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Short-term Fluctuations and Fluctuations in the Development Dynamics of Fixed Assets of Industry

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Abstract
The purpose of the article is to create a concise nonlinear mathematical model for analyzing the growth of fixed assets in a specific industry. The emergence of chaotic behaviour in economic systems was explored, focusing on fluctuations. The study employed methods such as systems analysis, correlation analysis, nonlinear dynamics, and differential equations. It was identified that sharp technological innovations as the primary drivers of short-term fluctuations impacting fixed asset development. The resulting nonlinear dynamic model allowed for flexible operation, transitioning between equilibrium, periodic, and chaotic states based on coefficient values for asset growth rates and time constants reflecting economic system dynamics.

Keywords
Nonlinear dynamic model; Strange attractor; Life cycle; Econometric models; Asset management.

Introduction

When studying fluctuations, it does not matter which indicator of economic activity is considered. Most macroeconomic variables that measure one or another type of income, expenditure or production, are changed to a large extent synchronously (Kurmanov, 2023). A more difficult task is to combine the causes of these fluctuations and the theory of economic fluctuations remains debatable and requires systematic study within the framework of the modern theory of dynamic systems (Mankiw, 2002; Ruelle & Tacens, 1971). The studies at the end of the 20th century revealed a wide variety of nonlinear systems dynamics and led to one of the most important discoveries in nonlinear dynamic systems, deterministic chaos with the generation of a "strange attractor" (Ruelle & Tacens, 1971; Li & Yorke, 1975). Currently, it is widely accepted that real dynamic systems are nonlinear, often exhibiting deterministic chaos. This chaotic behaviour can have detrimental effects, leading to "separation" and "accidents" in technical systems and "crises" in socio-economic, biological, and medical systems, etc. The deterministic chaos mode in the economic system occurs as fluctuations, which, at a certain frequency and amplitude lead to a "crisis", that is, business cuts economic activity down (Mankiw, 2002).

It is necessary to consider in more detail the issue of short-term economic fluctuations, which are also called business cycles in publications (Burlutski et al., 2017; Lu & Su, 2011). Researchers identified the main features of short-term fluctuations, which included: recessions and upswings of uneven duration and severity, the impact on the entire economy, their expansion and contraction, as well as their possible global impact. Four important monthly indicators used to date recessions were also identified: industrial production, total sales in production, wholesale and retail trade, non-farm employment, real household income after tax payment (Lu & Su, 2011). In the study conducted by (L. Lenart and M. Pipien (2013) two main reasons for the occurrence of short-term fluctuations were suggested:

- markets need time to reach an equilibrium price and quantity (companies rarely change prices, the produced quantity is not in equilibrium during the adjustment period, firms produce products to meet demand at current prices);
- changes in total costs at given prices affect the output level (when costs are low, the output will be below potential output, economy-wide changes in costs are the main causes of the output gap).

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There are self-regulation mechanisms in the economy and, eventually, firms adjust to the output gap, if costs are less than potential output (crisis gap), firms slow down the increase of their prices. However, if costs exceed potential output (expansive gap), firms raise prices and there is a potential inflationary pressure (Bashnyanyn, 2015). But the definition of the cause of fluctuations in the development of the fixed assets of the industry is not clearly justified, and the structural and logical model of the influence of short-term fluctuations on the development dynamics of fixed assets has not been defined. It is not clear whether the state has the ability to prevent periods of short-term fluctuations or influence their duration (Alieksieiev & Mazur, 2022). Therefore, considering the economy as a nonlinear dynamic system to study the impact of fluctuations on the dynamic development of fixed assets, to further improve approaches to their management is a topical scientific topic, an in-depth study of which will help increase the profitability and productivity of the industry (Senchukov, 2019). Here it needs nonlinear mathematical models that adequately describe the processes dynamics occurring in the economic system. Their system analysis is a much more complicated matter, but it is necessary in solving many problems of the economic development (Beisenbi, 2011; Beisenbi & Shuteeva, 2017; Beisenbi et al., 2017; Beisenbi & Beisembina, 2020).

Economic development is directly determined by the quantity and quality of fixed assets. Therefore, the main purpose of the presented article is to study the scenarios of the generation of the deterministic chaos mode with "strange attractor" (short-term fluctuations and fluctuations) in the economic system using the simplest nonlinear mathematical model of the fixed assets dynamics of one industry.

