THREE-POINT BENDING OF SEVEN LAYER BEAMS –
– THEORETICAL AND EXPERIMENTAL STUDIES

E. MAGNUCKA-BLANDZI¹, P. PACZOS², P. WASILEWICZ³, A. WYPYCH⁴

The subject of the analytical and experimental studies therein is of two metal seven-layer beam – plate bands. The first beam – plate band is composed of a lengthwise trapezoidally corrugated main core and two crosswise trapezoidally corrugated cores of faces. The second beam – plate band is composed of a crosswise trapezoidally corrugated main core and two lengthwise trapezoidally corrugated cores of faces. The hypotheses of deformation of a normal to the middle surface of the beams after bending are formulated. Equations of equilibrium are derived based on the theorem of minimum total potential energy. Three-point bending of the simply supported beams is theoretically and experimentally studied. The deflections of the two beams are determined with two methods, compared and presented.

Keywords: trapezoidally corrugated cores, orthotropic structures, layered plate-bands

1. INTRODUCTION


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2. **ANALYTICAL MODELLING OF SEVEN-LAYER BEAMS**

2.1. **THE FIRST BEAM – PLATE BAND**

The first beam – plate band (B-1) with a lengthwise trapezoidally corrugated main core and two crosswise trapezoidally corrugated cores of faces is shown in Fig. 1. The direction of the corrugation of the main core and the face cores are orthogonal. The beam measurements are as follows: \( L \) – length, \( b \) – width, \( t_{c1} \) – depth of the main core, \( t_{c2} \) – depth of facing cores, \( t_s \) – thicknesses of flat sheets.
Fig. 1. Scheme of the first beam – plate band (B-1)

The sizes of the trapezoidal corrugations of the main core and facing cores are shown in Fig. 2. The index $i=1$ refers to the main core, while the index $i=2$ refers to the face cores. The length of one pitch of the corrugation is $b_{0i}$, the base of the trapezoid is $b_{fi}$, and thickness of the corrugated sheet is $t_{0i}$.

Fig. 2. Scheme of trapezoidal corrugations of the main core ($i=1$) or face cores ($i=2$)

The analytical model of the beam is formulated with regard to the broken line hypothesis (Fig. 3). This hypothesis for multi-layer structures is described in detail by Magnucka-Blandzi et al [7] and Magnucki et al [8].
Displacements with consideration of this hypothesis are as follows:

- the upper sandwich facing

\[
(2.1) \quad u(x,z) = -t_{c1} \left[ \zeta \frac{dw}{dx} + \psi(x) \right], \text{ for } -\frac{1}{2} \leq \zeta \leq -\frac{1}{2},
\]

- the main corrugated core

\[
(2.2) \quad u(x,z) = -t_{c1} \zeta \left[ \frac{dw}{dx} - 2\psi(x) \right], \text{ for } -\frac{1}{2} \leq \zeta \leq \frac{1}{2},
\]

- the lower sandwich facing

\[
(2.3) \quad u(x,z) = -t_{c1} \left[ \zeta \frac{dw}{dx} + \psi(x) \right], \text{ for } \frac{1}{2} \leq \zeta \leq \frac{1}{2} + 2x_1 + x_2,
\]

where:

- \( x_1 = t_1/t_{c1}, \ x_2 = t_2/t_{c1} \) – dimensionless parameters,
- \( \zeta = z/t_{c1} \) – dimensionless coordinate,
- \( \psi(x) = u_1(x)/t_{c1} \) – dimensionless displacements,
- \( u_1(x) \) – displacements in the \( x \) direction and \( w(x) \) – deflection (Fig. 3).
Thus, the strains are as follows:

- the upper (the sign “+”)/lower (the sign “−”) sandwich facing

\[
\varepsilon_x = \frac{\partial u}{\partial x} = -t_{14} \left( \zeta \frac{d^2 w}{dx^2} \pm \frac{dw}{dx} \right), \quad \text{and} \quad \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{dw}{dx} = 0,
\]

