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THE IMPACT OF STOCHASTIC LEAD TIMES
ON THE BULLWHIP EFFECT – AN EMPIRICAL INSIGHT

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ABSTRACT
In this article, we review the research state of the bullwhip effect in supply chains with stochastic lead times. We analyze problems arising in a supply chain when lead times are not deterministic. Using real data from a supply chain, we confirm that lead times are stochastic and can be modeled by a sequence of independent identically distributed random variables. This underlines the need to further study supply chains with stochastic lead times and model the behavior of such chains.

KEYWORDS
supply chain, bullwhip effect, inventory policy, lead time demand, order-up-to-level policy, stochastic lead time, demand forecasting, lead time forecasting, lead time demand forecasting.

Introduction

Supply chains consist of firms (supply chain members) which act to deliver a product to the end-customers. Supply chain members optimize their objectives ignoring the efficiency of the supply chain, and this potentially results in a poor performance of the supply chain [1]. Thus, optimum local policies of members do not result in a global optimum of the chain, and they yield the tendency of replenishment orders to increase in variability as one moves upstream in the chain. Forrester [2] first formalized this effect in the middle of the twentieth century, and Procter & Gamble management coined the term "bullwhip effect". The bullwhip effect is recognized as one of the main inefficiencies because of its consequences that are (see, e.g., [3]): excessive inventory investment, poor customer-service levels, lost revenue, reduced productivity, more difficult decision-making, sub-optimal transportation, sub-optimal production, and so forth. Thus, the fundamental target of supply chain research is to identify the causes of the bullwhip effect, to quantify the

increase in order variability at each stage of the supply chain and offer methods to reduce this variability. In recent studies, the main causes of the bullwhip effect are given as (see, e.g., [4, 5]) demand forecasting, non-zero lead time, supply shortage, order batching, price fluctuation, and lead-time forecasting [6, 7]. To reduce bullwhip, one needs to identify all factors causing the bullwhip effect and to quantify their impact on the effect.

Many different theoretical models have been constructed to quantify the bullwhip effect. Jointly, these models assume deterministic lead times and study the influence of different methods of demand forecasting on the bullwhip effect, such as simple moving average, exponential smoothing, and minimum-mean-squared-error forecasts when demands are independent, identically distributed (i.i.d.) or constitute integrated moving-average, autoregressive processes or autoregressive-moving averages [8–15]. It follows from these contributions that lead time is a central parameter influencing the magnitude of the bullwhip effect. Non-deterministic lead times are investigated intensively in inventory sys-

tems [16–24]. Lead time can be treated as a parameter of lead-time demand (for an overview of models with varying and stochastic parameters see [25]). Most contributions consider the so-called exogenous lead times, where the assumption is that lead times do not depend on the system state, for example, when lead times are independent of the orders or the capacity utilization of a supplier. These contributions focus on the impact of lead time on the control parameters, the inventory levels, or the costs. So and Zheng [26] investigate the so-called endogenous lead times that is, lead times depending on the system. They show the amplification of the order variance using computer simulation. Moreover, stochastic lead time yields new problems such as the effect of the so-called order crossover which means that replenishments arrive in a different sequence than ordered (see, e.g., [27, 28] and the references therein). Wang and Disney's paper [27] investigates the proportional and order-up-to-level policies of replenishment in the presence of stochastic lead times and crossovers. Nielsen et al. [29] shows that order crossover is highly likely to occur in any reasonably structured supply chain following a replenishment strategy, and Nielsen et al. [29] shows that if order crossovers do occur, observed lead times no longer necessarily have i.i.d. structures. From this, one must recognize that supply chain management and the behavior of the bullwhip effect is, in fact, highly complex under stochastic lead times. Therefore, the main aim of this contribution is to review papers devoted to stochastic lead times in supply chains in the context of the bullwhip effect with a special emphasis on those which quantify the effect. We analyze real lead times of a real supply chain and confirm that the lead time can be modeled by a sequence of independent and identically distributed random variables as many models assume. In Table 1, we collect all the main articles that provide models of the bullwhip effect with stochastic lead times (except the famous works of Chen et al. [10, 11] where deterministic lead time is considered and some analyze the effect using simulation). The remainder of the paper is structured as follows. First, a discussion of the bullwhip effect is presented along with the common definition of it. Second, we present a brief study of real lead times in a supply chain documenting their nature and before concluding remarks are included.

