INFLUENCE OF A/D QUANTIZATION IN AN INTERPOLATED DFT BASED SYSTEM OF POWER CONTROL WITH A SMALL DELAY

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Abstract

Fast and accurate grid signal frequency estimation is a very important issue in the control of renewable energy systems. Important factors that influence the estimation accuracy include the A/D converter parameters in the inverter control system. This paper presents the influence of the number of A/D converter bits \(b\), the phase shift of the grid signal relative to the time window, the width of the time window relative to the grid signal period (expressed as a cycle in range \((CiR)\) parameter) and the number of \(N\) samples obtained in this window with the A/D converter on the developed estimation method results. An increase in the number \(b\) by 8 decreases the estimation error by approximately 256 times. The largest estimation error occurs when the signal module maximum is in the time window center (for small values of \((CiR)\)) or when the signal value is zero in the time window center (for large values of \((CiR)\)). In practical applications, the dominant component of the frequency estimation error is the error caused by the quantization noise, and its range is from approximately \(8 \times 10^{-10}\) to \(6 \times 10^{-4}\).

Keywords: photovoltaic system, frequency estimation, signal processing, A/D converter, control of power system.

1. Introduction

It is difficult to imagine our life without electricity. In recent years, renewable energy sources have become very popular around the world because of their high availability, “clean energy” production and ability for individuals to install their own systems [1]. The main reason for the popularity of these energy sources is because traditional energy sources will end one day and will have to be replaced.

Among the sources of renewable energy, solar energy is one of the most commonly used. Photovoltaic systems used for the solar energy conversion into electricity are constantly improved in the hardware and software [2–3]. One of the basic elements of the photovoltaic system is an inverter that is used to convert DC current and voltage obtained from photovoltaic cells to AC current and voltage with specified parameters. The energy produced by renewable energy systems (as well as the safety of the production) must fulfill the requirements defined in the respective standards and directives, such as IEC 61727-2002, IEEE Std 929-2000 and others [4–6].

One of the key issues for electricity production in photovoltaic systems is the grid signal parameters’ estimation. This allows the proper control of the inverter and improvements in the quality of the produced energy [7–9]. The main estimated parameter is grid signal frequency. There are several methods for the sinusoidal signal frequency estimation. These can be divided into spectral methods of analysis and other methods e.g., the Prony method, the transmission modeling method, and subspace methods [10–12]. The methods of the first
group are less accurate but fast, and the methods of the second group are more accurate but more computationally expensive. A method that allows very fast and accurate frequency estimation is proposed in [13] (the principle of operation is briefly discussed in Section 2).

The quality of the estimation is dependent not only on the method but also on the hardware. The poor A/D converter quality in the control system unit can result in a significant loss of the produced energy quality [14–16]. This paper presents the information necessary to determine the number of A/D converter bits for a given number of samples and width of the time window so that the estimation error of the method of [13] is at a certain level.

The remainder of this paper is organized as follows. In Section 2, the principle of operation for the frequency estimation method is presented. Section 3 contains a simulation study for the influence of the sliding time window’s phase on the frequency estimation error. Section 4 presents the results of research for the worst sliding window’s phase cases. Section 5 contains information on how to choose the number of A/D converter bits so that the estimation error is at certain level of accuracy. Finally, the conclusions are presented in Section 6.

2. The continuous tracking system with low delay

The fast, accurate and permanent (continuous tracking) parameters estimation of the grid signal is described in the time domain as:

\[ x(t) = \sum_{i=1}^{M} A_i \sin(2\pi f_i t + \phi_i), \]  

(1)
i.e., the sum of a finite number of sinusoidal oscillations, each of which is characterized by the amplitude \( A_i \), frequency \( f_i \) and phase angle \( \phi_i \) and is necessary for proper signal generation in the system of power control.

The fundamental frequency (for \( i=1 \)) is the most distorted in a deterministic manner (e.g., harmonics and interharmonics of the signal (for \( i=1 \))) or a random manner (e.g., white noise, “colored”, quantization noise caused by the A/D converter) (Fig. 1a). After sampling, the \( x(t) \) signal with a frequency \( f_s=1/T \) samples \( x_n=x(nT) \), where \( n=0,\ldots,N-1 \) are obtained.

