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Hysteresis Modeling Using a Preisach Operator

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***Abstract*—The aim of this paper is to present the analysis and modeling of the hysteresis phenomenon using a Preisach operator. The fundamentals of parameterized hysteresis modeling are introduced, by utilizing three probability density functions. Then, the Preisach operator and its characteristics are defined. Subsequently, results of simulations obtained by means of the aforementioned functions are presented and compared to the ones obtained by other authors.**

***Keywords*—Preisach operator, hysteresis, probability density function.**

I. Introduction

HISTORICALLY, the term hysteresis was used for the first time in association with the ferromagnetic phenomenon [1]. In ferromagnetism the matter remains in a magnetized state, despite the disappearance of the coerced magnetic field (a so-called spontaneous magnetization). Ferromagnetic materials constitute the group of materials, for which the relative magnetic permeability μ_r (the property, that determines the dependence of magnetic induction on the coerced magnetic field, with $\mu_r >> 1$ [2]). Ferromagnetic materials have the inner structure based on microareas that are in a permanent magnetized state - magnetic domains. These areas are provided with a certain magnetic momentum. Despite the dissapearance of the external magnetic field, the magnetic domains remain in a magnetized state, which has a crucial impact on creating a magnetic curve, called the hysteresis loop. In the latter, the magnetic induction B is not determined interchangeably by the coerced field H. In devices in which the multiple magnetization process occurs (i.e. the transformer core) the hysteresis is considered as a problem because it's surface area is proportional to the energy dissipation during one cycle of magnetization process. On the other hand, hysteresis can also be considered of great importance to the technology of non-volatile memory storage for computers, as it allows for creation of strong magnetic fields.

The hysteresis phenomenon has the following properties (see Fig. 1):

- Causality - the output value of B depends only on the past and present values of the input H.

- Monotonicity - the monotonic changes of H depend on monotonic changes of B.

- Presence of a major loop - the set of points on the (H, B) surface, that are placed between the curves create the major hysteresis loop. Their mutual relations influence the most distinctive feature of the hysteresis loop, which is the magnetic saturation condition. It is the invariability of the output signal for large values of the input signal.

- Minor loop closure - let's assume, that the input values H and B in a certain moment are such that the point (H, B) occurs on the major loop. If the input changes from a value $(\widetilde{H})$ and then comes back to the value H, then the output changes it's value to $(\widetilde{B})$ and then back to B. The other way of describing this dependence is the following: $(H, B) \rightarrow (\widetilde{H}, \widetilde{B}) \rightarrow (H, B)$.

- Energy dissipation - it can be observed, that the supplying energy needed to close the loop between two points (H_1, B_1) and (H_2, B_2) is proportional to the area embraced by the closed loop.

- Order preservation - the lines created by increasing input values on the (H, B) plane do not intersect. The similar phenomenon takes place when the input values are decreasing.

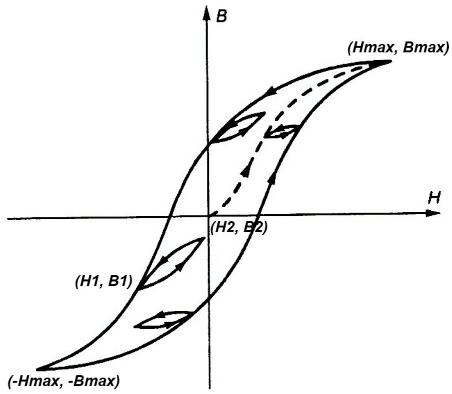

Fig. 1. Various magnetisation curves (major and minor loops) presented for an unspecified ferromagnetic material. The major hysteresis loop on the plane (H, B) is obtained by changing the value of the coerced magnetic field H from the value $-H_{max}$ to H_{max} and vice versa. There can be also be observed the phenomenon of the minor loop closure $(H_1, B_1) \rightarrow (H_2, B_2)$.

II. The Fundamentals of Mathematical Hysteresis Modeling

In order to define the term "hysteresis" properly, one must determine the character of quasistatic changes on the input of the system. It can be done by using a rate-independent operator. A relay (Fig. 2) is an electrical device, designed to cause an established sudden state change in one or many of output circuits, at the same time fulfilling certain input conditions.