The primary scientific novelty of this research lies in its fresh approach to economic analysis by treating the economy as a nonlinear dynamic system, diverging from traditional methods. It introduces a straightforward mathematical model for fixed asset dynamics that enables the examination of intricate economic behaviours. It establishes a previously unexplored link between fixed asset lifecycles and economic cycles, utilizing advanced mathematical techniques like nonlinear dynamics and bifurcation analysis in an innovative economic context. Importantly, the research clarifies the relationship between growth rates, time constants, and fluctuations, presenting a novel insight into how economic parameters relate to instability. The proposed model suggests potential applications in enhancing asset management and forecasting through its unique nonlinear modelling approach.

Materials and methods

The work was done based on the provisions of economic theory, economic analysis, lifecycle theory, as well as the system approach. To solve the set tasks, the following methods of scientific research were used: analysis and synthesis to study the development of economic cycles, correlation analysis to determine the connection between lifecycle of fixed assets and average economic cycles, the nonlinear dynamics method to build a dynamic model of the fixed assets development and their further study, and also the theory of differential equations (Brock, 2001; Blokdyk, 2020).

When conducting a structural and dynamic analysis of fixed assets horizontal, vertical and trend analyses are used. During the analysis, it is advisable to evaluate what part of all fixed assets are assets and determine their development trends over a certain period. If, upon availability of other constant factors, the share of active assets increases, then it can be concluded that the production capacity of the enterprise also increases. At the same time, it should be noted that an increase in the assets active part is not always an objective indicator of an increase in the production capacity of an enterprise due to the chosen evaluation of such assets and depreciation methods. It is also important to determine whether the active part of fixed assets is technically advanced. Determining the optimal ratio of active and passive (not directly involved in the process of production and provision of services) assets is an important condition for the effective use of assets. Therefore, it is necessary to evaluate all elements of material fixed assets and analyse their development reasons (Magnitsky, 2011; Broer & Takens, 2017).

Practice shows that fixed assets elements are changed in different ways. At the same time, it is important to analyse not only the entire fixed asset, but also the dynamics of all its elements. This information analysis is especially significant if a longer period is analysed. This is due to the fact that some fixed assets are rarely updated, as they can be used for a long time. Active elements of tangible fixed assets should be updated more often, however, as practice shows, in some cases the passive part of assets increases faster.

The analysis of changes in fixed assets should be connected with compositional, structural and dynamic analyses, since the change in any element of assets affects the overall results of the composition, structure and dynamics of the development of fixed assets of the industry. The analysis of changes in the fixed assets of the industry should be carried out based on the most important elements of such assets. It is important to determine the reasons upon which there have been significant changes in certain elements. Often, the changes in fixed assets are affected by the reconstruction, division or merger of an enterprise, the changes in the organizational structure and management system, improvement of production processes and forms of labour organization; restoration of worn and obsolete assets; the level of industrial specialization and cooperation; the change in the enterprise geographical location, etc.

It is very important to use fixed assets efficiently, that is, all of their elements should in a sense contribute to the production, increase the provided services or achieve other purposes of the enterprise. To assess the efficiency of using fixed assets, it is suggested to evaluate them using various comparative coefficients (Nicolis & Prigozhin, 1990).

Results and discussion

It is customary to refer to fixed assets both for an enterprise and for the industry as a whole, fixed production assets and fixed assets for non-production purposes. Within the framework of the set purposes in the present work, the main backgrounds for production purposes, which make up approximately 95-98% of the total value of the enterprise's fixed assets, and their lifecycle will be considered. Fixed assets for production purposes, in turn, are divided into an active part (working machines and mechanisms, vehicles, power machines and mechanisms, production implements) and a passive part (industrial buildings and structures, structures and transmission devices, household equipment) (Tamuleviñienĺ, 2019). Fixed assets lifecycle is presented in Figure 1.

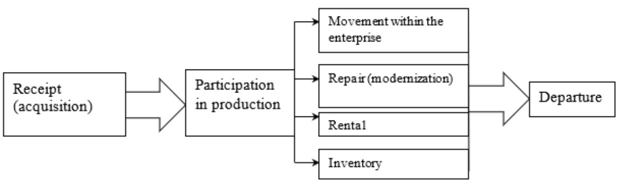

Fig. 1. Fixed assets lifecycle

In most works in the study of fixed assets lifecycle, they omit various options for their dynamic development in the process of creating finished products, such as movement within the enterprise, repair or modernization, rental and inventory (King, 2011; Fernandes & Pinto, 2018). But if consider fixed assets lifecycle from the perspective of economic cycles, then the occurrence short-term fluctuations may lead to a nonlinear development of fixed assets lifecycle of the industry and to fluctuations in the dynamics of their development. The active production assets lifecycle averages 10-15 years and this cycle can be compared with the economic cycle of average duration (Juglar cycle). To the reasons for economic cyclicality, C. Juglar added a delay in making investment decisions by launching new production facilities. In addition, this includes the time lag between the decrease in sales and the liquidation of operational capacities. As a result, C. Juglar cycles acquire additional duration (Tuyakova & Cheremushnikova, 2016). Fixed assets lifecycle is presented in the form of a diagram in Figure 2. The diagram clearly shows the main Juglar cycle phases, such as recovery, upswings, recession and depression.