Currently, renewable energy systems are often used to produce electricity, e.g., photovoltaic systems. A basic diagram of a photovoltaic system consisting of solar panels, inverter and control unit is shown in Fig. 1a. The DC signal obtained from the solar panels must be converted in a specific way to the AC signal using the inverter. There are several structures of inverter control units. Most of them are based on the grid signal parameter values obtained from the feedback [17–19]. Thus, the control algorithm adjusts the control signals (e.g., the frequency of the switching transistors in the inverter) so that the signal produced at the system output fulfills the quality standards [20–22].

One of the most important blocks in the control system is the DSP unit, which is responsible for the estimation of the grid signal parameters, such as \( f_1, A_1, \phi_1, THD \), and \( Q \) (Fig. 1b). The value of the signal frequency \( f_1 \) is used, e.g., in under/over frequency protection algorithms or in resonant controllers to shape the appropriate response characteristics [23]. The value of the signal amplitude \( A_1 \) is used e.g., in under/over voltage protection algorithms [24]. The value of the phase angle \( \phi_1 \) is mainly used to synchronize the voltage and current at the system output and in islanding protection algorithms [25]. For the accuracy of the entire system, it is essential to accurately estimate the frequency \( f_1 \) because most often, on this basis other parameters can be determined.

In [13], the method for calculating the grid signal frequency estimation in photovoltaic systems is presented, although it can be successfully applied in other renewable systems (and not only there). The method allows to determine the normalized grid signal frequency \( \lambda_1=f_1 T_0 \)
(where the width of the sliding time window $T_0$ expressed in the time unit is $T_0=NT$) by considering the impact of the conjugate component. The unit of $\lambda$ is Hz/Hz, which is marked as a bin, and $\lambda$ is referred to as cycle in range (CiR). The method uses the maximum decay sidelobes windows (for the window order $H=2$) and three relevant consecutive points of the spectrum $X(\lambda)$ for $\lambda = k-1, k, k+1$ (values $X_{k-1}, X_k, X_{k+1}$) around the main lobe. Developing the method for $H>1$, the frequency estimation is performed on the basis of:

$$ \Pi_1 - \lambda_1^2 \Pi_2 = 0, $$

where

$$ \Pi_1 = \begin{bmatrix} (2H-1)H & (2H-1) & X_{k-1} - X_k \\ -k^2 - H^2 & 2k & X_k \\ (2H-1)H & -(2H-1) & X_{k+1} - X_k \end{bmatrix}, $$

$$ \Pi_2 = \begin{bmatrix} 1 & -(2H-1) & X_{k-1} \\ 1 & 0 & X_k \\ 1 & (2H-1) & X_{k+1} \end{bmatrix}. $$

The accuracy of the frequency estimation is on the order of $5\cdot10^{-11}$ Hz/Hz for a 5-ms measurement window. In comparison, the relative errors of the other spectral methods are approximately $10^{-7}$ Hz/Hz. The possibility of using different windows from the discussed family has several advantages – it is possible to match the specific window data to the actual requirements of electricity generation. In addition, the discussed estimation method applies to one sinusoid, but it can be generalized to a multi-frequency signal.

The accuracy of the $f_1$ estimation is influenced by (Fig. 1a): the CiR width of the time window (relative to the period $T_1=1/f_1$), the signal phase angle $\varphi_1$ relative to the beginning of

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**Fig. 1.** The real-time inverter control unit in the photovoltaic system with a sliding time window: a) the general system scheme with the definition of the sliding window CiR width and the $x(t)$ signal angle phase $\varphi_1$ relative to the window beginning (or equivalently the angle phase $\varphi_1+0.5\text{CiR}$ relative to the window center); b) the DSP block diagram of the control unit along with the selected influence of the A/D converter quantization noise with a sampling frequency $f_s$; c) the algorithm to estimate the fundamental frequency $f_1$ of the grid signal $x(t)$. 

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the time window (or equivalently the phase $\varphi_i + 0.5CiR \times 2\pi$ [rad] of the signal relative to the center of the time window) and signal interferences, which may be deterministic disturbances or random noise.