- the main corrugated core

\[
\varepsilon_x = \frac{\partial u}{\partial x} = -t_{14} \zeta \left( \frac{d^2 w}{dx^2} - 2 \frac{dw}{dx} \right), \quad \text{and} \quad \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{dw}{dx} = 2\psi(x).
\]

The elastic strain energy of the beam – plate band (B-1) is a sum of the energy of particular layers

\[
U^{(B-1)}_e = U^{(i-i)}_e + U^{(i-2)}_e + U^{(i-o)}_e,
\]

where:

- the energy of the main corrugated core

\[
U^{(i-i)}_e = E_s b_t^3 \int_0^t \left[ \frac{1}{24} \bar{E}_e \left( \frac{d^2 w}{dx^2} \right)^2 - 4 \frac{d^2 w}{dx^2} \frac{d\psi}{dx} + 4 \left( \frac{d\psi}{dx} \right)^2 \right] + 2G^{(i-i)}_{xz} \left( \frac{\psi(x)}{t_{14}} \right)^2 \ dx,
\]

- the energy of the corrugated cores of facings

\[
U^{(i-2)}_e = E_s b_t^3 \int_0^t \left[ C_{ww}^{(i-2)} \left( \frac{d^2 w}{dx^2} \right)^2 - C_{u\psi}^{(i-2)} \frac{d^2 w}{dx^2} \frac{d\psi}{dx} + C_{\psi\psi}^{(i-2)} \left( \frac{d\psi}{dx} \right)^2 \right] \ dx,
\]

- the energy of the inner and outer sheets

\[
U^{(o)}_e = U^{(i-i)}_e + U^{(i-o)}_e = E_s b_t^3 \int_0^t \left[ C_{ww}^{(o)} \left( \frac{d^2 w}{dx^2} \right)^2 - C_{u\psi}^{(o)} \frac{d^2 w}{dx^2} \frac{d\psi}{dx} + 2x_i \left( \frac{d\psi}{dx} \right)^2 \right] \ dx.
\]

Thus, the elastic strain energy of the first seven-layer beam (2.6) is in the following form

\[
U^{(B-1)}_e = E_s b_t^3 \int_0^t \left[ \frac{1}{2} C_{ww}^{(B-1)} \left( \frac{d^2 w}{dx^2} \right)^2 - C_{u\psi}^{(B-1)} \frac{d^2 w}{dx^2} \frac{d\psi}{dx} + \frac{1}{2} C_{\psi\psi}^{(B-1)} \left( \frac{d\psi}{dx} \right)^2 \right] + 2G^{(B-1)}_{xz} \left( \frac{\psi(x)}{t_{14}} \right)^2 \ dx,
\]
where:
\[
C_{ww}^{(B-1)}, C_{wy}^{(B-1)}, C_{wy}^{(B-1)} \quad \text{dimensionless parameters of composite structures,}
\]
\[
G_{xz}^{(v_{-1})} \quad \text{dimensionless shear modulus of elasticity of the main corrugated core} \ [5],
\]
\[
E_s \quad \text{Young’s modulus.}
\]

The work of the load

\[
(2.11) \quad W = \int_0^l q w(x) dx,
\]
where:
\[
qu \quad \text{intensity of the transverse load.}
\]

The system of the equations of equilibrium – two differential equations derived based on the theorem
of minimum potential energy \( \delta(U_z^{(B-1)} - W) = 0 \), is in the following form

\[
(2.12) \quad C_{ww}^{(B-1)} \frac{d^2 W}{dx^2} - C_{wy}^{(B-1)} \frac{d \psi}{dx} = -\frac{M_s(x)}{E_b t_{c1}} \quad \text{and} \quad C_{wy}^{(B-1)} \frac{d^2 W}{dx^2} - C_{wy}^{(B-1)} \frac{d \psi}{dx} + 4C_{xz}^{(v_{-1})} \frac{\psi(x)}{t_{c1}} = 0.
\]

Three-point bending of the seven-layer beam – plate band (B-1) is shown in Fig. 4.

