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Research Paper

A Real-Time Key-Finding Algorithm Based on the Signature of Fifths

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The signature of fifths is a special kind of music representation technique enabling acquisition of musical knowledge. The technique is based on geometrical relationships existing between twelve polar vectors inscribed in the circle of fifths, which represent individual pitch-classes detected in a given composition. In this paper we introduce a real-time key-detection algorithm which utilizes the concept of the signature of fifths. We explain how to create the signature of fifths and how to derive its descriptors required by the algorithm, i.e., the main directed axis of the signature of fifths, the major/minor mode axis, the characteristic vector of the signature of fifths, the characteristic angle of the signature of fifths, and the angle of the major/minor mode. We performed a series of experiments to test the algorithm's effectiveness. The results were compared with those obtained using key-detection approaches based on key-profiles. All experiments were conducted using works composed by J.S. Bach, F. Chopin, and D. Shostakovich. The distinctive features of the presented algorithm, with respect to the considered key-detection approaches, are computational simplicity and stability of the decision-making process.

Keywords: music key-detection; tonality; music information retrieval; music classification.

1. Introduction

Tonality, or key-detection, algorithms utilize various techniques. The foundations of tonality analysis date back to the time of Pythagoras, who defined numerical relationships between consonant and dissonant sounds. Major input to the tonality analysis is also ascribed to Leonard Euler, who in his tone network, commonly referred to as Tonnetz, "tied together" the tones that make up the major and minor chords. The fifths and thirds (major and minor) intervals present in Tonnetz constitute the founding elements of the harmonic relationships among major and minor scales. Similar interval relationships are reflected in the Longuet-Higgins tonal maps (LONGUET-HIGGINS, 1962a; 1962b). Harmonic networks also constitute the basis for various 3D spiral array models (SHEPARD, 1982; CHEW, 2000; 2007). Such models allow us to associate individual pitch-classes present in a given music piece with particular locations on the spiral, enabling chord and key recognition (MAUCH *et al.*, 2010; OSMALSKYJ *et al.*, 2012; SIGTIA *et al.*, 2015; CHUAN, CHEW, 2005; 2007).

More sophisticated models for representing the content of musical works have recently been proposed (HARTE *et al.*, 2006; BERNARDES *et al.*, 2016; HERREMANS, CHEW, 2019; TYMOCZKO, 2006; 2011). They have been used, for example, to detect harmonic changes (HARTE *et al.*, 2006; BOULANGER-LEWANDOWSKI *et al.*, 2013; CHEN, SU, 2018; JACOBY *et al.*, 2015; HORI *et al.*, 2017; NI *et al.*, 2013; PEISZER *et al.*, 2008; WU, LI, 2018), represent chords geometrically for visualization (TYMOCZKO, 2006; 2011; CANCINO CHACÓN *et al.*, 2014; SAPP, 2001), and assess changes in distribution of pitch-classes present in compositions from different epochs (YUST, 2019). Such models have also been utilized in the algorithms for recognition of musical genres (Anglade *et al.*, 2010, Pérez-Sancho *et al.*, 2010) and have been applied to the evaluation of the chord structure (Bernardes *et al.*, 2016). Other applications of tonal models include generating structured music with constrained patterns, shaping the harmonic structure of musical pieces (Roig *et al.*, 2014), and assessment and creation of tension in composition fragments (Chapin *et al.*, 2010; Yang *et al.*, 2021). Naturally, such models also find utility in computer-aided composition software (Huang *et al.*, 2016; Sabathé *et al.*, 2017). In recent years, many tonal analysis solutions implementing artificial intelligence (Foscarin *et al.*, 2021; Dawson, 2018; Deng, Kwok, 2017) or machine learning (Korzeniowski, Widmer, 2017; Masada, Bunescu, 2017; McFee, Bello, 2017; Zhou, Lerch, 2015) techniques have been proposed.

There are many ways to represent the content of a musical work. One of the most popular representations is Euler's Tonnetz, which illustrates chord relationships of the harmonic triad in 2D space. The spiral array model of Chew (2000), which depicts chords in 3D space, is a more advanced approach. Other approaches introduce more dimensions, e.g., the tonal centroid 6D space of Harte *et al.* (2006), or the solution provided by Bernardes *et al.* (2016) which employs a space spanning as many as 12 dimensions. Increase in the dimensionality of the proposed models results from the constant quest for new ways to improve the accuracy of the analysis of musical works. However, improvement of computer analysis should not be sought only in the implementation of more and more complicated solutions. The signature of fifths is one example of a simple yet effective concept that can make a significant contribution to the development of algorithmic key-detection methods. Creation of the signature of fifths corresponding to a given musical piece (or its fragment) enables finding its key via analysis of the geometrical arrangement of the polar vectors comprising the signature. Details of this procedure are presented later in this paper.

The major-minor tonality is inherently associated with Western music. An important current research problem is algorithmic determination of a musical piece's key (Baumann, 2021; Bernardes *et al.*, 2017; Foscarin *et al.*, 2021; Korzeniowski, Widmer, 2018; Nápoles López *et al.*, 2019; 2020; Quinn, White, 2017; Toiviainen, Krumhansl, 2003). Input data used in key-finding algorithms is either in symbolic (e.g., MIDI, MusicXML) or audio (e.g., wav, mp3) format (Baumann, 2021; Chuan, Chew, 2005; Gebhardt *et al.*, 2018; Peeters, 2006; Papadopoulos, Peeters, 2012; Raphael, Stoddard, 2004; Weiss, 2013; Wu, Li, 2018). The computationally simplest key-detection approaches utilize the so-called key-profiles (Herremans, Chew, 2019; Korzeniowski, Widmer, 2017; Albrecht, Shanahan, 2013; Gomez, Herrera, 2004; Kania, Kania, 2019; Kania *et al.*, 2022; Krumhansl, Kessler, 1982; Krumhansl, 1990; Temperley, 2004; Temperley, Marvin, 2008). In the most basic scenario, determination of the key is based on searching for the maximum correlation coefficient of a given fragment of the analyzed composition with the 12 major and 12 minor key-profiles. It is still unclear which family of key-profiles and which fragment of a music piece are most appropriate for the analysis purposes. Sometimes the methods based on local-key estimation are also considered (Nápoles López *et al.*, 2020).

There are many families of key-profiles (Herremans, Chew, 2019; Korzeniowski, Widmer, 2017; Albrecht, Shanahan, 2013; Gomez, Herrera, 2004; Krumhansl, Kessler, 1982; Krumhansl, 1990; Temperley, 2004; Temperley, Marvin, 2008). They were created using a variety of methods, ranging from extensive experimental research based on cognitive psychology (Krumhansl, Kessler, 1982; Krumhansl, 1990) to computationally intensive statistical/probabilistic analyses (Aarden, 2003; Bellmann, 2005; Temperley, 2004) anchored in the theory of music. In some cases, creation of new key-profiles resulted from experiences with well-established key-profiles. For example, analysis of the Krumhansl–Kessler profiles inspired Temperley to propose a new family of key-profiles (Temperley, 2004; Temperley, Marvin, 2008). Temperley's proposal was backed with advanced models based on probabilistic reasoning. In some cases, key-profiles were created based on the analysis of numerous audio files (Chuan, Chew, 2014; Gomez, Herrera, 2004; Korzeniowski, Widmer, 2018). Particularly interesting are the key-profiles developed with the use of artificial intelligence techniques (Albrecht, Shanahan, 2013). Experiments have proven their high key-detection efficacy (Kania, Kania, 2019).

