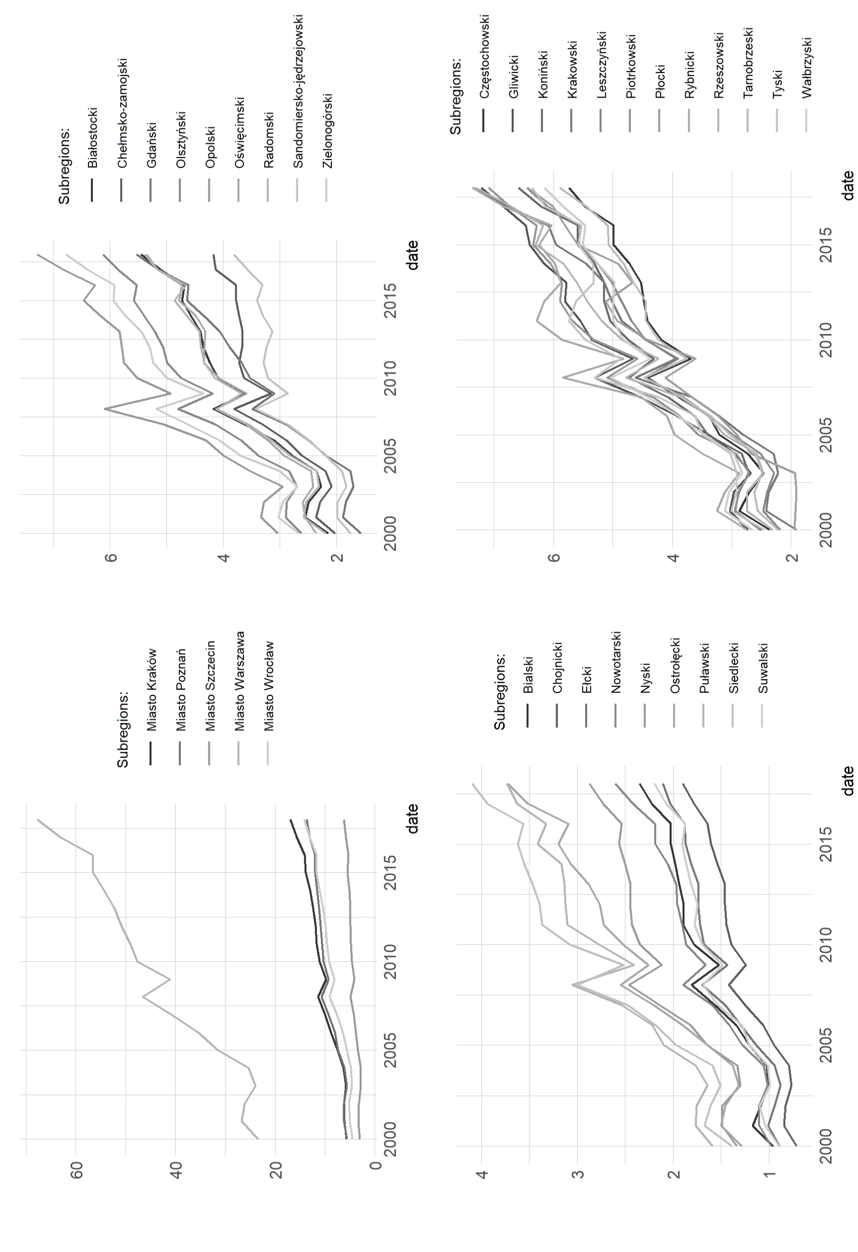


Figure 1: GDP in Polish subregions in years 2000-2018 (billion EUR) cont.

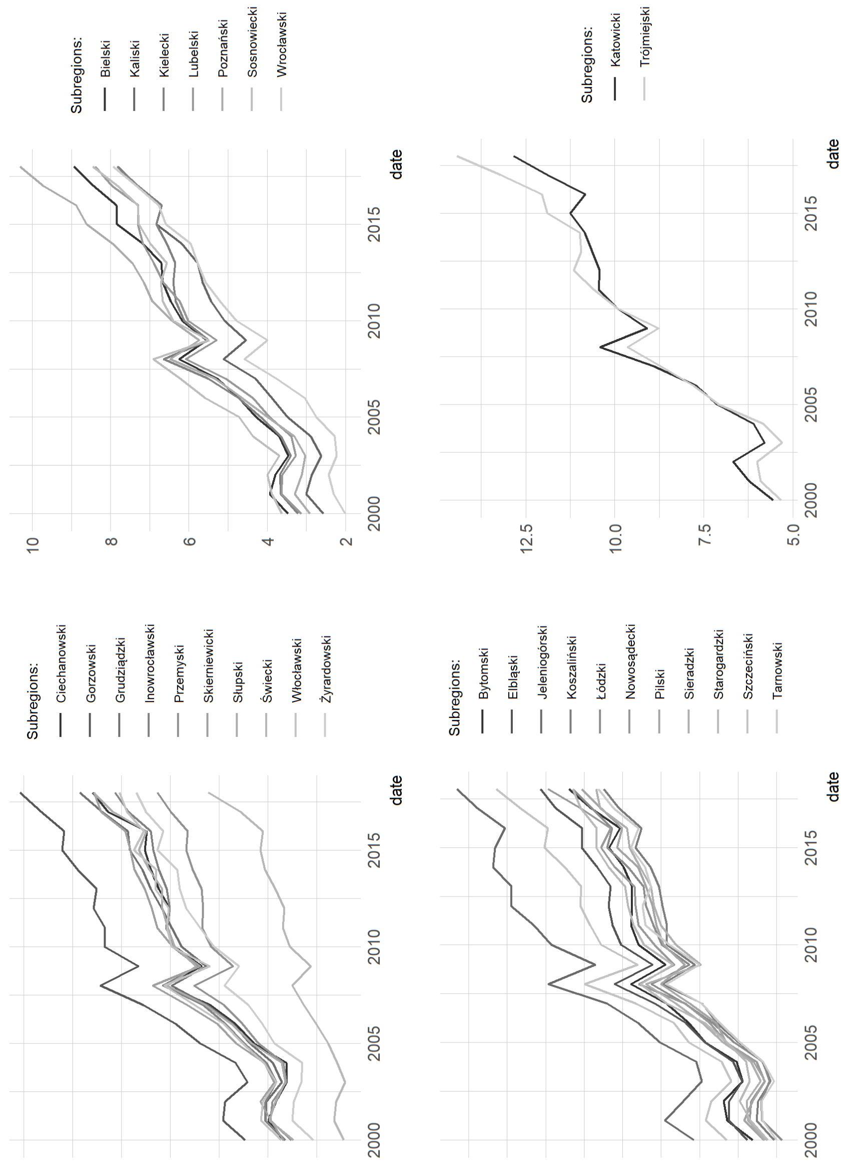


Figure 2: GDP in Poland in selected years (billion EUR)

