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An Adaptive Vibration Control Procedure Based on Symbolic Solution of Diophantine Equation

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In this paper, the adaptive control based on symbolic solution of Diophantine equation is used to suppress circular plate vibrations. It is assumed that the system to be regulated is unknown. The plate is excited by a uniform force over the bottom surface generated by a loudspeaker. The axially-symmetrical vibrations of the plate are measured by the application of the strain sensors located along the plate radius, and two centrally placed piezoceramic discs are used to cancel the plate vibrations. The adaptive control scheme presented in this work has the ability to calculate the error sensor signals, to compute the control effort and to apply it to the actuator within one sampling period. For precise identification of system model the regularized RLS algorithm has been applied. Self-tuning controller of RST type, derived for the assumed system model of the 4th order is used to suppress the plate vibration. Some numerical examples illustrating the improvement gained by incorporating adaptive control are demonstrated.

Keywords: adaptive control, vibration cancellation, Diophantine equation, self-tuning control, RLS algorithm.

1. Introduction

Conventional control system design is often not sufficient for controlling a process with parameters that are unknown or which may vary significantly during operation. Thus, the necessity for precise control leads to deriving various control theories. Adaptive control is one of those research fields that are emerging as important class of controller design. Recent advances in computational capabilities and modern digital signal processing techniques made the real-time implementation of adaptive control algorithms more practical. In the paper, this technique is used to suppress circular plate vibrations, a system assumed to be unknown. In practice, the vast majority of processes to be controlled are neither stationary nor linear systems and change their characteristics over time or when the set point changes. Those systems are, in fact, unknown.

Generally, adaptive control is a set of techniques for the automatic, on-line adjustment of control-loop regulators. Several approaches to solving this problem have been reported (Appolinario, 2009; Diniz, 2008; Isserman, 1982), however, the number of practical applications is still low, especially for high order systems. Among all the adaptive control algorithms there are the so-called *self-tuning controllers* (STC) which are a promising alternative for the still preferred classic methods of control and regulation.

A self-tuning controller can be very effective in providing the expected controller performance (Bobál, *et al.*, 2000; Leniowska, Kos, 2009; Macháček, Bobál, 2002). This technique is usually employed when the controlled process parameters are unknown, as it was assumed in case of the circular plate considered here. The STC controller is able to use the measured data to identify the model of the controlled process on-line and then adjust its coefficients according to the feedback from error signals. The area of adaptive control has close connections with *system identification* (Ljung, Söderström, 1983; Söderström, Stoica, 1994). The methods developed in system identification are widely used in adaptive control in order to estimate the unknown parameters of the system model in the closed loop by recursive identification algorithms. The choice of identification algorithm is made on the basis of a number of performance indices, such as convergence rate, computational complexity, error robustness, and numerical stability. The least mean square (LMS) and its normalized version (NLMS) have been used extensively in many applications and they can be described as the 'workhorses' of adaptive control (Appolinario, 2009; Latos, Pawelczyk, 2010) Unfortunately, they have slow asymptotic convergence rates (Appolinario, 2009), hence, in this paper, it was the Recursive Least Squares algorithm (RLS) that was applied to the identification problem.

The goal of this work is to describe an adaptive control procedure that simplifies the implementation and improves the performance of feedback active control on a complex structure where many sensors and actuators are needed to achieve acceptable vibration control performance. The basic philosophy is the recursive identification of the best model for the controlled process and the subsequent synthesis of the controller. This mean that the approach using STC controller is possible if we can derive appropriate formulas for on-line correction of controller coefficients (Bobál *et al.*, 2000; Leniowska, Kos, 2009). Several approaches to solving this problem have been reported, however, much effort has been devoted to meeting specific practical requirements. A more general approach is a self-tuning controller method proposed by Bobál and his colleagues (Bobál *et al.*, 2000; Macháček, Bobál, 2002) which contains detailed recipes, as well as appropriate formulas for STC controller adjustment. On the other hand, the transfer functions of the considered processes models are simple and of low orders (they do not exceed the third order), therefore they are inapplicable for more complex objects. In case of the considered planar structures, characterized by several resonance frequencies, this approach seems to be insufficient for effective vibration cancellation. Thus, in this paper more powerful formulas for tuning controller parameters are proposed, the ones which promise their successful implementation.