Although the correlational approach to music key-detection based on key-profiles has low computational complexity, it is possible to create simpler solutions. Reducing the number of multiplication operations is usually a good way to speed up an algorithm. In this respect, the key-detection algorithm based on the signature of fifths, presented in (Kania, Kania, 2019), is much simpler than its strictly correlational alternatives. The simplification results from the fact that the correlation coefficients are calculated only for two relative key-profiles, not all 24 of them. In (Kania *et al.*, 2021a), it was shown that the signature of fifths can also be utilized to determine the key signature of a given piece of music without calculating any correlation coefficient. The authors' proposed an algorithm that inspired the search for a simplified key-detection method and the study discussed in this paper. Computational simplification of the key-detection process is particularly important when it comes to solutions implemented in hardware, e.g., for instruments presenting musical notation in a real-time manner.

The aim of this paper is to present an original algorithm for real-time key determination based solely on the signature of fifths. The novelty of the proposed approach lies in the assessment of the structure of the signature of fifths. Essentially, the method boils down to the analysis of the geometrical relationships existing between the so-called characteristic vector of the signature of fifths and the pair of directed axes – the main directed axis of the signature of fifths and the major/minor mode axis. The algorithm takes a symbolic description of the piece as input. Therefore, its application is justified only in the context of equal-temperament tuning, i.e., when all keys are equivalent, so there are no better or worse-sounding keys, as in the case of mean-tone or non-temperament tuning. It is assumed that the analyzed pieces are tonal works, which is not always true, especially in contemporary music since the mid-19th century. The essence of the algorithm lies in searching for the most populous set of notes forming a pentatonic scale consisting of steps I, II, III, V, and VI in the major mode and I, III, IV, V, and VII in the Aeolian minor mode (both consist of any five neighboring keys in the circle of fifths). Additionally, the durations of pitch classes are analyzed, but no additional information is considered, such as the significance of individual notes in chords, which impacts listeners' perception.

In the next section of the article, the basic concepts behind the proposed key-detection algorithm are presented. In Sec. 3, the algorithm is thoroughly described. The ideas presented in Secs. 2 and 3 are supported with simple examples. Section 4 presents the results of the carried out experiments, along with a discussion focused on identifying the strengths and weaknesses of the proposed algorithm. The article ends with a summary of the conducted study.

2. Theoretical background

A musical piece can typically be represented with tones belonging to twelve pitch-classes: C, C♯/D♭, D, D♯/E♭, E, F, F♯/G♭, G, G♯/A♭, A, A♯/B♭, B. Let X be the set of durations of individual pitch-classes comprising a given fragment of a musical piece (1):

$$X = \{x_{\mathrm{C}}, x_{\mathrm{C}\sharp/\mathrm{D}\flat}, x_{\mathrm{D}}, x_{\mathrm{D}\sharp/\mathrm{E}\flat}, x_{\mathrm{E}}, x_{\mathrm{F}}, x_{\mathrm{F}\sharp/\mathrm{G}\flat}, x_{\mathrm{G}}, x_{\mathrm{G}\sharp/\mathrm{A}\flat}, x_{\mathrm{A}}, x_{\mathrm{A}\sharp/\mathrm{B}\flat}, x_{\mathrm{B}}\}. \tag{1}$$

The vector $\mathbf{K}$, which represents the normalized aggregate durations of pitch-classes corresponding to the analyzed fragment of music, is given by:

$$\mathbf{K} = [k_{\mathrm{A}}, k_{\mathrm{D}}, k_{\mathrm{G}}, k_{\mathrm{C}}, k_{\mathrm{F}}, k_{\mathrm{B}\flat}, k_{\mathrm{E}\flat}, k_{\mathrm{A}\flat}, k_{\mathrm{D}\flat}, k_{\mathrm{G}\flat/\mathrm{F}\sharp}, k_{\mathrm{B}}, k_{\mathrm{E}}], \tag{2}$$

where

$$k_i = \frac{x_i}{\max(x_{\mathrm{A}}, x_{\mathrm{D}}, x_{\mathrm{G}}, x_{\mathrm{C}}, x_{\mathrm{F}}, x_{\mathrm{B}\flat}, x_{\mathrm{E}\flat}, x_{\mathrm{A}\flat}, x_{\mathrm{D}\flat}, x_{\mathrm{G}\flat/\mathrm{F}\sharp}, x_{\mathrm{B}}, x_{\mathrm{E}})}, \tag{3}$$

and

$$i \in \{\mathrm{A}, \mathrm{D}, \mathrm{G}, \mathrm{C}, \mathrm{F}, \mathrm{B}\flat, \mathrm{E}\flat, \mathrm{A}\flat, \mathrm{D}\flat, \mathrm{G}\flat/\mathrm{F}\sharp, \mathrm{B}, \mathrm{E}\}.$$

The values comprising the vector $\mathbf{K}$ are ordered in accordance with the succession of the pitch-classes in the circle of fifths, beginning from the A tone and continuing counter-clockwise.

Definition 1 (Kania, Kania, 2019):

The signature of fifths is a set of twelve polar vectors $\{\mathbf{k}_i : i = \mathrm{A}, \mathrm{D}, \mathrm{G}, ..., \mathrm{E}\}$ whose coordinates (r_i, ϕ_i) fulfill the following conditions:

- the length of each vector is equal to the normalized corresponding component of a given pitch-class vector $\mathbf{K}$, i.e., $r_i = |\mathbf{k}_i| = k_i$;
- the direction of each vector is determined with the following relationship: $\phi_i = j \cdot 30°$, where $j = 0|_{i=A}$, $j = 1|_{i=D}$, and so on.

Example 1 (Kania, 2021):

Let us create the signature of fifths for the first bar of J.S. Bach's Prelude No. 1, BWV 846, whose musical notation was illustrated in Fig. 1.

Fig. 1. First bar of Bach's Prelude No. 1, BWV 846.

The vector $\mathbf{K}$ corresponding to the tones presented in Fig. 1 can be expressed as:

$$\mathbf{K} = [0 \;\; 0 \;\; 0.2 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0.9], \tag{4}$$

whereas the signature of fifths associated with the above vector is shown in Fig. 2.

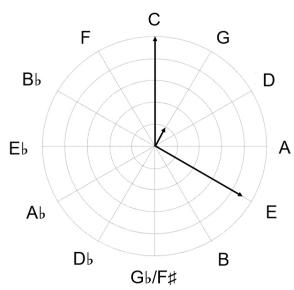


Fig. 2. Signature of fifths obtained for the fragment of the prelude shown in Fig. 1.