The relay responds to a change of a certain physical input value (i.e. voltage, current, temperature etc.) in a way that makes the output signal change its value in a step manner

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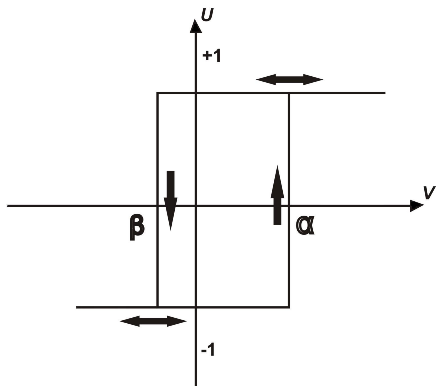

Fig. 2. The relay defined by a pair of threshold values (β, α). The output of the relay is +1, when the input is greater than α, -1 when the input is smaller than β and stays unaltered when the input stays in the (β, α) range.

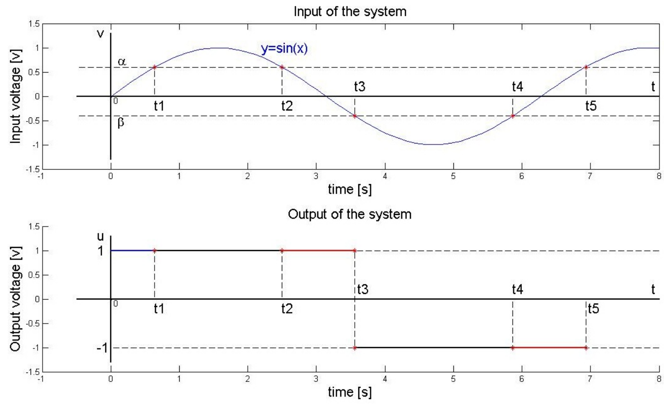

Fig. 3. Sine input signal, in which extreme values are greater than the threshold values α and β. For a set of instants $A_t = sup\{t_1, t_2\}$ the new extreme value is the latter value from this set. Thresholds of the input function for instant t_1 and t_2 describe the output state of the relay (in this case +1). The similar case refers to instants t_3 and t_4. It is acknowledged that the relay in case of the signals that are bigger than the predetermined threshold values is in the saturated state ("0"). b) The output of the system depends on the local threshold values of the input signal. The output of the relay in the instants t_1 and t_2 is +1. Because the state of the operator that precedes the instant t_1 is unknown, it is necessary to assume one of the two options. In this case it is assumed that the relay state is +1. Unknown is also the state of the relay in the range from t_2 to t_3, it is assumed however that it is determined by the latter state, that is +1. Similar case concerns the range $t_3 - t_4$.

after the input signal achieves a certain value. Typically it changes between two values: the high one (turn on) and the low one (turn off). Based on Fig. 2 it can be interpreted that if the input v has a value smaller than α, then the output u is "low", which means that the relay is off. During the increase of v, the output stays low untill v reaches the value of β. It is the moment, when the relay turns on. Subsequent increase of v doesn't cause any further changes. On the other hand, it's decrease causes that the input tries to achieve the value α again. The dependence of each operator's state on the previous ones is then obvious. Let $\alpha, \beta \in \mathbb{R}$ and $\beta \leq \alpha$, where α and β are a pair of threshold values. Let's assume, that $v(t)$ is measurable in the range of $[0, T]$ (where T is a period of input signal duration) and $u(t) = \zeta$, where $\zeta = \{-1, +1\}$. The output function of the relay presented in Fig. 2 is defined as follows [3]:

$$u(0) = \begin{cases} -1 & \text{when } v(t) \leq \beta \\ 1 & \text{when } v(t) \geq \alpha \\ \zeta & \text{when } \beta < v(t) < \alpha \end{cases} \tag{1}$$

with additional definitions:

$$A_t = \{t \in [0, T] : v(t) = \alpha \quad \text{or} \quad \beta\}, \tag{2}$$

where A_t is a set of switching instants and:

$$u(t) = \begin{cases} u(0) & \text{when } A_t = 0 \\ 1 & \text{when } A_t \neq 0 \quad \text{and } v(max(A_t = \alpha)) \\ -1 & \text{when } A_t \neq 0 \quad \text{and } v(max(A_t = \beta)) \end{cases} \tag{3}$$

The definition of the relay presented above shows that the current value of the output signal in the whole hysteresis loop depends on the history of changes of the input signal $v(t)$ (see Fig. 3).