![Fig. 4. Scheme of the three-point bending of the first beam (B-1)](image)

The system of two differential equations (2.12) is reduced to one differential equation in the following form

\[
(2.13) \quad \frac{d^2 \psi}{dx^2} - \left(\frac{k}{t_{c1}}\right)^2 \psi(x) = -\frac{q}{E_b t_{c1}} \frac{Q(x)}{t_{c1}}
\]
where:
\[
k = 2 \sqrt{\frac{C_{ww}^{(B-1)}G_{xz}^{(v_{-1})}}{C_{wy}^{(B-1)} - \left(C_{wy}^{(B-1)}\right)^2}} \quad \text{and} \quad C_q = \frac{C_{ww}^{(B-1)}}{C_{wy}^{(B-1)} C_{wy}^{(B-1)} - \left(C_{wy}^{(B-1)}\right)^2} \quad \text{dimensionless parameters,}
\]
\[
Q(x) = \frac{dM_s}{dx} \quad \text{the shear force.}
\]
The general solution of the equation (2.13) is in the form

\[

\psi(x) = C_1 \sinh \left( \frac{x}{t_{c_1}} \right) + C_2 \cosh \left( \frac{x}{t_{c_1}} \right) + \psi_p(x)

\]

where:

\( C_1, C_2 \) – integration constants, \( \psi_p = \frac{C_{w_p}^{(\beta-1)}}{8C_{w_p}^{(\beta-1)}G_{xz}^{(\beta-1)}} \frac{F}{E_b t_{c_1}^3} \) – particular solution.

Taking into account the boundary conditions for the half beam \( \frac{d\psi}{dx} \bigg|_{x=0} = 0 \) and \( \psi \left( \frac{L}{2} \right) = 0 \) the integration constants \( C_1 = 0 \) and \( C_2 = -\cosh^{-1} \left( \frac{kL}{2t_{c_1}} \right) \psi_0 \) are determined. Then, the dimensionless displacement (2.14) is in the following form

\[

\psi(x) = \left[ 1 - \cosh \left( \frac{x}{t_{c_1}} \right) \cosh^{-1} \left( \frac{kL}{2t_{c_1}} \right) \right] \psi_p.

\]

Substituting this function (2.15) and the bending moment \( M_p(x) = Fx/2 \) for \( 0 \leq x \leq L/2 \) to the first equation (2.12), and taking into account the boundary conditions for the half beam \( w(0) = 0 \) and \( dw/dx \bigg|_{x=L/2} = 0 \), the maximum deflection is determined, i.e. the deflection in the middle span of the beam in the following form

\[

w_{\text{max}}^{(\beta-1)} = \frac{L}{2} \left[ 1 + 3 \left( 1 - \frac{2t_{c_1}}{kL} \tanh \left( \frac{kL}{2t_{c_1}} \right) \right) \left( \frac{C_{w_p}^{(\beta-1)}}{8C_{w_p}^{(\beta-1)}G_{xz}^{(\beta-1)}} \left( \frac{t_{c_1}}{L} \right)^2 \right) \right] \frac{F}{48C_{w_p}^{(\beta-1)}E_b \left( \frac{L}{t_{c_1}} \right)^3}.

\]

### 2.2. THE SECOND BEAM – PLATE BAND
The second beam – plate band (B-2) with the crosswise trapezoidally corrugated main core and two lengthwise trapezoidally corrugated cores of faces is shown in Fig. 5. The direction of corrugation of the main core and the face cores are orthogonal. The sizes of the beam are analogous to the first beam.