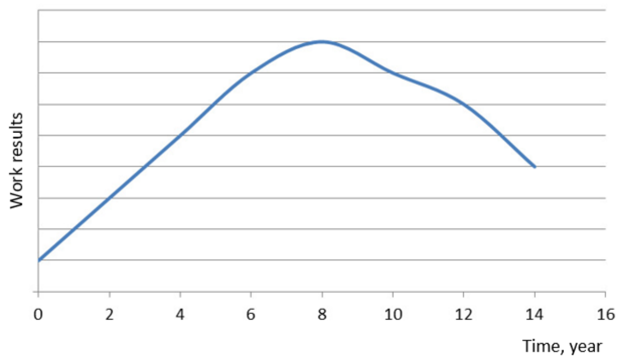

Fig. 2. Stages of fixed assets lifecycle are presented as an economic cycle

K.G. Liapis and D.D. Kantianis (2015) indicate that the main reasons for the occurrence of mid-term cycles are industrial transformations and new developments. However, it cannot be stated that the reasons are explained only in the technical direction, the exchange sphere, distribution and consumption also has an impact, therefore fixed assets lifecycle, as well as the economic cycle should be considered as a multivector process that depends on many factors leading to the nonlinear development of fixed assets of the industry.

Recently, a large number of approaches have been developed to manage fixed assets lifecycle. The main approaches should be considered and the operation principles of them should be distinguished. Enterprise Asset Management (EAM) is the managing process of physical assets lifecycle to maximise their use, which saves financial means, increase production quality and efficiency and protects health, safety and the environment. EAM focuses on maintaining, monitoring, evaluating, and optimizing assets from acquisition to disposal (Lutkevich, 2022). EAM is mainly used in industries that depend heavily on expensive and complex physical assets such as vehicles, factories and heavy equipment, more often this is done with specialised EAM software. It is often used for this purpose, as it can provide a holistic view of an organization's physical assets throughout their entire lifecycle (Facility Force, 2023). The known users include the oil and gas, shipbuilding, mining, energy, government, utilities, aerospace and defence industries. Asset lifecycle management (ALM) is one of the common features of the EAM software. There are four stages of asset lifecycle management, which include planning, procurement/acquisition, operation and maintenance, and disposal (Comparesoft, 2023). It covers the main stages of an asset's lifecycle from its initial design to planning for its production or construction, if it is a building or other infrastructure, including warranty management, decommissioning and disposal. ALM is a restricted approach to fixed assets managing and includes a set of interrelated iterative processes. During the design and assemble stage of a project, programs are managed to complete the project, at a much longer stage of operation and maintenance, the process focuses on asset performance management, portfolio assets and project management leads the overall investment portfolio in assets in accordance with the strategic purposes of the company (Sheilagh, 2021).

The fixed assets lifecycle model presented in this article has many more stages than the traditional model on which fixed assets lifecycle management approaches mentioned above are based (Hrabynska & Kosarchyn, 2022). The weaknesses of the traditional model with regard to short-term fluctuations (business processes) are not in what activities they cover, but in the lack of sufficient detail to see the main lifecycle stages that support these processes and how the processes interact (Kozyk et al., 2018). Based on the analysis, separate processes for operation, optimisation, maintenance and improvement have been identified, although they can be a part of the same organisational unit for operation and maintenance, the tasks and purposes of the people responsible for operating and maintaining the facility are completely different. Another point to note is that the same design, assembly, operation, optimisation, maintenance and improvement processes are used for all of the organisation's assets. This is an important point that is absent in the traditional model. While some of the individuals who perform these processes can be associated with a particular project or asset, the processes themselves are not specific for a project or plant (Alieksieiev et al., 2014). It is also should be noted that different asset classes undergo different lifecycle stages. The choice of commercial equipment, such as dump trucks and rolling stock, can be an important decision, but the project stage is primarily an acquisition stage, and not a complex design and assembly stage. Likewise, such stages as commissioning, trial period and overclocking are likely to be fast events that do not require particular attention (Medvedieva & Ahapova, 2020). But the managing of various production stages still requires significant attention. Similar comments can be made for facilities, information technologies (IT) and line assets, each of them goes through lifecycle stages in its own way with its own unique problems and requirements.

