Central European Journal of Economic Modelling and Econometrics

Real-Time Market Abuse Detection with a Stochastic Parameter Model

Radosław Cholewiński*

Submitted: 27.06.2009, Accepted: 25.12.2009

Abstract

This paper develops a new model of market abuse detection in real time. Market abuse is detected, as Minenna (2003) proposed, on the basis of prediction intervals. The model structure is based on the discrete-time, extended market model introduced by Monteiro, Zaman, Leitterstorf (2007) to analyze the market cleanliness. Parameters of the expected return equation are assumed, however, to be time-varying and estimated under the state-space framework using the extended Kalman filter postulated by Chou, Engle, Kane (1992) to capture the GARCH effect in returns. QML estimation is performed on intraday data; its utilization is proposed as an alternative to the continuous time modeling by Minenna (2003). This framework is generalized to the bivariate case which enables the analysis of daily open/close data. The paper also extends procedures of the statistical verification of the estimated state-space model to include the uncertainty arising from time-invariant parameters.

Keywords: Market abuse detection, insider trading, real-time analysis, time-varying parameters, uni- and bivariate GARCH processes

JEL Classification: C14, C32, C52, G14, G19.

*Noble Bank; e-mail: radoslaw.cholewinski@noblebank.pl.

1 Introduction

The issue of market abuse detection goes back at least to 1930s when US legal system prohibited insider trading, i.e. securities trading motivated by publicly unavailable, price-sensitive information (Meulbroek 1992). The term 'market abuse' refers, however, to a much broader scope of activities than insider trading exlusively, including in particular various forms of market manipulation. Market manipulation may be defined as distorting the price-setting mechanism of financial instruments (trade-based manipulation) or as disseminating false or misleading information (information-based manipulation). Such a 'broad' understanding of market abuse has been adopted by the legal system of the European Union. The definition of market abuse may be extended further to include e.g. brokerage frauds like 'front-running' (taking advantage of advance knowledge of pending orders).

The econometric literature has considered detection of market abuse, however, only since the early 1990's (e.g. Mitchell, Netter 1994). Financial econometrics, which has focused on insider trading, proposed a framework to identify the latter on the basis of security returns. A case of market abuse may be thus considered as an abnormal event, whose probability under standard market conditions is less than the significance level. The methodology to identify such events had been proposed much earlier by event studies. The problem, however, with the application of the event studies methodology is that it is *ex post* approach, which enables to detect market abuse cases regarding the past rather than the present. Moreover, an attempt to apply the event study methodology directly to the issue of market abuse puts too much emphasis on looking for a statistical 'proof' of the abuse, which is already suspected.

So far, only a handful of attempts have been made to develop the real-time fraud detection model as well as to utilize more data than the rates of return solely. Minenna (2003) proposed the continuous time model to detect market abuse on real-time basis. The model analyzes return rates, volumes and market structure characteristics (concentration measures), but relies on daily closing data only. The continuous time framework is intended to extrapolate closing values through to the entire next session. Such an approach might be justified when the only available data were the closing values. If it is not the case, the introduction (not necessarily trivial) of a continuous time model could be exchanged for the utilisation of intraday data. Moreover, the continuous time framework imposes some simplifying limitations on the model structure. That is particularly relevant in the Minenna's (2003) paper – there are no exogenous variables while all endogenous variables are assumed to follow an AR(1) process.

Another attempt to utilize data from more sources was made by Dubow and Monteiro (2006) who used a standard CAPM-like market model to measure market cleanliness. Abnormal events (in terms of security returns) are identified on the 'benchmark' basis, referring to the exogenous market performance. The same framework is applied to analyse trading volumes. The analysis, however, is conducted on *ex post* basis under the classical event study approach. Moreover, the model structure, though extended byMonteiro, Zaman, Leitterstorf (2007) to capture both autocorrelation and heteroskedasticity effects, may be still too restrictive to track the market evolution. This issue is of a particular importance in the case of emerging markets of CEE economies, although it has been not intensively discussed in the literature on the fraud detection models so far.

The aim of this paper is to develop a model that will both allow for a fraud detection on real-time basis and possibly closely replicate the evolving market structure. The latter requires a time-varying parameter model whilst the former requires intraday data as the model input. This framework may be generalized to analyze daily open/close data which is of a particular importance in identifying some manipulation cases.

The remainder of the paper is organized as follows: the model framework, including procedures to identify market abuse, is developed in Section 2. Estimation and statistical inference issues are discussed in Section 3. Section 4 presents some empirical results of the model's application to the Warsaw Stock Exchange and discusses model's ability to detect market abuse using the example of insider trading. Section 5 concludes.

2 Econometric framework

The basis for further analysis is the model proposed by Monteiro, Zaman, Leitterstorf (2007) to identify insider trading. The following model is an extension of the standard CAPM-like market model (see e.g. Alexander 2001) allowing for autocorrelation and heteroskedasticity in security returns:

$$R_t = \alpha + \beta R_{mt} + \rho R_{t-1} + \gamma R_{m,t-1} + \varepsilon_t,\ \varepsilon_t \sim \mathrm{NID}(0, \sigma_t^2) \tag{1}$$

$$\sigma_t^2 = \omega_0 + \omega_1 \varepsilon_{t-1}^2 + \omega_2 \sigma_{t-1}^2, \tag{2}$$

where R_t stands for the return on a given security between two subsequent closings, t and $t-1$, and R_{mt} is the return on the market portfolio in the same period. σ_t^2 denotes the conditional variance of the Gaussian white-noise error term ε_t. The conditional variance evolves according to a simple GARCH(1,1) equation.

The difference between the actual return, R_t, and its conditional expectation, $\alpha + \beta R_{mt} + \rho R_{t-1} + \gamma R_{m,t-1}$, defines the abnormal return. Following the event study methodology, Monteiro, Zaman, Leitterstorf (2007) identify insider trading when the average abnormal returns both around the day of information release ('announcement') and in the days preceding the information release ('pre-announcement period') are statistically significant as well as of the same sign. The approach by Monteiro, Zaman, Leitterstorf (2007) has, however, some disadvantages. Firstly, the event study methodology makes use of *ex post* analysis with prior knowledge about the moments of potential frauds ('announcements'). Thus, the econometrics is employed here to yield a kind of a 'statistical proof' of a suspected abuse, not to detect the abuse.

Secondly, this methodology can be applied to the detection of insider trading solely, as there is no prior knowledge of the timing of other frauds, like 'announcements' in the case of insider trading.

It may be worth stressing that although the above model captures both heteroskedasticity and autocorrelation, Monteiro, Zaman, Leitterstorf (2007) consider these two issues as phenomena that are of statistical rather than economic nature. This may be partially justified in the case of heteroskedasticity, since in the GARCH framework the realized volatility is mostly approximated as squared returns. As a consequence, the volatility analysis becomes equivalent to the return analysis. That is not the case of autocorrelation, which should not be present at all under the weak efficient market hypothesis. Autocorrelated security returns indicate a distortion of the market efficiency, which might be associated with some market abuse activities (mainly trade-based manipulation) unless the market had been inefficient prior to the study. Thus, autocorrelation analysis may offer a great deal of information for market abuse investigations.

Minenna (2003) proposed real-time fraud detection based on the predicted return intervals as an alternative to *ex post* analysis. Given R_t as the return on a given security, an alert about the possible market abuse would be generated once R_t fell outside the interval:

$$[\hat{R}_{t|t-1} - z_{\alpha/2} \cdot \hat{\sigma}_{\hat{R}_{t|t-1}},\ \hat{R}_{t|t-1} + z_{1-\alpha/2} \cdot \hat{\sigma}_{\hat{R}_{t|t-1}}], \tag{3}$$

where $\hat{R}_{t|t-1}$ is the forecast of R_t based on the data available up to $t-1$, $\hat{\sigma}_{\hat{R}_{t|t-1}}$ is the RMSE of that forecast and $z_{\alpha/2}, z_{1-\alpha/2}$ stand for the quantiles of the error term distribution of order $\alpha/2$ and $1-\alpha/2$ respectively.

The forecast of R_t is computed on the basis of some reference model, which needs to be adapted to the continuous-time framework. Minenna (2003) develops a continuous-time model for the security price which is estimated from daily closing data. Under the continuous-time framework, the model structure has to remain, however, relatively simple (in Minenna's model security prices are driven exclusively by the one-dimensional Wiener process). Such an approach could be advocated if the only available data were closing prices; when it is not the case a reasonable alternative is to use a less sophisticated (i.e. discrete-time) model and more (i.e. intraday) data.