We can define a number of directed axes of the circle of fifths $Y \to Z$, which connect two opposite pitch-classes. A given axis points from Y towards Z, where pair $(Y;Z) \in$ (C, F♯); (F, B); (B♭, E); (E♭, A); (A♭, D); (D♭, G); (F♯, C); (B, F); (E, B♭); (A, E♭); (D, A♭); (G, D♭). The value $[Y \to Z]$ is called the characteristic value of the directed axis $Y \to Z$. It is equal to $\mathbf{K}_R - \mathbf{K}_L$, where $\mathbf{K}_R$ and $\mathbf{K}_L$ are the sums of the lengths of the vectors comprising the signature of fifths, located on the right and left sides of the directed axis $Y \to Z$, respectively.

Definition 2 (Kania, Kania, 2019):

The directed axis of the signature of fifths $Y \to Z$, for which $[Y \to Z]$ assumes the maximum value is called the main directed axis of the signature of fifths (MDASF).

Definition 3:

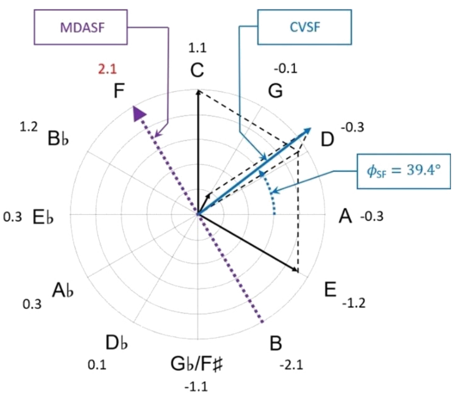
The polar vector obtained as the sum of vectors comprising the signature of fifths is called the characteristic vector of the signature of fifths (CVSF).

The position of the characteristic vector of the signature of fifths can be described with the Cartesian coordinates (x, y) depicting its end or by providing polar coordinates $(r_{\text{SF}}, \phi_{\text{SF}})$.

Definition 4:

The angle ϕ_{SF} is called the characteristic angle of the signature of fifths.

The MDASF, CVSF, and ϕ_{SF} corresponding to the signature of fifths presented in Fig. 2, are shown in Fig. 3. The outline of the plot illustrated in Fig. 3 was supplemented with the characteristic values associated with individual directed axes. The maximum value (2.1) indicates the direction of the MDASF, which is B → F in the considered case.



$[\text{B} \to \text{F}] = \text{K}_\text{R} - \text{K}_\text{L} = 1 + 0.9 + 0.2 = 2.1$

Fig. 3. Signature of fifths supplemented with the MDASF, CVSF, and ϕ_{SF}.

Let us assume that $Y \downarrow Z$ is the major/minor mode axis, which is perpendicular to the $Y \to Z$ axis. Its tip indicates the tone, which is 90° away in clockwise direction from the tone indicated by the $Y \to Z$ axis. Let us then depict the inclination of the major/minor mode axis $Y \downarrow Z$ with respect to the abscissa as the angle ϕ_1.

Definition 5:

The angle between the CVSF and the major/minor mode axis is called the major/minor mode angle, depicted with the symbol ϕ_m. It is calculated as $\phi_m = \phi_{\text{SF}} - \phi_1$.

Figure 4 helps to clarify the way in which the major/minor mode angle corresponding to the signature of fifths presented in Fig. 2 was obtained.

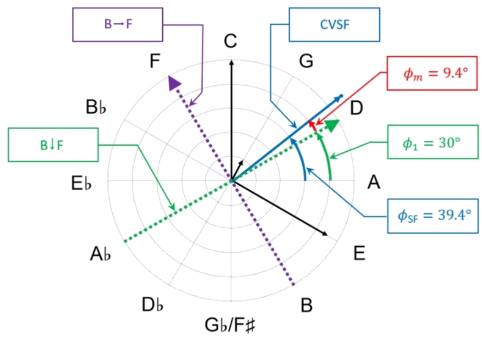


Fig. 4. Clarification of the way in which the angle of the major/minor mode ϕ_m is determined (for the signature of fifths presented in Fig. 2).

Determination of the key of a music composition becomes possible via computation of the major/minor mode angle ϕ_m, relating the direction of the major/minor mode axis and the direction of the characteristic vector of the signature of fifths.

3. Algorithm

In this section we present a real-time key-finding algorithm based on the signature of fifths. It is a simplified version of the algorithm shown by Kania and Kania (2019). It determines the major/minor mode of the analyzed piece of music via application of new descriptors of the signature of fifths, i.e., the major/minor mode axis, the characteristic angle of the signature of fifths, and the major/minor mode angle. The key-detecting procedure consists of several steps which lead to the calculation of the major/minor mode angle. In general, the value of this angle can be positive, negative, or zero. It is strictly associated with the key chosen from the obtained pair of two relative keys. A positive value indicates the major key whereas a negative value indicates the minor key. When the value of the major/minor mode angle is zero, i.e., the major/minor mode axis coincides with the characteristic vector of the signature of fifths, the analyzed fragment of the musical composition should be extended.

The proposed algorithm for the musical key-detection can be divided into the following steps:

1) Creation of the signature of fifths corresponding to the analyzed fragment of a music composition:
   - determination of the aggregate durations of individual pitch-classes within the considered fragment of music;
   - designation of the vector **K**, representing the normalized aggregate durations of individual pitch-classes.
2) Determination of the MDASF:
   - calculation of the characteristic values corresponding to all possible directed axes;
   - selection of the directed axis with the maximum characteristic value, i.e., MDASF;
   - if MDASF cannot be determined:
     a) increment the length of analyzed fragment;
     b) jump to the point no. 1.
3) Determination of the two relative keys associated with the obtained MDASF (one of which is the key of the analyzed fragment of music). The relative keys corresponding to a given MDASF are pointed by the MDASF rotated clockwise by 30°.
4) Determination of the major/minor mode axis and the angle it creates with the axis OX (ϕ_1). The major/minor mode axis is perpendicular to the MDASF, and its tip points to the tone which is 90° away, clockwise, from the tone indicated by the MDASF.
5) Determination of the CVSF and the angle it creates with the OX-axis (ϕ_{SF}).
6) Calculation of the angle of the major/minor mode (ϕ_m):
   - if $\phi_m = 0$:
     a) increment the length of the analyzed sample/fragment of music;
     b) jump to the point no. 1;
   - if $\phi_m > 0$, the analyzed piece of music is in the major key from the pair of the two previously obtained relative keys;
   - if $\phi_m < 0$, the analyzed piece of music is in the minor key from the pair of the two previously obtained relative keys.

The essence of the developed algorithm is illustrated resorting to example 2.

Example 2 (Kania, 2021):

Let us determine the keys of the first two preludes from the part I of J.S. Bach's collection "The Well-Tempered Clavier" [in German: Das Wohltemperierte Klavier], the first bars of which are shown in Fig. 5.

a)

b)

Fig. 5. First bars of the preludes from the part I of Bach's collection "The Well-Tempered Clavier": a) Prelude No. 1, written in C major, BWV 846; b) Prelude No. 2, written in C minor, BWV847.