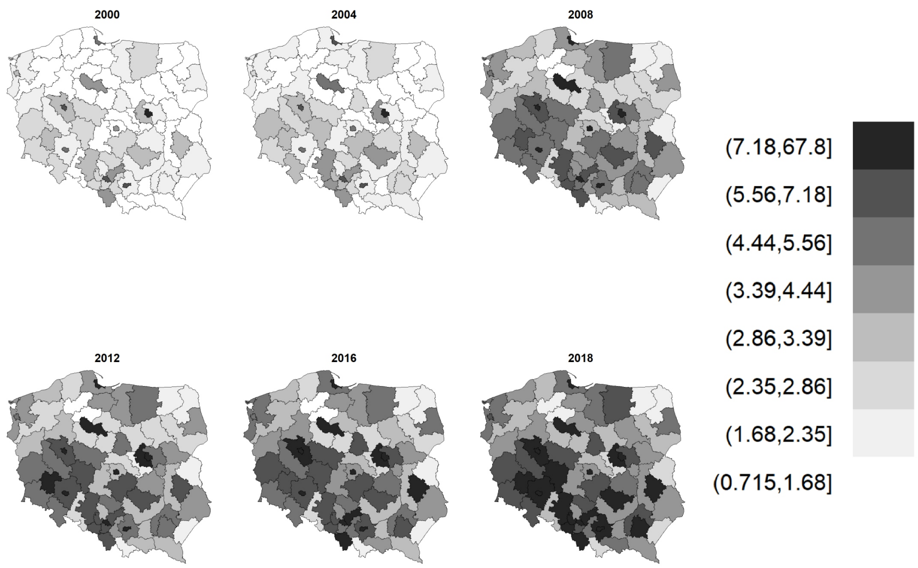


Figure 3: Unemployment rate in Poland in selected years

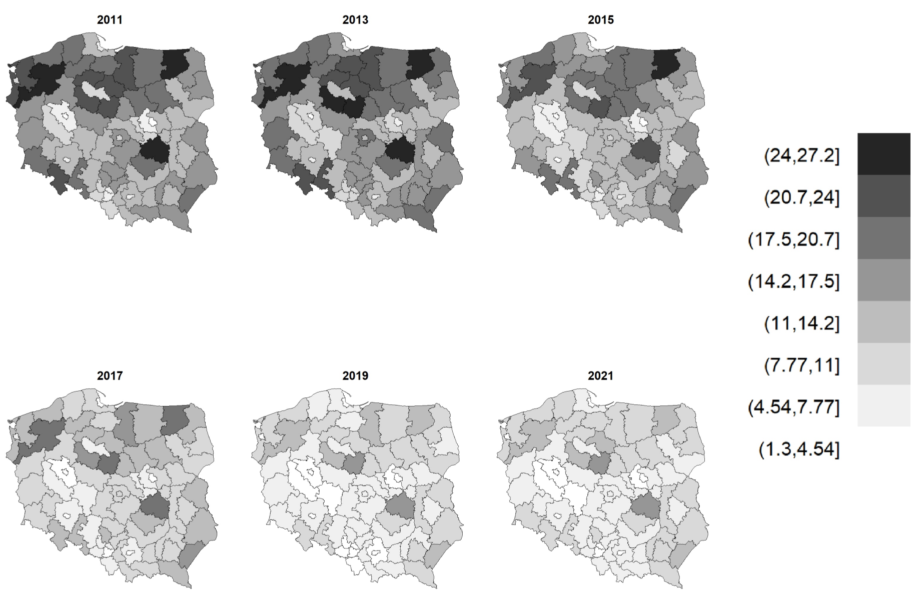


Figure 4: Unemployment rate in Polish subregions in years 2011-2021

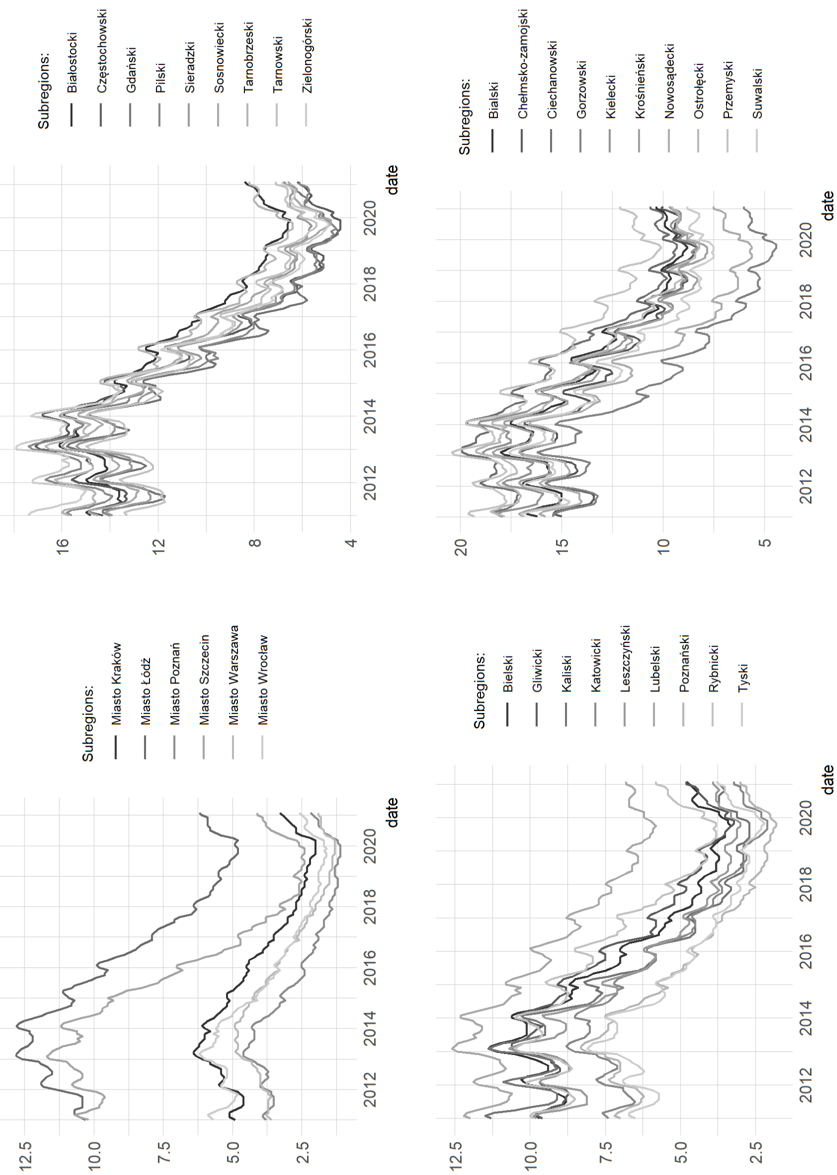
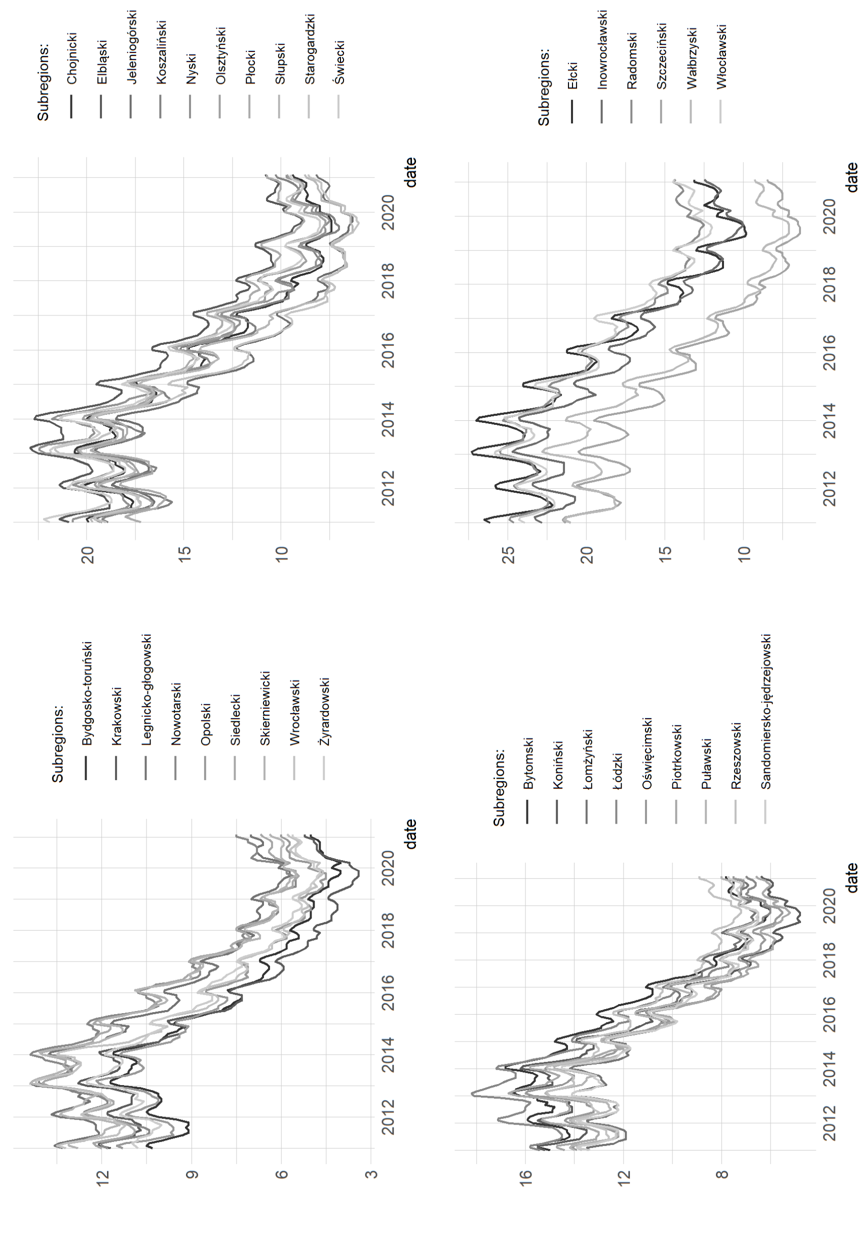


Figure 4: Unemployment rate in Polish subregions in years 2011-2021 cont.



years 2011-2021, changes in percentage points calculated vs the same month of the previous year). For each n, the variable $s_{t,n}$ constitutes a 2-state Markov chain with region-specific 1-period-ahead probabilities of remaining in state 1 denoted as $\mathbf{p_{11}}$, and likewise $\mathbf{p_{00}}$ for state 0 (with an obvious consequence of transition probabilities amounting to $\mathbf{1_N} - \mathbf{p_{11}}$ from 1 to 0 and $\mathbf{1_N} - \mathbf{p_{00}}$ from 0 to 1), ρ is a spatial autoregression parameter, $\mathbf{1_N}$ represents a column vector of ones, of length N, and $\odot$ is the Hadamard product.

This specification does not take on board some features proposed in the extant literature, as reviewed in the previous section, including multiple states, endogenous transition probabilities and heavy-tailed distributions. We motivate the choice of a relatively simple specification in a twofold manner: due to relatively large N and small T. The former was our intentional choice, the latter – a practical consequence thereof.

Following Kondo (2021), we set the following prior distributions, independent from one another and across $n = 1, ..., N$:

i) inverse gamma for σ_n^2: $IG(\frac{\underline{v}}{2}, \frac{\underline{\delta}}{2})$;

ii) two-dimensional normal for each pair $[m_{0,n}\ m_{1,n}]^T$: $MVN_2(\underline{\boldsymbol{\mu}}, \underline{\boldsymbol{\Sigma}})$;

iii) beta for $p_{00,n}$ and $p_{11,n}$: $p_{00,n}\ Beta(\alpha_{00}, \alpha_{01})$, $p_{11,n}\ Beta(\alpha_{11}, \alpha_{10})$;

iv) uniform for $\rho:\ U(1/\lambda_{min}(\mathbf{W}), 1)$, where $\lambda_{min}(\mathbf{W})$ is the lowest eigenvalue of $\mathbf{W}$, in line with the typical stationary bounds for the spatial autoregression parameter for a row-normalized $\mathbf{W}$ matrix (see Anselin and Florax, 1994).

Hyperparameters were set as follows for GDP: $\underline{v} = 6$, $\underline{\delta} = 100$, $\underline{\boldsymbol{\Sigma}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\underline{\alpha_{00}} = 8$, $\underline{\alpha_{01}} = 2$, $\underline{\alpha_{10}} = 2$, $\underline{\alpha_{11}} = 8$. For unemployment the hyperparameters were set as: $\underline{\delta} = 10$, $\underline{\alpha_{10}} = 1$, $\underline{\alpha_{11}} = 9$, and otherwise same as for the GDP model. Parameter $\underline{\mu}_0$ (and $\underline{\mu}_1$) as an element of $\underline{\boldsymbol{\mu}}$ was set as an average of the mean and the first (or third) quintile of the GDP dynamics series, and inversely in the case of the unemployment rate. These values were established on a region-by-region basis, and then the cross-regional average has been set as a prior for all regions.