The above alternative might be criticized, since intraday data indicates specific statistical features, as the autocorrelation in returns and in squared returns. The usually low though significant autocorrelation of low order may result from market microstructure effects, like bid-ask spreads, while higher-order autocorrelation may be associated with the well-known intraday periodicity phenomena (see Alexander 2001 for a review of stylized facts concerning intraday returns). The autocorrelation in squared returns happens in turn to be quite high and decays at a very slow pace, which suggests a higher-order GARCH should be employed. If these features are, however, common for all assets at the given market, then they could be implicitly incorporated into the reference model by including the market portfolio return as the explanatory variable.

Such a solution calls for the application of the model by Monteiro, Zaman, Leitterstorf (2007), which in addition captures the autocorrelation in returns. Under such a framework, the random term corresponds to abnormal returns rather than to 'raw' returns, and its intraday properties do not have to replicate those of 'raw' returns. In particular, if the higher-order autocorrelation of squared returns on individual assets might be explained by the analogous pattern of market portfolio squared returns (so it is common for all assets), then the variance structure of the random term (specific for a given asset) might evolve according to a GARCH process of a lower order. For simplicity, the GARCH(1,1) structure will be further assumed. This presumption needs obviously an empirical confirmation. Moreover, intraday data should be still adjusted in terms of time and volatility units, which is rather a technical issue, see e.g. Engle (2000).

The problem with the model of Monteiro, Zaman, Leitterstorf (2007) arises when computing one-period-ahead return forecasts in real time. The return on market portfolio is known at the same time as the returns on individual assets. Thus, the one-period-ahead return forecast cannot be computed *ex ante* and becomes available only when the actual return is observed. In the real-time framework based on prediction intervals it is however the difference between the actual return and its forecast which is necessitated to identify the abnormal return. This difference becomes known only after the actual return is observed, regardless the model consists of exogenous variables or not. Under the continuous time framework by Minnena (2003) this difference is calculated on a permanent (real-time) basis. Under the discrete time framework, which is postulated here, this difference can be calculated on an interval basis, depending on the choice of the time unit. The shorter the intervals, the more this approach resembles the continuous time framework. Owing to that fact, and since asset returns are real-time data, which is accessible immediately after the asset price is set, this approach will be still referred to as the real-time analysis. The maximum delay between potential market abuse and its discovery under such a framework will be bounded by the chosen time interval.

Utilizing intraday data should not, however, imply equally weighting all data points. In many financial markets, especially in the stock exchange, trading does not operate on a fully continuous basis. Discontinuous trading requires separate handling of opening and closing prices. Opening prices reflect all orders placed after the preceding closing while 'regular' intraday prices usually correspond to single orders. Closing prices are often determined during the fixing through an algorithm imitating the Walrasian auctioneer, which distinguishes them from ordinary market prices.

Separate handling of opening and closing data can be done relatively easily by a generalization of the model (1) – (2):

$$R_{0,t} = \alpha_0 + \beta_0 R_{0,mt} + \rho_0 R_{1,t-1} + \gamma_0 R_{0,m,t-1} + \varepsilon_{0,t} \tag{4}$$

$$R_{1,t} = \alpha_1 + \beta_1 R_{1,mt} + \rho_1 R_{0,t} + \gamma_1 R_{1,mt} + \varepsilon_{1,t} \tag{5}$$

$$\boldsymbol{\varepsilon_t} = [\varepsilon_{0,t}\ \varepsilon_{1,t}]' \sim \mathrm{N}(\mathbf{0}, \mathbf{H_t}), \tag{6}$$

where $R_{0,t}$, $R_{0,mt}$ denote the returns between the opening on day t and the preceding closing on the given security and on the market portfolio respectively. $R_{1,t}$ and $R_{1,mt}$ stand for the returns between the closing and the opening on day t on the security and the market portfolio respectively. $\mathbf{H_t}$ denotes the variance-covariance matrix of $\boldsymbol{\varepsilon}_\mathbf{t}$ which evolves according to a multivariate GARCH process of the general VECH form:

$$\mathrm{vech}(\mathbf{H_t}) = \mathbf{C} + \mathbf{A}\mathrm{vech}(\boldsymbol{\varepsilon}_\mathbf{t-1}\boldsymbol{\varepsilon}'_\mathbf{t-1}) + \mathbf{B}\mathrm{vech}(\mathbf{H_{t-1}}). \tag{7}$$

Although index t refers to different time units depending whether the model for intraday or open/close data is used, it will be left unchanged to avoid further complications of already complicated notation.

The original model of Monteiro, Zaman, Leitterstorf (2007), which is postulated here to analyze intraday data and then extended for opening and closing data, is estimated under two restrictive assumptions. Firstly, one has to assume the error term follows the Gaussian distribution, which is clearly not what is observed on financial markets. The model estimates would still be unbiased and consistent even if the underlying distribution was not Gaussian (the variance-covariance matrix of estimates needs, however, to be adjusted for the non-normality of the error term distribution). In the later part of the text no particular distribution of the error term will be assumed and only the first and the second central moment will be specified; this approach is commonly referred to as semi-parametric (see e.g. Engle 2000). Under that approach a problem arises when testing for the significance of abnormal returns, which are not Gaussian. That may be however solved easily using the well-established bootstrapping techniques, see e.g. Efron (1987). Secondly, the model parameters are assumed to be constant over the time. Numerous studies of CEE capital markets (Emerson, Hall, Zalewska-Mitura 1997; Zalewska, Hall 1999; Rockinger, Urga 2000; Worthington, Higgs 2003) show this assumption does not hold. Models that do not capture the market evolution may misidentify abnormal returns.

Therefore, parameters of models (1) − (2) and (4) − (7) should be treated as the realization of a stochastic process. Most research up to date (e.g. Chou, Engle, Kane 1992; Zalewska, Hall 1999; Rockinger, Urga 2000) suggest the random walk process as a plausible way to incorporate the time variation in model parameters:

$$\xi_t = \xi_{t-1} + \upsilon_t, \;\; \upsilon_t \sim \mathrm{D}(0, \sigma_\xi^2), \tag{8}$$

where ξ_t represents any structural parameter of those models and υ_t denotes the innovation to the process, which is independently distributed with a zero mean and variance σ_ξ^2. The capital letter D denotes some probability distribution. In the limit case when $\sigma_\xi^2 \to 0$, the parameter ξ_t becomes constant for all $t, t = 1, \ldots, T$. Note that parameters of GARCH equations (2) and (7) remain intact. Then, the complete model can be rewritten as:

$$R_t = \alpha_t + \beta_t R_{mt} + \rho_t R_{t-1} + \gamma_t R_{m,t-1} + \varepsilon_t,\ \ \varepsilon_t \sim \mathrm{D}(0, \sigma_t^2) \tag{9}$$

$$\xi_t = \xi_{t-1} + \upsilon_t, \upsilon_t \sim \mathrm{D}(0, \sigma_\upsilon^2),\ \ \xi \in \{\alpha, \beta, \rho, \gamma\} \tag{10}$$

$$\sigma_t^2 = \omega_0 + \omega_1 \varepsilon_{t-1}^2 + \omega_2 \sigma_{t-1}^2 \tag{11}$$

in the case of intraday data, and:

$$R_{0,t} = \alpha_0 + \beta_0 R_{0,mt} + \rho_0 R_{1,t-1} + \gamma_0 R_{0,m,t-1} + \varepsilon_{0,t} \tag{12}$$

$$R_{1,t} = \alpha_1 + \beta_1 R_{1,mt} + \rho_1 R_{0,t} + \gamma_1 R_{1,mt} + \varepsilon_{1,t} \tag{13}$$

$$\xi_t = \xi_{t-1} + \upsilon_t, \upsilon_t \sim \mathrm{D}(0, \sigma_\xi^2),\ \ \xi \in \{\alpha_0, \beta_0, \rho_0, \gamma_0, \alpha_1, \beta_1, \rho_1, \gamma_1\} \tag{14}$$

$$\boldsymbol{\varepsilon_t} = [\varepsilon_{0,t}\ \varepsilon_{1,t}]' \sim \mathrm{D}(\mathbf{0}, \mathbf{H_t}) \tag{15}$$

$$\mathrm{vech}(\mathbf{H_t}) = \mathbf{C} + \mathbf{A}\mathrm{vech}(\boldsymbol{\varepsilon_{t-1}\varepsilon'_{t-1}}) + \mathbf{B}\mathrm{vech}(\mathbf{H_{t-1}}) \tag{16}$$

in the case of open/close data. The MSE for one-step-ahead forecasts of R_t and $R_{0,t}, R_{1,t}$ should be then used to standardize residuals and find (through resampling) the estimates of error term distribution quantiles which are necessary for the construction of prediction intervals. Similarly, MSE of estimates of ρ_t and $\rho_{0,t}, \rho_{1,t}$ enables to test (using a simple t-statistic) the significance of autocorrelation in security returns. The next section discusses the estimation of above models and the computation of MSE's.