Supply chains and the bullwhip effect

A supply chain is considered as the system of organizations, people, activities, information, and resources involved in moving a product or service from suppliers to customers. More precisely in the physi-

cal sense, a supply chain consists of customers, retailers, warehouses, distribution centers, manufacturers, plants, raw material suppliers, and so forth. In the typical supply chain, the assumption is that every member of the chain possesses a storehouse and uses a certain stock policy (a replenishment policy) in its inventory control to fulfill its customer (a member of the supply chain which is right below) orders promptly. Commonly used replenishment policies are: the periodic review, the replenishment interval, the order-up-to level policy (out policy), (s, S) policy, the continuous review, the reorder point, and the proportional order-up-to-level policy (see, e.g., [30] and, for the last policy, [27] and the references therein). The order-up-to level policy is optimal in the sense that it minimizes holding costs and backlog costs if there are no crossovers [28, 31]. A member of a supply chain observes demands from the stage below and lead times from the stage above. Based on the previously observed demands and lead times and using a certain stock policy, each member of a chain places an order to its supplier. The phenomenon of the variance amplification in replenishment orders if one moves up in a supply chain is called the bullwhip effect [32, 33] for the definition and historical review). Munson et al. [34] asserts, "When each member of a group tries to maximize his or her benefit without regard to the impact on other members of the group, the overall effectiveness may suffer". The bullwhip effect is the major contributor of a supply chain inefficiency.

A very popular measure of the bullwhip effect is the ratio of variances, that is, if q is a random variable describing orders of a member of the supply chain to a member above and D is a random variable responsible for demands of the member below (e.g., q describes orders of a retailer to a manufacturer and D shows customer demands to the retailer) then the measure of performance of the bullwhip effect is the following:

$$
\begin{aligned}
BM &= \frac{Var(\text{orders})/\mathbb{E}(\text{orders})}{Var(\text{demands})/\mathbb{E}(\text{demands})} \\
&= \frac{Var q/\mathbb{E}q}{Var D/\mathbb{E}D},
\end{aligned}
\tag{1}
$$

where ($\mathbb{E}$ and *Var* mean expected value and variance of a given random variable. In many models, we have $\mathbb{E}q = \mathbb{E}D$. If the value of BM is greater than 1, then the bullwhip effect is observed in a supply chain. If BM is equal to 1 then there is no variance amplification whereas BM smaller than 1 indicates dampening which means that the orders are smoothed compared to the demands showing a push rather than pull supply chain. The net stock amplification of

Table 1
Articles on the impact of lead time on the bullwhip effect.

Article	Demands	Lead times	Forecasting
Chen et al. [10]	AR(1)	deterministic	moving average of demands
Chen et al. [11]	AR(1)	deterministic	Expon. Smoothing of demands
Chaharsooghi and Heydari [37]	deterministic	i.i.d.	–
Chatfield et al. [38]	AR(1)	i.i.d.	moving average of lead-time demands
Kim et al. [39]	AR(1)	i.i.d.	moving average of lead-time demands
Duc et al. [15]	AR(1) ARMA(1,1)	i.i.d.	the minimum-mean-squared–error forecast of demands
Fioriolli et al. [40]	AR(1)	i.i.d.	moving average of demands
Michna and Nielsen [6]	i.i.d.	i.i.d.	moving average of demands and lead times
Reiner and Fichtinger [41]	dependent	i.i.d.	moving average of demands and lead times
So and Zheng [26]	AR(1)	mutually dependent	the minimum-mean-squared–error forecast of demands
Wang and Disney [27]	ARMA(p,q)	i.i.d.	the minimum-mean-squared–error forecast of demands

a given supply chain member is another very important measure of the supply chain efficiency.

