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EXPONENTIAL SMOOTHING FOR MULTI-PRODUCT LOT-SIZING WITH HEIJUNKA AND VARYING DEMAND

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Abstract
Here we discuss a multi-product lot-sizing problem for a job shop controlled with a heijunka box. Demand is considered as a random variable with constant variation which must be absorbed somehow by the manufacturing system, either by increased inventory or by flexibility in the production. When a heijunka concept (production leveling) is used, fluctuations in customer orders are not transferred directly to the manufacturing system allowing for a smoother production and better production capacity utilization. The problem rather is to determine a tradeoff between the variability in the production line capacity requirement and the inventory level.

Keywords
Job shop, multi-product, varying demand, lot-sizing, heijunka. exponential smoothing.

Introduction

This paper is devoted to the topic of production lot-sizing for the case of uncertain but stationary demand. The demand is defined as a random variable with an estimated mean value and variation. Mean value and standard deviation do not shift over time, meaning no seasonal cycles or trend are considered. Demand fluctuations can be absorbed with an increased inventory level or by adding capacity flexibility in the production system [1]. Fluctuations in demand are often highly amplified and delayed throughout the production system, and can result in increased overtime or idle time [2–3]. Increased inventory levels decouple the production system from demand allowing production to stabilize at a fixed level. This approach is traditional and was the most popular some time ago. Increased inventories lead to the elongation of cycle times, increases in the size of production batches and reductions in flexibility [4]. On the other hand, demand fluctuations could be passed directly to the production system, which must then be flexible enough to meet customer requirements. In this case, production is organized in a make-to-order manner and in extreme case no inventory of finished products is required. This creates variable production schedules, which can be stressful and can lead to mistakes. Cycle time is short as finished products are transferred directly to the customer [5]. Usually it's very hard or even impossible to organize a production system that is completely flexible, as often this flexibility requires working overtime with an additional workforce and machining equipment. Thus an intermediate solution is required [6], a sort of compromise or tradeoff between inventory levels and flexibility.

Heijunka in recent years has gained popularity as a lean management tool for smoothing production, and as a result better control of the inventory of finished products [7]. The objective of heijunka is to avoid peaks and valleys in the production schedule.

In the case where demand is uncertain, two approaches are possible: demand leveling and production leveling. Demand leveling is the role of marketing and sales departments, while production leveling is done by the manufacturing department [7].

Let's focus here on production leveling, which in Japanese is called heijunka, a distribution of production volume and mix evenly over time [4]. Heijunka is a core concept that helps bring stability to a manufacturing process by converting uneven customer demand into an even and predictable manufacturing process. Heijunka is a form of cyclic scheduling. The cyclic schedules offers advantages both for shop floor activities and for planning. It may help to detect disturbances earlier and reduce set up time and costs [8].

Heijunka requires that the company has already introduced other lean management tools, such as: takt time, kanban planning, SMED (reduction of changeover and set-up times).

Introducing heijunka requires the determination of a base period called the EPEI (Every Part Every Interval) [6]. During the base period the whole mix of products has to be produced.

In literature, it has been demonstrated that Heijunka improves operational efficiency in several objectives related to flexibility, speed, cost, quality and level of customer service.

It is crucial to realize that a heijunka is the establishment of a finished goods inventory buffer that absorbs fluctuations in demand [9]. Although this may initially increase inventory, it is essential in creating stability on the shop floor. This stability enables a greater focus on the reduction of changeover times, which in turn lead to the reduction of batch sizes and inventory.

The goal of this study is to develop a model of lot-sizing for a production system controlled with a heijunka box under varying demand.

This paper is further organized as follows. The following Sec. 2 presents a literature review. The subsequent Sec. 3 describes a model statement followed then by a case study 4. Section 5 presents an analysis of the results. The final Sec. 6 concludes the paper with a summary and some perspectives.

Literature review

Heijunka is a concept developed by Toyota [7] for the automotive industry. The concept is widely universal and by now has been implemented in many different manufacturing environments, even in process industries [9] and [10]. A case study from a semi-process industry [10] has shown that heijunka (termed cyclical scheduling) helped to realize regularity in the continuous part of production. Its simplicity enabled coordination of the planning and control processes with the production processes. The positive effects realized through the application of heijunka included [9]: improved coordination across the entire value-stream, and the potential for reduced changeover times leading to reduced batch sizes and increased throughput.