![Fig. 5. Scheme of the second beam – plate band (B-2)](image)

The analytical model of the beam is formulated with regard to the broken line hypothesis (Fig. 6). Displacements with consideration of this hypothesis are as follows:

- the upper sheet

\[
v(y,z) = -t_{\zeta} \left[ \xi \frac{d\nu}{dy} + x_2 \phi(y) \right], \text{ for } -\left( \frac{1}{2} + 2x_1 + x_2 \right) \leq \xi \leq -\left( \frac{1}{2} + x_1 + x_2 \right),
\]

(2.17)

- the upper core of the face

\[
v(y,z) = -t_{\zeta} \left[ \xi \frac{d\nu}{dy} \right. \left. - \left[ \xi + \left( \frac{1}{2} + x_1 \right) \right] \phi(y) \right], \text{ for } -\left( \frac{1}{2} + x_1 + x_2 \right) \leq \xi \leq -\left( \frac{1}{2} + x_1 \right),
\]

(2.18)

- the main core with two inner sheets

\[
v(y,z) = -t_{\zeta} \frac{d\nu}{dy}, \text{ for } -\frac{1}{2} + x_1 \leq \xi \leq \frac{1}{2} + x_1,
\]

(2.19)
• the lower core of the face

$$v(y, z) = -t_{c1} \left\{ \zeta \frac{dw}{dy} \left[ \zeta - \left( \frac{1}{2} + x_1 \right) \right] \phi(y) \right\}, \text{ for } \frac{1}{2} + x_1 \leq \zeta \leq \frac{1}{2} + x_1 + x_2,$$

• the lower sheet

$$v(y, z) = -t_{c1} \left[ \zeta \frac{dw}{dy} - x_2 \phi(y) \right], \text{ for } \frac{1}{2} + x_1 + x_2 \leq \zeta \leq \frac{1}{2} + 2x_1 + x_2,$$

where:

$$\phi(x) = v_1(y)/t_{c2} \text{ – dimensionless displacements, } v_1(y) \text{ – displacements in the } y \text{ direction and } w(y) \text{ – deflection (Fig. 6).}$$

Fig. 6. Scheme of the deformation of a plane cross-section of the seven-layer beam (B-2)

Thus, the strains are as follows:
• the upper (the sign “+”) / lower (the sign “−”) sheets

\[ \varepsilon_y = \frac{\partial v}{\partial y} = -t_{c1} \left[ \zeta \frac{d^2 w}{dy^2} \pm x_2 \frac{d\phi}{dy} \right], \quad \text{and} \quad \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{d\psi}{dy} = 0, \]

• the upper (the sign “+”) / lower (the sign “−”) core of the face

\[ \varepsilon_y = \frac{\partial v}{\partial y} = -t_{c1} \left[ \zeta \frac{d^2 w}{dy^2} \pm \left( \frac{1}{2} + x_1 \right) \right] \frac{d\phi}{dy}, \quad \text{and} \quad \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{d\psi}{dy} = \phi(y), \]

• the main core with two inner sheets

\[ \varepsilon_y = \frac{\partial v}{\partial y} = -t_{c1} \zeta \frac{d^2 w}{dy^2}, \quad \text{and} \quad \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{d\psi}{dy} = 0. \]

The elastic strain energy of the beam–plate band \((B-2)\) is a sum of the energy of particular layers, the same as for the first beam \((2.6)\), therefore, it takes the following form

\[ (2.25) U_e^{(B-2)} = E_c a t_{c1} \int_0^L \left[ \frac{1}{2} C_{w\psi}^{(B-2)} \left( \frac{d^2 w}{dy^2} \right)^2 - C_{\psi w}^{(B-2)} \frac{d^2 w}{dy^2} \frac{d\phi}{dy} + C_{\psi\psi}^{(B-2)} \frac{d\psi}{dy} \right] + x_2 \tilde{G}_{yz}^{(\text{c-2})} \left( \frac{\psi(x)}{t_{c1}} \right)^2 \right] dx, \]

where:

\( C_{w\psi}^{(B-2)}, C_{\psi w}^{(B-2)}, C_{\psi\psi}^{(B-2)} \) – dimensionless parameters of composite structures, \( \tilde{G}_{yz}^{(\text{c-2})} \) – dimensionless shear modulus of elasticity of the main corrugated core [5].