In the direction of studying the fluctuations impact on the development of fixed assets, it is necessary to consider how short-term fluctuations affect the production in general, and, as a result, the development of fixed assets in the industry. There are four major factors that cause both long-term trends and short-term fluctuations in the market. These factors are government, international transactions, speculation and expectation, and supply and demand. Economic activity can influence market trends, for the better or for the worse. Government policy and geopolitical events are factors that can lead to either stability or instability in markets. Higher interest rates and taxes, for example, can deter spending and result in a contraction or a long-term fall in market prices. In the short term, these news releases can cause large price swings as traders and investors buy and sell in response to the information (Mitchell, 2022). Fixed assets are for long-term use and include land, buildings, leasehold improvements, equipment, machinery, and vehicles. Depreciation is listed with the cost of goods sold if the expense associated with the fixed asset is used in the direct production of inventory. Examples include the purchase of production equipment and machinery and a building that houses a production plant (John F. Kennedy Memorial Library, 2023).

With short-term fluctuations, output gaps occur between the actual and potential production of goods. The output gap G is the difference between the actual output of a product and its potential output in relation to the potential output at a certain period in time and is described by the following formula:

$$G = [(Y - Y*)/Y*] \times 100 \tag{1}$$

where: $Y*$ is a potential output (Schuman & Brent, 2005).

Potential output, in turn, is the maximum sustainable output rate that the industry can produce, also called output at full employment, using capital and labour at higher rates than usual and exceeding $Y*$ for a certain period of time. It is stated that the production potential output grows over time, while the actual output grows at a variable rate and reflects the growth rate $Y*$, which takes into account the variable rates of technical innovation, capital accumulation, weather conditions, etc. At the same time, the actual output is not always equal to the potential output (Blanchard, 2020). Based on the system analysis of a nonlinear dynamic mathematical model, it was determined that the development dynamics of fixed asset of a separate industry strikes with an unimaginable variety of behaviour types varying from simple equilibrium points to multiple periodic (fluctuations) or chaotic depending on the value of some generalised coefficient. Moreover, this complex behaviour is associated with the loss of stability in the development of a fixed asset and the economic system as a whole. The development dynamics of fixed assets of the i-th industry $x_i(t)$ of the economic system is described by the mathematical model of a multi-industry economy by a nonlinear system of differential equations presented in formula 1 (Beisenbi, 2019):

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = \frac{1}{T_{xi}} x_i(t) \Bigg[\varkappa(1-\eta) \left(1 - \frac{1}{P_i(t)} \sum_{j=1}^{n} a_{ij} p_j(t) + \frac{1-h_i}{1+r(t)}\right) \times (x_i(t))^{\alpha-1} (L_i(t))^{\beta} - \frac{r(t) F_i^k(t)}{x_i(t) P_i(t)} - (1-\gamma) \frac{w_i(t)}{x_i(t)} L_i(t) - \mu_i \Bigg], \quad i = 1, \dots, n \tag{2}$$

where: T_{xi} is the time constant of fixed assets that characterises the development dynamics of fixed assets of the i-th industry; $\varkappa_i$, α_i and β_i are the parameters of production function ($\alpha_i + \beta_i = 1$), respectively; η is the tax rate on the income of the industry; α_{ij} is the coefficient of material consumption of the i-th products type for the j-th product; $p_j(t)$ is the price of the j-th product; $p_i(t)$ is the price of products of the i-th industry; h_i is the share of the repaid loan from the loan amount received by the i-th industry; $r(t)$ is the interest rate on the loan capital; $L(t)$ is the number of employees in the i-th industry of the economy; μ_i is the retirement rate of fixed assets of the i-th industry of the economy; $F_i^k(t)$ is the amount of credit funds of the i-th industry invested in the production development; γ is the social insurance coefficient; $\underline{w}_i(t)$ is the wages level in the labour market.