The above-discussed elicitation can be regarded as pseudo-Bayesian rather than fully Bayesian, since the prior expected values of regime-switching means in eq. (1) are assigned on a data-driven basis. It must be stressed that priors of $\mathbf{m_0}$ and $\mathbf{m_1}$ constitute an important part of the economic identification scheme. They play a vital role in differentiating between both regimes and have to carry a balanced, medium-level information content (see Figure 9 and 10 for a visualization). This is because uninformative priors cannot support the differentiation between expansion and recession, and overly informative priors block identification of expansion or recession phases in regions that grow structurally faster or slower in the long-term perspective. As a result, moderately informative priors were chosen to support the model in capturing the difference between the regimes, but with prior density of $\mathbf{m_0}$ and $\mathbf{m_1}$ roughly covering the scope of the observed values for $\mathbf{y}$. Note that a similar effect could be achieved during the posterior simulation by the permutation sampler as proposed by Frühwirth-Schnatter (2001).

On the contrary, the priors for $\mathbf{p_{00}}$ and $\mathbf{p_{11}}$ appear as informative, favouring values higher than 0.5, and largely indicative of $\mathbf{p_{00}} < \mathbf{p_{11}}$. This elicitation closely follows the previous literature, motivating the choice with general frequency properties of business cycles: persistence of both expansion and recession phases as measured by quarterly or monthly data, but also a tendency of expansions to last longer than recessions. It should perhaps be stressed that prior information on $\mathbf{p_{00}}$ and $\mathbf{p_{11}}$ is a critical element of the model structure, allowing to obtain a reasonable economic reading of the results (including the fact that the probability estimates should belong to the $[0;1]$ interval).

For sampling from the posterior distribution, elements of the empirical strategy by Kim and Nelson (1998, 1999) have been applied, additionally modified by Kondo (2021) for the purpose of spatial analysis. The above references demonstrate that conditional posterior distributions are known in almost all cases, except for ρ. This raises the case for the Metropolis-within-Gibbs simulation, with Metropolis-Hastings algorithm applied to sample ρ in the last step of a Gibbs sampler iteration. The single g-th iteration of the Gibbs procedure runs as follows:

1. Set the initial value of $\boldsymbol{\theta}^{(g-1)}$, comprising all individual model parameters (i.e. all elements of $\mathbf{p_{00}}$, $\mathbf{p_{11}}$, $\mathbf{m_0}$, $\mathbf{m_1}$, σ^2, ρ), at arbitrary levels (for $g = 1$) or as values sampled in the previous step (for $g > 1$).

2. For each region $n = 1, ..., N$, sample a path of states $s_n^{(g)} = [s_{1,n} \dots s_{T,n}]^T$ using the multimove Gibbs sampler, as presented in detail by Kim and Nelson (1998):

   (a) For $t = 0$, set the probability of expansion and recession at long-term values consistent with $\mathbf{p_{00}}^{(g-1)}$ and $\mathbf{p_{11}}^{(g-1)}$.

   (b) Iterate forward with the Hamilton filter (Chib, 2001; Kondo, 2021) to compute probabilities of state 0 and 1 for each region n and period t, conditional on respective probabilities at $t-1$, data (from the beginning of the sample until t) and $\boldsymbol{\theta}^{(g-1)}$.

   (c) Use the multimove Gibbs sampler to draw states $s_{t,n}$, starting with the last period $t = T$ and then iterating backwards, conditionally on states in subsequent periods, data (for the whole sample) and $\boldsymbol{\theta}^{(g-1)}$.

3. For each region $n = 1, ..., N$, sample $p_{00,n}^{(g)}$ and $p_{11,n}^{(g)}$ from the conditional (on $\mathbf{s_n^{(g)}}$ from point 2) posterior distributions $Beta(\overline{\alpha}_{00,n}^{(g)}, \overline{\alpha}_{01,n}^{(g)})$ and $Beta(\overline{\alpha}_{11,n}^{(g)}, \overline{\alpha}_{10,n}^{(g)})$, where $\overline{\alpha}_{ij,n}^{(g)}$ denotes in each case the corresponding prior parameter augmented by a number of transitions from state i to j ($i, j \in 0; 1$) in $\mathbf{s_n^{(g)}}$ from point 2.

4. For each region $n = 1, ..., N$, draw $\sigma_n^{2(g)}$ from the conditional (on $\mathbf{s_n^{(g)}}$ from point 2, and other elements of $\boldsymbol{\theta}^{(g-1)}$) posterior distribution$IG(\frac{\overline{v}}{2}, \frac{\overline{\delta}_n}{2})$, where $\overline{v} = \underline{v} + T$ and $\overline{\delta}_n = \underline{\delta} + \Sigma_{t=1}^{T} \varepsilon_{n,t}^2$.

5. For each region $n = 1, ..., N$, draw $\begin{bmatrix} m_{0,n}^{(g)} & m_{1,n}^{(g)} \end{bmatrix}^T$ from the conditional (on $\mathbf{s_n^{(g)}}$ from point 2, $\sigma_n^{2(g)}$ from point 4 and other elements of $\boldsymbol{\theta}^{(g-1)}$) posterior distribution $MVN_2(\overline{\boldsymbol{\mu}}_n, \overline{\boldsymbol{\Sigma}}_n)$, where:

$$\overline{\boldsymbol{\Sigma}}_n = \left(\underline{\boldsymbol{\Sigma}}^{-1} + \sigma_n^{-2(g)} \begin{bmatrix} \mathbf{1_T} - \mathbf{s_n^{(g)}} & \mathbf{s_n^{(g)}} \end{bmatrix}^T \begin{bmatrix} \mathbf{1_T} - \mathbf{s_n^{(g)}} & \mathbf{s_n^{(g)}} \end{bmatrix} \right)^{-1}, \tag{2}$$

$$\begin{aligned} \overline{\boldsymbol{\mu}}_n = \quad & \overline{\boldsymbol{\Sigma}}_n \left[\underline{\boldsymbol{\Sigma}}^{-1} \underline{\boldsymbol{\mu}} + \sigma_n^{-2(g)} \begin{bmatrix} \mathbf{1_T} - \mathbf{s_n^{(g)}} & \mathbf{s_n^{(g)}} \end{bmatrix}^T \cdot \right. \\ & \left. \cdot \left(\begin{bmatrix} y_{n,1} \\ \vdots \\ y_{n,n} \end{bmatrix} - \rho^{(g-1)} (\mathbf{1_T} \otimes \begin{bmatrix} w_{n,1} & \dots & w_{n,n} \end{bmatrix}) \begin{bmatrix} y_{n,1} \\ \vdots \\ y_{n,n} \end{bmatrix} \right) \right]. \end{aligned} \tag{3}$$

6. Sample $\rho^{(g)}$ from an unknown posterior distribution, conditional on all elements of $\boldsymbol{\theta}^{(g)}$ drawn in the previous steps. The sampling distribution is simulated with the random walk Metropolis-Hastings algorithm, in which the candidates are generated by a truncated normal distribution with mean $\rho^{(g-1)}$. Variance has been adjusted to maintain the mean candidate acceptance probability from 0.2 to 0.4, and the upper and lower bounds correspond to the prior distribution of ρ.

It is straightforward to see from Equation (1) that, for each n, the model decomposes each region's cyclical position into three parts: (i) external: the spillover of other regions' business cycles (spatial autoregression), (ii) internal: n's own business cycle (Markov switching), (iii) other determinants (error term). Hyperparameters, prior distributions and initial values mentioned in this section of the paper are summarized in Tables 2 and 3 below.

4 Empirical results for Polish NUTS-3 regions

4.1 MCMC convergence

The posterior distribution has been simulated in two chains. The Gibbs sampler of $S = 20000$ iterations has been used (with a burn-in of $S_0 = 4000$), and for each of those, the Metropolis-within-Gibbs procedure for ρ had 10000 iterations (with a burn-in of 3000). In the case of GDP, for all parameters, potential scale reduction factors

Table 2: Prior distributions and initial values for dataset I (GDP)

Parameters	Prior distribution	Hyperparameters	Initial values
σ_n^2	$IG(\underline{\nu}/2, \underline{\delta}/2)$	$\underline{\nu} = 6; \underline{\delta} = 50$	$\sigma_n^{2,(0)} = 1$
$(m_{n,0}, m_{n,1})^T$	$MVN_2(\underline{\mu}, \underline{\mathbf{M}})$	$\underline{\mu} = (2;9)^T; \underline{M} = \mathbf{I}_2$	$(m_{n,0}^{(0)}, m_{n,1}^{(0)})^T = (1;10)^T$
$p_{n,00}$	$Beta(\underline{\alpha}_{00}, \underline{\alpha}_{01})$	$\underline{\alpha}_{00} = 8; \underline{\alpha}_{01} = 2$	$p_{n,00}^{(0)} = 0.8$
$p_{n,11}$	$Beta(\underline{\alpha}_{11}, \underline{\alpha}_{10})$	$\underline{\alpha}_{11} = 8; \underline{\alpha}_{10} = 2$	$p_{n,11}^{(0)} = 0.8$
ρ	$U(1/\omega_{min}, 1)$		$\rho^{(0)} = 0.5$

Table 3: Prior distributions and initial values for dataset II (unemployment rate)

Parameters	Prior distribution	Hyperparameters	Initial values
σ_n^2	$IG(\underline{\nu}/2, \underline{\delta}/2)$	$\underline{\nu} = 6; \underline{\delta} = 10$	$\sigma_n^{2,(0)} = 1$
$(m_{n,0}, m_{n,1})^T$	$MVN_2(\underline{\mu}, \underline{\mathbf{M}})$	$\underline{\mu} = (0.2, -1.3)^T; \underline{M} = \mathbf{I}_2$	$(m_{n,0}^{(0)}, m_{n,1}^{(0)})^T = (0.2, -1.3)^T$
$p_{n,00}$	$Beta(\underline{\alpha}_{00}, \underline{\alpha}_{01})$	$\underline{\alpha}_{00} = 8; \underline{\alpha}_{01} = 2$	$p_{n,00}^{(0)} = 0.8$
$p_{n,11}$	$Beta(\underline{\alpha}_{11}, \underline{\alpha}_{10})$	$\underline{\alpha}_{11} = 9; \underline{\alpha}_{10} = 1$	$p_{n,11}^{(0)} = 0.8$
ρ	$U(1/\omega_{min}, 1)$		$\rho^{(0)} = 0.5$

were equal to 1.00-1.03 (see Figures 5 and 6 for the Gelman-Rubin plots related to the critical parameter ρ).