Heijunka and Just in Sequence (JIS) were compared in a case study of BMW engine production [11]. Results from this study showed that heijunka outperformed JIS when used to smooth out the most extreme production values, with the remainder of production carried out with JIS.

Abdulmalek and Rajgopal [12] demonstrated a steel industry use of heijunka as one component of a lean transformation, together with: value stream mapping, SMED, 5S, JIT, TPM, cellular manufacturing.

Matzka et al. [13] modeled a heijunka-controlled kanban manufacturing system as a queuing network with synchronization stations in order to find optimal buffer capacities. Processing times were considered to be constant. The arriving demands were controlled and limited by a kanban loop.

A simulation based comparative study of kanban and heijunka controlled production systems was presented by Runkler [14] for an electronic circuit manufacturer. Heijunka was preferable in stable environments with demand and servicing rate history because then it yielded lower buffer levels and a higher average ability to deliver than Kanban.

A procedure for leveling low volume and high mix production was developed in [15]. Clustering techniques were used to subsume the large number of product types into a manageable number of product families. Production leveling was realized by scheduling actual production orders into the leveling pattern. The pattern is determined and kept constant for a defined period of time. In the case of deviations resulting from significant changes in product mix and/or customer demand, the leveling pattern needed to be adapted.

In this paper, we have developed an alternative approach to leveling: a smooth adjusting leveling pattern for every time period depending on inventory levels for all final products. Demand is known only for the forthcoming period, under the assumption that mean value of demand and the variation over a longer time remain unchanged.

Heijunka controlled production

Let's analyze the typical production system controlled by a heijunka and presented in Fig. 1. Demand is fulfilled from a supermarket. The term supermarket is from lean management vocabulary where it means an inventory with defined physical positions and volume for each type of product. The demand is not fulfilled directly from production but instead finished products are taken from the supermarket that is supplied by production. This is quite similar to a make-to-stock policy. Information about the products taken is transferred to a heijunka box. The heijunka box is another concept from lean management. It is a cyclical schedule which is divided into a grid of boxes. The columns represent a specific period of time. Rows represent the product types produced by the subsequent production. This type of organization is in-between make-to-stock and make-to-order. It is not a make-to-stock as production is not completely separated from demand and production lots are unequal to orders. It is also not make-to-order as demand is fulfilled instantly from the supermarket as long there is a sufficient inventory level. In the case of an inventory shortage the remaining part of demand is backordered and will be supplemented in the forthcoming period. The assumption is that the entire demand is realized and nothing is lost due to inventory shortages.

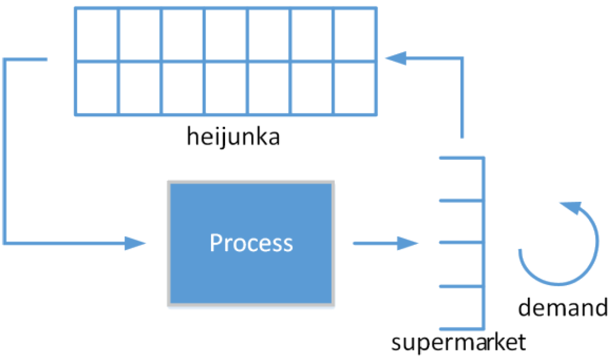

Fig. 1. Heijunka controlled production system.

An important notion closely related with heijunka is EPEI, i.e. Every Part Every Interval. EPEI defines the time interval during which every type of product has to be manufactured. EPEI is reflected in days or hours. From a lean management point of view, the shorter the EPEI, the better. A short EPEI means that changeover times were reduced and inventory levels could be decreased. EPEI is calculated by dividing the sum of changeover times for each product by the available time for changeovers per period.

Heijunka has gained attention in industry because in combination with other lean tools like SMED, it provides the methodology for efficient changeover in the manufacturing process. It allows producing in small batches, keeping low inventory levels and short production cycles. Usually EPEI is calculated and based on fixed lot-sizes for all products. Unless something happens, like changing of proportion or higher or lower demand, the lot-size stays unaffected. This approach works under the assumption that demand variation can be completely absorbed from the inventory. Otherwise a lot of backlogging emerges. The adaptive lot-sizing approach presented below is intended to capture part of the demand fluctuation in order to decrease inventory levels.

