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ANALYSIS OF BIAS OF MODAL PARAMETER ESTIMATORS

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Summary

This paper presents an analysis based on a mathematical model of a bias in modal parameter estimators of a machine tool. The analytically determined amplitude-frequency characteristics were disturbed by random noise. The modal parameter estimation process was based on individual characteristics, followed by the determination of a bias in those parameters.

Keywords: modal analysis, estimation, bias of estimator

Analiza obciążeń estymatorów parametrów modalnych

Streszczenie

W pracy prowadzono analizę obciążenia estymatorów parametrów modalnych na przykładzie modelu matematycznego obrabiarki. Wyznaczone analitycznie charakterystyki amplitudowo-częstotliwościo-we zakłócono szumem losowym. Dokonano estymacji parametrów modalnych na podstawie poszczególnych charakterystyk.Określono stopień ich obciążenia.

Słowa kluczowe: analiza modalna, estymacja, obciążenie estymatora

1. Introduction

Experimentation is the primary source of knowledge about dynamic properties of different types of objects: from simple machine components to complex mechatronic systems [1,2] and large buildings [3,4]. Information obtained during measurements may be used in many ways.

The most common is the construction of a modal model. The studied modal model consists of estimated poles (frequency and modal damping) and vibration modes. Based on this information it is possible to identify dangerous frequency ranges in which resonance phenomena can occur.

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It is also possible to identify those system components, known as "weak element", that are responsible for excessive vibration. Another way of using the information obtained from experiments is in updating computational models [5] mainly finite element models (FEMs) [2,6,7] usually using complex dedicated computer systems. In the case of multi-dimensional responsible structures such as buildings, bridges, pipelines and turbines, this information is also used in the development of diagnostic procedures and facility monitoring [7-10]. In each case, modal model parameters should be determined as accurately as possible. Any inaccuracy could affect the quality of FE models or result in false alarms.

Even a rough look at this field of research leads to the conclusion that objects tested in terms of their dynamics are characterized by specific variability, commonly known as the "noise of an object" [11]. This variability directly affects modal parameter estimators which can be considered as random variables regardless of the estimation method.

There is also a separate problem connected with measurement techniques and the influences of the choice of signal processing on the results of experiments [12-14]. Some authors [15,16] attempted to assess uncertainty in the estimation of modal parameters by analyzing the results of operational modal models. Other authors [17-19] have discussed the problem of random error assessment, but do not analyze the problem of bias in estimations.

This paper presents the results of numerical investigations aimed at analyzing the impact of random noise on bias in modal parameters.

2. Quality of estimators

In general, the relationship between the frequency characteristics of a real object and the parameters **p** used in the identification process is non-linear. Assuming that systematic errors were removed during the determination of experimental characteristics, this relationship can be summarized as follows:

$$z_i(\omega_j) = y_i(\omega_j, \mathbf{p}) + \varepsilon_j \tag{1}$$

where:$z_i(\omega_j)$–experimental values of the determined i-th characteristic of the object,**p**–values of the i-th frequency characteristic of the model whose components are nonlinear functions of parameters **p**,ε_j–random error.

The most common estimation method is the nonlinear least squares method. Non-linear **p** parameter estimators using the least squares method are usually biased [20]. Let $\hat{\mathbf{p}}$ denote the vector of parameter **p**estimations. The bias error of $\hat{\boldsymbol{p}}$ estimations is defined as the difference between the real value of the **p** parameter and the expected value of estimator$E(\hat{\mathbf{p}})$:

$$b(\mathbf{p}) = \mathbf{p} - E(\hat{\mathbf{p}}) \tag{2}$$

The analytical determination of the bias error of non-linear estimators is difficult. Therefore, this error is usually not determined; instead, the lower limit of random error of estimators is determined, based on sensitivity analysis. [20] However, since the bias error of nonlinear estimations can be much greater than the random scatter, it is appropriate to estimate the level of this error on the basis of a numerical experiment. In this paper, the procedure of estimation of characteristics which enabled the determination of the mean and variance was repeated for each estimator. This in turn made it possible to assess whether a difference existed between the real value of the parameter and its estimators, and thus whether the estimator is biased.