III. The Preisach Operator Definition

The Preisach operator is a "black box" model. It describes the way the system works in the sense, that it associates certain input signals with corresponding output signals. The Preisach model can be exceptionally accurate in a ferromagnetic field if the the sufficiently accurate modeling method and Preisach density function is assumed. It represents the ferromagnetic material structure, which is based on magnetic domains.

The coerced magnetic field causes the domains translocation on the microscopic level in a step manner. The magnetization direction of the magnetic domains is identical with the direction of the coerced field [2].

As it was stated in section II, there is a certain relation between the input and the output signals in hysteretic models. In order to obtain the major hysteresis loop, a sine signal is provided to the system input. Minor hysteresis loop can be acquired by providing a fading function to the system input.

Consider the relay described with the equations (1) and (3) and depicted in Fig. 2. The relay is defined by $R_{\beta,\alpha}$, where α and β are scalar values. The output $u_{\beta,\alpha}(t)$ of the relay $R_{\beta,\alpha}$ depends only on $v(t)$ and the output value in the time instant 0: $u_{\beta,\alpha}(0)$, as it is defined by $u_{\beta,\alpha}(t) = R_{\beta,\alpha}(v(t), u_{\beta,\alpha}(0))$. Let's consider a constant, piecewise-monotonic input function $v(t), t \in [0, T]$, where $u_{\beta,\alpha}(0) \in \{+1, -1\}$. The Preisach operator is defined by the weighted superposition of relays, also known as hysterons. For the input function $v(\cdot)$, the Preisach operator output is described by [4]:

$$u(t) = \iint_{\alpha \geq \beta} \mu(\beta, \alpha) \cdot R_{\beta,\alpha}(v(t), u_{\beta,\alpha}(0))\, \mathrm{d}\beta\, \mathrm{d}\alpha \tag{4}$$

where $\mu : S \to \mathbb{R}$ is the nonnegative, measurable function on a halfplane $S = \{(\beta, \alpha) : \alpha \geq \beta\}$, described further as the weight of the relay. This function describes the amount of hysterons for each pair (β, α). The set S is the Preisach halfplane (a triangle obtained by dividing the plane by the $\beta = \alpha$ line into two halves), while μ is the Preisach probability density function. For each point (β, α) belonging to S there exists interchangeably defined relay $R_{\beta,\alpha}$ and analogically -

for each relay there is a point on the plane. The behavior of the Preisach operator is determined by the equation:

$$u(t) = (v(t), u_{\beta,\alpha}(t)), t \in [0, T] \tag{5}$$

where $u_{\beta,\alpha}(t)$ is the primary state of the operator. Hence, the primary state of the Preisach operator is the set of initial conditions for each relay on S.

IV. The Preisach Operator Characteristics

During the calculation of the output of the Preisach operator, the state of relays accumulated on the Preisach plane changes at each instant. Therefore equation (4) can be subdivided into two integrals, over positive and negative sets of relays

$$u(t) = \iint_{S^+} \mu(\beta, \alpha)\, \mathrm{d}\beta\, \mathrm{d}\alpha - \iint_{S^-} \mu(\beta, \alpha)\, \mathrm{d}\beta\, \mathrm{d}\alpha \tag{6}$$

where S^+ and S^- are the areas on the Preisach plane, where respectively the positive and negative relays are gathered.

The equation (6) is used to prove the three basic properties of the Preisach operator: minor loop closure, congruency and rate-independence [5]. It is also used to prove the characteristic theorem for the Preisach operator. It states that for each hysteresis operator with the properties mentioned above, there exists the Preisach operator with the same input-output map, as the hysteresis operator [5], [6]. In other words, these properties are essential and sufficient for the hysteresis operator to be realized by the Preisach operator. The inversion of a Preisach operator is used to compensate the effects that the hysteresis has on the system. The general approach to control the systems with hysteresis consists in combining the inverse compensation with the feedback loop [7].