Model statement

Let us consider a forecast of demand d_i over the horizon N. The goal is to choose q_i, the quantities to be manufactured in period i, so as to satisfy all demands at minimal costs on one hand, and not to translate external fluctuations into internal production process on the other. Demand is met from a supermarket of capacity C_j for type $j$product. Total supermarket capacity is $\sum_{j=1}^{P} C_j$ and at any time the inventory level could not exceed the supermarket capacity for each type of product

$$\bigvee_j I_j \leq C_j. \tag{1}$$

EPEI is the period for producing every type of product, i.e. total available production time

$$EPEI = \frac{\sum_{j=1}^{P} S_j}{T - \sum_{j=1}^{P} avg(d_j) \cdot t_j}, \tag{2}$$

where S_j is the setup time for product j and T is the gross available time for production, $avg(d_j)$ is the historical average demand for product j.

Net production time R is equal to total production time minus the sum of changeover times for all products,

$$R = T - \sum_{j=1}^{P} S_j. \tag{3}$$

Thus sum of quantities q_{ij} produced in any time period multiplied by the manufacturing time for each product type have to be smaller or equal to R

$$\bigvee_i \left(R \geq \sum_{j=1}^{P} q_{ij} \cdot t_j \right). \tag{4}$$

In the case demand could not be fulfilled, a backlog occurs, to be satisfied in the following period:

$$b_{ij} = \max(d_{ij} + b_{i-1,j} - I_j; 0). \tag{5}$$

Quantities to be manufactured are calculated using exponential smoothing, known as the Holt model [16], as a technique to reduce variability in production lot-sizes.

$$q^*_{ij} = \alpha \cdot (C_j - I_j) + (1 - \alpha) \cdot q_{i-1,j}, \tag{6}$$

where $\alpha \in (0, 1)$ is a smoothing factor. α decides how important history is (q from the previous time period). The smaller the value α, the more history plays an important role.

In order to satisfy Eq. (4), to not exceed production capacity, the quantities to be manufactured have to be normalized:

$$q_{ij} = q^*_{ij} \cdot R \Bigg/ \sum_{j=1}^{P} q^*_{ij} \cdot t_j. \tag{7}$$

A list of all parameters is presented in Table 1.

Table 1
Notation.

i – time period index $i = 0, 1, 2, \ldots, N$
j – product index $j = 1, 2, 3, \ldots, P$
d_{ij} – number of products j demanded in period i
N – number of time periods
P – number of product types
q_{ij} – quantities of product j to be manufactured in period i
C_j – supermarket capacity for product j
S_j – setup time for product j
I_{ij} – inventory in units of product j at the end of period i
b_{ij} – backlog in units of product j at the end of period i
EPEI – period for producing every type of product
T – gross available time for production
R – net production time available for one EPEI period
t_j – production time of product j

The above-presented model is represented by an algorithm (Table 2) which calculates q_{ij} and b_{ij}. Firstly, input variables have to be initialized. The initial quantity of products to be manufactured q_{0j} is set to an averaged historical demand; initial backlog b_{0j} is set to 0 and inventory I_j to the maximum capacity C_j. The main loop (lines 8–27) runs over time index i, the inner loop (lines 9–21) runs over product index j. In line 10, the inventory is replenished by the quantity manufactured during the previous period. If the inventory is not sufficient to cover demand then the remaining part creates a backlog (lines 11–12). Quantities to be manufactured during the current period are calculated in lines 18 and 19 using Holt's exponential smoothing. Finally adjustments are done in lines 22–26 to assure that the production capacity could meet all manufacturing requests.

Table 2
Lot-sizing algorithm.

```
//initialization
1 get N, P, alpha
2 For j = 1 To P
3  get d(.,j), C(j), t(j)
4  b(0,j)=0
5  I(j)=C(j)
6  q(0,j)=avg(d(.,j))
7 Next j
//main loop
8 For i=1 To N
9  For j=1 To P
10    I(j)=I(j)+q(i-1,j)
11    If d(i,j)+b(i-1,j) > I(j) Then
12      b(i,j)=d(i,j)+b(i-1,j)-I(j)
13      I(j)=0
14   Else
15    b(i,j)=0
16    I(j)=I(j)-d(i,j)
17   End If
18   q(i,j)=alpha*(C(j)-I(j))+
19    +(1-alpha)*q(i-1,j)
20  sumQ=sumQ+q(i,j)*t(j)
21 Next j
//adjustment
22 If sum > R Then
23   For j=1 To P
24   q(i,j)=q(i,j)*R/sumQ
25   Next j
26 End If
27 Next i
```