A single issue related to convergence affects parameters related to one region, Inowrocławski, in the analysis of unemployment. Potential scale reduction factors for m_1 and m_0 fluctuate from 1.6 to 1.8, and do not tend to increase or decrease when the chain is extended (up to 30000) or shrunk (down to 10000). In that case, neither the extension of chains nor of the burn-in contributed to a decrease. We identify the potential underlying reason as related high spatial dimensionality of the study ($N = 73$, i.e. considerably higher than in the previous, comparable literature) that leads to a high degree of determination of some regions' fluctuation by external factors, and hence leaves m_1 and m_0 for that region weakly identified. Jointly with the time-consuming evaluation of posterior log-density in every iteration, leading to a generally slow Metropolis-Hastings computation, as well as the simulation-based evaluation of sampling density that prevented us from using a more efficient Hamilton Monte Carlo scheme, this might be indicative of turning to other model specifications (e.g. random effects across units) in future research.

For key parameter ρ, the distribution of which has been simulated in the nested Metropolis-Hastings procedure, we conduct an additional graphical traceplots

Figure 5: GDP: potential scale reduction factor for ρ

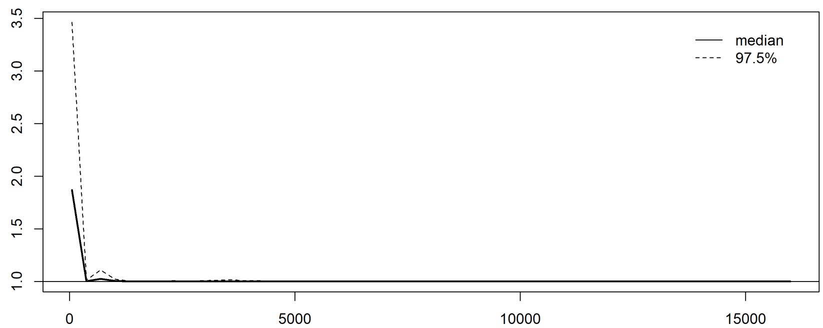
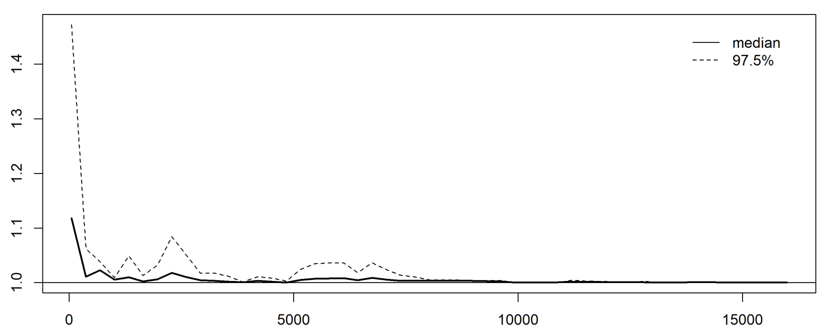


Figure 6: Unemployment: potential scale reduction factor for ρ



inspection (see Figures 7-8). We treat the patterns of missing trends and regular, normal-like density as reassuring.

Due to a high number of regions in the panel ($N = 73$), we decided to present further results for 8 selected NUTS-3 regions, one for each NUTS-1 region plus the capital city of Warsaw (see Table 4 for details). The remaining results are available upon request.

4.2 Amplitudes: expansion and recession means

Figures 9 and 10 present prior and posterior distributions of m_0 and m_1, interpretable as recession and expansion means respectively of GDP dynamics and unemployment rate, net of spatial interactions (Wy_t) and error terms (ε_t). For GDP, average regional posterior means amount to +0.33% for recession (ranging from -0.57 to +1.86) and +9.95% for expansion (ranging from +9.55 to +10.51). For unemployment, the analogous averages equal +0.15 p.p. for recession (max. +0.787) and -0.39 p.p.

Figure 7: MCMC convergence for ρ

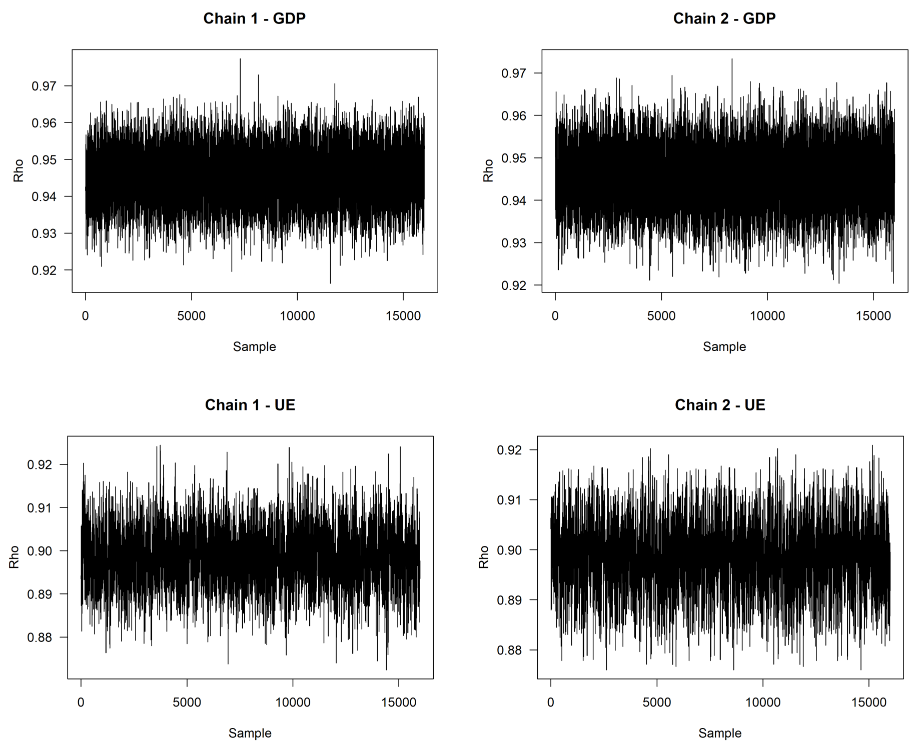
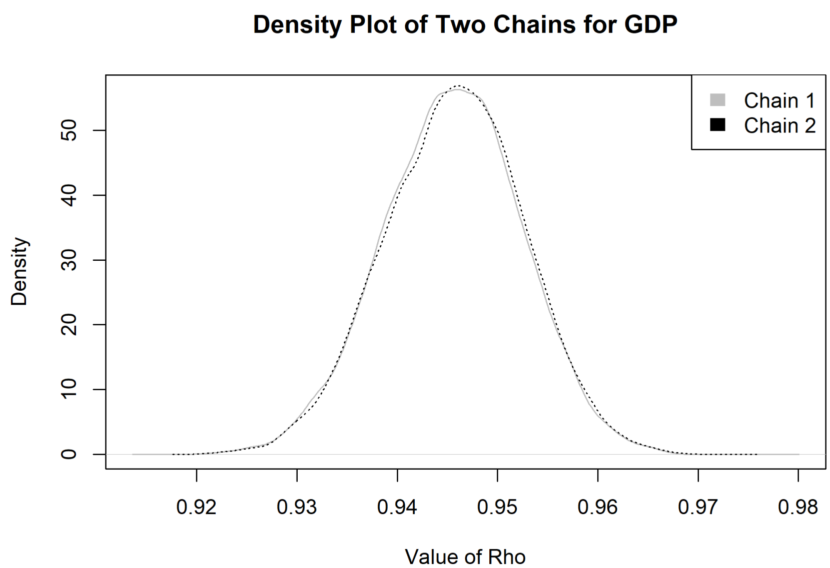


Table 4: Adherence of selected NUTS-3 regions to NUTS-2 and NUTS-1 entities

NUTS-3	NUTS-2	NUTS-1
Miasto Warszawa (Warsaw)	Warszawski capital	Mazowieckie voievodship
Żyrardowski	Mazowiecki regional	Mazowieckie voievodship
Miasto Kraków (Cracow)	Małopolskie	Południowy (South)
Poznański	Wielkopolskie	Północno-Zachodni (Northwest)
Opolski	Opolskie	Południowo-Zachodni (Southwest)
Chojnicki	Pomorskie	Północny (North)
Kielecki	Świętokrzyskie	Centralny (Central)
Białostocki	Podlaskie	Wschodni (East)

Figure 8: Comparison of the chains' density for ρ



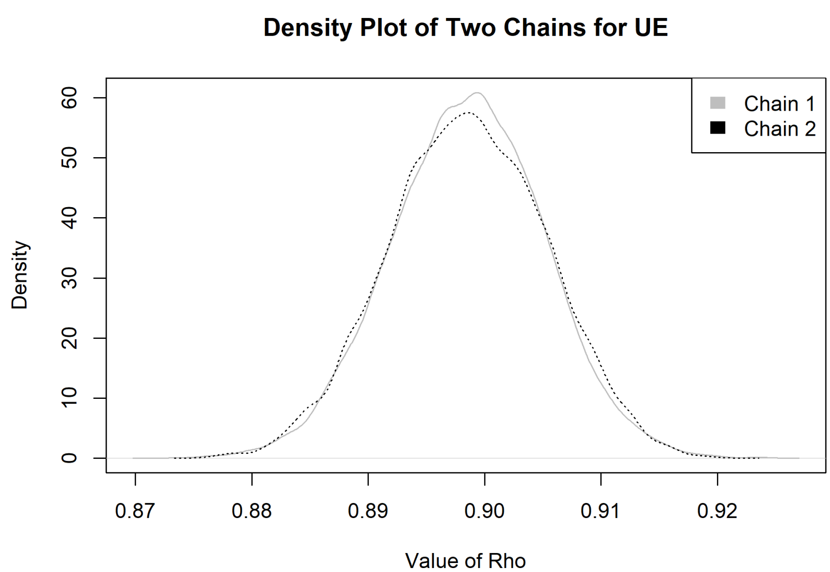


Figure 9: GDP: prior and posterior distributions of m_0 and m_1 for selected subregions