In the first step of the algorithm, after determining the aggregate durations of individual pitch-classes and calculating the vector **K**, the signature of fifths is created (step no. 1). Knowing the lengths and directions of the vectors comprising the signature of fifths, it is possible to determine the MDASF (step no. 2). Figure 6 presents the signature of fifths and the MDASF for the analyzed excerpts of preludes.

Knowing the direction of the MDASF, we determined the pairs of the potential keys of the analyzed pieces of music as well as the directions of the major/minor mode axes (steps no. 3 and 4). The potential keys are shown in Figs. 6a–c (marked in red). They are pointed out by the MDASFs rotated by 30° clockwise (step no. 3). In each case, the major/minor mode axis is perpendicular to the MDASF and points towards the tone which is 90° away, clockwise, from the tone indicated by the MDASF (step no. 4). The directions of the major/minor mode axes and the angles they create with the OX-axis, marked as ϕ_1, were shown in Fig. 6b. In the case of the Prelude No. 1, the angle ϕ_1 is equal to 30°, whereas for Prelude No. 2 it reaches 120°.

In the next phase of the procedure, the CVSF and the angle it creates with the OX-axis, denoted as ϕ_{SF}, are calculated (step no. 5). In the case of the Prelude No. 1, this angle is equal to 39.4°, whereas for the Prelude No. 2 it is 103.9° (Fig. 6c).

Knowing the angles ϕ_1 and ϕ_{SF}, we determined the value of the mode angle ϕ_m (step no. 6), which was then used to select one key from the previously obtained pair of relative keys. In the case of the Prelude No. 1, the value of angle ϕ_m is greater than zero ($\phi_m = 9.4°$), hence the key is C major. For Prelude No. 2, ϕ_m is smaller than zero ($\phi_m = -16.1°$), hence the key is C minor. In Fig. 6d, the keys detected using the proposed algorithm are marked in red. For readability, major keys are written in uppercase, whereas minor keys are written in lowercase. We apply this convention throughout figures and tables in the article.

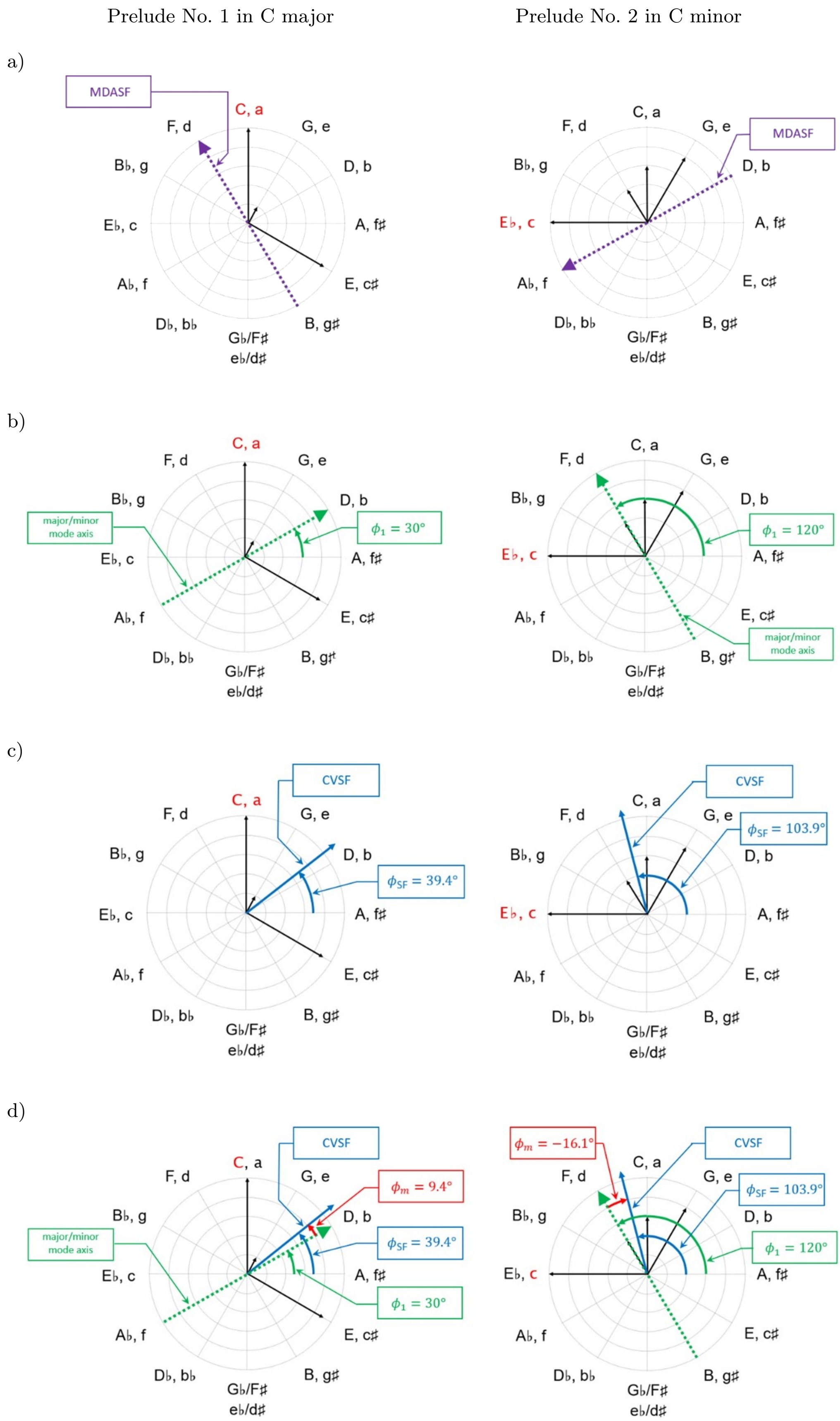


Fig. 6. Illustration of the successive steps of the proposed music key-detection algorithm based on Prelude No. 1 in C major and Prelude No. 2 in C minor, both from the part I of Bach's collection "The Well-Tempered Clavier."

4. Results and discussion

The aim of the conducted experiments was to compare the effectiveness of the developed algorithm with computationally simple key determination approaches based on correlation with well-known key-profiles. Four sets of preludes in 12 major and 12 minor keys were used in the experiments: the preludes by J.S. Bach from two collections of the "The Well-Tempered Clavier" – part I and part II (WTC I and WTC II), preludes by F. Chopin (Op. 28), and preludes by D. Shostakovich (Op. 87).

At first, we focused on determining the minimum number of notes needed for key-detection using the algorithm based on the signature of fifths. In each case the analysis was started with the minimum number of notes equal or greater than two. The analyzed fragment was extended by subsequent notes until it was possible to indicate the main directed axis of the signature of fifths (MDASF). All the constituent notes of any chords present were taken into account at once. For most of the analyzed works, the analysis process ended as soon as the main directed axis of the signature of fifths was determined for the first time. In a few cases, however, for which the major/minor mode axis coincided with the characteristic vector of the signature of fifths, the analyzed fragments were extended, and all the steps of the process reiterated.