3 Model structure and estimation

The model developed in the previous section, both in the uni- and bivariate case, may be estimated using the Kalman filter with some extension to capture the GARCH effect in disturbances (Chou, Engle, Kane 1992). Kalman filtering requires the model to be expressed in a state-space form (the following notation is based on Hamilton 1994):

$$\boldsymbol{\xi_{t+1}} = \mathbf{F}(\mathbf{x_t})\boldsymbol{\xi_t} + \mathbf{u_{t+1}} \tag{17}$$

$$\mathbf{y_t} = \mathbf{a}(\mathbf{x_t}) + [\mathbf{Z}(\mathbf{x_t})]'\boldsymbol{\xi_t} + \mathbf{v_t}, \tag{18}$$

where $\boldsymbol{\xi_t}$ denotes $r \times 1$ vector of state variables (time-varying parameters), $\mathbf{y_t}$ – $n \times 1$ vector of endogenous variables (security returns) and $\mathbf{x_t}$ – $k \times 1$ vector of exogenous or predetermined variables respectively (market portfolio returns, lagged security and market portfolio returns). Matrices $\mathbf{F}(\mathbf{x_t})$ and $\mathbf{Z}(\mathbf{x_t})$ as well as vector $\mathbf{a}(\mathbf{x_t})$ of dimension $r \times r$, $r \times n$ and $n \times 1$ respectively are functions of vector $\mathbf{x_t}$. Vectors of disturbances to both transition (17) and observation (18) equation are denoted by $\mathbf{u_{t+1}}$ and $\mathbf{v_t}$, with dimensions $r \times 1$ and $n \times 1$ respectively.
In the univariate case of intraday data ($n = 1$), vector $\mathbf{y_t}$ is reduced to scalar R_t

and matrix $\mathbf{Z}(\mathbf{x_t})$ equals simply $\mathbf{x_t} = [1\ R_{mt}\ R_{t-1}\ R_{m,t-1}]'$ provided all structural parameters are time-varying ($r = 4$). Otherwise, $\mathbf{Z}(\mathbf{x_t})$ is extracted from $\mathbf{x_t}$ by taking those terms, for which structural parameters are time-varying. Function $\mathbf{a}(\mathbf{x_t})$ is defined as a sum of products of the time-invariant structural parameters and the corresponding terms from $\mathbf{x_t}$. If $r = 4$, $\mathbf{a}(\mathbf{x_t})$ simplifies to zero. Random vector $\mathbf{v_t}$ simplifies in the univariate case to the error term, ε_t.
In the bivariate case of open/close data ($n = 2$), vector $\mathbf{y_t}$ equals $[R_{0,t}\ R_{1,t}]'$ while $\mathbf{Z}(\mathbf{x_t})$ is the following matrix:

$$\begin{bmatrix} 1 & R_{0,mt} & R_{1,t-1} & R_{1,m,t-1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & R_{1,mt} & R_{0,t} & R_{0,mt} \end{bmatrix}',$$

unless some structural parameters are constant. If so, $\mathbf{Z}(\mathbf{x_t})$ is extracted from (3) by taking those rows of that matrix, for which structural parameters are time-varying. Function $\mathbf{a}(\mathbf{x_t})$ is defined *per analogiam* as a sum of products of the time-invariant structural parameters and the corresponding elements of (3). If there are no time-invariant parameters, function $\mathbf{a}(\mathbf{x_t})$ is simply zero. Random vector $\mathbf{v_t}$ in the bivariate case is equivalent to $[\varepsilon_{0,t}\ \varepsilon_{1,t}]$.
In both cases of intraday and open/close data, the random walk structure of (8) implies $\mathbf{F}$ is the $r \times r$ identity matrix:

$$\mathbf{F} = \mathbf{I}. \tag{19}$$

Under the textbook approach, both random vectors $\mathbf{u_{t+1}}$ and $\mathbf{v_t}$ of dimensions $r \times 1$ and $n \times 1$ are normally distributed conditional on the data available in the preceding period, $\mathcal{I}_{t-1} = \{\mathbf{y_{t-1}}, \mathbf{x_{t-1}}, \ldots, \mathbf{y_1}, \mathbf{x_1}\}$, and the current value of $\mathbf{x_t}$. Under the semi-parametric approach proposed in this paper the analytical form of the probability distribution function is not specified explicitly whilst the parameters of the distribution, i.e. the first moment and the second central moment, are assumed to be known:

$$\left[\begin{array}{c|c} \mathbf{u_{t+1}} & \\ \mathbf{v_t} & \mathbf{x_t}, \mathcal{I}_{t-1} \end{array}\right] \sim \mathrm{D}\left(\begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \begin{bmatrix} \mathbf{Q}(\mathbf{x_t}) & \mathbf{0} \\ \mathbf{0} & \mathbf{H}(\mathbf{x_t}) \end{bmatrix} \right), \tag{20}$$

where matrices $\mathbf{Q}(\mathbf{x_t})$ and $\mathbf{H}(\mathbf{x_t})$ of dimension $r \times r$ and $n \times n$ respectively are both functions of $\mathbf{x_t}$.
Matrix $\mathbf{H}(\mathbf{x_t})$ represents the conditional variance-covariance matrix of the disturbances in the observation equation (18) which under the GARCH framework depends on the past disturbances rather than $\mathbf{x_t}$. In the univariate case $\mathbf{H}(\mathbf{x_t})$ equals simply σ_t^2. In the bivariate case it will be referred to shortly as $\mathbf{H}$. $\mathbf{Q}(\mathbf{x_t})$ is the variance-covariance matrix of disturbances in the transition equation (innovations to time-varying parameters) which is constant over the time, $\mathbf{Q}(\mathbf{x_t}) \equiv \mathbf{Q}$. Provided disturbances are independent, $\mathbf{Q}$ is diagonal.
Standard Kalman filter algorithm, see e.g. equations 13.8.6 – 13.8.9 in Hamilton (1994), may be then applied do obtain estimates of unknown time-varying parameters $\boldsymbol{\xi}_\mathbf{1}, \ldots, \boldsymbol{\xi}_\mathbf{T}$. This algorithm, however, does not conform with the GARCH effect

in disturbances in the observation equation. Chou *et al.* (1992) proposed therefore to extend the standard Kalman filter algorithm by a slightly modified GARCH equation:

$$\sigma_t^2 = \omega_0 + \omega_1 e_{t-1}^2 + \omega_2 \sigma_{t-1}^2, \tag{21}$$

where the lagged error term ε_{t-1} was replaced by the lagged residual e_{t-1}. Under the Kalman filter framework, this residual denoted by $\mathbf{\hat{v}_{t|t-1}}$ is defined as the difference between the one-step-ahead forecast of $\mathbf{y_t}$ and its actual value. Although the analysis by Chou *et al.* (1993) considers the univariate model, the obvious generalization would handle the bivariate case as well:

$$\mathrm{vech}(\mathbf{H_t}) = \mathbf{C} + \mathbf{A}\mathrm{vech}(\mathbf{e_{t-1}e'_{t-1}}) + \mathbf{B}\mathrm{vech}(\mathbf{H_t}), \tag{22}$$

where $\mathbf{e_{t-1}}$ denotes the two-dimensional counterpart of e_{t-1}.
Equation (22) involves, however, 21 parameters to be estimated. A more parsimonious parametrization would be the diagonal VECH model with 9 unknowns (see e.g. Alexander 2001); this model unfortunately does not guarantee the variance-covariance matrix to be both positive-definite and stationary. These two restrictions may be imposed with the BEKK model by Baba, Engle, Kraft and Kroner (1995) which extends the number of parameters to 11. The latter may be decreased to 7 if the model is estimated in the reduced, diagonal form, which may be somehow restrictive.
In the author's opinion, a plausible solution to the trade-off between the flexibility of the model structure and the parsimony of the parametrization is the CCC (constant conditional correlation) model, restricting the conditional correlation between the disturbances to be constant over time:

$$\mathbf{H_t} = \begin{bmatrix} \sigma_{0,t}^2 & \varrho \cdot \sigma_{0,t}\sigma_{1,t} \\ \varrho \cdot \sigma_{0,t}\sigma_{1,t} & \sigma_{1,t}^2 \end{bmatrix} \tag{23}$$

$$\begin{bmatrix} \sigma_{0,t}^2 \\ \sigma_{1,t}^2 \end{bmatrix} = \mathbf{C} + \mathbf{A} \odot \begin{bmatrix} e_{0,t}^2 \\ e_{1,t}^2 \end{bmatrix} + \mathbf{B} \odot \begin{bmatrix} \sigma_{0,t-1}^2 \\ \sigma_{1,t-1}^2 \end{bmatrix}, \tag{24}$$

where $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are here 2×1 vectors. $\odot$ denotes element-by-element multiplication. Matrix $\mathbf{H_t}$ is positive-definite for all $\varrho \in [-1, 1]$ if only $\sigma_{0,\tau}^2 \neq \sigma_{1,\tau}^2$. To ensure the variance-covariance matrix to be stationary one has only to check whether the following inequality holds:

$$\mathbf{A} + \mathbf{B} \leq \mathbf{1}. \tag{25}$$

If ϱ is not statistically significant and set to zero, the CCC model becomes equivalent to the two simultaneously estimated GARCH processes.
The Kalman filter assumes all time-invariant parameters of the state-space model to be known. As this is here not the case, those are estimated through the numerical

optimization of the log-likelihood function, which takes the following form:

$$\ell = \sum_{t=1}^{T} \log f_{\mathbf{Y_t}|\mathbf{X_t},\mathcal{I}_{t-1}}(\mathbf{y_t}|\mathbf{X_t},\mathcal{I}_{\mathbf{t-1}}) = \tag{26}$$

$$= \sum_{t=1}^{T} -\frac{n}{2}\log(2\pi) - \frac{1}{2}\log|\mathbf{W_t}| - \frac{1}{2}\hat{\mathbf{v}}_{\mathbf{t}}'\mathbf{W_t}^{-1}\hat{\mathbf{v}}_{\mathbf{t}}. \tag{27}$$

Filter residuals are denoted here according to the general notation of the state-space model by $\hat{\mathbf{v}}_{\mathbf{t}}$. Of course, $\hat{\mathbf{v}}_{\mathbf{t}}$ simplifies to e_t in the case of intraday data and is equivalent to $\mathbf{e_t}$ for open/close data. $\mathbf{W_t}$ denotes the MSE of the one-step-ahead forecast of $\mathbf{y_t}$.

The likelihood function is then maximized numerically with respect to $\boldsymbol{\theta}$. If both random vectors $\mathbf{u_{t+1}}$ and $\mathbf{v_t}$ were conditionally normal, the estimated vector of model parameters would follow a normal distribution as well. Watson (1989) shows that under some general conditions this distribution is asymptotically normal even if the underlying distribution of disturbances is not Gaussian. This semi-parametric estimation method is known as Quasi-Maximum Likelihood (QML). Collect all unknown model parameters in $\boldsymbol{\theta}$ and denote its estimate by $\hat{\boldsymbol{\theta}}$, then:

$$\sqrt{T}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}) \xrightarrow{L} \mathrm{N}\left(\mathbf{0}, [\varphi_{\mathbf{2D}}\varphi_{\mathbf{OP}}^{-\mathbf{1}}\varphi_{\mathbf{2D}}]^{-1}\right), \tag{28}$$

where:

$$\boldsymbol{\varphi}_{\mathbf{2D}} = \operatorname{plim} -\frac{1}{T}\sum_{t=1}^{T} \left.\frac{\partial^2 \log f(\mathbf{y_t}|\mathbf{x_t},\mathcal{I}_{t-1};\boldsymbol{\theta})}{\partial\boldsymbol{\theta}\,\partial\boldsymbol{\theta}'}\right|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}} \tag{29}$$

$$\boldsymbol{\varphi}_{\mathbf{OP}} = \operatorname{plim} \frac{1}{T}\sum_{t=1}^{T} [\mathbf{h_t}(\boldsymbol{\theta})][\mathbf{h_t}(\boldsymbol{\theta})]' \tag{30}$$

$$\mathbf{h_t}(\boldsymbol{\theta}) = \left.\frac{\partial \log f(\mathbf{y_t}|\mathbf{x_t},\mathcal{I}_{t-1};\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}\right|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}}. \tag{31}$$

Sums on the RHS of equations (29) – (30) can be interpreted as the Hessian of ℓ with respect to $\boldsymbol{\theta}$ and its approximation by the outer product of gradients respectively. It follows from (28):

$$\mathbb{E}(\hat{\boldsymbol{\theta}}) = \boldsymbol{\theta} \tag{32}$$

$$\mathbb{V}(\hat{\boldsymbol{\theta}}) = \frac{1}{T}\cdot[\boldsymbol{\varphi}_{\mathbf{2D}}\boldsymbol{\varphi}_{\mathbf{OP}}{}^{-1}\boldsymbol{\varphi}_{\mathbf{2D}}]^{-1}. \tag{33}$$

Standard errors of individual elements of $\hat{\boldsymbol{\theta}}$ are computed as square-roots of diagonal elements of $\mathbb{V}(\hat{\boldsymbol{\theta}})$. Their significance may be tested with a simple t-statistic. Since the limit distribution of $\hat{\boldsymbol{\theta}}$ is Gaussian, the test statistic follows a t distribution with

$T-k$ degrees of freedom in the limit, where k is the length of $\boldsymbol{\theta}$. For large T the distribution of $\hat{\boldsymbol{\theta}}$ converges to the normal distribution.

Note, however, that under both non-Gaussian disturbances and time-varying parameters, Kalman filter becomes suboptimal in terms of MSE associated with the one-step-ahead forecast of $\mathbf{y_t}$. Most research on the Kalman filter application to financial market models has paid no particular attention to that issue, which may be explained by the fact there is no general alternative to estimate such models.

Having the above model estimated, one still has to compute the MSE associated both with estimates of state variables and one-step-ahead forecast of $\mathbf{y_t}$, which is crucial for identifying both significant autocorrelation and abnormal returns. Standard formulas for the MSE of the estimate of $\boldsymbol{\xi}_\mathbf{t}$ based on $\mathcal{I}_{t-1}$, $\hat{\boldsymbol{\xi}}_{\mathbf{t|t-1}}$, and the forecast of $\mathbf{y_t}$ conditional on $\mathcal{I}_{t-1}$ and $\mathbf{x_t}$, $\hat{\mathbf{y}}_{\mathbf{t|t-1}}$, do not account for uncertainty of model parameters, which are assumed to be known in the basic Kalman filter framework. Hamilton (1986) proposed to decompose the MSE of $\hat{\boldsymbol{\theta}}$, denoted by $\mathbf{P_{t|t-1}}(\hat{\boldsymbol{\theta}})$ in two parts: one, that would be present if the model parameters were known with certainty, $\mathbf{P_{t|t-1}}$, which is just the standard Kalman filter MSE of $\hat{\boldsymbol{\xi}}_{\mathbf{t|t-1}}$, and the other, which is due to the uncertainty in $\boldsymbol{\theta}$. Hamilton (1986) proves that:

$$\mathbf{P_{t|s}}(\hat{\boldsymbol{\theta}}) = \mathbf{P_{t|s}}(\boldsymbol{\theta}) + \mathbb{E}\{[\hat{\boldsymbol{\xi}}_{\mathbf{t|s}}(\boldsymbol{\theta}) - \hat{\boldsymbol{\xi}}_{\mathbf{t|s}}(\hat{\boldsymbol{\theta}})][\hat{\boldsymbol{\xi}}_{\mathbf{t|s}}(\boldsymbol{\theta}) - \hat{\boldsymbol{\xi}}_{\mathbf{t|s}}(\hat{\boldsymbol{\theta}})]'\}. \tag{34}$$

The second element of the above sum may be evaluated by the Monte-Carlo integration, see Hamilton (1986), (1994) for details. Then, the significance of any element of $\boldsymbol{\xi_t}$ can be tested by the t-statistic.