Let N_S be the level of the net stock of a supply chain member and D be demands observed from its downstream member (customers or a retailer) then the following measure

$$NSM = \frac{Var(\text{net stock})}{Var(\text{demands})} = \frac{Var(N_s)}{VarD} \qquad (2)$$

is also considered as a critical performance measure. In many models, it is assumed that the costs are proportional to $\sqrt{\mathbf{Var}(\text{orders})}$ and $\sqrt{\mathbf{Var}(N_s)}$. Under this assumption, the order-up-to-level replenishment policy is optimal in that it minimizes costs if lead times do not cross over. However, the proportional order-up-to-level policy outperforms the out policy if there are crossovers [26]. Order crossover is the phenomenon of orders being received in a different sequence than they are placed. Several newer studies [29, 35] show that there is a significant likelihood of this occurring. Likewise, from Nielsen et al. [36] we know that there is a significant impact on the lead-time demand uncertainty due to this.

Establishing real lead-time behavior

Despite a number of contributions underlining that lead times are one of the main causes of the bullwhip effect, limited literature exists investigating actual lead-time behavior. Most research to date focuses on lead-time demand. Added to this focus it often assumed that lead times are constant or that lead times are independent identically distributed (i.i.d.). To support the assumptions used in references [6, 15, 38, 39], that is, that lead times are i.i.d. – the lead-time behavior from a manufacturing company is analyzed as an example. The data used is 6,967 orders for one product varying in quantity ordered over a period of two years (481 work days) in a manufacturing company. On average, 14.5 orders are received per day in the period; each order is to an individual customer in the same geographical region. The following two tests are used to test whether lead times are, in fact, i.i.d.

1. Autocorrelation (see, e.g., [42]) for independence of the lead-times: This is done on the average lead time per day as the individual orders cannot be ordered in time periods smaller than one day.

2. Kolmogorov-Smirnov test (see, e.g., [43]) is applied. The test is a widely used robust estimator for identical distributions [43]. The method (as seen in Fig. 1) relies on comparing samples of lead times and using the Kolmogorov-Smirnov test to determine whether these pairwise samples are identical. In this research, a 0.05 significance level is used. The ratio of pairwise comparisons that pass this significance test is the output from the analysis. To see the level of stability different sample sizes are used to determine if the lead times can be assumed to be similarly distributed in smaller time periods. This allows one to determine whether it is fair to sample previous lead-time observations to estimate lead-time distribution for planning purposes.

For a detailed account of the method, please refer to Nielsen et al. [44].

An autocorrelation (top) and partial autocorrelation (bottom) plots are found in Fig. 2 which shows that the average lead times per day can, for all practical purposes, be considered mutually independent.

There may be some minor indications that the average lead time on a given day depends slightly on recent average lead times observed in the set. However, the correlation coefficients are small (approximately 0.1), and the penalty for assuming independence seems slight in this case.

$$SL_1 = \{t | t \in [1,Z]\} \quad \text{Vs.} \quad SL_2 = \{t | t \in [Z+1,2Z]\}$$

$$SL_3 = \{t | t \in [2, Z+1]\} \quad \text{Vs.} \quad SL_4 = \{t | t \in [Z+2, 2Z+1]\}$$