Illustrative example

Let's analyze the proprieties of the lot-sizing algorithm presented in the previous section. We use a simple example of a pull production system with a supermarket of finished products and a heijunka box (see Fig. 1). Let's assume that three types of final products are manufactured: A, B and C. The historic average weekly demand is 450, 270 and 180 respectively for products A, B and C. Production time t is 2 min, 3 min and 1 min respectively for products A, B and C. Number of time periods N is 200. Parameters are presented in Table 3.

Table 3
Parameters.

Parameters	A	B	C
avg(d)	450	270	180
C	900	540	360
I at time = 0	450	270	180
t	2 min	3 min	1 min
S	50 min	30 min	20 min
d:case 1	N(450,22.5)	N(270,18.5)	N(180,9)
d:case 2	N(450,45)	N(270,27)	N(180,18)
d:case 3	N(450,90)	N(270,54)	N(180,36)

Customer demand is fulfilled from supermarket in equal length cycles. If in a particular cycle a demand could not be meet by the amount of products available in the supermarket ($d_{ij} > I_j$), the unfulfilled demand is backordered (b_{ij}) on the next cycle until it is satisfied.

In every cycle all types of products are manufactured. This is assured by the heijunka box. The heijunka box is thus divided into 3 slots, one for each final product. Production lot-size, i.e. quantity of product to be manufactured in a heijunka slot is determined by the proposed algorithm (Table 2), with the restriction that the total amount of time necessary for production could not exceed the net available working time per cycle.

The time period is one week. EPEI is equal to 1 what means that every type of product is manufactured during a week period. The results presented below can be easily extended to the case when EPEI creates shorter cyclic schedules. In this case, demand will be divided into smaller chunks and the manufacturing system will supply the inventory more often too.

To illustrate advantages of the proposed lot-sizing algorithm an experimental design was proposed.

First, we wanted to examine how it behaved with various levels of demand fluctuation. We have randomly generated three test cases drawn from a normal distribution with the means presented in Table 3, and standard deviations of 5%, 10% and 20% of the mean value. For all three cases we tested α parameter at 5 levels: 0.1, 0.2, 0.4, 0.6 and 1.

Figures 2 and 3 present demand and quantities to be manufactured for 10% and 20% standard deviations from mean values. One can observe in both cases with exponential smoothing that quantities to be manufactured, especially when $\alpha = 0.1$, have smaller changeability. This is also shown in Table 5.

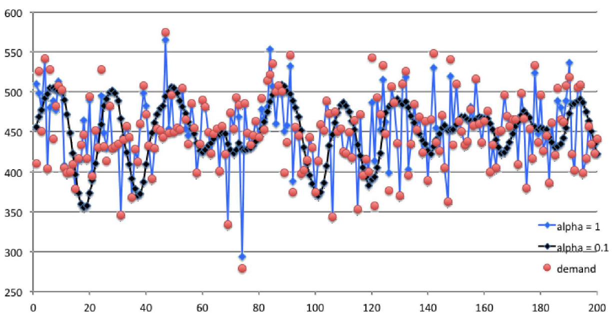

Fig. 2. Quantities to be manufactured with demand at 10%.

Table 4 presents differences (deltas) between two consecutive quantities to be manufactured for all products together, with the same differences for the original demand. The deltas of quantities to be manufactured are 6–7 times smaller on average than the delta for demand. This shows that the developed algorithm fulfills its role and avoids passing demand fluctuations to the manufacturing process.

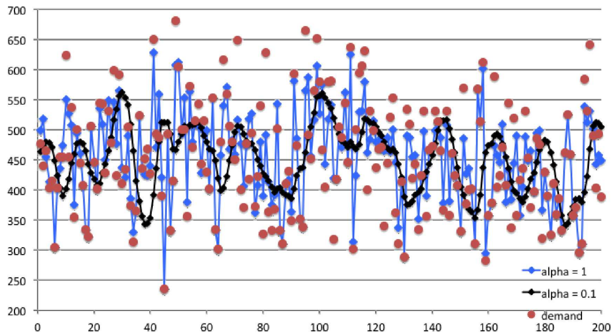

Fig. 3. Quantities to be manufactured with demand at 20%.