The determined minimum numbers of notes needed to identify keys for different pieces of music are shown in Fig. 7. The vertical axis corresponds to the number of notes needed to detect the piece's key, while the horizontal axis represents the numerical identifiers of the analyzed preludes. Correctly detected keys are marked with blue diamonds, whereas incorrectly detected ones are marked with orange triangles.

The analysis of the results indicates that the effectiveness of the developed algorithm, understood as the ratio of the number of correctly detected keys to the total number of analyzed works, was 85.4 %. In the case of Bach's preludes, it was 89.6 %, whereas for Shostakovich's and Chopin's preludes the effectiveness reached 83.3 % and 79.2 %, respectively.

Figure 8 shows the effectiveness of the developed algorithm and the average number of notes needed to detect the key for each of the analyzed sets of works. Detection of the key was possible after analysis of 6.7 notes, on average (considering all collections). Excluding the significantly different result obtained for Shostakovich's 14th prelude (207 notes), the average number of notes for key-detection was 4.6 notes. The fewest notes were needed for Bach's works – 3.7 notes, on average. In the case of Chopin's compositions, the key was found after 4.4 notes, on average, and in the case of Shostakovich's works, 15 notes were required, on average. Again, after rejecting Shostakovich's Prelude No. 14, the average number of notes required to detect the key dropped to 7. It should be emphasized that the specificity of the musical notation often limits opportunities to use a small number of notes, as there are compositions which start with chords comprised of multiple notes. For exam-

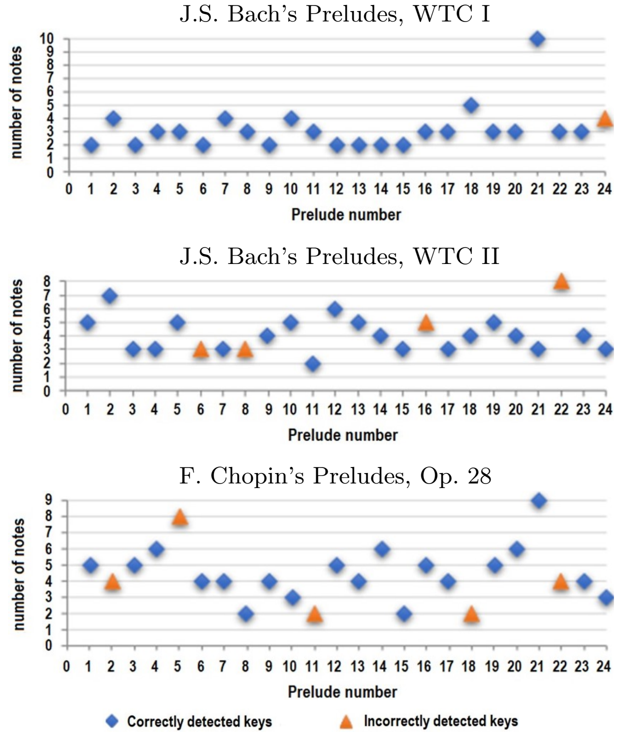


Fig. 7. Minimum number of notes required to detect the key using the proposed algorithm for individual preludes, where blue diamonds and orange triangles represent, respectively, the correctly and incorrectly detected keys (in the case of the Prelude No. 14 by Shostakovich the algorithm was able to correctly detect the key after analysis of 207 notes).

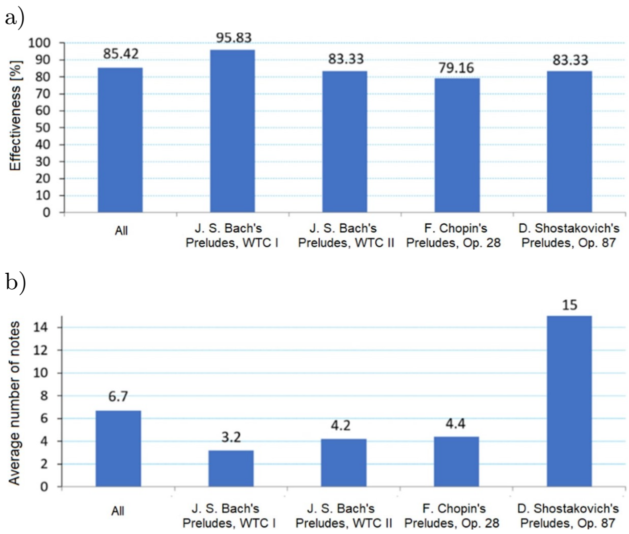


Fig. 8. Results of the music key-detection using the proposed algorithm based on the signature of fifths: a) effectiveness of the algorithm; b) average number of notes needed to detect the key.

ple, if a given piece of music starts with a five-note chord, it is not possible to perform the analysis for 2, 3 or 4 notes.

For further assessment, we compared the proposed key-detection algorithm with correlational approaches based on key-profiles. We chose to perform this comparison because key-profile methods in a certain way resemble the algorithm based on the signature of fifths. The way of determining the main directed axis of the signature of fifths can be associated with assigning the weight 1 to tones located on one side of the axis, and −1 to the tones located on its other side. This procedure resembles assignment of appropriate weights to particular tones, as in the considered key-profile approaches. As part of the assessment, we also accounted for the each method's computational complexity. The key-detection algorithm based on the signature of fifths is much simpler in this respect, as there is no need for calculating the correlation coefficients, hence no need for multiple (costly) multiplication operations. This is particularly important when it comes to hardware implementation of the key-detection process, e.g., in System on Chip (SoC) solutions.

In the conducted experiments, we used three sets of key-profiles: Krumhansl–Kessler (Krumhansl, Kessler, 1982; Krumhansl, 1990), Temperley (Temperley, 2004), and Albrecht–Shanahan (Albrecht, Shanahan, 2013). For each of the analyzed preludes the key was determined based on a short fragment taken from the beginning of a given composition. For the considered key-profile approaches, the key was detected after analysis of very short fragments of music, even ones comprised of just two notes. However, in many cases, extending the analyzed fragment of the piece resulted in the change of the previously detected key. For the algorithm based on the signature of fifths, the key was detected later, as typically more notes were needed to determine the MDASF. However, usually the determined key was stable and did not change with extension of the musical fragment.

Let us analyze in detail the results of the key-detection process for all the considered scenarios. We compared the minimum number of notes for which all key-detection approaches were able to indicate the key. The number of notes needed to determine the key was different for individual preludes but was always equal to the number of notes required by the algorithm based on the signature of fifths (due to the specificity of the performed comparison, as explained earlier in this article). Results for individual preludes are shown in Table 1. Correctly and incorrectly detected keys are illustrated for all considered key-detection approaches.

Table 1. Results obtained for different key-detection approaches, given the minimum number of notes for which all the considered methods were able to indicate the key.