3. Assessment of bias in parameter estimators

Calculations of estimators were performed for a simplified machine tool model, whose 3D view and its kinematic structure, are shown in Fig. 1. Analysis concerned the characteristics of a milling machine table in selected directions, as a result of well-developed resonances clearly seen in frequency characteristics.

a) b)

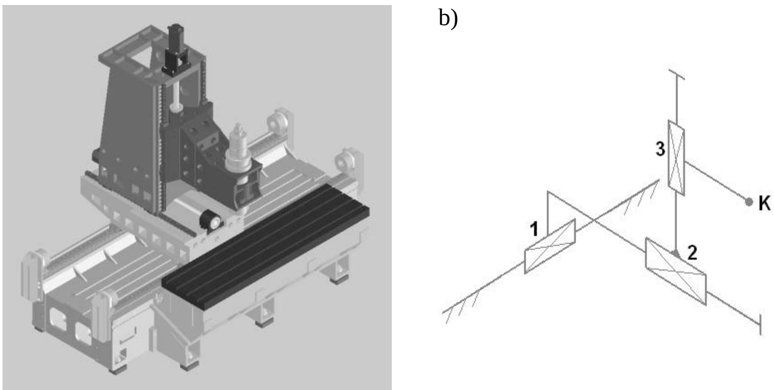

Fig. 1. Machine model: a) 3D model, b) kinematic diagram

With a model of the mass - damping – spring) system (**MDS**) written in the form of a matrix equation:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{f} \tag{3}$$

where the matrices$\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$are diagonal matrices of inertia, damping and stiffness, respectively; column vectors $\ddot{\mathbf{x}}$, $\dot{\mathbf{x}}$, $\mathbf{x}$ with dimensions (Nx1) are time-dependent vectors that describe the motion parameters of an object; and vector $\mathbf{f}$ with the dimension (Nx1) is a time-dependent vector of external force; then it is possible to transform the formula into a state space. Such a transformation allows finding a solution of equation (3) for systems with viscous damping, causing changes of their dimensionality. For the forced vibrations, those equations can be rewritten in the form (4):

$$\mathbf{A}\dot{\mathbf{u}}(\mathrm{t}) + \mathbf{B}\mathbf{u}(\mathrm{t}) = \begin{bmatrix} \mathbf{f}(\mathrm{t}) \\ \mathbf{0} \end{bmatrix} \tag{4}$$

Then using the transformation of coordinates, may be described by the equation:

$$\dot{\mathbf{u}}(\mathrm{t}) = \mathbf{\Psi}'\mathbf{q}(\mathbf{t}) \tag{5}$$

where:$\mathbf{\Psi}'$–complex modal matrix with the dimension 2Nx2N

The equation (4) can be expressed as:

$$\mathbf{A}\mathbf{\Psi}'\dot{\mathbf{q}}(\mathrm{t}) + \mathbf{B}\mathbf{\Psi}'\mathbf{q}(\mathbf{t}) = \begin{bmatrix} \mathbf{f}(\mathrm{t}) \\ \mathbf{0} \end{bmatrix} \tag{6}$$

Multiplying both sides of equation (6) by the matrix $\mathbf{\Psi}'$and using its orthogonality, equation (6) can be written as:

$$\mathrm{diag}(\mathbf{a}_\mathbf{r})\dot{\mathbf{q}}(\mathrm{t}) + \mathrm{diag}(\mathbf{b}_\mathbf{r})\mathbf{q}(\mathbf{t}) = (\mathbf{\Psi}')^\mathrm{T} \begin{bmatrix} \mathbf{f}(\mathrm{t}) \\ \mathbf{0} \end{bmatrix} \tag{7}$$

This equation shows 2N decoupled sets of equations, each of which can be written as:

$$\dot{\mathrm{q}}_\mathrm{r}(\mathrm{t}) - \mathrm{s}_\mathrm{r}\mathrm{q}_\mathrm{r}(\mathrm{t}) = \frac{1}{\mathrm{a}_\mathrm{r}} + (\mathbf{\Psi}')^\mathrm{T} \begin{bmatrix} \mathbf{f}(\mathrm{t}) \\ \mathbf{0} \end{bmatrix} \tag{8}$$

where:

$$\mathrm{s}_\mathrm{r} = -\omega_\mathrm{r}\xi_\mathrm{r} + \mathrm{i}\omega_\mathrm{r}\sqrt{1-\xi_\mathrm{r}{}^2} \tag{9}$$