The system of the equations of equilibrium – two differential equations based on the theorem of minimum potential energy \( \delta (U_e^{(B-2)} - W) = 0 \), are in the following forms

\[ (2.26) C_{w\psi}^{(B-2)} \frac{d^2 w}{dy^2} - C_{\psi w}^{(B-2)} \frac{d\phi}{dy} = -\frac{M_e(y)}{E_c a t_{c1}} \quad \text{and} \quad C_{w\psi}^{(B-2)} \frac{d^2 w}{dy^2} - 2C_{\psi w}^{(B-2)} \frac{d^2 \phi}{dy^2} + 2x_2 \tilde{G}_{yz}^{(\text{c-2})} \frac{\psi(y)}{t_{c1}} = 0. \]
This system of the equations of equilibrium is analogically solved as for the first beam. Thus, for three-point bending of the seven-layer beam – plate band (B-2) shown in Fig. 7, the deflection in the middle span of the beam is in the following form

\[ w_{\text{max}}^{(B-2)} = \frac{L}{2} \left\{ 1 + \frac{1}{kL} \tanh \left( \frac{kL}{2t_{t1}} \right) \left[ \frac{C_{wph}^{(B-2)}}{C_{ww}^{(B-2)}x_zG_{zz}^{(B-2)}} \left( \frac{t_{t1}}{L} \right)^2 \right] \right\}^{\frac{1}{2}} \frac{F}{48C_{ww}^{(B-2)}E_a \left( \frac{L}{t_{t1}} \right)^3} \]

where:

\[ k = \sqrt{\frac{2C_{ww}^{(B-2)}x_zG_{zz}^{(B-2)}}{2C_{ww}^{(B-2)}C_{ww}^{(B-2)} - C_{wph}^{(B-2)}}} \] is dimensionless parameter.

Fig. 7. Scheme of the three-point bending of the second beam (B-2)

3. EXPERIMENTAL TESTS OF THREE-POINT BENDING OF SEVEN-LAYER BEAMS

3.1. THE FIRST BEAM – PLATE BAND

The view of the first steel beam (B-1) located in the ZWICK test machine is shown in Fig. 8.
The scheme of the support of this beam in the test machine is shown in Fig. 9.

![Fig. 9. The scheme of the support of the first beam (B-1) in the test machine](image)

The sizes and Young’s modulus of the first steel beam (B-1) are as follows: \( L = 700 \text{ mm} \), \( b = 120 \text{ mm} \), \( t_s = 0.6 \text{ mm} \), \( t_{c1} = t_{c2} = 11.0 \text{ mm} \), \( b_{01} = b_{02} = 40.0 \text{ mm} \), \( b_{f1} = b_{f2} = 10.0 \text{ mm} \), \( t_{01} = t_{02} = 0.6 \text{ mm} \), \( E = 2 \times 10^5 \text{ MPa} \). Thus, the analytical linear dependence (2.16) between the deflection and the load-force for the three-point bending of this beam is in the form:

\[
\frac{w_{\text{max}}^{(B-1)}}{k_{B-1}^{\text{(analyt)}}} \quad \text{where the stiffness of the beam is}
\]

\[
k_{B-1}^{\text{(analyt)}} = 1.862 \frac{\text{kN}}{\text{mm}}
\]

Hence, the load-force:

\[
F^{(\text{analyt})} = w_{\text{max}}^{(B-1)} \cdot k_{B-1}^{\text{(analyt)}}
\]

The experimental dependence (the solid line) between the load-force and the deflection of the three-point bending of this beam is shown in Fig. 10. Additionally, the analytical linear dependence (the broken line) for this beam is presented with a view to compare both methods.
The analytical solution for the first beam in the linear dependence form (the broken line) approaches the experimental curve (the solid line).

Example values of the analytical and experimental dependences \( F(w_{\text{max}}) \) are specified in the Table 1.