The aim of this study is to answer the question of how quantization (caused by using the A/D converter) influences the estimation accuracy and how to optimally select the number of the A/D converter bits $b$ (Fig. 1b) using the discussed method (Fig. 1c). Therefore, it is assumed in further analysis that the quantization noise of the A/D converter is the dominant source of signal interferences. It is also worth noting that the low-pass filter or band-stop filter that is visible in Fig. 1c (the denoising digital filter) can remove a number of signal interferences that influence the accuracy of the $f_1$ estimation, but it has no effect on the level of quantization noise brought by the A/D converter. The selection of the number of bits $b$ depends on the parameter $CiR$ and $\varphi_1$ (equivalently $\varphi_i + 0.5CiR \times 2\pi$), and it must take into account the following limitations:

– the value of $CiR$ cannot be in practice greater than the maximum trip time defined in the standards (Tab. 1), which means that the $CiR$ value should not exceed the 2-2.5 value (50-33.3 ms); in practice, the $CiR$ value should be smaller to shorten the response time of the system for the appearance of system operation irregularities, but the smaller the $CiR$ value is, the greater the influence of the interferences (including quantization noise) is on the $f_1$ estimation accuracy and other parameters.

– the angle phase $\varphi_1$ (or $\varphi_i + 0.5CiR \times 2\pi$) in the continuous tracking system of the AC signal parameters changes its value in the full range (from 0 to $2\pi$ rad), and how the angle phase influences the $f_1$ estimation accuracy should be determined. In particular, the least favorable cases should be determined for the estimation accuracy.

In the case of the multi-frequency signal (which occurs in the grid), proper filtration is important before the estimation (denoising digital filter in Fig. 1c). Because of the limited trip times, the filtration must also be conducted very fast to maintain a good performance.

<table>
<thead>
<tr>
<th>Voltage range [%]</th>
<th>Maximum trip time [s]</th>
<th>Voltage range [%]</th>
<th>Maximum trip time [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V &lt; 50$</td>
<td>0.100</td>
<td>$V &lt; 50$</td>
<td>0.100</td>
</tr>
<tr>
<td>$50 \leq V &lt; 88$</td>
<td>2.000</td>
<td>$50 \leq V &lt; 85$</td>
<td>2.000</td>
</tr>
<tr>
<td>$110 \leq V &lt; 137$</td>
<td>2.000</td>
<td>$110 \leq V &lt; 135$</td>
<td>2.000</td>
</tr>
<tr>
<td>$V \geq 137$</td>
<td>0.033</td>
<td>$V \geq 135$</td>
<td>0.050</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Frequency range [Hz]</th>
<th>Maximum trip time [s]</th>
<th>Frequency range [Hz]</th>
<th>Maximum trip time [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$59.3 &gt; f &gt; 60.5$</td>
<td>0.100</td>
<td>$49 &gt; f &gt; 51$</td>
<td>0.200</td>
</tr>
</tbody>
</table>

Normal operation:
- $88 \% (105.6 \text{rms}) \leq V \leq 110 \% (132 \text{rms})$
- $59.3 \text{Hz} \leq f \leq 60.5 \text{Hz}$

3. Influence of the sliding time window’s phase

The photovoltaic system in Fig. 1 operates in real time and in continuous mode, i.e., the sliding of the time window is performed with a small step. The last $N$ samples of the signal are stored in the DSP memory because they are needed to calculate the current frequency estimate. The frequency value is updated every 4 sampling periods (167 $\mu$s) [13]. The high influence on the systematic error $\delta f_1$ has $\varphi_1$ and $CiR$ (but also $b$, $N$, $k$ and $H$). This dependence can be reduced to a value equal to $\varphi_1 + 0.5CiR \times 2\pi$ [rad], which is the signal angle phase relative to the sliding window center and not the sliding window’s beginning.
The systematic error plot depends on the signal phase shift (without quantization) and has local maxima relative to the sliding window center: one smaller for \( mt + \pi/2 \) (\( m \) – integer) and one larger – the global maximum – for \( mt \) (Fig. 2a). For integer \( CiR \) values (1 or 2), the systematic error is equal to 0 (in practice the error is non-zero because of the finite arithmetic precision used). The maximum of the systematic error calculated for \( \varphi_1 = 0, ..., 2\pi \) at a given \( CiR \) value is inversely proportional to \( N^4 \) (Fig. 2b). The minimum of the systematic error can be obtained for \( k = 1 \) for \( CiR < 2.08 \) and \( k = 2 \) for \( CiR \geq 2.08 \). Increasing the number of samples at the constant \( CiR \) value reduces this error. Analogous to the Cramer-Rao bound behavior for the frequency estimator, the systematic error significantly increases when the \( CiR \) value is smaller than ca. 0.8 bin.