V. The Preisach Density Function Identification

In order to use this type of the model it is necessary to use a certain Preisach density function (which is the case in all parametric methods). In this paper three such functions were tested: factorized Lorentzian, Gauss-Gauss and lognormal-gaussian [8]. These functions are the most often used in modeling hysteresis with the Preisach operator. The unknown parameters of the density functions must be estimated (identified).

a) Factorized-Lorentzian distribution function

$$p(\alpha, \beta) := N[1 + (\frac{\alpha - H_0}{\sigma \cdot H_0})^2]^{-1}[1 + (\frac{\beta + H_0}{\sigma \cdot H_0})^2]^{-1} \tag{7}$$

b) Gauss-Gauss distribution function

$$p(\alpha, \beta) := Nexp[-\frac{(\frac{\alpha - \beta}{2} - H_0)^2}{2\sigma^2 \cdot H_0^2}]exp[-\frac{(\frac{\alpha+\beta}{2})^2}{2\sigma^2 \cdot H_0^2}] \tag{8}$$

c) Lognormal-Gauss distribution function

$$p(\alpha, \beta) := N(\frac{2}{\alpha - \beta})exp[-\frac{ln^2(\frac{\alpha - \beta}{2H_0})}{2\sigma^2}]exp[-\frac{(\alpha + \beta)^2}{8\sigma^2 \cdot {H_0}^2}] \tag{9}$$

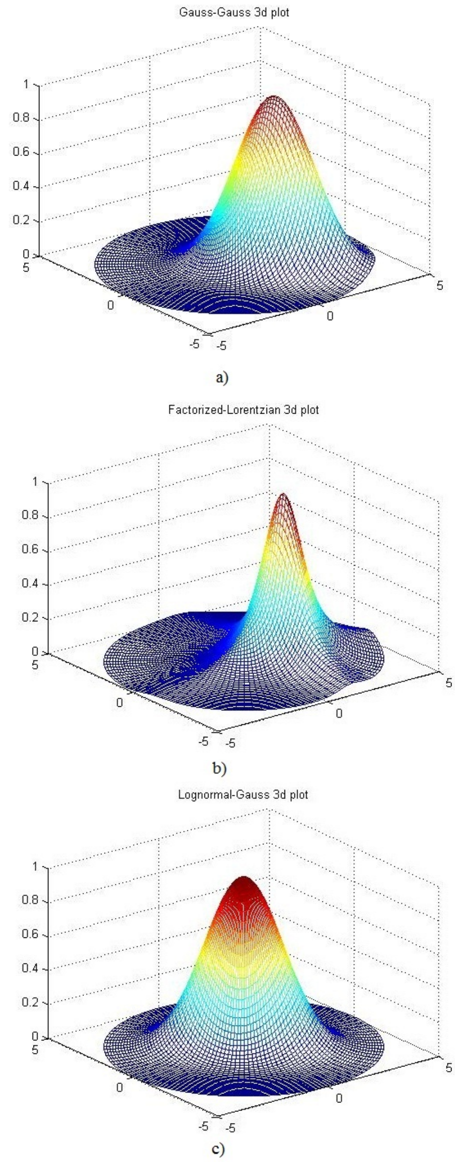

Fig. 4. An example of probability density functions, achieved for σ = 1 and H_0 = 1. a) Gauss-Gauss density function; b) Factorized-Lorentzian density function; c) Lognormal-gauss density function.

where N is a normalization factor, σ is a standard deviation and H_0 is a bias field. In this case the normalization factor is used to increase the number of relays on the Preisach plane. In fact, it increases the size of the hysteresis loop that is obtained on the system output. The maximum values of H_0 and σ are unknown.

The integration of the Preisach plane is a two-dimensional, numerical process that produces result for each pair of (β, α) values. The most popular method of integrating the half-plane (a triangle), along the $\alpha = \beta$ line is the least squares method. Unfortunately, it requires a significant amount of time and computer resources, depending on the degree of accuracy. Therefore, an attempt to improve the process of integration of Preisach plane was made. It consists in integrating the transformed distribution functions in one dimension only, which is less computationally demanding. The transformations of the original functions were made using the Gauss-Gauss and factorized-Lorentzian functions. The obtained results were compared to the ones from the least squares method and its more generalized version - the characteristic function of the integrated area.