Piece	Bach's Preludes, WTC I				Bach's Preludes, WTC II				Chopin's Preludes, Op. 28				Shostakovich's Preludes, Op. 87			
No.	SF	KK	T	AS	SF	KK	T	AS	SF	KK	T	AS	SF	KK	T	AS
1	✓	✓	✓	✓	✓	✓	✓	✓	✓	×	✓	✓	✓	✓	✓	✓
2	✓	✓	✓	✓	✓	×	✓	✓	×	×	×	×	✓	✓	✓	✓
3	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	×	×	✓
4	✓	✓	✓	✓	✓	✓	✓	✓	✓	×	×	✓	✓	✓	✓	✓
5	✓	✓	✓	✓	✓	✓	✓	✓	×	×	×	×	✓	✓	✓	✓
6	✓	✓	✓	✓	×	✓	✓	✓	✓	✓	✓	✓	×	×	×	✓
7	✓	✓	✓	✓	✓	×	×	✓	✓	×	✓	✓	✓	✓	×	✓
8	✓	✓	✓	✓	×	✓	✓	×	✓	✓	✓	✓	✓	✓	✓	✓
9	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
10	✓	✓	✓	✓	✓	✓	✓	✓	✓	×	×	✓	✓	✓	✓	✓
11	✓	✓	✓	✓	✓	×	×	✓	×	×	×	×	✓	✓	✓	×
12	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	×	✓	✓	✓
13	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
14	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	×	×	✓
15	✓	✓	✓	✓	✓	✓	✓	✓	✓	×	✓	×	×	✓	✓	×
16	✓	✓	✓	✓	×	✓	✓	×	✓	×	✓	✓	✓	✓	✓	✓
17	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
18	✓	✓	✓	✓	✓	✓	✓	✓	×	×	×	×	✓	✓	✓	✓
19	✓	✓	✓	✓	✓	✓	×	✓	✓	×	×	✓	✓	✓	✓	✓
20	✓	✓	✓	✓	✓	✓	✓	×	✓	✓	✓	✓	×	✓	✓	✓
21	✓	✓	✓	✓	✓	✓	✓	✓	✓	×	×	✓	✓	✓	✓	✓
22	✓	✓	✓	✓	×	✓	✓	✓	×	×	×	×	✓	✓	✓	✓
23	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
24	×	×	✓	✓	✓	✓	✓	✓	✓	×	×	✓	✓	✓	✓	✓

SF – the method using the signature of fifths, KK – the correlation approach based on the Krumhansl–Kessler key-profiles, T – the correlation approach based on the Temperley key-profiles, AS – the correlation approach based on the Albrecht–Shanahan key-profiles, ✓ – correctly detected key, × – incorrectly detected key.

Table 2. Effectiveness of determining the key in particular groups of preludes and using different key-detection approaches.

Collection	SF [%]	KK [%]	T [%]	AS [%]
Bach's Preludes, WTC I	95.83	95.83	100	100
Bach's Preludes, WTC II	87.50	87.50	91.67	87.50
Chopin's Preludes, Op. 28	83.33	45.83	62.50	83.33
Shostakovich's Preludes, Op. 87	83.33	87.50	87.50	95.83
All Preludes	85.42	79.17	85.42	91.67

SF – the method using the signature of fifths, KK – the correlation approach based on the Krumhansl–Kessler key-profiles, T – the correlation approach based on the Temperley key-profiles, AS – the correlation approach based on the Albrecht–Shanahan key-profiles.

Table 2 presents a synthetic summary of the results. It lists the effectiveness of each of the considered key-detection approaches in different sets of preludes. The results of the effectiveness obtained for the set of all preludes are also given (last row).

Analyzing the results illustrated in Tables 1 and 2, one can get the impression that, in terms of the effectiveness, the algorithm using the signature of fifths does not differ significantly from the correlational approaches implementing key-profiles (the same or greater effectiveness was achieved with Albrecht–Shanahan key-profiles). Moreover, the algorithm utilizing the signature of fifths in the majority of cases required a greater number of notes to indicate a piece's key. In some cases, it was the only method that indicated the wrong key, e.g., for Prelude No. 22 from Bach's "The Well-Tempered Clavier" – part II – or Prelude No. 20 by Shostakovich. However, this algorithm does exhibit some unique and advantageous properties.

In order to show the distinctive features of the algorithm based on the signature of fifths, let us first inspect the Prelude No. 14, Op. 87, by Shostakovich, whose fragment is presented in Fig. 9.

At the beginning of this Prelude, in its left-hand part, one can notice many repeating tones of B♭. It is also worth mentioning that up to the point marked with the index 5, where the note G♭ appears in the right-hand part, there are only three tones present in the composition (E♭, D♭, B♭). Determination of the MDASF (Fig. 10e) is not possible until reaching that point on the staff. Knowing the MDASF, one can determine the direction of the major/minor mode axis as well as the angle $\phi_m = -36.6°$ (Fig. 10f). The negative value of the angle ϕ_m indicates the minor key mode – in the considered case it is e♭ minor. The signatures of fifths corresponding to the increasingly longer fragments of the prelude (starting from its beginning to a given index) are presented in Fig. 10.

The proposed algorithm correctly identified the key after 207 notes. Table 3 presents the results of key-detection obtained for all the approaches considered, based on the analysis of fragments 0–1, 0–2, 0–3, 0–4, and 0–5.

Analyzing the results presented in Table 3, we can notice that the key-profile approaches need fewer notes to detect the key. Unfortunately, the initial indications are often incorrect and tend to vary as the

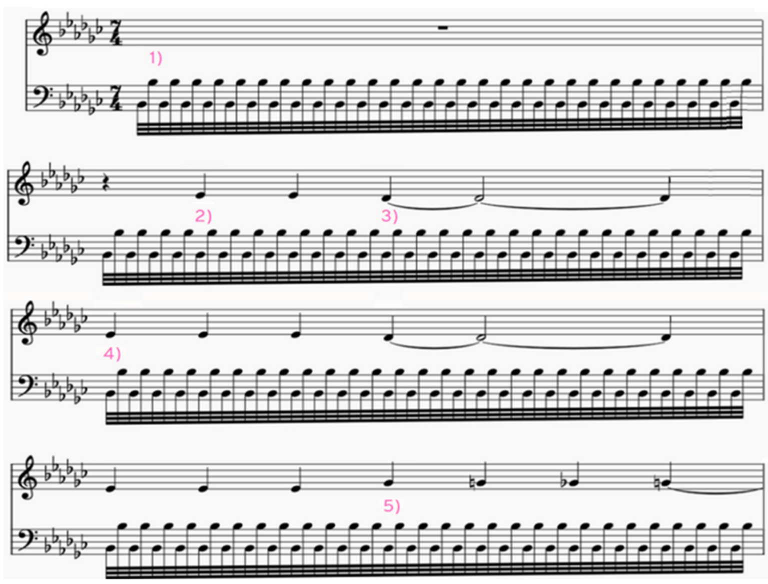


Fig. 9. Initial fragment of the Prelude No. 14, Op. 87, by Shostakovich.

