Optimization of two-component armour

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Abstract. The paper presents research on optimization of two-layer armour subjected to the normal impact of the 7.62x54 B32 armour piercing (AP) projectile. There were analysed two cases in which alumina $\text{Al}_2\text{O}_3$ was supported by aluminium alloy AA2024-T3 or armour steel Armox 500T. The thicknesses of layers were determined to minimize the panel areal density whilst satisfying the constraint, which was the maximum projectile velocity after panel perforation. The problem was solved through the utilization of LS-DYNA, LS-OPT and HyperMorph engineering software. The axisymmetric model was applied to the calculation in order to provide sufficient discretization. The response of the alumina alloy, armour steel and projectile material was described with the Johnson-Cook model, while the one of the alumina with the Johnson-Holmquist model. The study resulted in the development of a panel optimization methodology, which allows the layer thicknesses of the panel with minimum areal density to be determined. The optimization process demonstrated that the areal density of the lightest panel is 71.07 and 71.82 kg/m$^2$ for $\text{Al}_2\text{O}_3$-Armox 500T and $\text{Al}_2\text{O}_3$-AA2024-T3, respectively. The results of optimization process were confirmed during the experimental investigation.

Key words: optimization, composites, numerical simulations, ballistic protection.

1. Introduction

One of the main criteria used for comparing armours providing the same level of protection is areal density. A lower panel mass allows a reduction in fuel consumption and preserves the mobility of the vehicle on which it is mounted. One of the methods used to decrease the areal density of the armour is the employment of a two-layer system, in which a hard layer is supported by a plastic one. One layer is designed to erode the projectile, while the other, softer layer, absorbs the kinetic energy of the projectile by plastic deformation [1]. The first ceramic-metal armour was proposed by Wilkins et al. [2]. Many descriptions in the literature indicate the improved efficiency of two-component systems over monolithic metal armours [3–5]. Among the most popular dual hardness panels, there are solutions consisting of alumina supported by the aluminium alloy or steel.

A number of studies can be found in the literature regarding the optimization of two-component armour systems, which generally refer to ceramic/metal armours. The optimization problem has been solved in the literature by both modelling and experimental approaches. The modelling approaches predominantly include analytical methods, which are typically simplified from complex realistic cases. The analytical optimization of ceramic/metal armours is mainly based on the Florence model [6]. The problem of determining the structure of a two-component armour with specific areal density that provides the maximum ballistic limit velocity was considered by Hetherington [7]. Wang and Lu [8] studied a similar problem, in which the total thickness of the armour rather than the areal density was given. Shi and Grow [9] investigated the problem of a two-component armour in which both the total thickness and the areal density were limited. Ben-Dor et al. [10] studied the problem of maximization of the ballistic limit velocity for given areal density or total thickness and minimization of the areal density or the total thickness for the given impact velocity, using an updated version of the Florence model.

The authors have been investigating the optimal thicknesses of armour layers using a coupling of numerical methods, such as the finite element method (FEM) with optimization tools. The paper presents an approach that approximates the objective function with a neural network and then searches for its optimum by applying a hybrid adaptive simulated annealing algorithm (ASA). The study concerned the minimization of the areal density of a two-component armour protecting against a 7.62x54 B32 AP projectile.

2. Problem description

The optimization of a two-layer armour subjected to the normal impact of a 7.62x54 B32 AP projectile was carried out. Calculations were performed for two cases, which differed with the material of the second layer. The target diameter was 50 mm, and the initial velocity of the projectile was equal to 854 m/s. The geometry of the projectile and armour are shown in Fig. 1. The aim of the study was to determine the layer thicknesses providing protection against the assumed projectile. The initial configuration and ranges of layer thicknesses are listed in Table 1.

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Fig. 1. Configuration of studied problem

Table 1
Ranges of design variables and their initial values

<table>
<thead>
<tr>
<th>Variant</th>
<th>Range of thickness [mm]</th>
<th>Initial thickness [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Al₂O₃ Armox 500T</td>
<td>6–10</td>
<td>8</td>
</tr>
<tr>
<td>2 Al₂O₃ Armox 500T</td>
<td>3–7</td>
<td>5</td>
</tr>
</tbody>
</table>