Indication of the nonlinear function through formula (3):

$$f_i(p_i, x_i, L_i \underline{w}_i, r, k) = \varkappa(1-\eta) \left(1 - \frac{1}{P_i(t)} \sum_{j=1}^{n} a_{ij} p_j(t) + \frac{1-h_i}{1+r(t)}\right) \times (x_i(t))^{\alpha_i - 1} (L_i(t))^{\beta_i} - \frac{r(t) F_i^k(t)}{x_i(t) P_i(t)} - (1-\gamma) \frac{\underline{w}_i(t)}{x_i(t)} L_i(t) \tag{3}$$

The nonlinear function was replaced in (2) and obtain the following expression of formula (4):

$$f_i(p_i, x_i, L_i \underline{w}_i, r, k) = \varkappa(1-\eta) \left(1 - \frac{1}{P_i(t)} \sum_{j=1}^{n} a_{ij} p_j(t) + \frac{1-h_i}{1+r(t)}\right) \times (x_i(t))^{\alpha_i - 1} (L_i(t))^{\beta_i} - \frac{r(t) F_i^k(t)}{x_i(t) P_i(t)} - (1-\gamma) \frac{\underline{w}_i(t)}{x_i(t)} L_i(t), \tag{4}$$

The first approximation was represented relatively to $x(t)$ and equation (2) at fixed values $p_j(t) = p_{j0}$, $p_i(t) = p_{i0}, x_i(t) = x_{i0}$, $L_i(t) = L_{i0}$, $r(t) = r_0$, $F_i^k(t) = F_{i0}^k$, $\underline{w}_i(t) = w_{i0}$ as formula (5):

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\alpha}{T} x \left(1 - \frac{\gamma_1}{\alpha} x\right) \tag{5}$$

where in formulas (6) and (7):

$$T = T_{xi}, \quad x = x_i(t), \quad \alpha = \beta - \mu_i,$$
$$\beta = \varkappa_i (1-\eta) \left(1 - \frac{1}{p_{i0}} \sum_{j=1}^{n} a_{ij} p_{j0} + \frac{1-h_i}{1+r_0}\right) (x_{i0})^{\alpha_i - 1} (L_{i0})^{\beta_i} - \frac{r_0 F_{i0}^k}{x_{i0} P_{i0}} - (1-\gamma) \frac{w_{i0}}{x_{i0}} L_{i0} \tag{6}$$

$$\gamma_1 = (\alpha_i - 1) \varkappa(1-\eta) \left(1 - \frac{1}{p_{i0}} \sum_{i=1}^{n} a_{ij} p_{j0} + \frac{1-h_i}{1+r_0}\right) (L_{i0})^{\beta_i} (x_{i0})^{\alpha_i - 2} + \frac{1}{2} \frac{r_0 F_{i0}^k}{p_{i0} (x_{i0})^2} + (1-\gamma) \frac{\underline{w}_{i0}}{2 (x_{i0})^2} \tag{7}$$

The logistic equation (5) with $\Delta t = 1$ can be represented as a one-dimensional mapping in formula (8):

$$x_{n+1} = \left(1 + \frac{\alpha}{T}\right) x_n \left(1 - \frac{\gamma_1}{\alpha} x_n\right) \tag{8}$$

To analyse the generation scenario of short-term fluctuations and fluctuations we study the complex behaviour of the one-parameter mapping (8) by gradually increasing the parameter α/T in the range from $\alpha \le T$ to $\alpha > 3T$ and the changes in the dynamics of this point mapping will be monitored. The quadratic mapping $\varphi\colon : R \to R$, is considered in formula (9), where:

$$\begin{aligned}\varphi(x,\alpha,\gamma_1,T) &= \frac{T+\alpha}{T}\left(1-\frac{\gamma_1}{\alpha}x\right)\\ &= \frac{T+\alpha}{T}\frac{\gamma_1}{\alpha}x\left(\frac{\alpha}{\gamma_1}-1\right)\end{aligned} \tag{9}$$

where: $T > 0$, $\alpha > 0$, $\gamma_1 > 0$.

This quadratic mapping (9) depends on the α, γ_1, T parameters. The authors are interested in the behaviour functions $\varphi(x,\alpha,\gamma_1,T)$ on the $\left[0, \dfrac{\alpha}{\gamma_1}\right]$ interval. The diagrams of all these functions intersect the x-axis at the $x = 0$, $x = \dfrac{\alpha}{\gamma_1}$ points. The global maximum of the functions $y = \varphi(x,\alpha,\gamma_1,T)$ is reached at the point $x = \dfrac{\alpha}{2\gamma_1} : \varphi(x,\alpha,\gamma_1,T) = \dfrac{T+\alpha}{T}\dfrac{\alpha}{4\gamma_1}$. The stability of the fixed points of the mapping φ^k, $k \ge 1$, where $\varphi^k = \varphi\left(\varphi^{k-1}\right)$, $\varphi^0 = I$ is the identity mapping will be studied. Firstly, the case $k = 1$ should be considered. From the relation $x = \varphi(x,\alpha,\gamma_1,T)$ the next formula is (10):

$$x\left(1-\frac{T+\alpha}{T}\frac{\gamma_1}{\alpha}\left(\frac{\alpha}{\gamma_1}-x\right)\right) = 0 \tag{10}$$

where (11):

$$x_1 = 0, \qquad x_2 = \frac{\alpha}{T+\alpha}\frac{\alpha}{\gamma_1} \tag{11}$$