To the author's knowledge, economic literature has so far ignored the need to adjust the MSE of $\hat{\mathbf{y}}_{\mathbf{t|t-1}}$, henceforth $\mathbf{W_t}$, for the uncertainty of $\boldsymbol{\theta}$. That can be done in a following way. It follows from the definition of $\mathbf{W_t}$ that:

$$\begin{aligned}\mathbb{E}\{[\mathbf{y_t} - \hat{\mathbf{y}}_{\mathbf{t|t-1}}][\mathbf{y_t} - \hat{\mathbf{y}}_{\mathbf{t|t-1}}]'\} = \mathbb{E}\{[\mathbf{x_t}(\mathbf{a}-\hat{\mathbf{a}}) + \mathbf{x_t}(\boldsymbol{\xi}_\mathbf{t} - \hat{\boldsymbol{\xi}}_{\mathbf{t|t-1}}) + \mathbf{u_t}]\}\times \\ \times[\mathbf{x_t}(\mathbf{a}-\hat{\mathbf{a}}) + \mathbf{x_t}(\boldsymbol{\xi}_\mathbf{t} - \hat{\boldsymbol{\xi}}_{\mathbf{t|t-1}}) + \mathbf{u_t}]'\} = \mathbf{x_t}\mathbb{E}\{[\mathbf{a}-\hat{\mathbf{a}}][\mathbf{a}-\hat{\mathbf{a}}]'\}\mathbf{x}'_\mathbf{t} + \\ \mathbf{x_t}\mathbb{E}\{[\boldsymbol{\xi}_\mathbf{t} - \widehat{\boldsymbol{\xi}_{\mathbf{t|t-1}}}][\boldsymbol{\xi}_\mathbf{t} - \hat{\boldsymbol{\xi}}_{\mathbf{t|t-1}}]'\}\mathbf{x}'_\mathbf{t} + \mathbf{x_t}\mathbb{E}\{[\mathbf{a}-\hat{\mathbf{a}}][\boldsymbol{\xi}_\mathbf{t} - \hat{\boldsymbol{\xi}}_{\mathbf{t|t-1}}]'\}\mathbf{x}'_\mathbf{t} + \\ \mathbf{x_t}\mathbb{E}\{[\boldsymbol{\xi}_\mathbf{t} - \hat{\boldsymbol{\xi}}_{\mathbf{t|t-1}}][\mathbf{a}-\hat{\mathbf{a}}]'\}\mathbf{x}'_\mathbf{t} + \mathbb{E}[\mathbf{u_t}\mathbf{u}'_\mathbf{t}] = \\ = \mathbf{x_t}\mathbb{V}(\hat{\boldsymbol{\theta}}_\mathbf{CP})\mathbf{x}'_\mathbf{t} + \mathbf{x_t}\mathbf{P_{t|t-1}}(\hat{\boldsymbol{\theta}})\mathbf{x}'_\mathbf{t} + \mathbf{x_t}\mathbf{G_t}(\hat{\boldsymbol{\theta}})\mathbf{x}'_\mathbf{t} + \mathbf{H_t}.\end{aligned} \tag{35}$$

$\mathbb{V}(\hat{\boldsymbol{\theta}}_{CP})$ stands for the MSE of time-invariant paramaters entering the observation equation (18), which is constructed by selecting the corresponding rows and columns from the $\mathbb{V}(\hat{\boldsymbol{\theta}})$ matrix. $\mathbf{P_{t|t-1}}(\hat{\boldsymbol{\theta}})$ is given by (34) and $\mathbf{G_t}(\hat{\boldsymbol{\theta}})$ is defined as:

$$\mathbf{G_t}(\hat{\boldsymbol{\theta}}) = \mathbf{K_t}(\hat{\boldsymbol{\theta}}) + [\mathbf{K_t}(\hat{\boldsymbol{\theta}})]' \tag{36}$$

$$\mathbf{K_t}(\hat{\boldsymbol{\theta}}) = \mathbb{E}\{[\mathbf{a}-\hat{\mathbf{a}}][\boldsymbol{\xi}_\mathbf{t} - \hat{\boldsymbol{\xi}}_{\mathbf{t|t-1}}]'\}. \tag{37}$$

Unknown matrix $\mathbf{K_t}(\hat{\boldsymbol{\theta}})$ may be estimated together with $\mathbf{P_{t|t-1}}(\hat{\boldsymbol{\theta}})$ by the Monte-Carlo integration, which should save some time on computations. The MSE of $\hat{\mathbf{y}}_{\mathbf{t|t-1}}$

is then used to standardize filter residuals which are subsequently bootstrapped to identify abnormal ones.

4 Empirical results

The models given by equations (9) – (11) and (12) – (16) were estimated for three stocks listed on the Warsaw Stock Exchange. These stocks were selected on the following criteria:

- previously disclosed manipulation cases,
- high intensity of insiders' trading (not to be confused with insider trading),
- mergers and acquisitions, which are often accompanied by informed dealing of securities.

The individual securities will be named in the remainder of the text as A, B and C (detailed information on them is available from the author upon request). Data on historical quotes (daily as well as intraday) of these securities is available *inter alia* on the website of 'Parkiet', the popular Polish financial daily (`http:\\www.parkiet.com`). Intraday data comprises hourly quotes from 1st June 2006 to 31st January 2007; this period was chosen according to the data availability and the comparability of data for individual stocks. The hourly frequency is a result of balancing between possibly short intervals between data points and minimization of data points with no trades (zero returns). For each trading day there are seven observations from 10:00 to 16:00 (totally 1183 observations).
Data on opening (9:30) and closing prices (16:20) prices covers the period from the 2nd January 2006 to 31st August 2008 (667 trading days). Similarly to the case of intraday data, this period was chosen due to the availability and the comparability of data.
Both models for intraday and open/closing data were estimated numerically in the R environment using the Fortran-based algorithm of Broyden-Fletcher-Goldfarb-Shanno (L-BFGS-B). This algorithm enables to impose linear restrictions on parameters, which may be necessary to ensure the stationarity with respect to the GARCH process. Initial values for GARCH processes were set by fitting univariate GARCH processes to residuals from OLS regressions of individual observation equations. Estimates of these regressions were used as initial values for time-invariant parameters. Initial values for $\mathbf{Q}$ were set to the variances of time-varying parameters (in first differences) coming from rolling regressions of individual observation equations. Matrix $\mathbf{Q}$ was factorized on the basis of Cholesky decomposition in order to accelerate the computations and to ensure the estimated matrix is positive definite (Hamilton 1994). Since $\mathbf{Q}$ is diagonal, variance estimates are simply squares of corresponding elements of the estimated Cholesky matrix.

Tables 1 − 2 present estimation results. In the case of intraday data, for each security at least one parameter of the observation equation was not significant at 0.10 level regardless of whether it was set to be time-varying or time-invariant. Among other parameters of the observation equation at least one was time-varying. An error made when applying the constant parameter model to detect market abuse events may be, as evidence suggests, substantial. For all securities, the variance of ε_t was heteroskedastic, though its persistence varied among securities.

Table 1: Estimation results for intraday data. Standard errors are given in parentheses, t statistics in square brackets and p-values in braces, sqrt(·) denotes the algebraic square root. Italics placed for sqrt(·) indicate estimates of time-invariant parameters.

	A	B	C
sqrt(σ^2_α)	-0.0116 (0.0077) [-1.5022] {0.1331}		
sqrt(σ^2_β)	0.1621 (0.0876) [1.8500] {0.0643}	0.1460 (0.0938) [1.5575] {0.1194}	
sqrt(σ^2_ρ)	*-0.1593* *(0.0349)* *[-4.5668]* *{0.0000}*	0.0065 (0.0019) [3.4718] {0.0005}	*-0.1432* *(0.0531)* *[-2.6969]* *{0.0070}*
sqrt(σ^2_γ)		*0.5264* *(0.2054)* *[2.5626]* *{0.0104}*	-0.0074 (0.0033) [-2.2495] {0.0245}
ω_0	3.5786 (3.4411) [1.0400] {0.2984}	10.259 (6.8174) [1.5048] {0.1324}	0.9972 (0.6652) [1.4990] {0.1339}
ω_1	0.1187 (0.1700) [0.6982] {0.4851}	0.0798 (0.0319) [2.5012] {0.0124}	0.0529 (0.0205) [2.5819] {0.0098}
ω_2	0.6972 (0.2980) [2.3397] {0.0193}	0.8391 (0.0713) [11.7633] {0.0000}	0.9054 (0.0484) [18.6868] {0.0000}

Interestingly, both residuals and squared residuals from the intraday model were almost uncorrelated at any order, although the input data showed substantial higher-order autocorrelation with respect to both 1-hour returns and squared 1-hour returns. Only in the case of C there were indices of some autocorrelation in squared residuals, though on the frontier of statistical significance. The presumption that the inclusion of the market portfolio return into the reference model should extract the specific features of intraday data, justifying the simple GARCH(1,1) variance structure of the random term, appears to be correct.