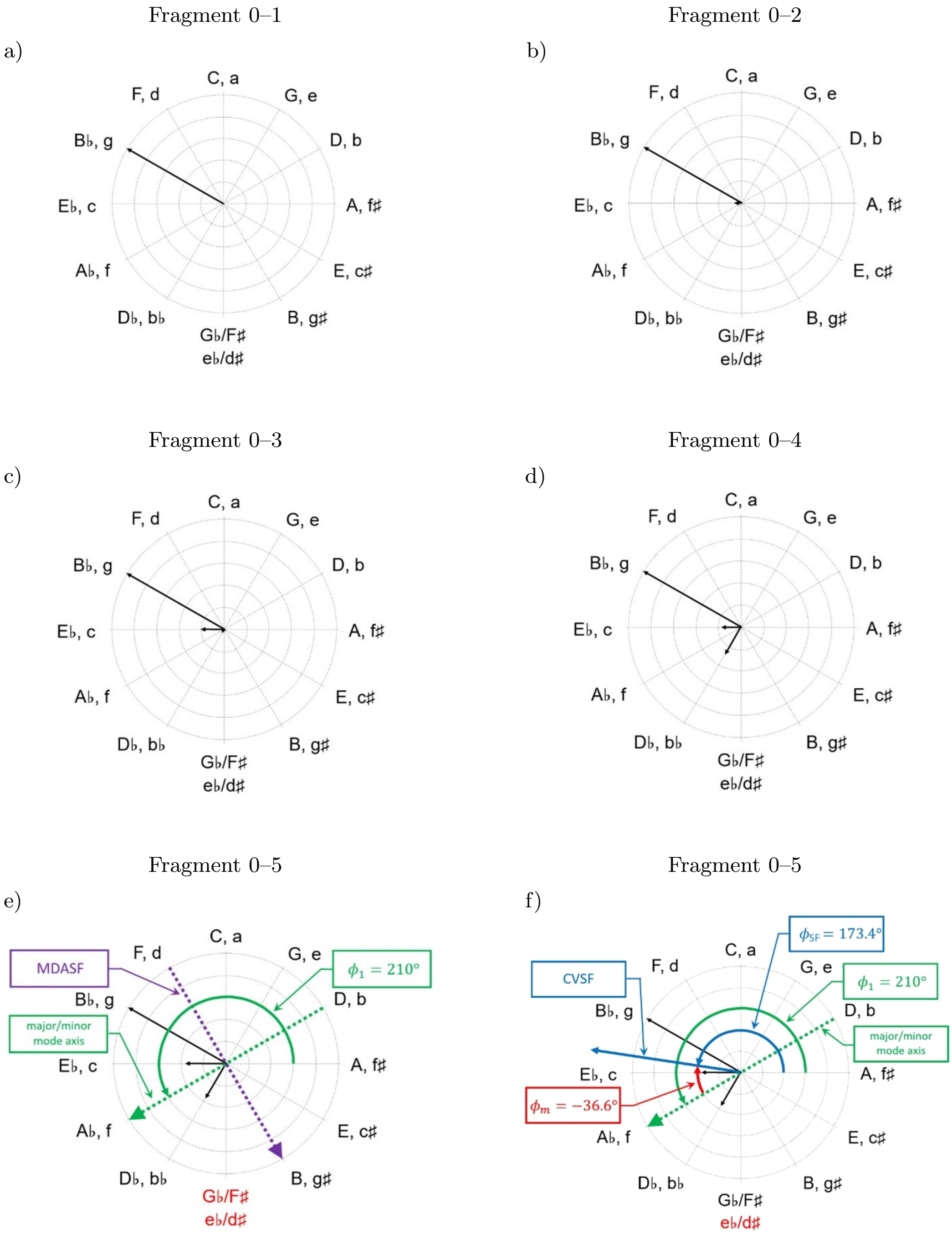


Fig. 10. Signatures of fifths corresponding to increasingly longer fragments of Prelude No. 14, Op. 87, by Shostakovich, shown in Fig. 9 (up to the point marked with index 5).

Table 3. Summary of the key-detection results obtained for the fragment of the prelude shown in Fig. 10, using the algorithm based on the signature of fifths (SF) as well as the considered approaches implementing the key-profiles of Krumhansl–Kessler (KK), Temperley (T), and Albrecht–Shanahan (AS).

Method	Analized fragment				
	0–1	0–2	0–3	0–4	0–5
SF	?	?	?	?	e♭
KK	B♭	B♭	b♭	b♭	B♭
T	b♭	b♭	b♭	b♭	b♭
AS	B♭	e♭	b♭	e♭	e♭

length of the analyzed music fragment increases. For the key-profiles of Krumhansl–Kessler and Temperley, the obtained keys were incorrect for all considered fragments – B♭ major (B♭) or B♭ minor (b♭). This result can be explained by the dominance of the sound B♭, which is the tonic of the indicated keys. The Albrecht–Shanahan key-profile approach detected various keys, among which was the correct one, i.e., E♭ minor (e♭). The algorithm using the signature of fifths needed more notes than the key-profile approaches. However, it indicated the key only when the MDASF was determined, and hence the detected key was usually correct.

Another distinctive feature of the key-detection algorithm proposed in this paper is the stability of the decision-making process, understood as low susceptibility to changes in the detected key as the length of the analyzed music fragment increases. This feature can easily be illustrated by the Prelude No. 21, Op. 28, by Chopin, the initial fragment of which is shown in Fig. 11.

Fig. 11. Initial fragment of the Prelude No. 21, Op. 28, by Chopin.

Table 4 presents the values of r_i, which represent the lengths of vectors making up the signatures of fifths calculated based on the aggregate durations of individual pitch-classes for a given number of notes (starting from the beginning of the prelude). The values from Table 4 were used to create the signatures of fifths shown in Fig. 12.

Table 4. Lengths of vectors representing the signatures of fifths corresponding to the Prelude No. 21, Op. 28, by Chopin, obtained for different numbers of initial notes.

Pitch-class	Number of notes							
	2	3	5	7	9	11	13	15
C						0.29	0.29	0.25
C♯/D♭								
D					0.17	0.14	0.29	0.38
D♯/E♭				0.2	0.17	0.14	0.14	0.13
E			0.25	0.2	0.17	0.14	0.14	0.13
F	1	1	1	1	1	1	1	1
F♯/G♭								
G			0.25	0.2	0.17	0.14	0.14	0.13
G♯/A♭								
A				0.2	0.17	0.14	0.14	0.13
A♯/B♭	1	0.33	0.25	0.2	0.33	0.29	0.43	0.38
B								

In Table 5, the results of the key-detection based on the signature of fifths were juxtaposed with the results

Table 5. Key-detection results obtained for the Prelude No. 21 by Chopin, Op. 28, using the algorithm based on the signature of fifths (SF) as well as the considered approaches implementing the key-profiles of Krumhansl–Kessler (KK), Temperley (T), and Albrecht–Shanahan (AS).

Method	Number of notes							
	2	3	5	7	9	11	13	15
SF	?	?	?	?	B♭	B♭	B♭	B♭
KK	F	F	F	F	F	F	d	d
T	f	f	f	F	F	F	B♭	B♭
AS	b♭	b♭	F	F	B♭	F	B♭	B♭

obtained for the correlational approaches utilizing key-profiles.

The results in Table 5 indicate that key-detection algorithm based on the signature of fifths offers greater stability. The key indicated after analysing the 9th note does not change as the length of the fragment increases further because the direction of MDASF does not change, whereas the sign of the angle ϕ_m experiences only insignificant variations (Figs. 12e–h). The keys found using the key-profile approaches changed at least once. This example clearly illustrates that the algorithm based on the signature of fifths requires more notes to determine the key, but once the key is detected the decision is usually correct and less prone to changes.