Using further transformations, described in detail in the paper by [21] one can determine an equation for the vector of state variables:

$$\begin{bmatrix} \bar{\mathbf{X}} \\ \mathrm{i}\omega\bar{\mathbf{X}} \end{bmatrix} = \sum_{r=1}^{2N} \begin{bmatrix} \boldsymbol{\phi}_r \\ \boldsymbol{\phi}_r s_r \end{bmatrix} \left(\frac{1}{\mathrm{i}\omega - s_r} \right) \boldsymbol{\phi}_r{}^{T} \bar{\mathbf{F}} \tag{10}$$

and then, on this basis, the expression for amplitude:

$$\bar{\mathbf{X}} = \sum_{r=1}^{2N} \boldsymbol{\phi}_r \left(\frac{1}{\mathrm{i}\omega - s_r} \right) \boldsymbol{\phi}_r{}^{T} \bar{\mathbf{F}} \tag{11}$$

Receptance$\alpha_{jk}(\omega)$, defined as the displacement in coordinate j induced by excitation acting in the direction k, in the absence of excitations in other directions, is shown by the following relationship:

$$\alpha_{jk}(\omega) = \frac{\bar{\mathbf{X}}_j}{\bar{\mathbf{F}}_k} = \sum_{r=1}^{2N} \frac{\phi_{jr}\phi_{kr}}{\mathrm{i}\omega - s_r} \tag{12}$$

Because the eigenvalues and eigenvectors are complex values, equation (12) can be written as:

$$\alpha_{jk}(\omega) = \frac{\bar{\mathbf{X}}_j}{\bar{\mathbf{F}}_k} = \sum_{r=1}^{N} \left(\frac{\phi_{jr}\phi_{kr}}{\mathrm{i}\omega - s_r} + \frac{\phi_{jr}{}^{*}\phi_{kr}{}^{*}}{\mathrm{i}\omega - s_r{}^{*}} \right) \tag{13}$$

Introducing the determinations of modal residues, frequencies and modal damping, this equation can be transformed into a form used in this article as an approximation model:

$$\begin{aligned} \alpha_{jk}(\omega) = \sum_{r=1}^{N} \Bigg(& \frac{{}_{r}A_{jk}}{\omega_r \xi_r + \mathrm{i}\left(\omega - \omega_r \sqrt{1 - \xi_r{}^{2}} \right)} \\ & + \frac{{}_{r}A_{jk}{}^{*}}{\omega_r \xi_r + \mathrm{i}\left(\omega + \omega_r \sqrt{1 - \xi_r{}^{2}} \right)} \Bigg) \end{aligned} \tag{14}$$

It was assumed that bias assessment will concern amplitude of frequency characteristic. Solving the eigenvalue problem, the analytical values of modal parameters and the amplitude-frequency characteristics for different masses in 150 points in the frequency range from zero to 30 Hz, were determined – Fig. 2.