In order to transform the Gauss-Gauss function, a substitution based on a gaussian error function was performed:

$$\begin{array}{l} erf(\beta) = \int_{-1}^{\beta} e^{-y^2} \mathrm{d}\alpha = \int_{0}^{\beta+1} e^{-(\alpha-1)^2} \mathrm{d}\alpha(*) \\ t = \alpha + 1, \mathrm{d}t = \mathrm{d}\alpha, (*) = \int_{0}^{\beta+1} e^{-t^2} \mathrm{d}\alpha = \frac{\sqrt{\pi}}{2} \end{array} \tag{10}$$

thus obtaining the transformed Gauss-Gauss function:

$$\begin{array}{c} p(\alpha,\beta) := N \cdot e^{\frac{-1}{2\sigma^2}} \cdot (\int_{-1}^{\beta} (e^{\frac{-\beta}{8\sigma^2 H_0^2}} \cdot \frac{1}{2} \cdot \sqrt{8\pi\sigma^2 H_0^2} \cdot \\ \cdot (erf(\frac{\beta}{\sqrt{8\pi\sigma^2 H_0^2}}) - erf(\frac{-1}{\sqrt{8\pi\sigma^2 H_0^2}})) \cdot e^{\frac{-\beta}{8\sigma^2 H_0^2}} \cdot e^{\frac{-\beta \cdot H_0}{8\sigma^2 H_0^2}} \cdot \\ \cdot \frac{1}{2} \cdot \sqrt{8\pi\sigma^2 H_0^2} \cdot (erf(\frac{\beta}{\sqrt{8\pi\sigma^2 H_0^2}}) - erf(\frac{-1}{\sqrt{8\pi\sigma^2 H_0^2}}))) \end{array} \tag{11}$$

A similar process was performed in the case of lorentzian density function transformation. Here the following substitution was used:

$$t = (\frac{\alpha - H_0}{\sigma \cdot H_0}) \tag{12}$$

thus obtaining the following transformed function:

$$\begin{array}{c} p(\alpha,\beta) := N \cdot \sigma \cdot H_0 \cdot (\int_{-1}^{\beta} [1 + (\frac{\beta + H_0}{\sigma \cdot H_0})^2]^{-1} \mathrm{d}\beta) \cdot \\ \cdot [arctg(\beta) - arctg(-1)] \end{array} \tag{13}$$

An easier method of integrating the Preisach plane uses the characteristic function of the integration area. This method consists in multiplication of the probability density function by the characteristic function, that assumes one of two values:

$$X_D = \begin{cases} 1 & \text{for } (\alpha,\beta) \in D \\ 0 & \text{for } (\alpha,\beta) \notin D \end{cases} \tag{14}$$

where D is a function domain (all of the values gathered in the triangle of the Preisach plane).

According to (14), the value of the characteristic function outside of the triangle of the Preisach plane is zero. Therefore this area is negligible. In this case the area in which positive relays are collected, can be calculated from the equation:

$$\int_D \rho(\alpha,\beta) \mathrm{d}\alpha \cdot \mathrm{d}\beta = \int_{R^2} X_D \rho(\alpha,\beta) \mathrm{d}\alpha \cdot \mathrm{d}\beta \tag{15}$$

where $\rho(\alpha,\beta)$ is any probability density function.

VI. Proposed Preisach Operator Model

For the purposes of this paper a program in Matlab was written to efficiently calculate the hysteresis loops for the given sine input signal. The process of Preisasch plane integration was based on quad functions, provided by the programming environment. The number of iterations in which the program executes the integrations was associated with the number of samples constructing the input signal. In each iteration the program executes an algorithm that computes the output of the Preisach operator based on the current shape of the Preisach Memory Curve. It is a single curve on a plane (β, α) that determines the state of the Preisach operator in each instant, by dividing the plane into areas where positive and negative relays are gathered [5]. Therefore, if the input signal was constructed out of 100 samples, the program calculates the system output for 100 samples as well. In each iteration a current value of the output is calculated. The sum of the output values, obtained in iterations creates a hysteresis loop depending on the input signal. If the latter is sinusoid, the answer of the system is a major hysteresis loop. If the signal is a fading function, the answer is a minor hysteresis loop. Depending on the given Preisach probability density function provided by the user, the hysteresis loop varies in shape. Thus the user controls the size and shape of the output hysteresis loop, by setting the parametrized values, such as σ and H_0. The process of integrating the Preisach plane, which is based on the equation (6), focuses on calculating the difference between the part of the plane, in which positive relays are accumulated, and the one where negative relays are accumulated.