<table>
<thead>
<tr>
<th>( w_{\text{max}} ) [mm]</th>
<th>0.25</th>
<th>0.50</th>
<th>0.75</th>
<th>1.00</th>
<th>1.152</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F^{(\text{Exp})} ) [kN]</td>
<td>0.496</td>
<td>0.963</td>
<td>1.425</td>
<td>1.875</td>
<td>2.145</td>
</tr>
<tr>
<td>( F^{(\text{Analyt})} ) [kN]</td>
<td>0.466</td>
<td>0.931</td>
<td>1.397</td>
<td>1.862</td>
<td>2.145</td>
</tr>
</tbody>
</table>

The relative difference between the analytical \( F^{(\text{Analyt})} \) and experimental \( F^{(\text{Exp})} \) values of load-forces is below 5%.

3.2. THE SECOND BEAM – PLATE BAND

The view of the second steel beam (B-2) located in the ZWICK test machine is shown in Fig. 11.
The scheme of the support of this beam in the test machine is shown in Fig. 12.

The sizes and Young’s modulus of the second steel beam (B-2) are as follows: \( L = 660 \text{ mm} \), \( a = 120 \text{ mm} \), \( t_s = 0.6 \text{ mm} \), \( t_{c1} = t_{c2} = 11.0 \text{ mm} \), \( b_{01} = b_{02} = 40.0 \text{ mm} \), \( b_{f1} = b_{f2} = 10.0 \text{ mm} \), \( t_{01} = t_{02} = 0.6 \text{ mm} \), \( E = 2 \times 10^5 \text{ MPa} \). Thus, the analytical linear dependence (2.27) between the deflection and the load-force of the three-point bending of this beam is in the form

\[
\frac{w_{\text{max}}^{(B-2)}}{k_{B-2}^{(\text{analyt})}} = F^{(\text{analyt})},
\]

where the stiffness of the beam \( k_{B-2}^{(\text{analyt})} = 1.303 \frac{\text{KMN}}{\text{mm}} \). Hence, the load-force

\[
F^{(\text{analyt})} = w_{\text{max}}^{(B-2)} k_{B-2}^{(\text{analyt})}.
\]

Example values of the analytical and experimental dependences \( F(w_{\text{max}}) \) are specified in the Table 2.

<table>
<thead>
<tr>
<th>( w_{\text{max}} ) [mm]</th>
<th>1.00</th>
<th>1.25</th>
<th>1.50</th>
<th>1.75</th>
<th>2.00</th>
<th>2.421</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F^{(\text{Exp})} ) [kN]</td>
<td>1.437</td>
<td>1.779</td>
<td>2.105</td>
<td>2.412</td>
<td>2.715</td>
<td>3.154</td>
</tr>
<tr>
<td>( F^{(\text{Analyt})} ) [kN]</td>
<td>1.303</td>
<td>1.629</td>
<td>1.955</td>
<td>2.280</td>
<td>2.606</td>
<td>3.154</td>
</tr>
</tbody>
</table>

The relative difference between the analytical \( F^{(\text{Analyt})} \) and experimental \( F^{(\text{Exp})} \) values of load-forces for the second beam is below 10% and it exceeds the value for the first beam.
The experimental dependence (the solid line) between the load-force and the deflection of the three-point bending of this beam is shown in Fig. 13. Additionally, the analytical linear dependence (the broken line) for this beam is presented with a view to compare both methods.

![Experimental and Analytical Force-Deflection Curves](image)

Fig. 13. The experimental and analytical force-deflection curves of the three-point bending (B-2)

4. CONCLUSIONS

The analytical and experimental studies of the three-point bending of seven-layer beam – plate bands led to the following statements:

- the formulated analytical models for both beams (B-1) and (B-2) based on the assumed hypothesis for deformation of the plane cross-sections (Fig. 3 and Fig. 6) is positively verified by the experiments,
- the systems of the equations of equilibrium (2.12) and (2.26) refer to the general bending of seven-layer beam – plate bands, and also the buckling problems,
- analytically calculated stiffness of these two beams provide lower estimations as compared to experimental results (Fig. 10 and Fig. 13),
- the elaborated analytical models for seven-layer beams may be applied to analytical modelling of thin-walled seven-layer structures.