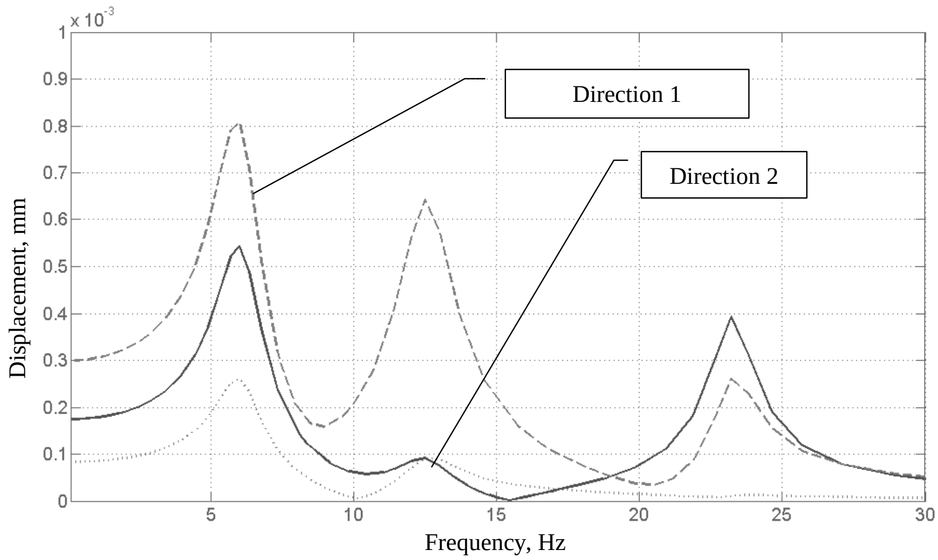

Fig. 2. Amplitude vs frequency characteristics of the system

It was assumed that assessment of estimator bias will be made on the basis of amplitude-frequency characteristics in three directions, obtained on the basis of time series, which are responses to initial conditions for individual variables. The analytical characteristics were interfered with by random noise with a normal distribution, and with an expected zero value and two values of the standard deviation (5% and 10%) of the amplitude. This result was based on a series of 20 characteristics for each of the masses in the estimation of modal parameters. Thus 20 values of each estimated modal parameter were obtained.Results of estimations of parameters $\hat{\mathbf{p}}$, standard deviations S_p, coefficients of variation $\hat{\vartheta}$ and the relative bias error (in %) of parameters compared to analytical values, are shown in Table 1 and Table 2.

Table 1. Results of parameter estimations for the characteristics in direction 1

Resonance frequency, Hz	**Parameter**	**Analytical values**	**Noise level, %**	**Mean $\hat{p}$**	**Standard deviation S_p**	$\hat{\vartheta}$	**Absolute error,%**
6.05	ξ_1	5,156	5	5,135	0,186	0,036	0,42
			10	4,884	0,500	0,097	5,3
	ω_1	37,965	5	37,927	0,148	0,004	0,10
			10	37,795	0,386	0,010	0,45
12.64	ξ_2	5,927	5	5,876	0,119	0,020	0,85
			10	5,686	0,225	0,379	4,07
	ω_2	78,726	5	78,660	0,187	0,002	0,08
			10	78,563	0,360	0,004	0,2
23,26	ξ_3	4,690	5	4,607	0,111	0,236	1,77
			10	4,469	0,306	0,651	4,71
	ω_3	145,869	5	145,838	0,118	0,001	0,02
			10	145,789	0,224	0,001	0,05

Table 2. Results of parameter estimations for the characteristics in direction 2

Resonance frequency, Hz	**Parameter**	**Analytical values**	**Noise level, %**	**Mean $\hat{p}$**	**Standard deviation S_p**	$\hat{\vartheta}$	**Absolute error, %**
6.05	ξ_1	5,156	5	5,105	0,196	0,038	1,00
			10	4,986	0,441	0,855	3,30
	ω_1	37,965	5	37,896	0,164	0,004	0,18
			10	37,709	0,285	0,007	0,68
12.64	ξ_2	5,927	5	5,765	0,147	0,025	2,73
			10	5,860	0,550	0,093	9,14
	ω_2	78,726	5	78,820	0,185	0,002	0,10
			10	79,204	0,511	0,006	0,60
23.26	ξ_3	4,690	5	4,605	0,139	0,029	1,81
			10	4,371	0,291	0,062	6,79
	ω_3	145,869	5	145,780	0,085	0,001	0,06
			10	145,647	0,202	0,001	0,15

4. Discussion of results

By analyzing the results of bias calculations for modal parameter estimations one can see clearly that they are not negligible, especially for modal damping coefficients. In extreme cases the relative bias error reaches 7%.

Additionally, bias errors are strongly dependent on the level of amplitude of individual resonances in a given characteristic and on the level of random errors. At a double rise in standard deviation of the random error, the bias error increases up to 12 times.

Hence it is necessary to analyze and estimate bias errors based on estimated parameters, for example in the manner proposed by this paper. Different values of bias errors in modal estimations for individual vibration modes determined using different frequency characteristics, indicate a need for a method of determination of global parameters that would reflect this fact.

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