Table 4
Changes of quantity to be manufactured.

SD	alpha		delta Q A	delta Q B	delta Q C	delta D A	delta D B	delta D C
5%	0.1	avg	4	3	2	25	22	11
		max	13	12	6	98	72	32
	0.2	avg	5	5	2			
		max	18	18	8			
	0.4	avg	8	8	4			
		max	28	34	13			
	0.6	avg	12	11	5			
		max	40	43	17			
	1	avg	24	21	11			
		max	98	72	32			
10%	0.1	avg	9	3	3	51	30	19
		max	28	13	9	206	104	82
	0.2	avg	10	6	4			
		max	32	21	15			
	0.4	avg	17	11	7			
		max	71	38	23			
	0.6	avg	24	15	10			
		max	126	47	41			
	1.0	avg	46	28	19			
		max	191	87	82			
20%	0.1	avg	14	8	7	101	61	38
		max	50	29	29	335	292	137
	0.2	avg	20	12	8			
		max	73	53	34			
	0.4	avg	33	19	13			
		max	142	83	54			
	0.6	avg	44	26	19			
		max	179	101	72			
	1.0	avg	76	47	37			
		max	311	163	129			

The resulting stability of production plans causes greater predictability of the process, reducing the number of errors, making it easier to plan staffing, maintenance, etc.

Table 5
Average and standard deviation for quantities to be manufactured.

SD	alpha	Avg Q A	Avg Q B	Avg Q C	SD QA	SD Q B	SD Q C
5%	0.1	452	270	180	16	12	7
	0.2	452	270	180	15	13	6
	0.4	452	270	180	16	14	7
	0.6	452	270	180	17	15	7
	1	452	270	180	22	19	9
10%	0.1	450	270	180	35	14	13
	0.2	450	270	180	30	15	10
	0.4	450	270	180	32	18	11
	0.6	450	269	180	34	20	13
	1	450	270	180	43	24	17
20%	0.1	452	271	177	52	37	29
	0.2	453	271	177	55	38	24
	0.4	453	271	178	61	38	24
	0.6	452	271	178	65	40	26
	1	453	271	178	76	46	33

Table 6
Average and standard deviation for inventory levels.

SD	alpha	Avg I A	Avg I B	Avg I C	MinI A	Min I B	Min I C
5%	0.1	547	330	220	439	209	151
	0.2	548	329	220	441	258	184
	0.4	548	329	220	441	266	190
	0.6	548	329	220	441	268	194
	1	548	329	220	441	272	194
10%	0.1	544	327	218	285	187	114
	0.2	547	328	218	369	227	164
	0.4	548	329	219	402	243	176
	0.6	548	329	219	423	251	173
	1	547	328	219	423	253	179
20%	0.1	508	305	211	98	0	0
	0.2	503	302	207	179	69	24
	0.4	507	304	207	151	142	48
	0.6	506	304	207	148	128	77
	1	504	303	207	153	129	74

At the same time for all cases, average quantities to be manufactured and inventories remain at the same level, as shown in Tables 5 and 6. When analyzing the results presented in Table 6, it is clear that when demand fluctuations are small, the inventory level can be kept low. Minimal inventory levels are not much lower than average levels. In the case when demand variations are high and the smoothing coefficient $\alpha = 0.1$, minimal inventory levels fall down to 0, i.e. there were some shortages and a backlog appeared.

Conclusions

This paper presents a novel approach to lot-sizing management for manufacturing systems controlled with a heijunka. Exponential smoothing allows the adjustment of quantities to be manufactured in an environment with fluctuating demand. As has been shown in the case study, demand fluctuations are not passed directly do the manufacturing system, taking away stress from production and simplifying shop floor management. Production planning becomes more predictable. At the same time inventory levels can be keep low in order to reduce storage costs.

The proposed approach is a compromise between flexibility and stability. Assignment of the smoothing parameter α depends on the demand fluctuation and the readiness to backlog.

This work can be a starting point for optimization of production lot-sizing in regards to demand fluctuation, desired production system flexibility, inventory costs and backlog potential.

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