It is necessary to use the A/D converter to convert the analog signal to a digital signal. Unfortunately, in the process, undesirable quantization noise is present, and it is the greater than the smaller number of A/D converter bits \( b \) (the number of bits on which the conversion result is stored in the U2 code).

Computer simulations were performed in the Matlab environment, and they include the study of the influence of \( N, CiR \) and \( \varphi_1 + 0.5CiRx2\pi \) on the grid signal frequency estimation in two cases: 1) the A/D converter with an infinite number of bits was used (no quantization, the number of bits \( b \rightarrow \infty \)) and 2) the A/D converter with a finite number of bits \( b \geq 8 \) was used.

The accuracy of the estimation is affected by the grid signal phase shift relative to the time window, and the position of the estimation error maxima is fixed relative to the time window center i.e., the phase \( \varphi_1 + 0.5CiRx2\pi \) (Fig. 2a). For small values of \( CiR \) (less than 1), the largest estimation error occurs when the signal value is zero in the time window center (case \( \varphi_1 + 0.5CiRx2\pi = 0, \pi, 2\pi, ... \)). For larger values of \( CiR \) (above 1), the situation is reversed (Fig. 3). The accuracy of the estimation is also affected by the number of A/D converter bits \( b \), the value of \( CiR \) and the number of samples \( N \) (Fig. 3). The estimation error decreases when the values of these parameters increase. The choice of \( b \) is of particular importance. An increase in the number of bits by 8 causes a decrease in the estimation error of approximately \( 2^{14} = 256 \) times. The signal of the systematic error is periodic. Its shape is associated with the measurement window shifting and the grid signal periodicity.

Further studies describe the accuracy analysis of the frequency estimation method for the largest error case in the entire range of the grid signal phase shift for given \( b, CiR, k \) and \( H \).
parameters. It is the worst estimation accuracy case, and it is the most interesting case for practical applications of this method.

![Graphs showing the influence of A/D quantization on frequency error](image)

Fig. 3. The influence of the number of A/D converter bits $b$ on the relative frequency error for:

- a) $N=64$
- b) $N=256$
- c) $N=1024$
- d) $N=4096$

4. The worst sliding window phase case

From a practical point of view, the most interesting case is the case of the worst estimation error depending on the grid signal phase (the maximum estimation error that may occur using the discussed method). The maximum level of the estimation error for the given $H$ value depends on the number of A/D converter bits $b$ and the Cir value, $N$ value and $k$ value (Fig. 4). The curves in Fig. 4 have a number of local maxima and minima because of the random nature of the errors caused by the quantization noise. It applies to those cases for which the dominant element is the error caused by the quantization noise, e.g., for $b=8$. 


The quantization of the grid signal causes the \( k \) value to shift from 1 to 2, for which the estimation error is the smallest (without quantization, this value for \( H=2 \) is 2.08). Errors due to quantization reduce this level. For \( N=32,…,4096 \) and \( b=8,…,30 \), the threshold value varies in the range from 1.75 to 1.99 (Tab. 2). The higher the \( b \) value, the smaller the effect of quantization on the threshold. This means that the influence of the systematic error on the estimation accuracy increases, while the influence of the error due to quantization decreases. For the practical use of the method (for \( N \geq 128 \) and \( b=8,…,24 \)), it can be assumed that the threshold value is 1.8.

Tab. 2. Recommended CiR thresholds for the change \( k \) from 1 to 2 depending on \( N \) and \( b \).

<table>
<thead>
<tr>
<th>( N )</th>
<th>8</th>
<th>10</th>
<th>12</th>
<th>14</th>
<th>16</th>
<th>18</th>
<th>20</th>
<th>22</th>
<th>24</th>
<th>26</th>
<th>28</th>
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<tr>
<td>32</td>
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<td>1.76</td>
<td>1.77</td>
<td>1.82</td>
<td>1.88</td>
<td>1.95</td>
<td>1.98</td>
<td>1.99</td>
<td>1.99</td>
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<td>1.99</td>
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<td>1.82</td>
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<td>1.92</td>
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<td>1.99</td>
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<tr>
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<td>1.76</td>
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<td>1.93</td>
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<td>1.87</td>
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<tr>
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5. Choose the number of bits for the A/D converter

The main aim of this paper is to show the influence of the number of A/D converter bits \( b \) in the system in Fig. 1 on the accuracy of the grid signal frequency estimation using the method from [13]. The estimation result is also influenced by the previously defined \( N \), CiR, \( H \) and \( k \) parameters. If the values of these parameters are known, the value of \( b \) can be set so that the estimation error is at a specific level. It is important from a practical point of view where to begin the experiment the A/D converter should be used with a specified number of bits.