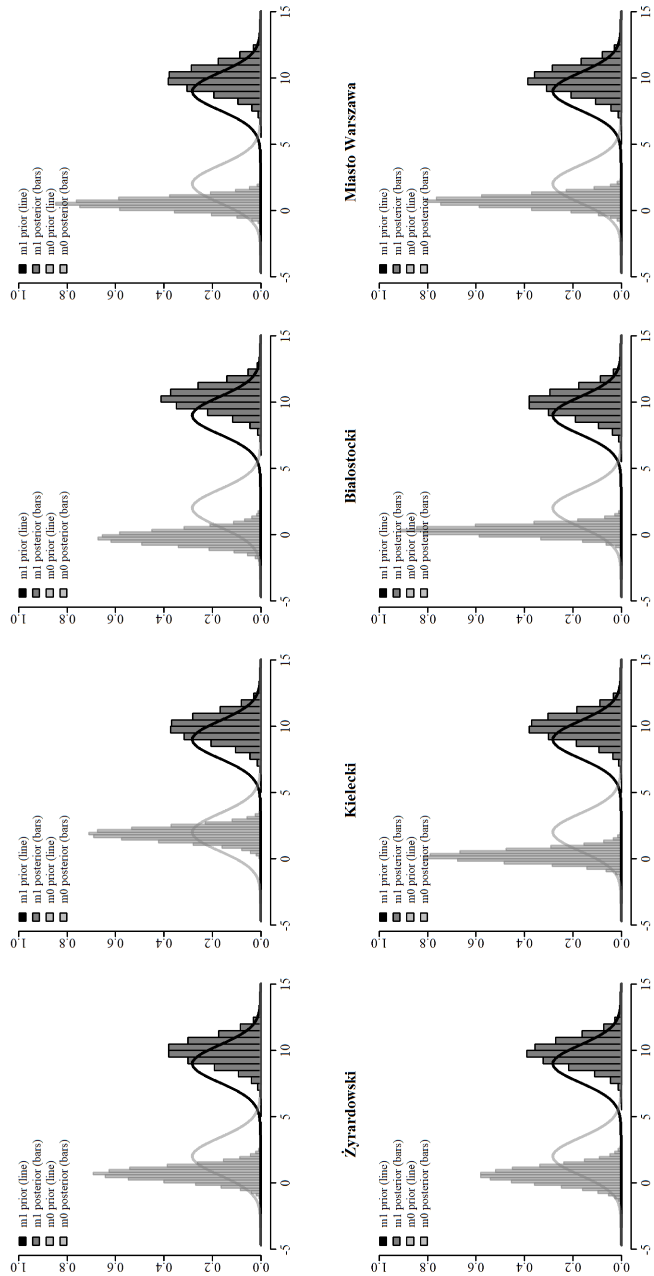
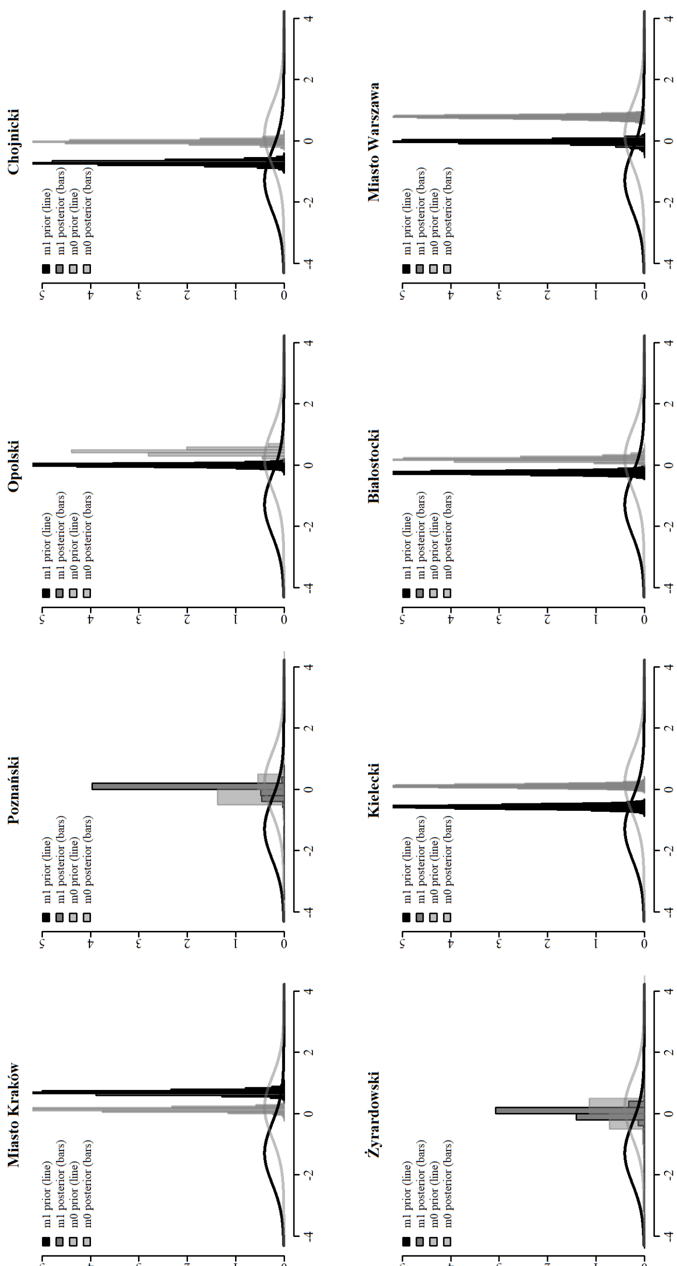


Figure 10: Unemployment: prior and posterior distributions of m_0 and m_1 for selected subregions



for expansion (min. -1.396). The posterior densities of regime-specific variable means shrink considerably as compared to the priors, for both GDP and unemployment.
For unemployment changes, the difference between expansion and recession means for an average region is tiny in the economic terms, and as compared to GDP dynamics. This should not come as a surprise, given the institutional setup of the Polish labour market. An alternative explanation entails some degree of cross-regional employment mobility across the regions in the occurrence of asymmetric shocks, since the NUTS-3 regions are small enough for agents to avoid the related residence mobility. Interestingly, with less informative priors of m_0 and m_1 than in the case of GDP dynamics, posterior denisities for some regions (see Miasto Kraków in Figure 10, and 11 out of 72 other regions) do not preserve the property $m_0 \leq m_1$. Since this is a just-identifying condition for the two regimes, their interpretation should be inverse under such circumstances, in spite of the slight difference in prior densities. For this reason, the best (worst) performing regions for unemployment during recession (expansion) exhibit $m_0 < 0$ ($m_1 > 0$), e.g. Ostrołęcki subregion at -0.401 p.p. (Trójmiejski subregion at +0.821 p.p.).

4.3 Timing: transition probabilities

Figures 11 and 12 present the prior and posterior distributions of parameters p_{11} and p_{00}, i.e. the probabilities of remaining in expansion and recession (respectively) one period ahead. Higher values imply a higher long-term prevalence of a given state, and obviously a lower chance of transition into the other phase. For GDP, noticeable patterns that emerge in this case are higher posterior means for p_{00} (0.91) than for p_{11} (0.78), suggesting longer NUTS-3 level recessions than expansions. (Note, however, different values of m_0 and m_1 over regions that re-define expansion and recession from region to region).
This is no longer the case for unemployment, with posterior means of p_{11} ranging between 0.93 and 0.95 in terms of average regional posterior mean and only slightly exceeding the means for p_{00}. Also note the similarity of posterior distributions across all regions.

4.4 Synchronization: business cycles within and between NUTS-2 regions

The probabilities of expansion and recession for a given period, depicted in Figures 13 and 14, have been computed for every NUTS-3 region using posterior means of the parameters. For the sake of tractability, these figures contain only the expansion probability (which amounts to one minus the recession probability). The time series for NUTS-3 regions are represented by lines in different colors and are further grouped according to their NUTS-2 adherence for better visibility (see Table 4 for examples of the grouping).

Figure 11: GDP: prior and posterior distributions of p_{11} and p_{00} for selected subregions