VII. Simulations

In order to analyze the usefulness of the parametric methods in modeling the Preisach hysteresis loops, it is necessary to compare the calculated simulations with experiments obtained experimentally. With the help of Henze and Rucker [9] publications, concerning co-coated Fe_2O_3 ferromagnetic material, the authors were able to compare the methods used in simulations with results acquired experimentally, by constructing a physical system coupled with the actual ferromagnetic material. In Fig. 5 three examples of major hysteresis loops, generated using the (7), (8) and (9) density functions are presented. They are compared to already measured major loop, obtained experimentally. The assumed values of unknown parameters (σ and H_0) are as follows (according to [9]):

Factorized-Lorentzian distribution: $\sigma = 0.614152$, $Ho = 0.427471$

Gauss-Gauss density distribution: $\sigma = 0.582933$, $Ho = 0.425094$

Lognormal-gaussian distribution: $\sigma = 0.601153$, $Ho = 0.454492$

Results obtained for the major hysteresis loop when using a set of transformed probability density functions (11), (13) and the characteristic function of integration area are identical. Thus Fig. 5 can be applied to each case. It can be stated from Fig. 5, that the parametric method of modeling using a Preisach operator is not very accurate. In the presented case, the best results were obtained with methods based on Gaussian distribution functions, which provide better approximation of the major hysteresis loop than the Lorentzian distribution. The coercivity problem can also be observed. When using parametric methods, a hysteresis model is obtained, which does not reflect the actual phenomenon accurately. It can be seen clearly when we overlap the hysteresis from simulations on the obtained from the physical system. In such a case the minor loops also won't overlap. Fig. 6 and Fig. 7 present simulations of such loops obtained using both Preisach plane integration methods. The loops calculated for each of the

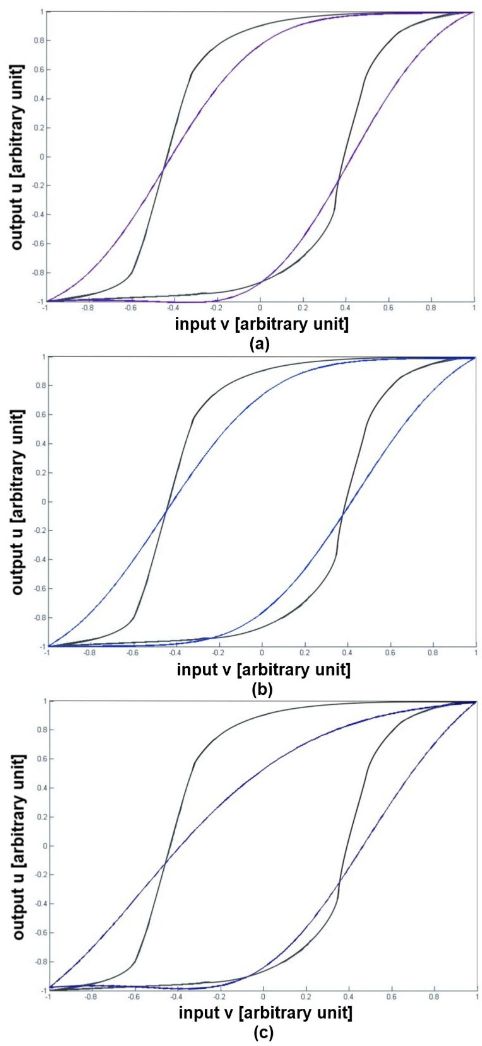

Fig. 5. Major hysteresis loop obtained experimentally (black-colored line) and the loop calculated using the probability density functions: a) Gauss-Gauss for parameters $N = 1, \sigma = 0.582933, Ho = 0.425094$; b) lognormal-gaussian for $N = 1, \sigma = 0.601153, Ho = 0.454492$; c) factorized-Lorentzian for $N = 1, \sigma = 0.614152, Ho = 0.427471$.