3. Optimization fundamentals

The issue of optimization can be simplified to determination of the best admissible solutions of a given problem taking into consideration the assumed criterion of quality [11]. The optimization problem is composed of the following elements: the set of design variables (design parameters), the objective function and the constraints. Its solution consists in the identification of a set of design variables that ensure the minimization of the objective function:

$$\min f(x).$$

Satisfying the constraints:

$$h_k(x) = 0, \quad k = 1, 2, \ldots, l,$$

$$g_j(x) \leq 0, \quad j = 1, 2, \ldots, m.$$  

where $f$, $g$ and $h$ are functions of independent variables $x_1$, $x_2$, $x_3$, ..., $x_n$. The function $f$, referred to as the cost or objective function, identifies the quantity to be minimised or maximised. Functions $g$ and $h$ are constraint functions representing the design restrictions. The variables collectively described by the vector $x$ are often referred to as design variables or design parameters. In the considered case the design parameters were the thicknesses of both layers. As it was previously mentioned, the panel areal density was assumed to be the objective function, whose minimum was sought. A limitation was the projectile velocity after panel perforation, which could not exceed 10% of its initial value. The character of the constraint was based on the assumption that the panel was mounted onto a vehicle hull, which provides the primary protection.

Method of problem solution. The optimization calculations were performed using the coupling of LS-DYNA, LS-OPT and HyperMorph. The optimization toolchain is depicted in Fig. 2.

The LS-OPT package was used to define the optimization problem, monitor the computation and analyse the results. LS-OPT generated sets of design variables (layer thicknesses), for which HyperMorph created meshes of the numerical models. Then, appropriate calculations were conducted in LS-DYNA software with the finite element method. The LS-OPT package searched for the optimal solution employing response surface model RSM, which replaces the actual objective and response functions. In the considered case, the response surface model, which was constructed based on an appropriately selected set of numerical simulations, was a neural network with radial basis functions RBF. The structure of the neural network is shown in Fig. 3 [12]. In calculation, a neural network made of three different layers was utilized. The input layer was linear and a number of its neurons resulted from a number of design variables. A single hidden layer consists of nonlinear neurons described with radial functions. The linear output layer is composed of one neuron, which superimposes signals from the hidden layer giving representation of the object and constraint functions.

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Fig. 2. Optimization toolchain
An approximation function built on the basis of the neural network, depicted in Fig. 3, has the following form (4):

\[ Y(x, W) = W_0 + W_1 g_1 + W_2 g_2 + \cdots + W_H g_H = W_0 + \sum_{h=1}^{H} W_h g_h, \]  

where \( x = (x_1, \ldots, x_n) \) represents vector of design variables, \( W \) is vector of neural network weights, \( W_{hn} = (W_{h1}, \ldots, W_{hn}) \) is vector of hidden layer weights – location of \( h^{th} \) RBF centre, \( W_{h0} \) controls the smoothness of \( h^{th} \) RBF, \( W_h = (W_1, \ldots, W_h) \) is the vector of output layer weights – weight of \( h^{th} \) RBF, \( W_0 \) – is bias of approximation function \( Y \) and \( g_h \) denotes RBF.

Radial functions change radially around the selected centre, each of which corresponds to only a local region of design space [13]. Example of RBF is Hardy’s function formulated as follows:

\[ g_h(x_1, \ldots, x_n) = \frac{1}{\sqrt{r^2 + W_{h0}^2}}, \]  

where \( r \) distance between the input vector of design variables \( x_k = (x_1, \ldots, x_n) \) and the centre of \( h^{th} \) RBF \( W_{hn} = (W_{h1}, \ldots, W_{hn}) \) in \( n \)-dimensional space calculated as:

\[ r = \sqrt{\sum_{k=1}^{n} (x_k - W_{hkk})^2}. \]  

Parameters of the neural network are determined during its training. The learning procedure of RBF network consists of three stages that include: choice of the centres of the hidden radial basis neurons \( W_{hn} \), choice of parameter \( W_{h0} \) – smoothness of the radial function for each hidden neuron, determination of the weight factors between hidden and output layer \( W_h \) [14]. In the presented case, the panel areal density and results of numerical simulations such as projectile velocity after panel perforation were used for training. An example of the response surface based on a neural network with two radial functions in one-dimensional space is shown in Fig. 4.

The minimum of the so-defined objective function (4) was found by applying a hybrid ASA algorithm. The hybrid algorithm is a combination of two optimization algorithms: ASA and a leapfrog optimizer for constrained minimization (LFOPC). The adaptive annealing algorithm is a stochastic procedure that makes it possible to find the basin of the objective function in which the global minimum is located [15]. The established solution is the starting point for the gradient-based leapfrog algorithm, which enables the quick and accurate determination of the global optimum [16].

**Numerical model description.** Numerical calculations were performed with the non-linear finite element code LS-DYNA, which is a commonly used tool for solving problems associated with shock wave propagation, blasts and impacts. An explicit integration scheme was used to solve the equation of motion. An axisymmetric model was built in order to ensure sufficient discretization.