Thus, the points x_1, x_2 are the fixed points of the operator φ, hence the fixed points of the operator φ^k for all $k \ge 1$. Moreover, from $\varphi\left(\dfrac{\alpha}{\gamma_1}\right) = 0$ and $\varphi(0) = 0$ it follows that $\varphi\left(\dfrac{\alpha}{\gamma_1}\right) = 0$, $k \ge 1$. Since $\varphi(x,\alpha,\gamma_1,T) < 0$ at $k < 0$ and $x > \dfrac{\alpha}{\gamma_1}$. Therefore, for $x \ge \dfrac{\alpha}{\gamma_1}$ there cannot be the fixed point of the operator φ^k for all $k \ge 1$. The function φ on the interval $\left[0, \dfrac{\alpha}{2\gamma_1}\right]$ increases from zero to the maximum value $\dfrac{T+\alpha}{T}\dfrac{\alpha}{4\gamma_1}$ and on the interval $\left[\dfrac{\alpha}{2\gamma_1}, \dfrac{\alpha}{\gamma_1}\right]$ decreases from $\dfrac{T+\alpha}{T}\dfrac{\alpha}{4\gamma_1}$ to zero. Thus, at $\alpha \le 2T$ (formula (12)):

$$\begin{aligned}\left|\varphi^1(x,\alpha,\gamma_1,T)\right|_{x=0} &= \left|\frac{T+\alpha}{T}\right| > 1\\ \left|\varphi^1(x,\alpha,\gamma_1,T)\right|_{x=\frac{\alpha}{y_1}} &= \left|\frac{T-\alpha}{T}\right| < 1\end{aligned} \tag{12}$$

Therefore, the point $x_1 = 0$ is not stable, because $\left|\varphi^1(x,\alpha,\gamma_1,T)\right|_{x=0} > 1$, and the other fixed point x_2 will be stable (attractive), because $\left|\varphi^1(x,\alpha,\gamma_1,T)\right|_{x=\frac{\alpha}{y_1}} \le 1$. At $\alpha = 2T$ the point x_2 is still attractive though $\left|\varphi^1(x,\alpha,\gamma_1,T)\right|_{x=0} = 1$. It is considered when $\alpha > 2T$ the mapping (4) undergoes the bifurcation: the fixed point x_2 becomes unstable, because $\left|\varphi^1(x_2,\alpha,\gamma_1,T)\right| = \left|\dfrac{T-\alpha}{T}\right| > 1$ and instead of it the stable twofold cycle appears, defined by the relation in formula (13):

$$\begin{aligned}x = \varphi^2(x) &= \left(\frac{T+\alpha}{T}\right)^2 x\left(\frac{\gamma}{\alpha}\right)^2\left(\frac{\alpha}{\gamma}-x\right)\\ &\times\left(\frac{\alpha}{\gamma}-\frac{T+\alpha}{T}\frac{\gamma}{\alpha}x\left(\frac{\alpha}{\gamma}-x\right)\right)\end{aligned} \tag{13}$$

Since the fixed points φ are the fixed points for φ^2, therefore (14):

$$\begin{aligned}&\left(\frac{T+\alpha}{T}\right)^2\frac{\gamma^2}{\alpha^2}\left(\frac{\alpha}{\gamma}-x\right)\\ &\qquad\left(\frac{\alpha}{\gamma}-\frac{T+\alpha}{T}x\frac{T+\alpha}{T}\frac{\gamma}{\alpha}x^2\right)-1\\ &\qquad = d\left(x_2-x\right)\left(x^2+ax+b\right)\end{aligned} \tag{14}$$

From the polynomials equality of have (15):

$$\begin{aligned}d &= \frac{(T+\alpha)^3}{T}\frac{\gamma^3}{\alpha^3}\\ \alpha &= -\frac{2T+\alpha}{T+\alpha}\frac{\alpha}{\gamma}\\ b &= \frac{(2T+\alpha)T}{(T+\alpha)^2}\left(\frac{\alpha}{\gamma}\right)^2\end{aligned} \tag{15}$$