The purpose of this paper is to describe a novel version of the self-tuning controller (STC) that addresses practical issues arising when control is implemented on a planar structure as a plate. The approach is based on the recursive identification of the 4th order model made with the use of regularized RLS algorithm and the subsequent synthesis of the controller with RST structure of the 3rd order. The coefficients of this controller are calculated using explicit formulas obtained by solving *Diophantine equation* symbolically with the Maple software. The detailed scheme in the case of 4th order system model is described step-by-step and simulations of plate vibration cancelation are included.

2. Identification of the system

The first step, the determination of the structure of a parametric system model, is an important one to do before going on to design an adaptive control algorithm. The capabilities of the adaptive control depend on the faithfulness with which the model represents the system. For small displacements the input/output model of a structure can be reasonably represented by a linear finite difference model. If the plate displacement is large or the control effort becomes too great, the input/output model may become nonlinear. Even for nonlinear systems, the linear finite difference model can be shown to be a reasonable approximation over a small region of interest.

The structure of the model in use, commonly called the Auto-Regressive moving average model with exogenous input (ARX), is shown below (Ljung, Söderström, 1983; Söderström, Stoica, 1994):

$$y(k) = z^{-d}\frac{\mathbf{B}(z^{-1})}{\mathbf{A}(z^{-1})}u(k) + \frac{1}{\mathbf{A}(z^{-1})}\mathrm{Z}(k), \tag{1}$$

where z^{-1} is a unit delay operator, k is a well-accepted form for representing a discrete time index and

$$\mathbf{A}(z^{-1}) = 1 + a_1 z^{-1} + \ldots + a_{nA} z^{-nA}, \tag{2}$$

$$\mathbf{B}(z^{-1}) = b_1 + b_2 z^{-1} \ldots + b_{nB} z^{-nB+1}. \tag{3}$$

The least square method can be applied to the standard model of regression. Therefore, the ARX model of the controlled system is as follows:

$$y(k) = \boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{\varphi}(k) + \mathrm{Z}(k), \qquad \text{for each} \quad k, \tag{4}$$

where

$$\boldsymbol{\theta} = [a_1, \ldots, a_{nA}, \;\; b_1, \ldots, b_{nB}]^{\mathrm{T}} \tag{5}$$

is the estimated vector of the model parameters and

$$\boldsymbol{\varphi}(k) = [-y(k-1), \ldots, -y(k-nA), \; u(k-d), ..., u(k-d-nB+1)]^{\mathrm{T}} \tag{6}$$

is the regression vector collecting the system output and input variables.

The mathematical model (4) can be derived with the use of any variant of LMS algorithms proposed in literature (Appolinario, 2009; Diniz, 2008; Ljung, Söderström, 1983; Söderström, Stoica, 1994). The two most important performance criterions of adaptive algorithms used in identification are the convergence speed and the misadjustment. Classical identification methods are related to the steady-state mean square error (MSE) criterion. It is well known that MSE decreases when the step-size is reduced, similarly, the convergence speed increases when the step-size is raised. By optimally selecting the step-size during the adaptation, we can obtain both fast convergence rate and low steady-state mean square error. Another feature that should be noticed in adaptive control algorithms is their computational complexity. Several adaptive controllers with a fixed or variable step-size have been proposed to reduce the computational complexity. However, as a rule, there is a tradeoff between the attainable convergence speed, misadjustment values and computational complexity. Taking into account all the relevant aspects of the issue, the widely used Recursive Least Squares (RLS) algorithm (Ljung, Söderström, 1983) was adopted in this paper. As the problem is ill-posed, to reduce possible instability of the algorithm, its regularized version (Leniowska, Kos, 2009; Tikhonov, Arsenin, 1997), has been chosen.

Experimental data have been acquired in similar way as in previously described papers (Leniowska, 2008, 2009), with sampling time of 0.0001 sec (10 kHz) on the multi-channel system working under real-time operating system (RTAI) by four strain gauges located along the plate radius, as can be seen in Fig. 1. Figure 2 shows the graph of power spectrum recorded signal response.