The Johnson-Cook (JC) constitutive model was used to describe the behaviour of the aluminium alloy, armour steel and projectile material. This model is typically applied in the study of explosive metal forming, armour perforation and impacts, i.e., situations that are accompanied by high strain rate deformations. The flow stress in the constitutive relation is expressed as [17]:

\[ \sigma_y = (A + B \varepsilon^m) (1 + C \ln \dot{\varepsilon}_y) (1 - (T^*)^n), \]  

where \( \varepsilon \) is the equivalent plastic strain, \( \dot{\varepsilon} \) is the plastic strain-rate, and \( A, B, C, n, m \) are material constants. The normalised strain-rate and temperature in Eq. (7) are given in the following forms:

\[ \dot{\varepsilon}_* = \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}, \]
\[ T_* = \frac{(T - T_0)}{(T_m - T_0)}, \]

where \( \dot{\varepsilon}_0 \) is the quasi-static threshold strain rate, \( T_0 \) is the reference temperature, and \( T_m \) is the melt temperature.

Pressure occurring during panel perforation is much larger than the yield stress of ductile materials comprising the model, which behave like a compressible liquid. This state is defined as the hydrodynamic regime, and requires a state equation for the determination of the constitutive model. In the constructed model, the Gruneisen equation of state (EOS) was used. The pressure for the compressed material is defined as [18]:

\[ p = \frac{\rho_0 C^2 \mu \left[ 1 + \frac{(1 - \gamma_0)}{2} \mu \frac{a}{\mu + 1} \right]}{\left[ 1 - (S_1 - 1)\mu - S_2 \frac{\mu^2}{\mu + 1} - S_3 \frac{\mu^4}{(\mu + 1)^2} \right]} + (\gamma_0 + a\mu) E, \]

and for expanded materials:

\[ p = \rho_0 C^2 \mu + (\gamma_0 + a\mu) E, \]  

where \( C \) is the bulk speed of sound, \( \rho_0 \) is the initial density, \( \gamma_0 \) is the Gruneisen gamma, \( a \) is the first order volume correction to \( \gamma_0 \), \( S_1, S_2, S_3 \) are the coefficients of the slope of
the shock wave velocity – particle velocity curve, and $E$ is internal energy, $\mu = (\mu/\rho_0) - 1$. The material data for the JC model and the Gruneisen EOS applied in this work are listed in Table 2 [19–23].

Table 2  
JC model and Gruneisen EOS parameters for core, jacket, aluminium alloy and armour steel

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Unit</th>
<th>Core</th>
<th>Jacket AA2024-T3</th>
<th>Armox 500T</th>
</tr>
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<tr>
<td>Source</td>
<td>N/A</td>
<td>N/A [19, 20] [21] [22, 23] [19, 20]</td>
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<tr>
<td>mass density</td>
<td>RO</td>
<td>g/cm³</td>
<td>7.85</td>
<td>8.8</td>
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<tr>
<td>shear modulus</td>
<td>G</td>
<td>GPa</td>
<td>79.6</td>
<td>44</td>
<td>28.6</td>
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<tr>
<td>Young’s modulus</td>
<td>E</td>
<td>GPa</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
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<tr>
<td>Poisson’s ratio</td>
<td>PR</td>
<td>-</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
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<tr>
<td>JC:</td>
<td>A</td>
<td>GPa</td>
<td>1.576°</td>
<td>0.112</td>
<td>0.369</td>
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<tr>
<td></td>
<td>B</td>
<td>GPa</td>
<td>2.906°</td>
<td>0.505</td>
<td>0.684</td>
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<tr>
<td></td>
<td>N</td>
<td>-</td>
<td>0.117°</td>
<td>0.42</td>
<td>0.73</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>-</td>
<td>0.00541</td>
<td>0.009</td>
<td>0.0083</td>
</tr>
<tr>
<td></td>
<td>M</td>
<td>-</td>
<td>0.87</td>
<td>1.68</td>
<td>1.7</td>
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<tr>
<td></td>
<td>TM</td>
<td>K</td>
<td>1800</td>
<td>1030</td>
<td>775</td>
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<tr>
<td></td>
<td>TR</td>
<td>K</td>
<td>293</td>
<td>293</td>
<td>293</td>
</tr>
<tr>
<td></td>
<td>EPSO</td>
<td>l/s</td>
<td>1</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>CP</td>
<td>J/kgK</td>
<td>450</td>
<td>376</td>
<td>875</td>
</tr>
<tr>
<td>Gruneisen EOS:</td>
<td>C</td>
<td>m/s</td>
<td>4570</td>
<td>3720</td>
<td>5382</td>
</tr>
<tr>
<td></td>
<td>S1</td>
<td>-</td>
<td>1.49</td>
<td>1.328</td>
<td>1.338</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>-</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>-</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
<td></td>
<td>GAMAO</td>
<td>-</td>
<td>1.930</td>
<td>1.657</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>A</td>
<td>-</td>
<td>0.5</td>
<td>0.48</td>
<td>0</td>
</tr>
<tr>
<td>JC FAILURE:</td>
<td>D1</td>
<td>-</td>
<td>0.0356°</td>
<td>0.54</td>
<td>-0.07</td>
</tr>
<tr>
<td></td>
<td>D2</td>
<td>-</td>
<td>0.0826°</td>
<td>4.89</td>
<td>1.02</td>
</tr>
<tr>
<td></td>
<td>D3</td>
<td>-</td>
<td>-2.5°</td>
<td>3.03</td>
<td>-1.62</td>
</tr>
<tr>
<td></td>
<td>D4</td>
<td>-</td>
<td>0</td>
<td>0.014</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>D5</td>
<td>-</td>
<td>0</td>
<td>1.12</td>
<td>0</td>
</tr>
</tbody>
</table>