The new fixed points are the roots of the quadratic equation (16):

$$x^2+ax+b = 0 \tag{16}$$

The discriminant D (17) is calculated:

$$\begin{aligned}D &= \left(\frac{\alpha}{\gamma}\right)^2\left(\frac{2T+\alpha}{T+\alpha}\right)^2 - 4\frac{(2T+\alpha)T}{(T+\alpha)^2}\left(\frac{\alpha}{\gamma}\right)^2\\ &= \left(\frac{\alpha}{\gamma}\right)^2\frac{(2T+\alpha)(\alpha-2T)}{(T+\alpha)^2}\end{aligned} \tag{17}$$

Whence it follows that $D < 0$ at $\alpha < 2T$, therefore, at $T < \alpha < 2T$ the fixed points of the function $\varphi^2(x)$ are only the points $x_1 = 0$ and $x_2 = \frac{\alpha}{T+\alpha}\frac{\alpha}{\gamma}$ If $\alpha \geq 2T$, then $D \geq 0$ and (18):

$$x_{3,4} = \frac{\alpha}{2\gamma}\frac{2T+\alpha}{T+\alpha}\left(1 \mp \sqrt{\frac{\alpha - 2T}{\alpha + 2T}}\right) \quad (18)$$

Note that at $\alpha = 2T$ three fixed points coincide $x_2 = x_3 = x_4 = \frac{\alpha}{T+\alpha}\frac{\alpha}{\gamma}$. The function derivatives $\varphi^2(x)$ at these points $\varphi^2(x_3)$ and $\varphi^2(x_4)$ do not exceed unity in absolute value:

$$\begin{aligned} \left|\varphi^2(x_3)\right| = \Bigg| & \left(\frac{T+\alpha}{T}\right)^2 \left(\frac{\gamma}{\alpha}\right)^2 \left(\frac{\alpha^2}{\gamma^2}\right. \\ & - 2\frac{\alpha}{\gamma}x - 2\frac{T+\alpha}{T}\frac{\alpha}{\gamma}x + 6\frac{T+\alpha}{T}x^2 \\ & \left.\left. - 4\frac{T+\alpha}{T}\frac{\gamma}{\alpha}x^3\right)\right|_{x=x_3}\Bigg| \leq 1 \end{aligned} \quad (19)$$

and:

$$\begin{aligned} \left|\varphi^2(x_4)\right| = \Bigg| & \left(\frac{T+\alpha}{T}\right)^2 \left(\frac{\gamma}{\alpha}\right)^2 \left(\frac{\alpha^2}{\gamma^2}\right. \\ & - 2\frac{\alpha}{\gamma}x - 2\frac{T+\alpha}{T}\frac{\alpha}{\gamma}x + 6\frac{T+\alpha}{T}x^2 \\ & \left.\left. - 4\frac{T+\alpha}{T}\frac{\gamma}{\alpha}x^3\right)\right|_{x=x_3}\Bigg| \leq 1. \end{aligned} \quad (20)$$

This means that for the function $\varphi^2(x)$ they are attractors, therefore the corresponding second-order cycle of the function $\varphi(x)$ is an attractor. At $\alpha \cong 2T$ the cycle doubled, a second-order cycle emerged from the first-order cycle, and the attraction property passed to this new cycle, such values of the α parameter are called bifurcation points. What happens with a further increase in the parameter α? Here it is necessary to consider already three functions $\varphi(x)$, $\varphi^2(x)$ and $\varphi^4(x)$. The last of these functions is (in x) an 8th degree polynomial. Its fixed points up to the value of the parameter $\alpha \leq \sqrt{6T}$ are only four fixed points of the function $\varphi^2(x)$. At $\alpha \leq \sqrt{6T}$, the function derivatives $\varphi^2(x)$ at the points x_1, x_2, x_3 and become equal to 1, and with further growth of α pairs of the function fixed points $\varphi^4(x)$ appear near each of them. For the $\varphi^2(x)$ function, these points form two stable cycles of the second-order, and for the function $\varphi(x)$ they form a stable cycle of the fourth-order. At $\alpha > (2.54\ldots)T$ this cycle becomes unstable, and it is replaced by the stable cycle of the period 8, etc.