Fig. 1. Circular plate with sensors and actuators.

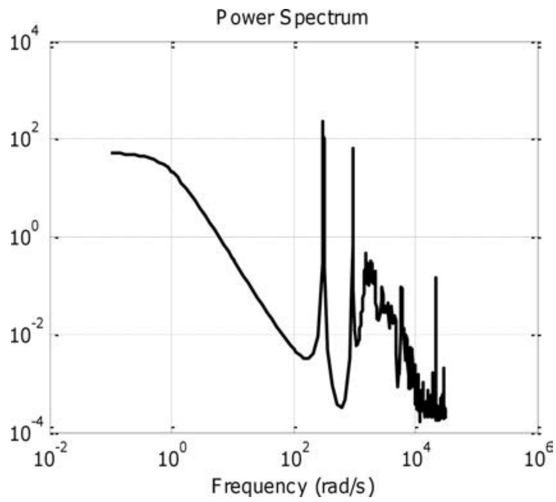

Fig. 2. The power spectrum of open-loop system.

The result of system identification for the off-line ARX model of high order is shown in Fig. 3. This model was used to generate discrete output in performed simulations of vibration cancelation.

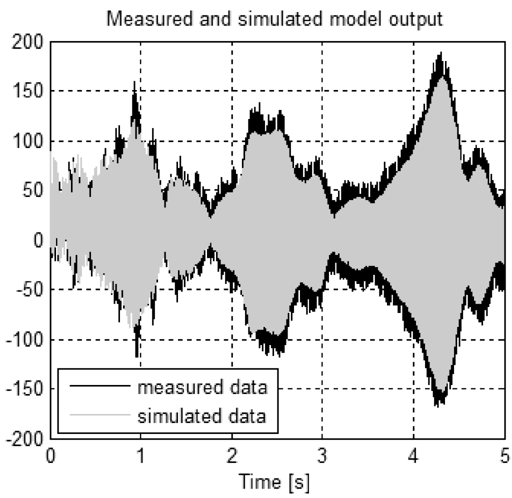

Fig. 3. The experiment data and ARX model response.

3. Adaptation algorithm

In order to reduce the effects of the disturbance on the system, one must devise the way to make future plant outputs work. It is desirable to express the future outputs as a linear combination of past plant outputs and past control efforts.

The main task of the designed adaptation controller is minimizing the error signal $e(k)$ which is calculated as the difference between the desired setpoint $r(k)$ and the output of the system. In considered case, the most desired value of the reference signal $r(k)$ equals zero (vibration cancellation).

For the object described by ARX model (1)–(3) we seek a controller transfer function expressed as the ratio of polynomials $Q(z^{-1})/P(z^{-1})$, which is adjusted, so that the closed-loop system shown in Fig. 4 can fit the desired criterion.

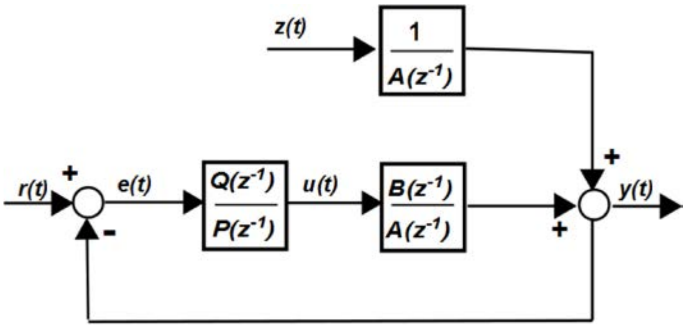

Fig. 4. Scheme of the system.