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Rys. 12. Schemat podparcia belki drugiej (B-2) w maszynie badawczej

Fig. 13. The experimental and analytical force-deflection curves of the three-point bending (B-2)
Rys. 13. Doświadczalna i analityczna zależność siła-ugięcie dla trzy-punktowego zginania (B-2)

Tab. 1. Example values of deflections and load-forces (Fig. 10)
Tab. 1. Przykładowe wartości ugięć i obciążeń-siły (Rys. 10)

Tab. 2. Example values of deflections and load-forces (Fig. 13)
Tab. 2. Przykładowe wartości ugięć i obciążeń-siły (Rys. 13)
TRZY-PUNKTOWE ZGINANIE BELEK SIEDMIO-WARSTWOWYCH – BADANIA TEORETYCZNE
I DOŚWIADCZALNE

Słowa kluczowe: rdzenie pofałdowane trapezowo, konstrukcje ortotropowe, pasma płytyowe

STRESZCZENIE:

Przedmiotem pracy są dwie stalowe cienkościenne siedmio-warstwowe belki – pasma płyty z trapezowo
pofałdowanymi dwoma rdzeniami okładzin oraz rdzeniem głównym. Obie belki różnią się między sobą kierunkami
pofałdowań rdzeni. Belka pierwsza (B-1) (Rys. 1) posiada rdzeń główny pofałdowany jest wzdłuż jej długości,
a rdzenie okładzin pofałdowane poprzecznie.

Natomiast belka druga (B-2) (Rys. 2) posiada rdzeń główny pofałdowany poprzecznie, a rdzenie okładzin pofałdowane
wzdłuż jej długości.

Kierunki pofałdowania rdzenia głównego w obu belkach są prostopadle do kierunków pofałdowań rdzeni okładzin. Rdzeń
główny połączony jest z rdzeniami okładzin za pośrednictwem cienkich blach-pasm płaskich. Warstwy zewnętrzne są
również cienkimi blachami-pasami płaskimi. Siedmio-warstwowa struktura tych belek jest więc niejednorodna.
Właściwości czterech stalowych cienkich blach-pasm są izotropowe, natomiast właściwości rdzeni postaci
pofałdowanych trapezowo cienkich blach są ortotropowe, a ich sztywności na rozciąganie, zginanie i ścignanie

\[
\frac{g_{11}}{g_{12}} \quad \text{Analit.}
\]

\[
\frac{g_{16}}{g_{12}}
\]

dla przyjętych – zmierzonych wymiarów badanych belek. Następnie, wykonane stalowe belki – pasma płytywle badano doświadczalnie w maszynie wytrzymałościowej. Otrzymano stąd zależności obciążenie – ugięcie \( F(w_{\max}) \) w postaci wykresów dla każdej belki. W celu porównania wyników otrzymanych z obu metod, naniesiono na wykresy doświadczalne, otrzymane z maszyny wytrzymałościowej, wykresy – linie proste wyznaczone analitycznie (Rys. 10 oraz Rys. 13). Dodatkowo w Tabeli 1 i Tabeli 2 zestawiono dla wybranych wartości ugięć odpowiadające im wartości obciażeń – sił wyznaczonych doświadczalnie i analitycznie. Stwierdzono zgodność otrzymanych wyników z obu metod. Różnice między wartościami sił wyznaczonych obiema metodami są mniejsze od 5% dla belki pierwszej oraz mniejsze od 10% dla belki drugiej. Ponadto, rozwiązanie analityczne daje dolne oszacowanie wartości obciążeń. Wynika stąd, że belki rzeczywiste charakteryzuje większa sztywność niż wynikałoby to z rozwiązania analitycznego.

Przedstawione w pracy badania analityczne i doświadczalne zginania siedmiowarstwowych belek o strukturze cienkościennej są badaniami podstawowymi. Szczególne znaczenie mają tu opracowane modele analityczne obu belek. Przegląd literatury wskazuje na aktualność tematyki badawczej dotyczącej wytrzymałości i stateczności konstrukcji warstwowych.