Table 2: Estimation results for open/close data. Standard errors are given in parentheses, t statistics in square brackets and p-values in braces, sqrt$(\cdot)$ denotes the algebraic square root. Italics placed for sqrt$(\cdot)$ indicate estimates of time-invariant parameters. Lower-case indexed letters a, b, c stand for the corresponding elements of $\mathbf{A}, \mathbf{B}, \mathbf{C}$.

	A	B	C		A	B	C
sqrt($\sigma^2_{\alpha_0}$)	*0.1951* *(0.0752)* *[2.5934]* *{0.0095}*		0.0917 (0.0357) [2.5706] {0.0102}	sqrt($\sigma^2_{\alpha_1}$)	*-0.1628* *(0.0963)* *[-1.6908]* *{0.0909}*		0.0309 (0.0143) [2.1588] {0.0309}
sqrt($\sigma^2_{\beta_0}$)	0.0458 (0.0239) [1.9163] {0.0553}	-0.0691 (0.0165) [-4.1901] {0.0000}	0.0245 (0.0151) [1.6244] {0.1043}	sqrt($\sigma^2_{\beta_1}$)	0.0296 (0.0176) [1.6799] {0.093}	0.0340 (0.0148) [2.3017] {0.0214}	*0.5689* *(0.0893)* *[6.3683]* *{0.0000}*
sqrt($\sigma^2_{\rho_0}$)	0.0026 (0.0013) [1.9184] {0.0551}			sqrt($\sigma^2_{\rho_0}$)	*-0.5816* *(0.0944)* *[-6.1582]* *{0.0000}*	*-0.3391* *(0.0699)* *[-4.8501]* *{0.0000}*	0.0315 (0.0099) [3.1888] {0.0014}
sqrt($\sigma^2_{\gamma_0}$)	*0.1908* *(0.0708)* *[2.6958]* *{0.007}*	0.1730 (0.0572) [3.0255] {0.0025}	*0.1746* *(0.0577)* *[3.0274]* *{0.0025}*	sqrt($\sigma^2_{\gamma_0}$)	*0.4524* *(0.1872)* *[2.416]* *{0.0157}*	*0.7410* *(0.2322)* *[3.1911]* *{0.0014}*	0.0213 (0.0075) [2.8582] {0.0043}
c_1	0.2457 (0.1179) [2.0836] {0.0372}	4.7672 (2.0984) [2.2719] {0.0231}	0.0796 (0.1175) [0.6775] {0.4981}	c_2	0.5089 (1.1041) [0.4609] {0.6449}	2.9670 (1.8749) [1.5825] {0.1135}	0.2640 (0.3347) [0.7888] {0.4302}
a_1	0.0283 (0.0214) [1.3246] {0.1853}	0.5677 (0.4934) [1.1507] {0.2498}	0.0670 (0.0642) [1.0424] {0.2972}	a_2	0.1287 (0.1465) [0.8786] {0.3796}	0.0776 (0.0325) [2.3907] {0.0168}	0.0782 (0.0640) [1.2214] {0.2219}
b_1	0.8851 (0.0431) [20.5172] {0.0000}	0.2423 (0.2396) [1.0112] {0.3119}	0.8885 (0.1202) [7.3927] {0.0000}	b_2	0.8042 (0.2891) [2.7814] {0.0054}	0.8028 (0.0904) [8.8843] {0.0000}	0.8754 (0.1201) [7.2907] {0.0000}

In the case of open/close data, from five up to eight parameters in the observation equation were statistically significant at 0.10 level; about the half of them were time-varying. In particular, the coefficients associated with the return on the market portfolio changed (with one exception) over time. Graphs 1 – 7 provide the trajectories of time-varying coefficients. The structure of the GARCH process differed between securities in terms of both significance and persistence; in no case, however, was the conditional correlation significant at 0.10 level.

The estimated models were used to generate alerts about possible market abuse events. An alert about abnormal returns was generated each time the realized return rate fell outside the 95%-confidence interval for the one-step-ahead forecast, while an alert about significant autocorrelation was generated each time when the t-ratio exceeded 1.64 in absolute terms which corresponds to the p-value of 10%. In the case of intraday data, the number of abnormal returns for all securities slightly exceeded 5%, which would be implied by the significance level. Statistically significant autocorrelation was discovered only in returns of B; it lasted for about 2/3 of the sample length. That may be attributed to some manipulation practices like generating buy or sell signals of technical analysis (such a case came to light shortly thereafter).

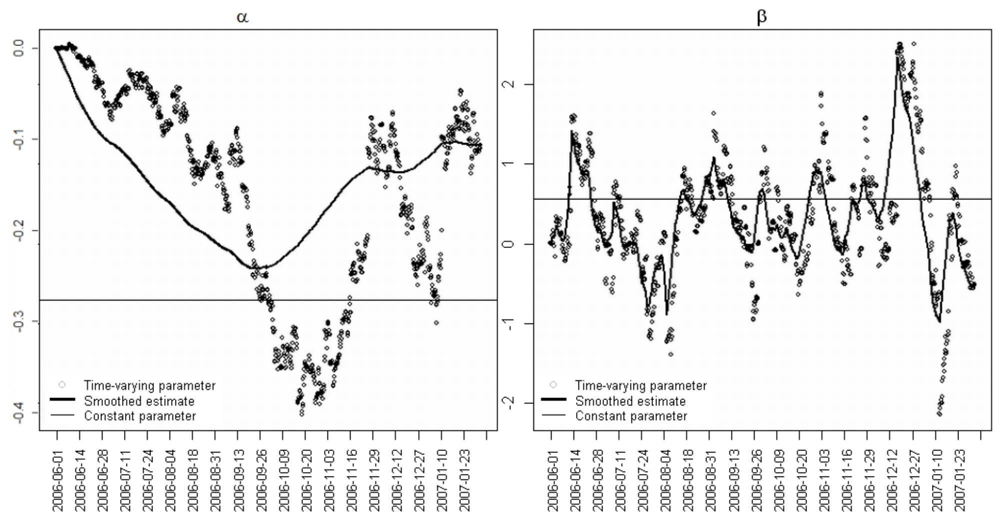

Figure 1: Estimates of time-varying parameters – intraday data of A.

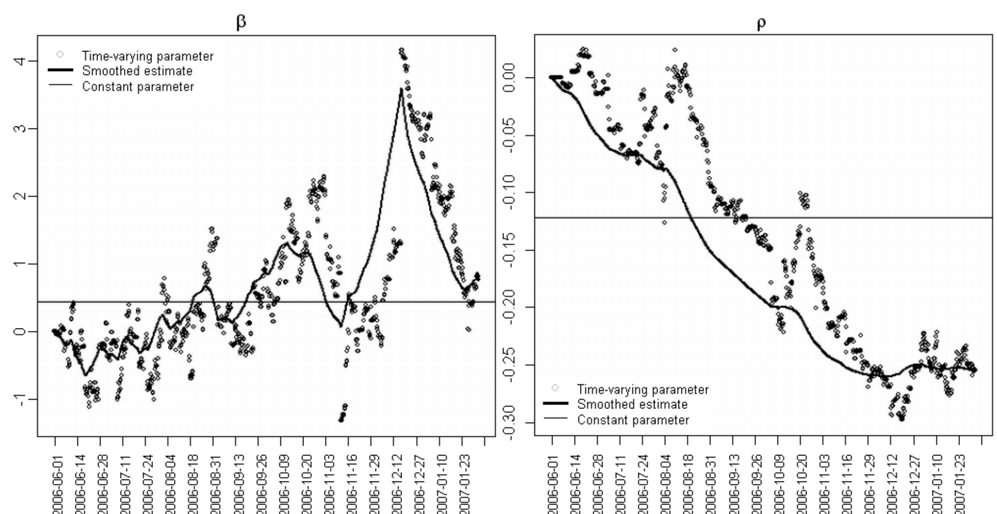

Figure 2: Estimates of time-varying parameters – intraday data of B.