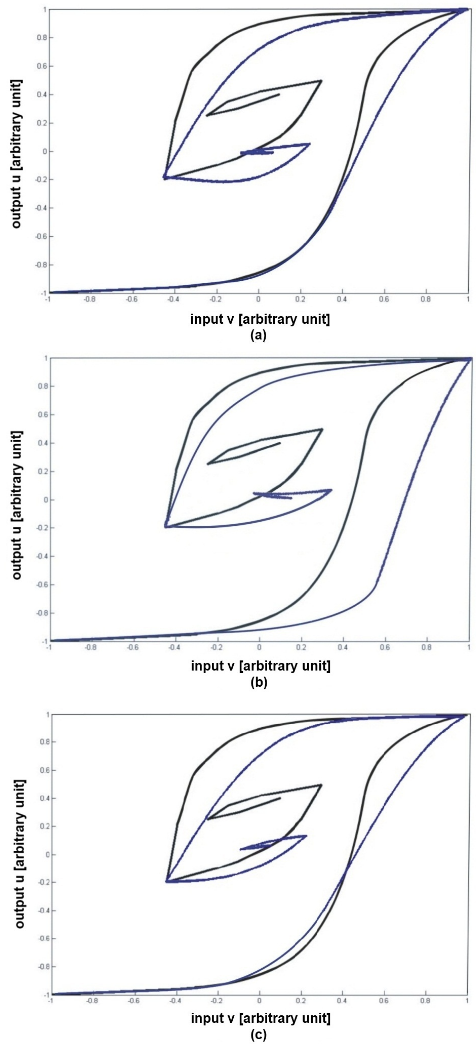

Fig. 6. The minor hysteresis loop obtained experimentally (black-colored line) and a loop obtained using the probability density functions (blue-colored line): a) Gauss-Gauss for parameters $N = 1, \sigma = 0.582933, Ho = 0.425094$; b) lorentzian for $N = 1, \sigma = 0.614152, Ho = 0.427471$; c) lognormal-gaussian for $N = 1, \sigma = 0.601153, Ho = 0.454492$. The calculations were made using the transformed probability density functions.

probability distributions clearly differ from loops obtained experimentally.

The other problem is that the twice-measurable density function has to be solved numerically. This is a complex, computationally demanding task. Analyzing Fig. 6 and Fig. 7 it can be seen that, although the results have been obtained using two different integration methods, the results differ only slightly. It is hard to determine which integration method is mathematically more effective. The program was examined in terms of estimating the time of its execution. The measurements were performed on a computer equipped with 2GHz Pentium(R) T4200 Dual-Core processing unit. The system input signal was constructed using 1600 samples. The acquired results are presented in Fig. 8. The time of program execution for increasing threshold value using transformed density functions and characteristic function of the integration area is visible there.

As it is observed in Fig. 8, measurements done using the transformed functions (11) and (13) are less time costly than the ones using the least squares method. The latter takes around 400% longer to calculate the integral of the same area as transformed functions method. The characteristic function of integration area is the fastest method of integrating the Preisach plane. It is about 20% percent quicker than the method based on transformed functions. Unfortunately, it is also the least accurate method. Therefore it may be insufficient to accuratly model the histeresis phenomenon.

VIII. Conclusions

The paper presented various methods of calculating the Preisach operator for modeling the hysteresis phenomenon. As the latter requires two-dimensional integration, which is time consuming, fast and accurate methods must be identified.