Considering the structure of control loop, one can get transfer function of closed system as:

$$G_R = \frac{Y(z^{-1})}{W(z^{-1})} = \frac{B(z^{-1})Q(z^{-1})}{A(z^{-1})P(z^{-1}) + B(z^{-1})Q(z^{-1})}. \tag{7}$$

Controller synthesis consists in solving linear polynomial, the so-called *Diophantine equation*, of the general form $AX + BY = C$. Firstly, the main task is to choose the location of the poles of the dominator of Eq. (7), which takes a form of the *Diophantine equation*:

$$A(z^{-1})P(z^{-1}) + B(z^{-1})Q(z^{-1}) = D(z^{-1}). \tag{8}$$

By appropriate selection of coefficient of the polynomial $D(z^{-1})$ one can fix the desired poles locations for closed-loop system as

$$D(z^{-1}) = \sum_{i=0}^{nd} d_i z^{-i} = d_0 + \sum_{i=1}^{nd} d_i z^{-i}. \tag{9}$$

The purpose of the method is to design a controller, so that all poles of the closed-loop system assume prescribed values, providing stability and good performance. Generally, we seek a controller of order n, satisfying the polynomial Diophantine equation (8):

$$G_R = \frac{Q(z^{-1})}{P(z^{-1})} = \frac{q_0 + q_1 z^{-1} + \ldots + q_n z^{-n}}{1 + p_1 z^{-1} + \ldots + p_n z^{-n}}. \tag{10}$$

In theory, if the system is controllable, the poles and zeros can be placed anywhere to improve closed system performance. It can be done analytically by solving the linear system of equation with $(m+n) \times (m+n)$ non-singular Sylvester matrix. In practice, there are numerical difficulties for solving a higher-order Diophantine equation (Peeters, Hanzon, 1998), and for this reason it is extremely hard to solve it in real-time applications.

One way to overcome this problem is to apply explicit formulas if they can be obtained by solving the equation symbolically. There are various approaches to the Sylvester equations in controller companion form (Peeters, Hanzon, 1998). The efficient *symbolic* solution of this problem very much depends on the algebraic structure of the expressions that constitute the entries of the matrices and vectors. In this paper the author used methods based on Cramer's Rule (Peeters, Hanzon, 1998) for solving linear systems, which was successfully applied with the use of Maple software.

For the considered system the digital controller of order $n = 3$ and with RST structure was applied. It can be described by transfer function as shown below

$$G_R = \frac{Q(z^{-1})}{P(z^{-1})} = \frac{q_0 + q_1 z^{-1} + q_2 z^{-2} + q_3 z^{-3}}{1 + p_1 z^{-1} + p_2 z^{-2}}. \tag{11}$$

It was also assumed, that the system in analysis is represented by discrete transfer function:

$$G_O = \frac{B(z^{-1})}{A(z^{-1})} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4}} \tag{12}$$

with denominator of order $m = 4$. This assumption provides that the estimated model has at least two resonant frequencies. For the assumed values of the system and controller orders, the linear equation with Sylvester matrix has a form:

$$\begin{bmatrix} 1 & 0 & 0 & \mathrm{b}_0 & 0 & 0 & 0 \\ \mathrm{a}_1 & 1 & 0 & \mathrm{b}_1 & \mathrm{b}_0 & 0 & 0 \\ \mathrm{a}_2 & \mathrm{a}_1 & 1 & \mathrm{b}_2 & \mathrm{b}_1 & \mathrm{b}_0 & 0 \\ \mathrm{a}_3 & \mathrm{a}_2 & \mathrm{a}_1 & \mathrm{b}_3 & \mathrm{b}_2 & \mathrm{b}_1 & \mathrm{b}_0 \\ \mathrm{a}_4 & \mathrm{a}_3 & \mathrm{a}_2 & 0 & \mathrm{b}_3 & \mathrm{b}_2 & \mathrm{b}_1 \\ 0 & \mathrm{a}_4 & \mathrm{a}_3 & 0 & 0 & \mathrm{b}_3 & \mathrm{b}_2 \\ 0 & 0 & \mathrm{a}_4 & 0 & 0 & 0 & \mathrm{b}_3 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ q_0 \\ q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}, \tag{13}$$

where x_i for $i = 0, 1, 2, 3, 4, 5, 6$ denotes appropriate poles location. If the target is zero vibration level, the polynomial $D(z^{-1})$ should be as simple as possible. This condition is valid for $D(z^{-1}) = 1$. The solution of Diophantine equation obtained in symbolic form allows to calculate the coefficients of the designed regulator on-line, during the process control. As a result of solving linear system equations (13) one should get unknown parameters of digital controller described by (11): q_0, q_1, q_2, q_3, p_1, p_2, which allow to calculate the output control signal $u(k)$:

$$u(k) = q_0 y(k) + q_1 y(k-1) + q_2 y(k-2) + q_3 y(k-3) - p_1 u(k-1) - p_2 u(k-2). \qquad (14)$$