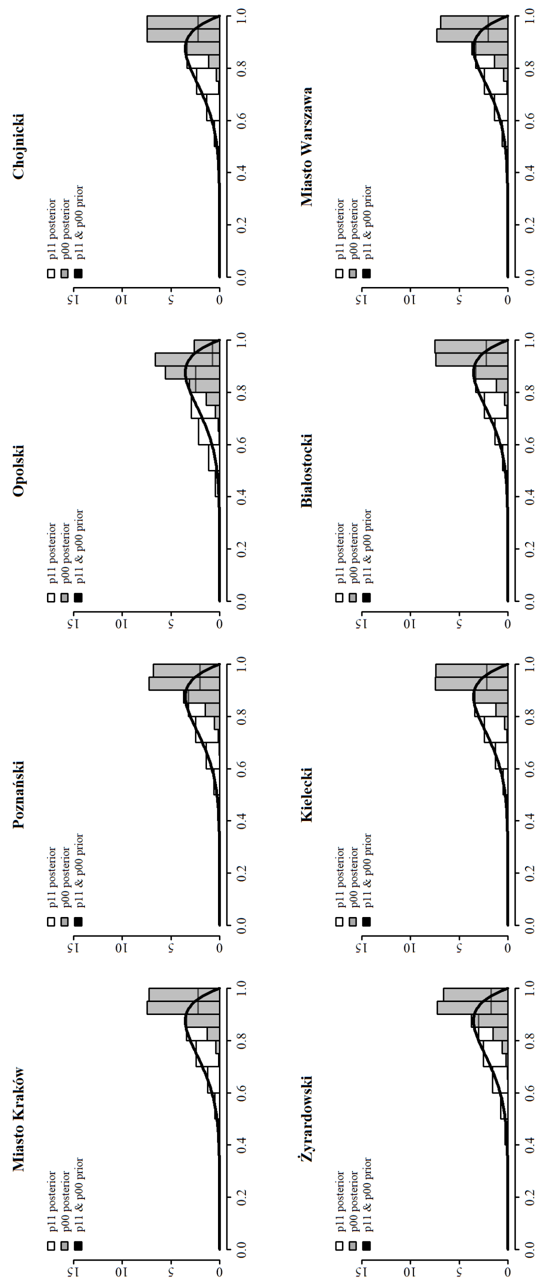
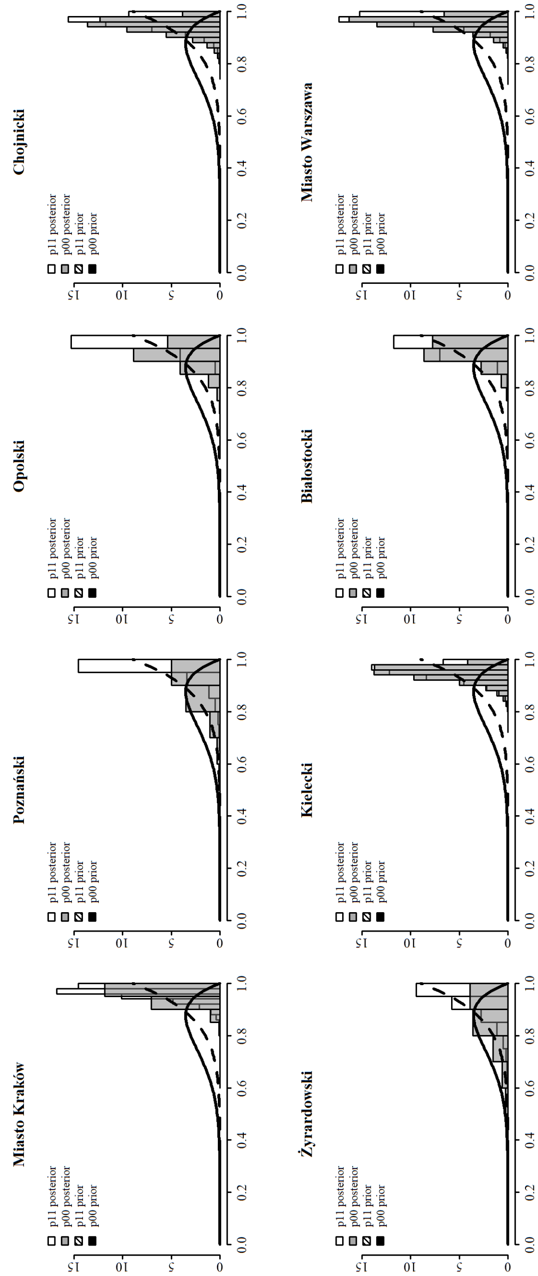


Figure 12: Unemployment: prior and posterior dist. of p_{11} and p_{00} for selected subregions



The model for GDP largely fails to clearly differentiate between own business cycle phases in individual NUTS-3 regions (Figure 13). For most of the time, a number of regions (including almost all from the NUTS-2 areas: Lubuskie, Świętokrzyskie, Warmińsko-mazurskie and Zachodniopomorskie) are clearly indicated as in recession. This means that the volatility in GDP dynamics has been interpreted as spatial spillovers from other regions. This largely explains the phenomenon of prior-posterior overlap for expansion means. Also, note that most of the high expansion probabilities have been observed in 3 voivodships, dominant in economic size and dynamics: Mazowieckie, Dolnośląskie and Śląskie. One might read this as an indication that these voivodships serve as business cycle gravity centers, and other voivodships inherit their business cycles through upstream or downstream connections rather than exhibit their own.

The reading of the unemployment analysis results (Figure 14) is completely different: the model is highly conclusive in finding the turning points, as well as both phases, in most NUTS-3 regions. This major difference most likely stems from the different panel dimensionality for the two variables in question. With the same number of regions ($N = 73$), the frequency of the data is annual for GDP and monthly for unemployment, yielding a material difference in T-dimension for both variables. Yet, given the sluggish nature and limited economic meaning of the unemployment rate in Poland, one should not easily reject the results for GDP and focus on unemployment instead.

As regards the business cycle synchronization between regions, evidence can be found with both variables. First, the GDP analysis suggests some within-voivodship (NUTS-2) synchronicity in the case of 3 voivodships where autonomous cycles have been detected (say, business cycle setters) in the form of 2 humps located in the beginning and in the middle of the sample. Since the rest are interpreted by the model as, say, business cycle takers, one can imagine that Pearson correlation analysis will generally exhibit a high degree of correlation, driven both by the idiosyncratic component and by spatial spillovers. Second, the probabilities in Figure 14 suggest that the unemployment data is too noisy for such conclusions, but some evidence of synchronicity can be found between Pomorskie, Warmińsko-mazurskie and Lubuskie voivodships.

4.5 Spatial interactions: impact of other regions' cycles

The posterior distributions of spatial autoregression parameter ρ are presented in Figure 15. Remarkably, the posterior means amount to 0.946 (GDP) and 0.899 (unemployment), and the HPDIs of +/- 2-3 percentage points cover almost entire posterior probability, in spite of a highly uninformative prior distribution. A direct reading of this result suggests that a 1 p.p. rise in the GDP dynamics (unemployment rate) in all other regions translates into a 0.946 p.p. (0.899 p.p.) change in the home region. According to this result, the spatial links across voivodships have turned out to be a material, and sometimes dominant, driver of individual regions' GDP and

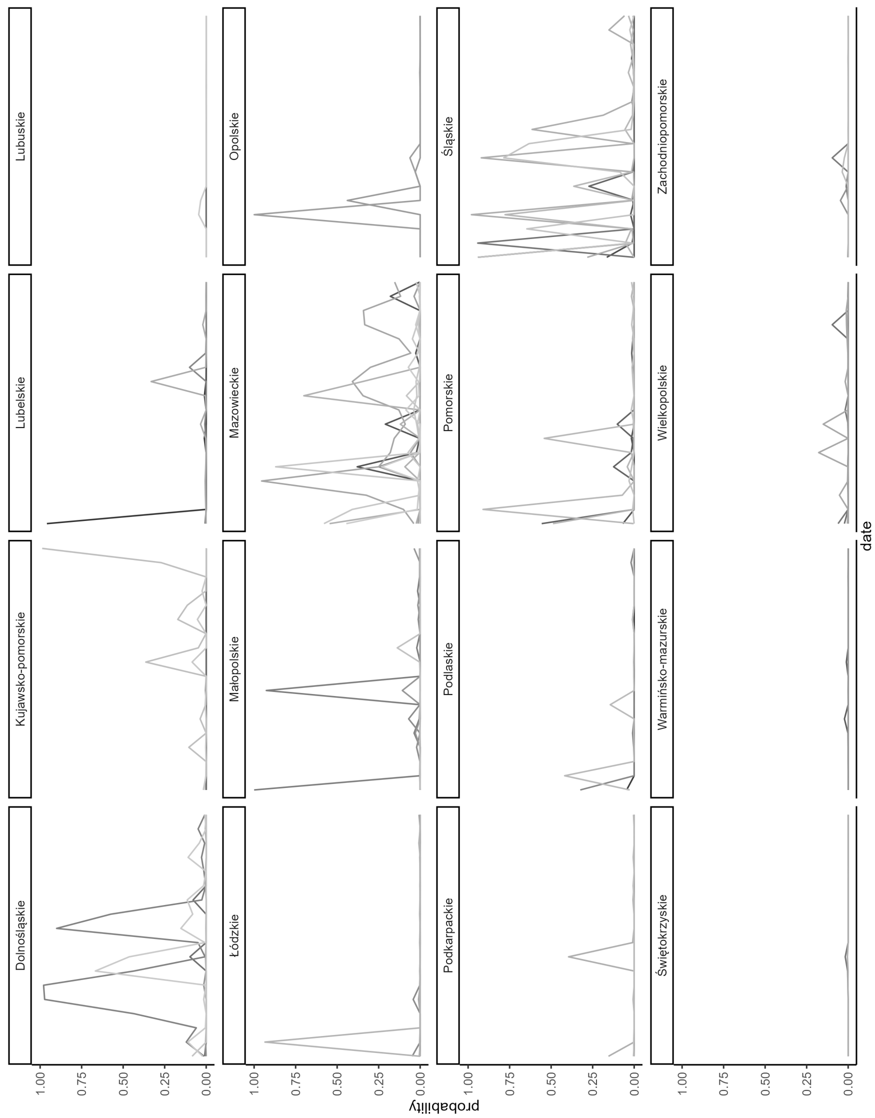
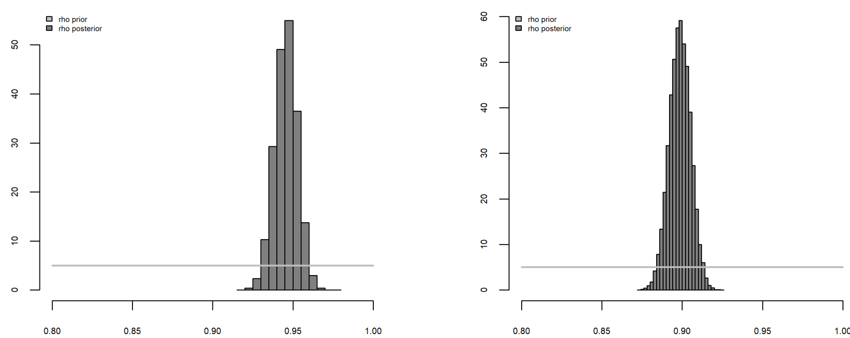


Figure 13: GDP: expansion probabilities computed for posterior means, grouped by NUTS-2 (colors vary based on NUTS-3 regions)

Figure 14: Unemployment: expansion probabilities computed for posterior means, grouped by NUTS-2 regions (colors vary based on NUTS-3 regions)

unemployment rate changes. This however should not be viewed as a surprise in a unitary, highly integrated country.