In summary, we can state the following:

– the proposed algorithm is computationally simple and easy to implement, as it does not require complex calculations;
– the key-detection algorithm based on the signature of fifths is competitive with the correlational approaches using key-profiles, especially if one wants to determine the key from a very short fragment of music;
– the algorithm using the signature of fifths usually needs a larger number of notes to determine the key than its key-profile alternatives, but once the key is detected the decision does not tend to change as the length of the analysed music fragment increases.

5. Conclusion

Development of multimedia systems is inextricably linked with methods enabling acquistion of musical knowledge. Currently, when almost all songs are only a few mouse clicks away, the problem for listeners is selection of music. Nowadays, many listeners use software applications which are able to suggest songs suitable for a given person. Such applications have become an integral part of the music industry.

Classification of music can be facilitated by various types of signal quantification and feature extraction techniques. Criteria of selection may include the

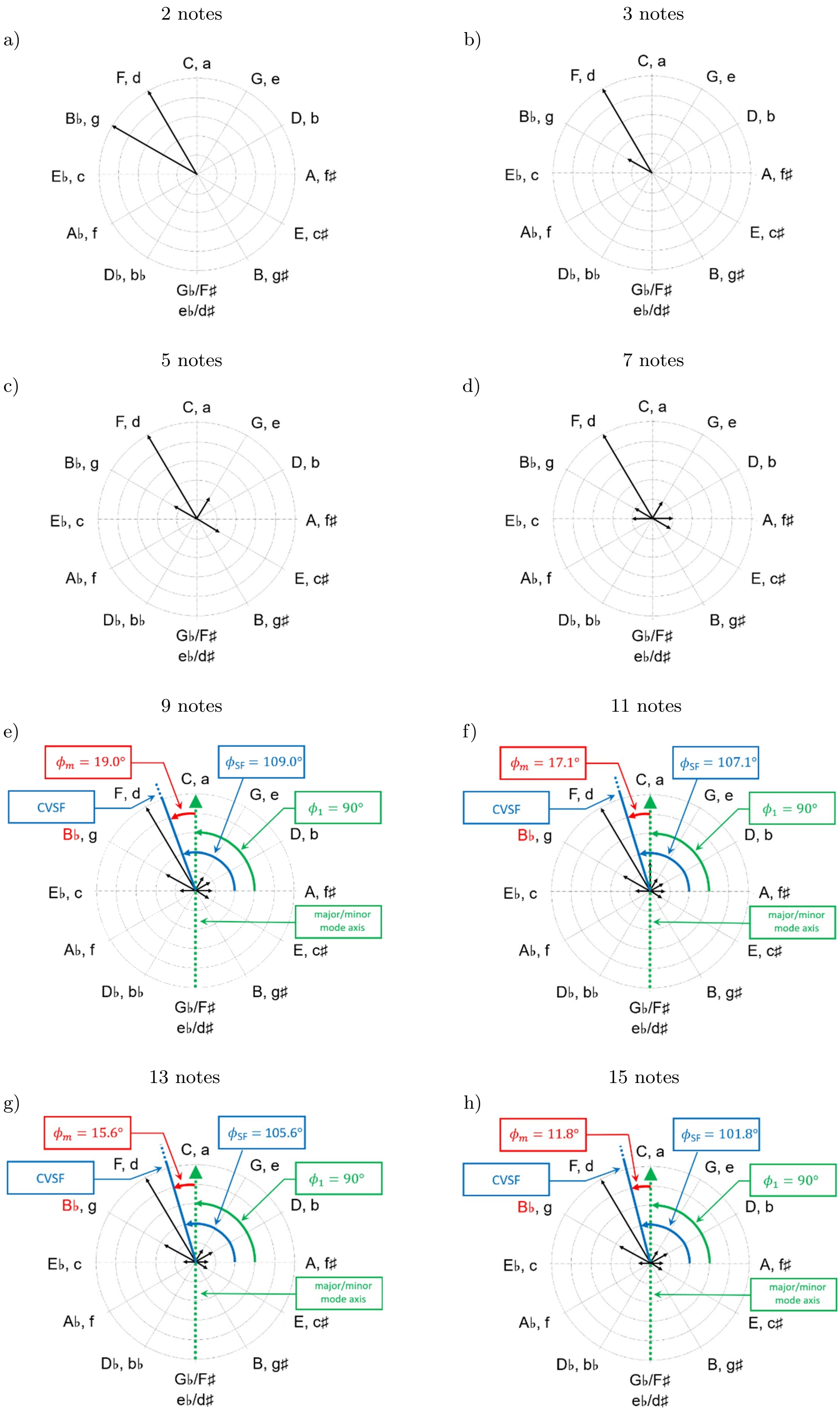


Fig. 12. Signatures of fifths obtained for different lengths of the analyzed fragments of the Prelude No. 21, Op. 28, by Chopin.

style, genre, character or key of a piece. The algorithm presented in this paper could be applied in such classification applications.

In this paper, we presented a novel algorithm enabling determination of the key of a musical piece based on the analysis of its signature of fifths. The algorithm is a simplified version of the method discussed by Kania and Kania (2019). The simplification boils down to the determination of the major/minor mode of the analyzed piece via application of new descriptors of the signature of fifths, i.e., the major/minor mode axis, the characteristic angle of the signature of fifths, and the major/minor mode angle. The effectiveness of the algorithm was tested on a collection of 96 preludes comprised of two sets of 24 preludes "The Well-Tempered Clavier", BWV 846-869, by Bach, 24 preludes Op. 28 by Chopin, and 24 preludes Op. 87 by Shostakovich. Each set of preludes, individually, covered all possible keys.

The main advantage of the proposed key-detection algorithm is the stability of its decision-making process, i.e., low sensitivity to changes of the indicated key as the length of the analyzed fragment of music is increased. This feature clearly distinguishes the method from the tested correlational key-detection approaches based on key-profiles. Another advantage is its conceptual as well as computational simplicity. The latter advantage facilitates the method's implementation in hardware, e.g., real-time presentation of musical notation on electronic displays. Calculations required to obtain the key signature with this method can be limited to execution of addition and comparison operations, which are convenient in terms of hardware implementation (only these two operations are needed to determine MDASF). Implementation of the proposed algorithm in a microprocessor system or SoC uses minimal resources, smaller than those required by the considered correlation-based approaches (using tonal profiles), in case of which many multiplication and division operations need to be performed.

The signature of fifths provides means for effective realization of the key-detection process. The effectiveness of the proposed algorithm, tested on the whole set of 96 preludes, was over 85 %. The correct key was detected after the analysis of 6.7 notes, on average.

The concept of the signature of fifths creates new opportunities in the area of music information retrieval. In addition to key determination, it has already been shown that the coefficients quantifying the variability of the signatures of fifths in time can be useful as feature coefficients in music classification processes (Kania *et al.*, 2021b; Łukaszewicz, Kania, 2022).

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