The acquired formulas are not very complex computationally – for instance, two of them are shown below:

```
p1 = (-b34*a1-b1*b32*a4*b2+b3*b23*a3*a1+b32*a22*b2*b1
     -b32*a2*b22*a1+a4*b23*a2*b1+b33*b1*a3+b33*a12*b2-a4*b24*a1
     -2*b32*b1*a3*a1*b2-2*b12*b3*a4*b2*a2+3*b1*b3*a4*b22*a1
     -b3*b22*a3*a2*b1-a4*b12*a3*b22+b2*a32*b12*b3+b2*a42*b13)/den;

p2 = b3*(a42*b13+a32*b12*b3-a4*b12*a3*b2-2*a4*b12*b3*a2
     -2*a3*a1*b1*b32+3*a4*b1*b3*a1*b2-a2*a3*b1*b2*b3+b32*a22*b1
     +a4*b1*b32+a4*b1*a2*b22+b33*a12+b3*a1*b22*a3-b32*a2*a1*b2
     -a4*b23*a1+b32*a3*b2-a4*b22*b3-b33*a2)/den;
```

where the following substitutions have been applied:

```
den = a42*b14+a32*b13*b3-a4*b13*a3*b2-2*a4*b13*b3*a2
      -2*b32*a3*b12*a1+3*a4*b12*b3*a1*b2-b3*a2*b12*a3*b2
      +b32*a22*b12+2*a4*b12*b32+a4*b12*a2*b22+b33*a12*b1
      +b3*a1*b22*a3*b1-b32*a2*b1*a1*b2-a4*b1*b23*a1+3*b32*a3*b1*b2
      -4*a4*b1*b22*b3-2*b33*a2*b1-b33*a1*b2-a3*b23*b3+b34
      +a4*b24+b32*a2*b22;

      a12=a1*a1;    a13=a12*a1;    a22=a2*a2;
      a32=a3*a3;    a33=a32*a3;    a42=a4*a4;

      b12=b1*b1;    b13=b12*b1;    b14=b13*b1;
      b22=b2*b2;    b23=b22*b2;    b24=b23*b2;
      b32=b3*b3.
```

Concluding, the adaptive control procedure described here consists of the following steps:

Step 1: The identification algorithm forms the ARX system model of the 4th order. The coefficients of the polynomials $A(z^{-1})$ and $B(z^{-1})$ are computed by regularized RLS algorithm according to formulas shown in (Leniowska, Kos, 2009).

Step 2: The parameters of the polynomial $P(z^{-1})$ and $Q(z^{-1})$ are calculated using explicit formulas obtained by solving Diophantine equation symbolically.

Step 3: The control law is computed according to expression (14) and applied to the system.

Some numerical examples which demonstrate the improvement gained by incorporating the described adaptive control are shown below.

4. Simulations results and conclusions

In order to obtain the best possible agreement with a real object, which makes the implementation more realistic, firstly, on a basis of the acquired measurements, a very high order model of considered plate was determined. Subsequently, to examine the feasibility of derived formulas for self-tuning controller, in order to generate output signals very similar to real object dynamic characteristics, the attained model has been applied in simulations.

Figures below show results of simulations of the adaptive control procedure with the use of STC controller of 3rd order, for different input signals, obtained with Simulink/Matlab computer program. To examine the effect of the STC controller on plate vibration suppression, the model was first subjected to a sinusoidal signal with constant amplitude and frequency of 100 Hz and 1000 Hz.