The method using the Preisach probability density function has significant advantages. The system response depends

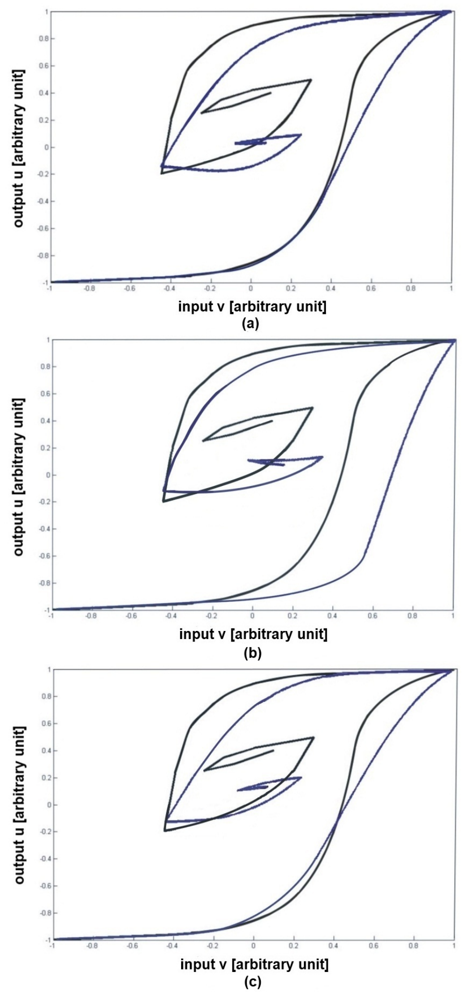

Fig. 7. The minor hysteresis loop obtained experimentally (black-colored line) and a loop obtained using the probability density functions (blue-colored line): a) Gauss-Gauss for parameters $N = 1, \sigma = 0.582933, Ho = 0.425094$; b) lorentzian for $N = 1, \sigma = 0.614152, Ho = 0.427471$; c) lognormal-gaussian for $N = 1, \sigma = 0.601153, Ho = 0.454492$. The calculations were made using the characteristic function of the integration area.

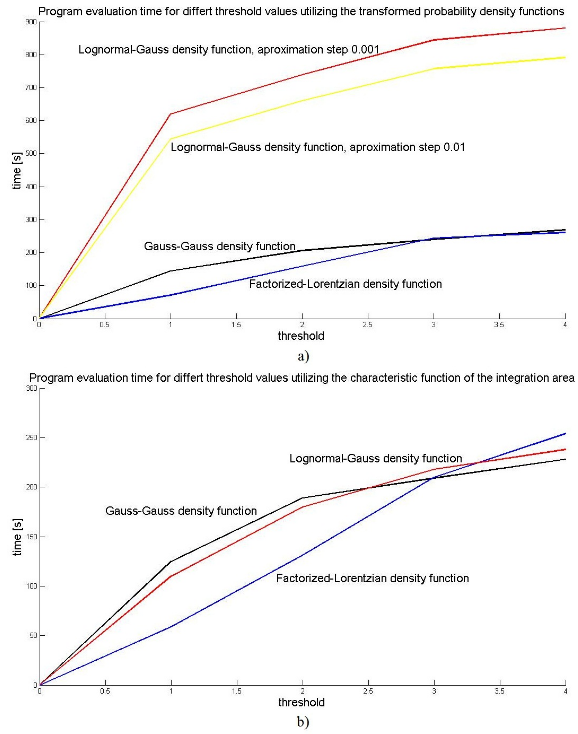

Fig. 8. Program evaluation time dependence on increasing value of threshold using a) three transformed probability density functions; b) characteristic function of integration area. Black color marks the Gauss-Gauss density function, blue - Lorentz density function and red marks the lognormal-gaussian density function.

only on the character of the input signal, while any additional measurements are redundant. However, considering results presented in the paper, it is obvious that parametrical methods of Preisach hysteresis modeling are insufficient for fully portraying the actual hysteresis measurement conditions. Therefore it is rational to focus on more precise methods of modeling (perhaps nonparametric methods) which do not assume any type of density function. Another option is to find density functions that are more accurate than the ones presented in the paper.

It is also important to select the proper method of integrating the Preisach plane. Simpler methods, using the characteristic function of the integration area are faster comparing to the methods using transformed density functions. The great advantage of the presented methods is that they can be used for any density function. It is important, as some probability density functions (like lognormal density function) are impossible to transform.

Regardless the selected integration method, it is necessary to say that the time spent on calculating areas over and under the Preisach Memory Curve is long and requires large processor toil. Therefore reducing the amount of measurement points for the input signal should be considered. It may have a negative effect on the precision of the calculated hysteresis loop. The compromise between the time of calculations and accuracy should be found.

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