$$\ldots \qquad \qquad \ldots$$

$$SL_{2J-1} = \{t | t \in [J, Z+J-1]\} \quad \text{Vs.} \quad SL_4 = \{t | t \in [Z+J, 2Z+J-1]\}$$
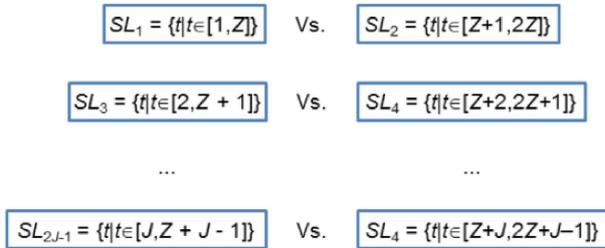
Fig. 1. Sample and comparison procedure using Kolmogorov-Smirnov test for pairwise comparisons.
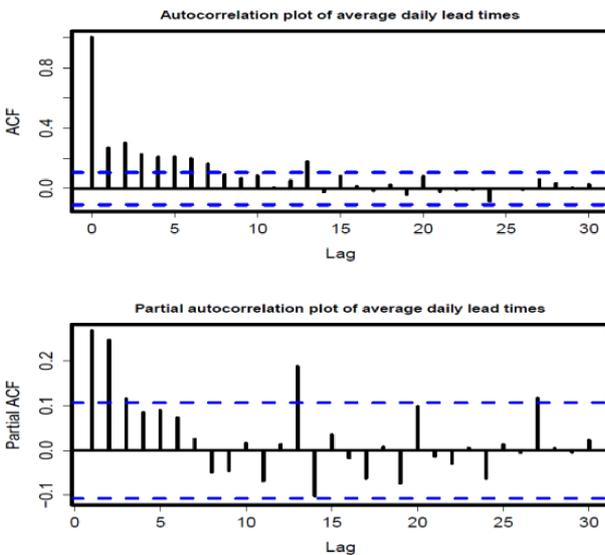

Fig. 2. Top: auto correlation plot of average lead time per day; bottom: partial auto correlation plot of average lead time per day.

Figure 3 shows that even for large samples (500 observations compared with the following 500 observations) most of the comparisons are found to be statistically similar on a 0.05 or better level. This supports the assumptions that the lead times are, in fact, identically distributed. The overall conclusion is that in the examined case it is not wrong to assume that lead times are in fact i.i.d. The investigation also underlines that it is a grave oversimplification to assume that lead times are constant for individual orders. There is also no guarantee that lead times are in fact i.i.d. in any and all contexts.
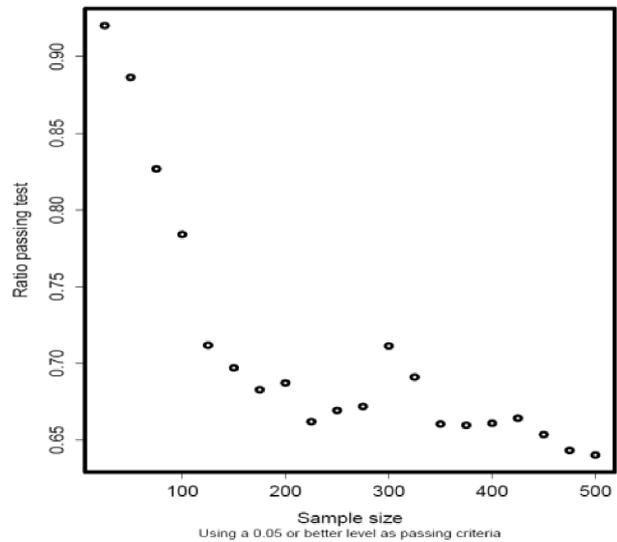

Fig. 3. Sample and comparison procedure using Kolmogorov-Smirnov test for pairwise comparisons.

Concluding remarks

This research aims to present findings from the current state of supply-chain research with an emphasis on the bullwhip effect under stochastic lead times. The literature established that lead times and their behavior have a significant impact on the performance of supply chains regarding the bullwhip effect. Likewise, we can establish that there is a significant body of evidence supporting that lead times behave in a stochastic manner and that this behavior influences the performance of supply chains in the form of increased bullwhip effect.

Several avenues of future research seem to be potentially fruitful. The first is establishing the impact of stochastic lead times on complex supply chains. We note that most contributions to the field of supply chain research and stochastic lead times have focused on two echelon systems. A second avenue would be to obtain better data and conduct more studies of actual lead times in real supply chains. These studies would be able to support the future modeling of supply chains.

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