It can be seen in Fig. 5 that the uncontrolled plate response vibrates significantly, while switching on the STC controller (after 4 sec) causes plate vibrations to be reduced about 90% in case of low frequency disturbances (100 Hz). However, when the disturbance signal of 1000 Hz is applied (Fig. 6), the vibration cancelation is worse and does not exceeded 30%. This result is caused by the problems with stable estimation of the assumed model if the frequency of disturbance is higher. Figures 5b and 6b contain plots of calculated on-line 4th order model coefficients which are adjusted to the incoming signal error.

In case, when the excitation signal frequency increases too rapidly, the calculation of the control signal can be more difficult and may lead to an increase in amplitude and even the loss of stability of the whole system. In the Fig. 7 the chirp signal was used to excite plate vibrations. The excitation signal frequency increased linearly during 100 seconds up to 400 Hz. It can be observed that the estimation of the model coefficients (Fig. 7b) is more difficult with ris-

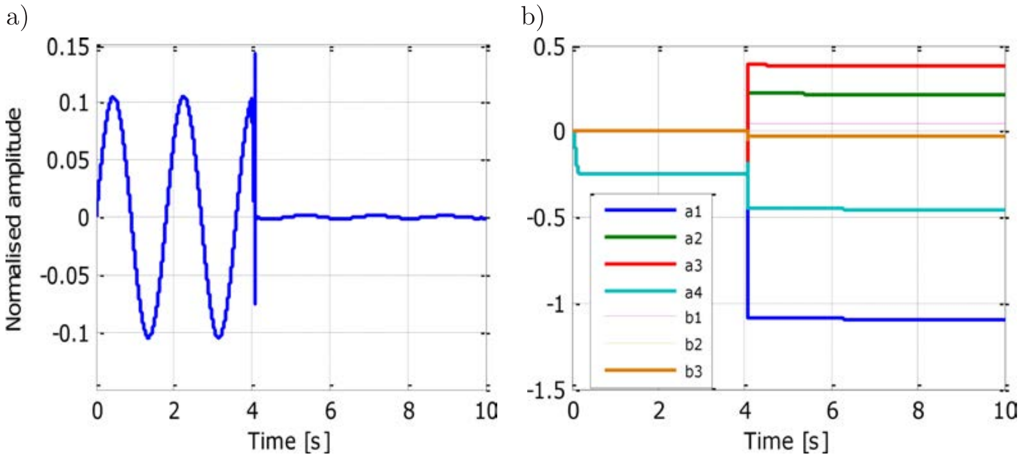

Fig. 5. Response of open-loop (0–4 s) and closed-loop (4–10 s) system for *sin* signal of 100 Hz; a) plate response; b) model coefficients.

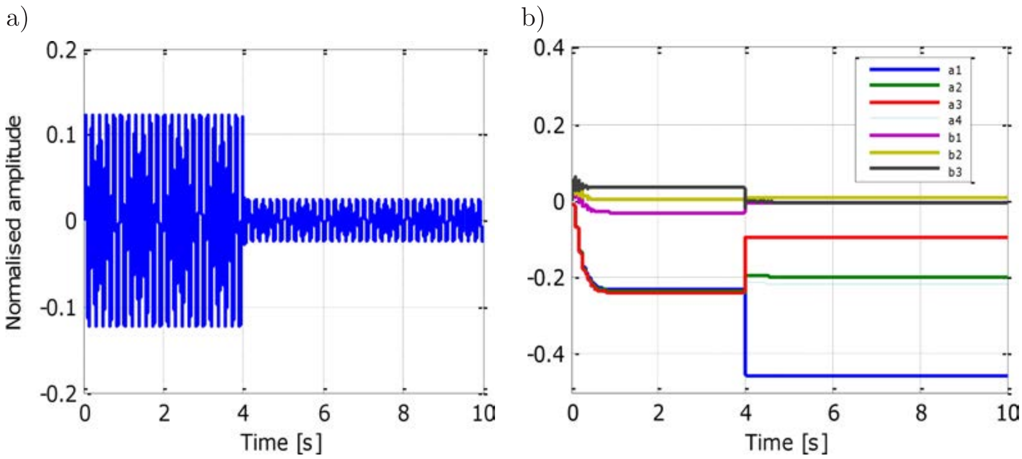

Fig. 6. Response of open-loop (0–4 s) and closed-loop (4–10 s) system for *sin* signal of 1000 Hz; a) plate response; b) model coefficients.

ing frequency (after about 70 s), and the quality of identification process has strongly degraded. It has substantial influence on STC parameters calculation and may be the reason why the controlled system becomes too sensitive or even not stable.