Figure 15: Distributions of ρ parameter for GDP and unemployment, respectively

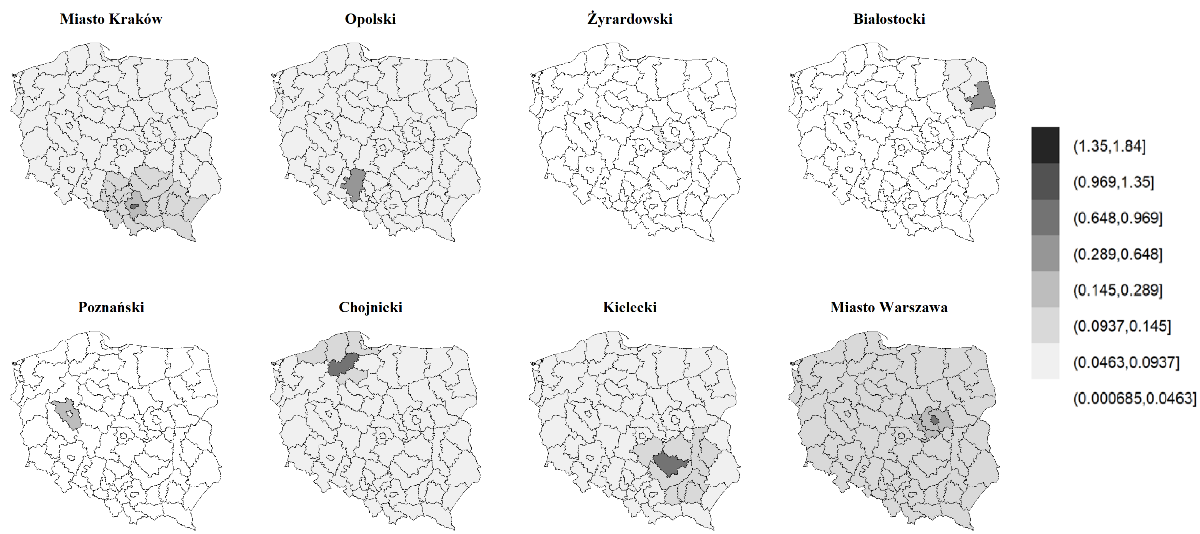


To visualize the differential scope of cross-regional impacts, we simulate the impacts on GDP and unemployment of a transition from recesssion to expansion, in every NUTS-3 region under detailed inspection (note that such transitions have not been found, in some cases). Figures 16 and 17 present the related spatial multipliers. The results are locally concentrated or spill over through the country, depending (mainly) on the scale of impulse, i.e. the size of the difference between μ_0 and μ_1 and (to a lesser extent) on the slight differences in connectivity of regions, implied by the W matrix. Note that both metropolitan areas under inspection (Miasto Warszawa, Miasto Kraków), as well as the ring around a third metropoly (Poznański), impact the other regions to a higher extent than more peripheral regions (Żyrardowski, Opolski, Kielecki, Chojnicki and, especially, Białostocki). This conclusion does not depend on the variable in question, since the positive impact of recession-to-transition probability ranks from highest in region Poznański, Miasto Kraków and Miasto Warszawa to lowest in regions Kielecki, Chojnicki and Białostocki for both GDP and unemployment dynamics.

The model for unemployment can be further used to decompose the fluctuations for each region into 3 parts: (i) local (related to $\mathbf{m_0} \odot (\mathbf{1_N} - \mathbf{s_t}) + \mathbf{m_1} \odot \mathbf{s_t}$), (ii) external (spilled over from other regions via $\rho\mathbf{W}\mathbf{y_t}$) and (iii) residual ($\boldsymbol{\varepsilon_t}$). This decomposition has been effected by assigning a state-specific constant to each region (when the probability of a given state exceeds 0.5), then computing the local business cycle as $\mathbf{m_0} \odot (\mathbf{1_N} - \mathbf{s_t}) + \mathbf{m_1} \odot \mathbf{s_t}$, along with the systematic part of the model as $(\mathbf{I} - \rho\mathbf{W})^{-\mathbf{1}} \cdot (\mathbf{m_0} \odot (\mathbf{1_N} - \mathbf{s_t}) + \mathbf{m_1} \odot \mathbf{s_t})$. Then, the share of the residual component has been evaluated as one minus the R-squared of a region-specific

Figure 16: GDP: response of individual regions' to a recession-to-expansion transition in a given region

Figure 17: Unemployment: response of individual regions to a recession-to-expansion transition in a given region



time-series regression of $\mathbf{y_t}$ on the systematic part of the model, and the rest has been allocated between the local and the external component in proportion to the R-squared from the regression of the systematic part on the local part. Table 5 presents the top-performing regions in terms of the local business cycle contributions (top cycle setters), whereas Table 6 – the regions representing the most passive behaviour as business cycle takers.

Table 5 shows that 6 out of 12 regions classified as the top cycle setters include big metropolies, and 5 of them being these metropolies alone (Miasto Poznań, Miasto Warszawa, Trójmiejski including Gdańsk, Miasto Kraków and Miasto Wrocław). Turning this table so as to rank the most passive business cycle takers, one obtains Table 6 mostly populated with peripheral regions. It contains one region where a NUTS-2 metropoly is mixed with peripheral areas (Rzeszowski), and the rest are NUTS-3 entities attributable to relatively small cities (Biała Podlaska, Żyrardów, Puławy, Bielsko-Biała, Ostrołęka, Tarnów, Skierniewice) or a ring of periphery around Poznań (Poznański region, where most of the variance remained unexplained). Figures 18 and 19 depict the spatial visualization of the shares of the local business cycle and external business cycle spillovers in a region's activity.

Table 5: Decomposition of unemployment fluctuations: NUTS-3 regions with the highest contribution of local fluctuations (business cycle setters)

NUTS-3	Local	External	Residual
Miasto Poznań	0.746	0.011	0.242
Miasto Warszawa (Warsaw)	0.713	0.010	0.277
Trójmiejski	0.711	0.014	0.275
Starogardzki	0.690	0.006	0.304
Ełcki	0.679	0.005	0.316
Miasto Kraków	0.671	0.014	0.315
Wałbrzyski	0.667	0.007	0.326
Warszawski zachodni	0.665	0.150	0.186
Elbląski	0.644	0.013	0.343
Częstochowski	0.626	0.008	0.367
Miasto Wrocław	0.603	0.007	0.391
Lubelski	0.581	0.019	0.401

Figure 18: Share of local business cycle in a region's activity

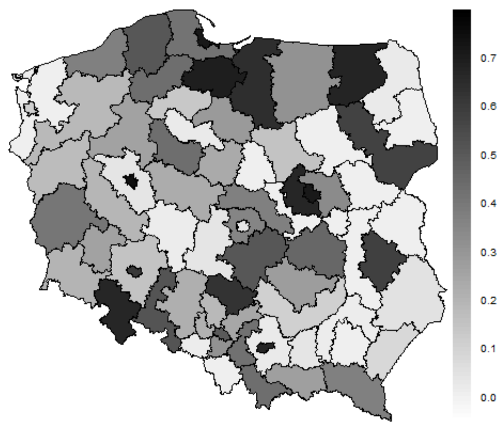


Figure 19: Share of external business cycle spillovers in a region's activity

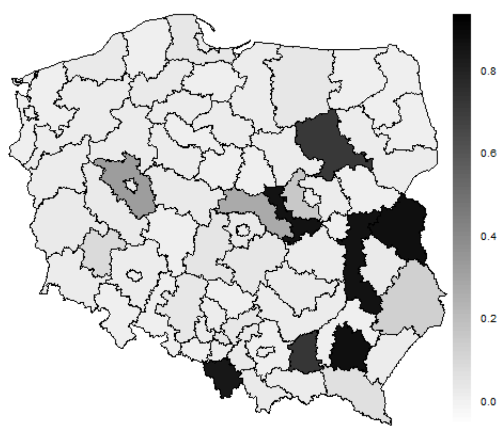


Table 6: Decomposition of unemployment fluctuations: NUTS-3 regions with the highest contribution of external fluctuations (business cycle takers)

NUTS-3	Local	External	Residual
Bialski	0.000	0.874	0.126
Rzeszowski	0.000	0.874	0.126
Żyrardowski	0.000	0.865	0.134
Puławski	0.000	0.862	0.138
Bielski	0.000	0.839	0.161
Tarnowski	0.034	0.724	0.242
Ostrołęcki	0.004	0.707	0.289
Poznański	0.038	0.323	0.639
Skierniewicki	0.365	0.268	0.368

5 Conclusions

We investigate the regional business cycle properties of GDP dynamics and unemployment rate changes, in a NUTS-3 regional granulation for Poland. The model includes three sources of variation: a 2-state Markov switching idiosyncratic business cycle of each region, a spatial interaction with other regions, as well as a random component, and we decompose the variance of each variable accordingly. In line with the extant literature, to handle the challenge of statistical identification and constrained parameter space (including probabilities), we use Bayesian methods, simulating the posterior distribution with a Metropolis-within-Gibbs procedure.

The novelty of this study, on top of the geographical extension of the previously proposed methodologies to the Polish data, is threefold. First, we investigate two variables: GDP and unemployment. The latter data is monthly, and the use of a relatively large-T dataset allows to take additional insights into the usability of the model in various data environments. Second, as opposed to the extant literature, we apply the model to a panel abundant in small regions ($N = 73$), which revealed some limitations related to the proposed setup (mainly computational constraints) and suggested the need to develop alternative, robust specifications for such applications. Thirdly, we use the model to put each of these small regions on a continuum from business cycle setters to business cycle takers.