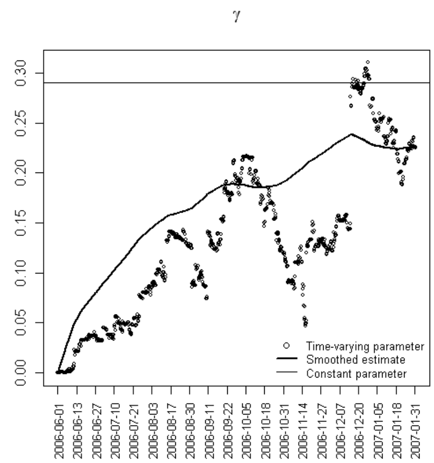

Figure 3: Estimates of time-varying parameters – intraday data of C.

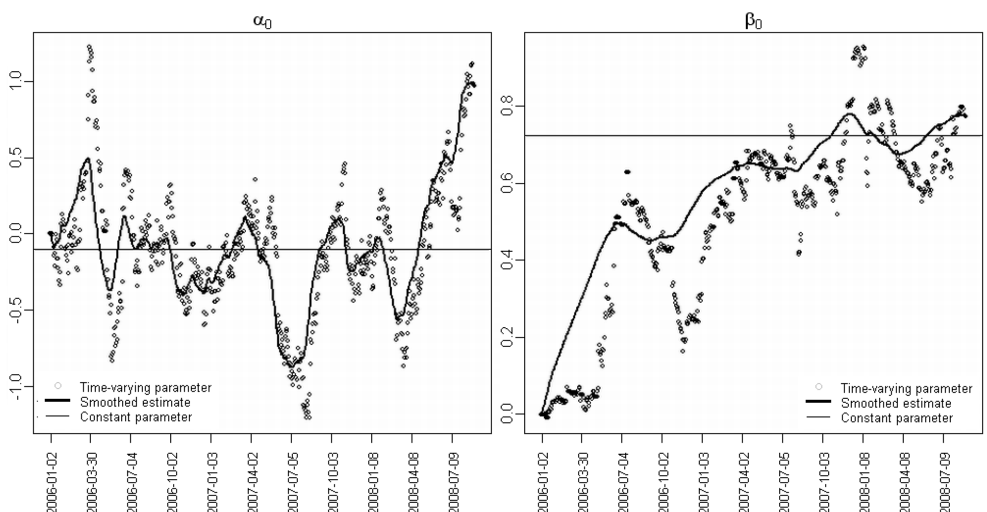

Figure 4: Estimates of time-varying parameters – open/close data of C.

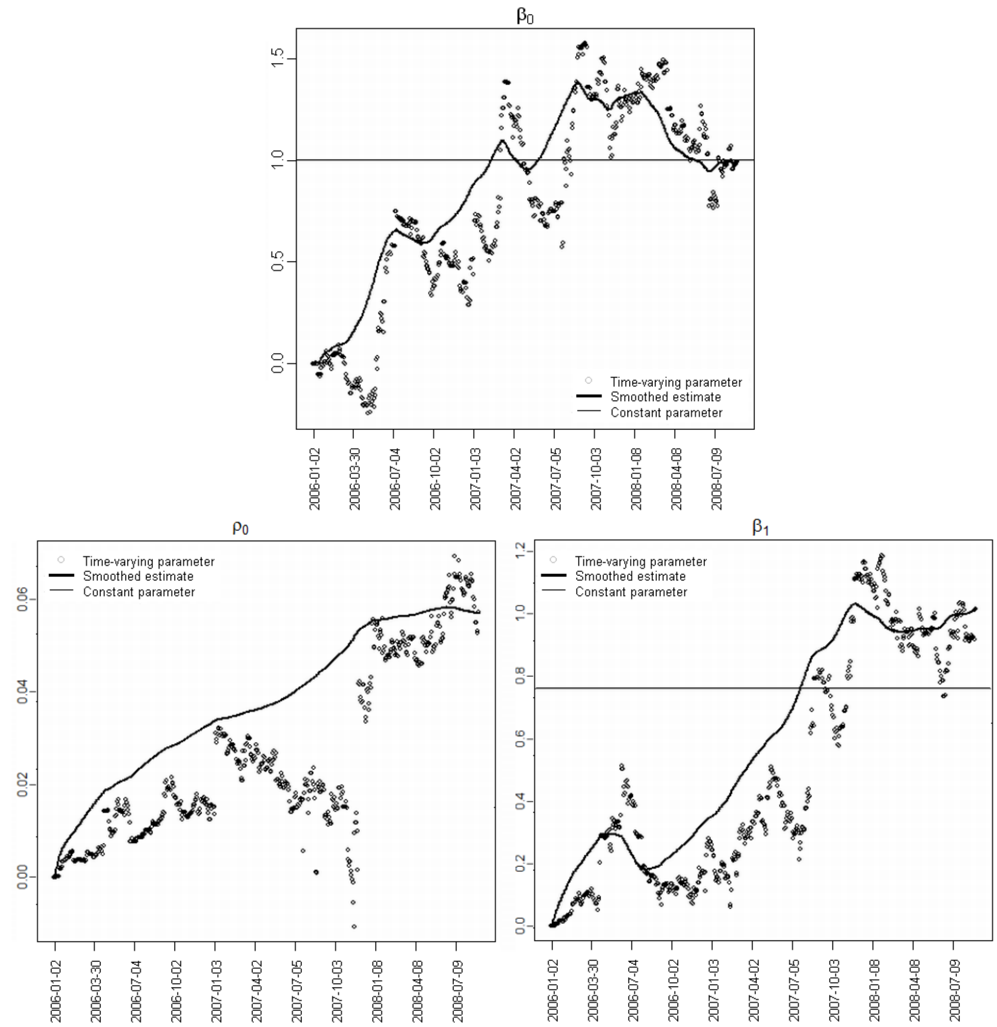

Figure 5: Estimates of time-varying parameters – open/close data of A.