The main aim of the paper was to design the adaptive procedure with the self-tuning controller of 3rd order for circular plate vibration suppressing. In this procedure the 4th order ARX system model is used and STC type controller was derived to design the control law. The performed simulations show that the designed adaptive procedure causes substantial reduction of the plate vibrations for the excitation signals that are not fast-variable in time.

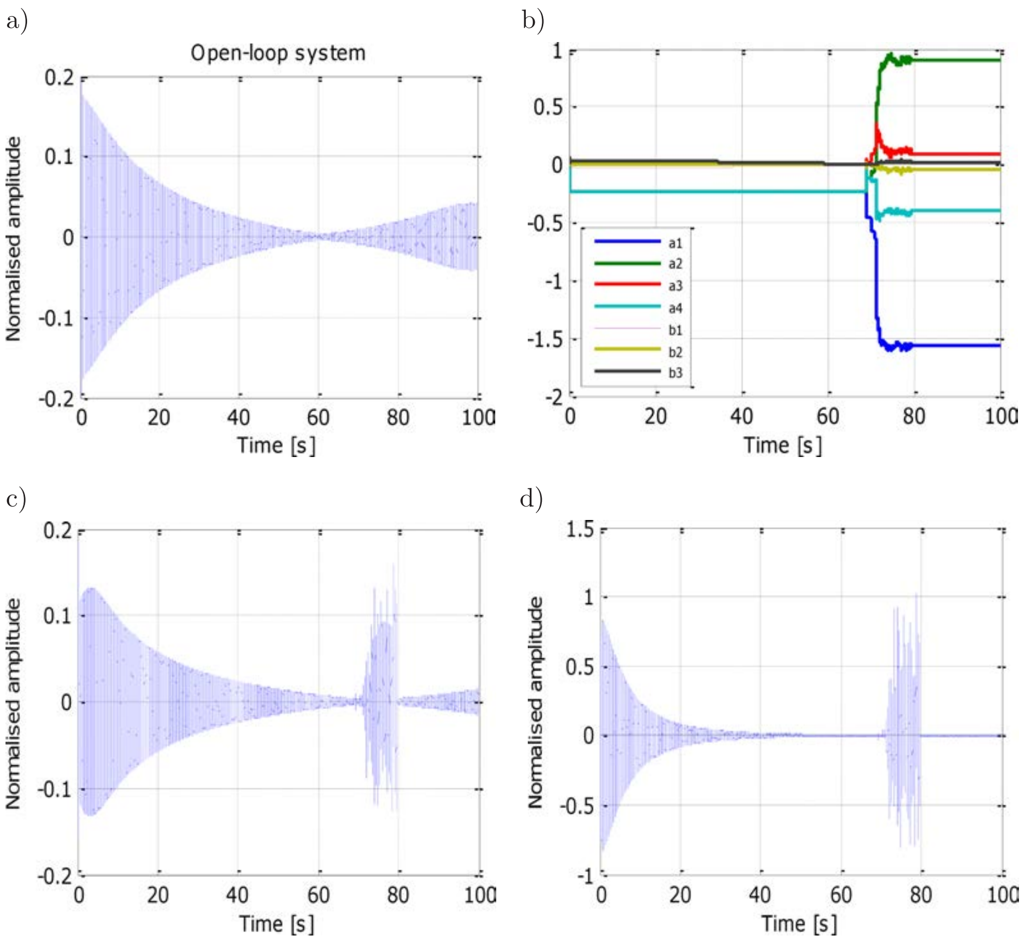

Fig. 7. Response of system for *chirp* signal of 0–400 Hz; a) open-loop plate response; b) model coefficients; c) closed-loop plate response; d) control signal.

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