The mechanism of further development of events is similar: at a certain value of the parameter $(2.54\ldots)$ $T < \alpha < \alpha_\infty = (2.56699\ldots)T$ (another bifurcation point) a stable cycle of order 2^{n-1} of the function $\varphi(x)$ splits into two, which form the cycle of order 2^n, $(n = 1, 2, \ldots)$. In this case, the "old" cycle loses the attractor properties, transferring them to the new cycle, and α_n is finite. Moreover, it turns out that $\alpha_n < 3T$. Thus, the function $\varphi(x)$ has cycles of order 2^n at all natural n. The successive bifurcations of doubling periods of the attracting cycle of mapping (7) occur up to the value $\alpha = \alpha_\infty = T(2.5699\ldots)$, at which the attracting cycle reaches an infinitely long period, and cycles of periods 2^n, $n = 1, 2, \ldots$, will be repulsive. The quadratic mapping (7) in this case does not have cycles of other periods. At $\alpha_\infty < \alpha \leq 4T$ the mapping has cycles with any period, including aperiodic trajectories. Such trajectories during the successive iterations will wander in an irregular, chaotic manner inside a quadrilateral of length $\frac{\alpha}{\gamma}$ and width $\frac{T+\alpha}{T}\frac{\alpha}{4\gamma}$ Thus, at $\alpha < \alpha_\infty$ mapping (7) has a single stable cycle of period 2^n, which, apart from a set of measure zero, attracts all points from the segment $\left[0, \frac{\alpha}{\gamma}\right]$. When $\alpha > \alpha_\infty$, the dynamics of mapping (7) becomes more complex. In this case, there are aperiodic trajectories that are not attracted to cycles. The sequence of values α_n, at which period doubling bifurcations are observed, satisfies the simple law (21):

$$\frac{\alpha_n - \alpha_{n-1}}{\alpha_{n+1} - \alpha_n} = \delta = 4.6692\ldots \quad (21)$$

The number δ is the Feigenbaum universal constant. It shows that the sequence of doubling bifurcations is universal. From the above conducted analysis of the fixed asset development model, it follows that fixed assets and the economic system as a whole is developed without fluctuations as long as the conditions $\alpha \leq 2T$ are met. Under certain relations between the values of the growth rate of fixed assets $\alpha : \alpha_1 < \alpha_2 < \ldots < \alpha_n < \ldots$, and time constant T the interval $(2T < \alpha < \alpha_\infty = T * 2.5699\ldots)$ corresponds to the infinite sequence of bifurcations, each of which leads to cycles of a higher order with a period doubling at each successive bifurcation. The values are accumulated near some special value α_∞, after which orbits with an "infinite period" are obtained, that is, a strongly pronounced chaotic behaviour. Ultimately, the entire space of the dynamical system states, defined by the area of a quadrilateral of width $\frac{T+\alpha}{4T}\frac{\alpha}{\gamma}$ and length $\frac{\alpha}{\gamma}$ turns out to belong to a single chaotic attractor characterised by instability and sensitivity to initial conditions. All this ultimately explains the short-term fluctuations and fluctuations that occur in the economic system.

Conclusions

The conducted analysis of scientific works has shown insufficiently deep study of the influence of short-term fluctuations on the dynamic development of fixed assets. This work proposes consideration of the fixed assets lifecycle of the industry as an economic cycle of mid-duration, which made it possible to develop a nonlinear dynamic model of the development dynamics of fixed assets and their fluctuations. To explain the causes of short-term fluctuations and fluctuations in the work, the country's economy is considered as a nonlinear dynamic system. The economic fluctuation is analysed based on the simplest nonlinear dynamic model of the development of fixed assets. Despite its apparent simplicity, the dynamic model exhibits a wide range of behaviours, including simple equilibriums, multiple periodic patterns, and chaotic states. This variation depends on two key factors: the coefficient α, representing the true growth rate of fixed assets, and the time constant T, characterizing the economic system's dynamics.

The work analysis of the suggested dynamic model of the development of fixed assets showed that the economic system and fixed assets, in particular, are developed without hesitation until the true growth rate of fixed assets becomes less than or equal to the twofold time constant. The developed model made it possible to clarify the causes of short-term fluctuations and fluctuations occurring in the economic system. The relations between the values of the growth rate of fixed assets and the time constant are determined, leading to an infinite sequence of bifurcations, each of which excites cycles of a higher order with a period doubling with each successive bifurcation as a result of which the pronounced chaotic behaviour of the system is observed. The model suggested in the work will allow improving the existing approaches to managing the fixed assets lifecycle, taking into account the occurrence of short-term fluctuations that affect the dynamics of their development. Further studies development in this direction will make it possible to predict changes and fluctuations occurring in the economic system, which in turn will lead to an increase in the efficiency of using fixed assets in the industry.

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