As for the empirical findings, the main one is a high value of spatial autoregression coefficient, amounting to 0.899-0.945 depending on the variable, and suggesting an important role of cross-regional interactions in shaping the regional business cycles on the NUTS-3 level. The systematic part of the idiosyncratic component is of secondary importance in most regions, where no local turning points have been detected in the analysis of GDP dynamics, and the bulk of variability can be regarded as inherited from other regions. As for unemployment, 7 out of 73 regions have been

identified as ones where external spillovers (i.e. variability originating in other regions) are responsible for more than 50% of variance.
Unemployment analysis finds turning points in most of the regions. Although the labour market adjustment is more sluggish than the product market adjustment, this result has been possible to achieve since the unemployment time series carried far more observations in the T-dimension than in the case of the GDP dataset. However, little can be concluded from the parallel analysis of GDP dynamics, since 18 annual observations are far insufficient to correctly identify the states. The GDP model tended to identify the business cycle switching in no more than three voivodships, all top performers in economic development and per capita GDP.
The results obtained for both variables may differ not only on the grounds of sample size. As can be concluded from Figures 1 and 4, the windows of data availability overlap only partly. Moreover, one has to bear in mind that the cyclical behaviour of unemployment does not exactly follow the GDP fluctuations, and that the labour market institutions (e.g. definition of the unemployed) imply some measurement error between the economic concept and the statistical measurement of the unemployment.
The NUTS-3 level findings have turned out to be more informative than NUTS-2 ones can potentially be, in terms of identifying the role of metropolitan areas. The cities of Warsaw, Poznań, Tricity (subregion Trójmiejski, including Gdańsk) and Wrocław, along with some NUTS-3 entities in Śląskie voivodship, appear to play the role of business cycle setters.
Due to both (i) the low T-dimension of the GDP panel and (ii) the limited role of unemployment in business cycle analysis, the results of this study should be interpreted with caution. They could, however, suggest areas where there is room for methodological improvement. One of them is a more robust specification, better fit for large-N problems. Further questions arise around the prior elicitation guidelines of means for the recession and expansion periods, so as to strike the balance between model flexibility and the correct inequality relationship between the posterior means. In line with the spatio-temporal modelling literature, one should also consider enriching the model dynamics beyond the Markov-switching process to include some form of temporal autoregression, and handle the econometric sources thereof. Bottom line, the indicative results achieved here suggest that the nation-wide business cycle dynamics covers a potentially rich network of structures (which we closed in a single spatial autoregression parameter), the understanding of which can be of particular importance for both regional policy planning, as well as regional and national forecasting.

Acknowledgements

The authors are grateful to the Editor and anonymous Referee for helpful comments. The usual disclaimer applies. The replication package (data and R codes) is available at: https://github.com/AndrzejToroj/CEJEME_2023.

References

[1] Anselin L., Florax R., (1994), Small sample properties of tests for spatial dependence in regression models: some further results, Regional Res. Inst. Research Paper, Morgantown, WV: Regional Research Institute, 9414.

[2] Artis M., Dreger C., Kholodilin K., (2011), What drives regional business cycles? The role of common and spatial components, *The Manchester School* 79(5), 1035–1044.

[3] Beraja M., Hurst E., Ospina J., (2016), The Aggregate Implications of Regional Business Cycles, NBER Working Papers, 21956.

[4] Bernardelli M., Dędys M., (2015), Przełącznikowe modele Markowa w analizie synchronizacji cykli koniunkturalnych, *Roczniki Kolegium Analiz Ekonomicznych SGH* 39(14), 213–227.

[5] Chauvet M., Hamilton J., (2005), Dating Business Cycle Turning Points, NBER Working Papers, 11422.

[6] Chauvet M., Piger J., (2003), Identifying business cycle turning points in real time, *Federal Reserve Bank of St. Louis. Review* 85(2), 47–61.

[7] Chib S., (2001), *Markov chain Monte Carlo methods: computation and inference*, Chapter 57, [in:] Handbook of Econometrics, vol. 5, [eds.:] J. Heckman, E. Leamer, Elsevier, Amsterdam, 3569–3649.

[8] Clements M., Krolzig H., (1998), A Comparison of the Forecast Performance of Markov-switching and Threshold Autoregressive Models of US GNP, *The Econometrics Journal* 1(1), 47–75.

[9] Diebold F., Lee J., Weinbach G., (1994), Regime switching with time-varying transition probabilities, *Nonstationary Time Series Analysis and Cointegration*, 283–302.

[10] Diebold F., Rudebusch G., (1996), Measuring Business Cycles: A Modern Perspective, *The Review of Economics and Statistics* 78(1), 67–77.

[11] Drozdowicz-Bieć M, (2008), Regionalne cykle koniunkturalne. Doświadczenia światowe – implikacje dla Polski, *Barometr Regionalny* 3(13), 5–15.

[12] Dueker M., (1997), Markov Switching in GARCH Processes and Mean-Reverting Stock-Market Volatility, *Journal of Business and Economic Statistics* 15(1), 26–34.

[13] Fatas A., (1997), EMU: Countries or Regions? Lessons from the EMS Experience, CEPR Discussion Papers, 1558.

[14] Filardo A., Gordon S., (1998), Business cycle durations, *Journal of Econometrics* 85(1), 99–123.

[15] Frankel J., Rose A., (1998), The Endogeneity of the Optimum Currency Area Criteria, *The Economic Journal* 108(449), 1009–1025.

[16] Frühwirth-Schnatter S., (2001), Markov chain Monte Carlo Estimation of Classical and Dynamic Switching and Mixture Models, *Journal of the American Statistical Association* 96, 194–209.

[17] Frühwirth-Schnatter S., Kaufmann S., (2008), Model-Based Clustering of Multiple Time Series, *Journal of Business & Economic Statistics* 26, 78–89.

[18] Hamilton J., (1989), A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle, *Econometrica* 57(2), 357–384.

[19] Hamilton J., Owyang M., (2012), The Propagation of Regional Recessions, *The Review of Economics and Statistics* 94(4), 935–947.

[20] Hansen B., (1992), The Likelihood Ratio Test Under Nonstandard Conditions: Testing the Markov-Switching Model of GNP, *Journal of Applied Econometrics* 7(S1), S61–S82.

[21] Harding D., Pagan A., (2006), Synchronization of cycles, *Journal of Econometrics* 132(1), 59–79.

[22] Kim C., Murray C., (2002), *Permanent and transitory components of recessions*, [in:] Advances in Markow-Switching Models, Studies in Empirical Economics, [eds.:] J. D. Hamilton, B. Raj, Physica Heidelberg, 19–39.

[23] Kim C., Piger J., Startz R., (2004), Estimation of Markov Regime-Switching Regression Models with Endogenous Switching, Federal Reserve Bank of St. Louis, Working Papers, 15.

[24] Kim C. J., Nelson C., (1998), Business cycle turning points, a new coincident index, and tests of duration dependence based on a dynamic factor model with regime switching, *Review of Economics and Statistics* 80(2), 188–201.

[25] Kim C. J., Nelson C., (1999), Has the U.S. economy become more stable? A Bayesian approach based on a Markov-switching model of the business cycle, *Review of Economics and Statistics* 81(4), 608–616.

[26] Kondo K., (2021), Spatial Dependence in Regional Business Cycles: Evidence from Mexican States, Discussion Paper Series Research Institute for Economic and Business Administration Kobe University, DP2015-27, available at `https://www.rieb.kobe-u.ac.jp/academic/ra/dp/English/DP2015-27.pdf`.

[27] Kośko M., Kwiecień M., Stempińska J., (2016), Przełącznikowe modele Markowa (MS) – charakterystyka i sposoby zastosowań w badaniach ekonomicznych, *Roczniki Kolegium Analiz Ekonomicznych SGH* 40, 479–490.

[28] Krolzig H., (1997), *Markov-Switching Vector Autoregressions: Modelling, Statistical Inference, and Application to Business Cycle Analysis*, Berlin, Springer.

[29] Krugman P., (1990), Increasing Returns and Economic Geography, NBER Working Papers, 3275.

[30] Lawrence J. C., Trabandt M., Walentin K., (2021), Involuntary unemployment and the business cycle, *Review of Economic Dynamics* 39, 26–54.

[31] Marelli E., (2007), Specialisation and Convergence in European Regions, *European Journal of Comparative Economics* 4(2), 149–178.

[32] Owyang M., Piger J., Wall H., (2005), Business Cycle Phases in U.S. States, *The Review of Economics and Statistics* 87(4), 604–616.

[33] Owyang M., Wall H., (2006), Regional Vars and the Channels of Monetary Policy, Federal Reserve Bank of St. Louis, Working Paper Series.

[34] Podgórska M., Decewicz A., (2001), Modele Markowa w analizie wyników testu koniunktury, *Prace i Materiały Instytutu Rozwoju Gospodarczego SGH* 70, 119–156.

[35] Shibaev S., (2016), Recession propagation in small regional economies: Spatial spillovers and endogenous clustering, Queen's Economics Department, Working Paper, 1369.

[36] Sims C., Zha T., (2006), Were There Regime Switches in U.S. Monetary Policy?, *The American Economic Review* 96(1), 54–81.

[37] Stock J., Watson M., (1989), New Indexes of Coincident and Leading Economic Indicators, *NBER Macroeconomics Annual* 4, 351–409.

[38] Stock J., Watson M., (1991), *A Probability Model of the Coincident Economic Indicators*, Chapter [in:] The Leading Economic Indicators: New Approaches and Forecasting Records, [eds.:] G. Moore, K. Lahiri, Cambridge University Press, 63–90.

[39] Torój A., (2020), *Metody ekonometryczne, statystyczne i matematyczne w modelowaniu zjawisk społecznych*, Tom II: Metody ekonometryczne w makro- i mikroekonomii, chapter Regionalne cykle rentowności polskich przedsiębiorstw: bayesowska analiza przestrzennego modelu przełącznikowego, Oficyna Wydawnicza SGH, 181–196.

[40] Wall H., (2007), Regional Business Cycle Phases in Japan, *Federal Reserve Bank of St. Louis. Review* 89(1), 61–76.

[41] Warżała R., (2011), Analiza koniunktury gospodarczej w ujęciu regionalnym na przykładzie województwa warmińsko-mazurskiego, *Studia Regionalne i Lokalne* 3(45), 81–99.

[42] Warżała R., (2014): Regionalne cykle koniunkturalne w Polsce w latach 2000-2014, *Zeszyty Naukowe Uniwersytetu Szczecińskiego. Studia i Prace Wydziału Nauk Ekonomicznych i Zarządzania* 35(2), 529–550.

[43] Warżała R., (2015), Specialization and business cycles fluctuations of Polish regions, *Managerial Economics* 16(2), 175–188.