Theoretically, the \( b \) value can be chosen so that the error caused by the grid signal quantization was comparable to the systematic error (for \( b\to\infty \)). Therefore, the total error (shown in Fig. 5a as a red dashed line) would be no greater than twice the systematic error (Fig. 5a). This means the choice of \( b \) determines whether the quantization maintains the total error at approximately the systematic error level. In practical applications, where \( N \) is e.g.,
512, the error caused by the use of conventional A/D converters (8-24 bits) is several times greater than the systematic error (Fig. 5b). Theoretically, the $N$ value can be decreased, but because of other factors, the value should be sufficiently large.

The total error $|\tilde{f}_1|$ is the sum of two components: the systematic error component $|\tilde{\delta}_{s,f_1}|$ (with the ideal sampling, $b \to \infty$) and the component $|\tilde{\delta}_{b,f_1}|$ caused by the quantization noise by using the A/D converter with a finite number of bits $b$:

$$\max_{\varphi} |\tilde{f}_1| = \max_{\varphi} |\tilde{\delta}_{s,f_1}| + \max_{\varphi} |\tilde{\delta}_{b,f_1}|,$$

where the rough upper limits of both components of the total error can be estimated using the following equations (Fig. 6):

$$\max_{\varphi} |\tilde{\delta}_{s,f_1}| \leq \begin{cases} 
\frac{40}{N^4 CiR^2} & \text{for } CiR < 0.5 \\
\frac{10}{N^4 CiR^4} & \text{for } 0.5 \leq CiR \leq 1, \\
\frac{10}{N^4 CiR^{0.8}} & \text{for } CiR \geq 1
\end{cases}$$

$$\max_{\varphi} |\tilde{\delta}_{b,f_1}| \leq \begin{cases} 
1.5 \cdot \frac{1}{\sqrt[4]{2^b N \cdot SNR_b}} & \text{for } CiR \leq 1.2 \\
0.87 \cdot \frac{1.1}{\sqrt[3]{N \cdot SNR_b}} & \text{for } CiR > 1.2
\end{cases}$$

Introduced here, $SNR_b = 1.5 \times 2^b$ is a well-known basic parameter that defines the $b$-bit A/D converter process dynamics. It is defined as the ratio of the sampled sinusoidal signal power to the power of the quantization noise that is modeled by a random variable with a uniform distribution.
6. Conclusions

The photovoltaic system consists of three main components: the solar panels, inverter and control unit. The quality of the generated signal is influenced by the accuracy of the grid signal frequency estimation.

This paper presents research on the dependence of the accuracy of the developed frequency estimation method [13] on the number of bits $b$, the A/D converter in the DSP system, the grid signal phase shift relative to the time window, the time window width relative to the grid signal period expressed as the $CiR$ parameter and the number of samples $N$ obtained in the time window. It is shown (Fig. 3) that the largest estimation error occurs when the signal module maximum is in the time window center (for small values of $CiR$ – below 1) or when the signal value is zero in the time window center (for large values of $CiR$). For these cases (Fig. 4), the equations to estimate the error depending on the $CiR$ value were determined. It is also shown in Tab. 2 that the $CiR$ value should change the $k$ value from 1 to 2 (for most practical cases, the threshold is 1.8). The upper bound of the estimation error is shown in (5)-(7), which demonstrates that the estimation error is the sum of two components: the systematic error, which is inversely proportional to the $N^4$ value, and the error caused by quantization noise, which is inversely proportional to the square root of the product of the $N$ value for the $SNR$ ratio used for the A/D converter. In practical applications ($N=32,\ldots,4096; b=8,\ldots,24$), the dominant component of the frequency estimation error is the error caused by the quantization noise. For $CiR > 1.2$, it is approximately $8\times10^{-10} \ldots 6\times10^{-4}$. For $CiR \leq 1.2$, it significantly increases when the $CiR$ values decrease and is inversely proportional to $CiR^3$ (Fig. 6, Eqn. 7).

The results allow for the optimal selection of the number of A/D converter bits in the control unit for the renewable energy system and the estimation of the error level for the grid signal frequency estimation.

References