**$^a$** designated based on conducted experimental tests.

Boundary conditions introduced in the model provided the panel fixing. A support in the simulation was implemented by removing the degrees of freedom from some of the nodes located on the rear surface of each plate.

An initial condition in the model was that the velocity of the projectile, which was set to be equal to 854 m/s.

**Discussion of optimization results.** The approximation of the objective and the constraint function is depicted in Figs. 5 and 6, respectively. As a measure of the accuracy of the response surface, the root mean square (RMS) error between the predicted and computed values was adopted:

$$\text{RMS} = \sqrt{\frac{1}{P} \sum_{i=1}^{P} (\hat{y}_i - y_i)^2},$$  

where $P$ is a number of numerical experiments, $y_i$ the results of the numerical simulations, and $\hat{y}_i$ the predicted values with the response surface model. The error of objective function RMS, depending on the variant, ranged from 0.000025 to 0.000394 kg/m². In contrast, the RMS error for the constraint
function ranged from 22.9 to 77.4 m/s. In the case of the objective function, the response surface model based on neural network maps the panel areal density precisely as evidenced by the negligible small RMS error. Panel areal density, as the sum of the products of the layer thicknesses and their volume density, is a linear dependence, which neural networks learn fast and efficiently. The constraint function approximation with the neural network is not as accurate as the objective function. Discrepancies between the predicted and computed values of constraint come from, among other things, its non-linear character.

![Fig. 5. Approximation of objective function](image)

![Fig. 6. Approximation of constraint function](image)

### Table 4
Results of two-layer armour optimization

<table>
<thead>
<tr>
<th>Variant</th>
<th>Optimal thickness [mm]</th>
<th>Residual projectile velocity [m/s]</th>
<th>Areal density [kg/m²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Al₂O₃ Armox 500T</td>
<td>Al₂O₃ Armox 500T</td>
<td>7.35</td>
</tr>
<tr>
<td>2</td>
<td>Al₂O₃ AA2024-T3</td>
<td>Al₂O₃ AA2024-T3</td>
<td>11.09</td>
</tr>
</tbody>
</table>
Despite the reduced properties of alumina, the two-component panel still shows predominance in relation to the panel made entirely of Armox 500T steel, which, according to the experimental and numerical study of Kilic and Ekici [28], provides protection against a 7.62×51 mm NATO AP projectile. Their studies reveal that even a panel with the areal density equal to 55 kg/m² is able to arrest the projectile. Shi and Grow [9] also determined that a two-component panel (Al₂O₃/AA6061) with the areal density of 53 kg/m² protects against a 7.62×51 mm NATO AP projectile. The reason of the difference is a low tensile strength of alumina used in the optimization process which is one of the most critical parameter deciding on ceramic effectiveness at the time of impact. Despite the reduced properties of alumina, the two-component panel still shows predominance in relation to the panel made entirely of Armox 500T steel, which, according to the experimental and numerical study of Kilic and Ekici [28], provides protection against a 7.62×54 B32 AP projectile with thickness and areal density greater than 13 mm and 100 kg/m², respectively.

4. Conclusions

The study resulted in the development of a panel optimization methodology, which allows the determination of the layer thicknesses of a panel with minimum areal density meeting the constraint in the form of the maximum velocity of the projectile after panel perforation. The conducted optimization process demonstrated that the areal density of a two-component panel is 71.07 and 71.82 kg/m² for Al₂O₃-Armox 500T and Al₂O₃-AA2024-T3, respectively. At the same time it was noted that tensile strength of ceramic is one of the key factors deciding on effectiveness of double-layer armour.

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REFERENCES

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