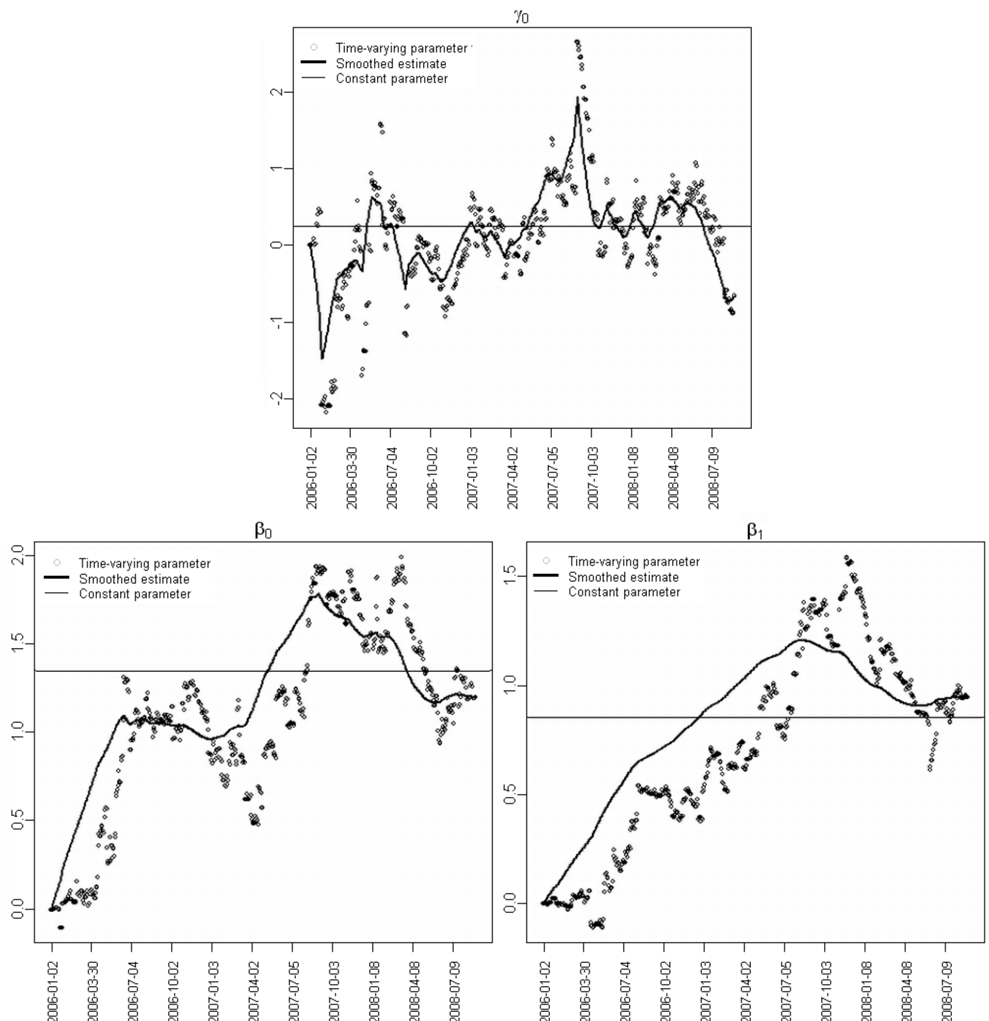

Figure 6: Estimates of time-varying parameters – open/close data of B.

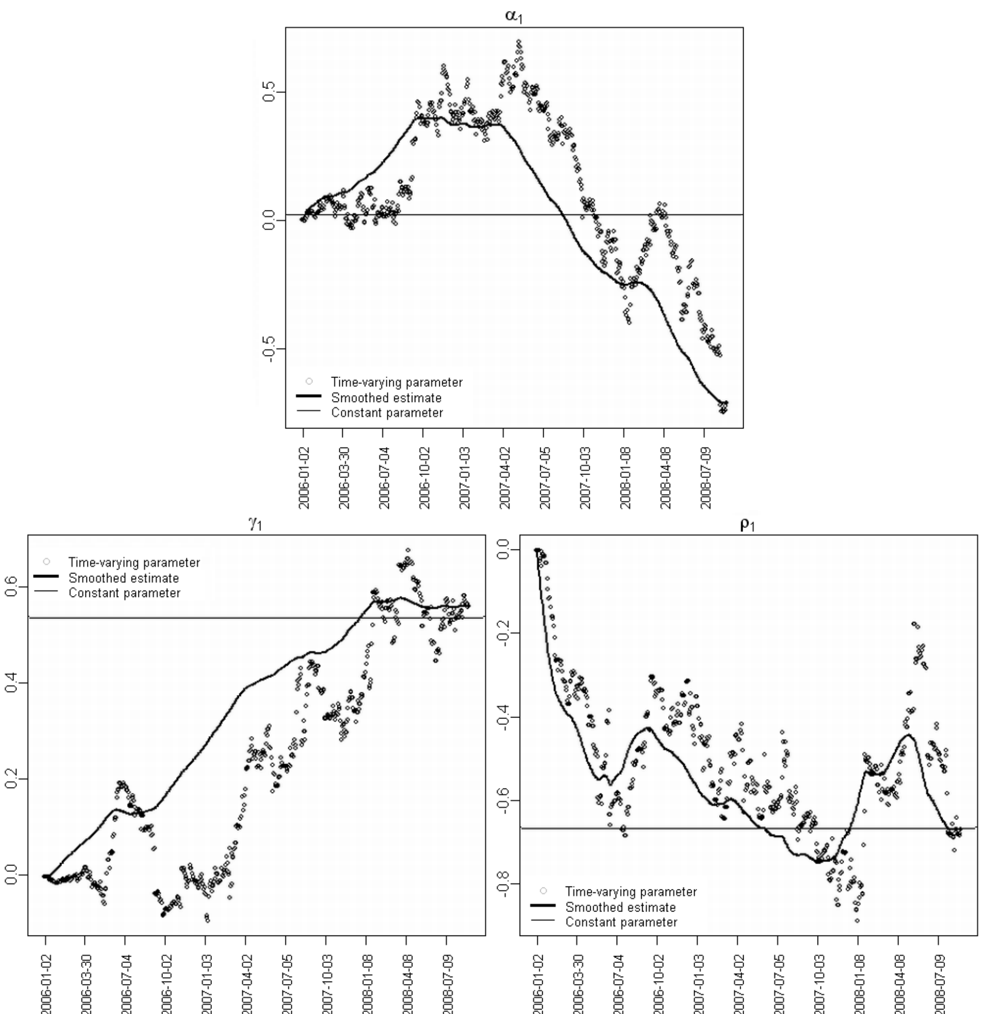

Figure 7: Estimates of time-varying parameters – open/close data of C (cont.).

To avoid the sample bias, the results were compared with those of the model reestimated on the first half of the sample, with no change in the model structure. The comparison yields a mixed picture. Whilst for one security (B) the performance of the full-sample model was superior to the half-sample model (more alerts were generated in both parts of the sample), for others the opposite held or the performance was similar. Details on that comparison are presented in Table 3.

In the case of open/close data, the number of abnormal returns for all stocks exceeded

Table 3: Alerts generated by the model for intraday data. 'Missing alerts' refer to those generated by one model and not generated by the other model.

Number of alerts in a half-sample	Model estimated on half sample			Model estimated on full sample		
Stock A	1st	2nd	Total	1st	2nd	Total
Generated						
Returns	30	38	68	25	33	58
Autocorrelation	0	0	0	0	0	0
Missing						
Returns	1	0	1	6	5	11
Autocorrelation	0	0	0	0	0	0
Stock B	1st	2nd	Total	1st	2nd	Total
Generated						
Returns	30	22	52	37	25	62
Autocorrelation	0	464	464	0	533	533
Missing						
Returns	8	5	13	1	2	3
Autocorrelation	0	69	69	0	0	0
Stock C	1st	2nd	Total	1st	2nd	Total
Generated						
Returns	29	16	45	39	21	60
Autocorrelation	0	0	0	0	0	0
Missing						
Returns	10	5	15	0	0	0
Autocorrelation	0	0	0	0	0	0

10% level implied by the 5% significance level and the doubled number of endogenous variables. For B abnormal returns on the opening were dominant, which may indicate the marking-the-close manipulation. For C, the relation between abnormal returns on the closing and on the opening was balanced, whilst in the case of A abnormal returns on the closing dominated over those on the opening. Moreover, there was evidence of time-varying, significant autocorrelation in returns of C, otherwise absent in intraday data (note that open/close data covers the entire period when intraday data was available). Time-varying autocorrelation in returns of A, which was present in the intraday data, disappeared in the open/close data analysis.

Similarly to the case of intraday data, the model for open/close data was reestimated on the first half of the sample. Contrary to the intraday data case, models estimated on the half sample generated more alerts in both parts of the sample, which indicates some overfit of the full-sample model. See Table 4 for details.

Abnormal return alerts may be applied in a straightforward manner to detect insider trading. While alerts following price-sensitive information releases just confirm

Table 4: Alerts generated by the model for open (Op)/close (Cl) data. 'Missing alerts' refer to those generated by one model and not generated by the other model.

Number of alerts in a half-sample	Model estimated on					
	half sample			full sample		
Stock A	1st	2nd	Total	1st	2nd	Total
Generated						
Returns Op	19	18	37	18	17	35
Returns Cl	21	36	57	15	21	36
Autocorrelation Op	0	0	0	0	0	0
Autocorrelation Cl	0	0	0	0	0	0
Missing						
Returns Op	1	0	1	2	1	3
Returns Cl	1	0	1	7	15	22
Autocorrelation O	0	0	0	0	0	0
Autocorrelation C	0	0	0	0	0	0
Stock B	1st	2nd	Total	1st	2nd	Total
Generated						
Returns Op	20	17	37	21	13	34
Returns Cl	19	16	35	18	15	33
Autocorrelation Op	0	0	0	0	0	0
Autocorrelation Cl	0	0	0	0	0	0
Missing						
Returns Op	7	3	10	6	7	13
Returns Cl	4	1	5	5	2	7
Autocorrelation Op	0	0	0	0	0	0
Autocorrelation Cl	0	0	0	0	0	0
Stock C	1st	2nd	Total	1st	2nd	Total
Generated						
Returns Op	21	13	34	24	13	37
Returns Cl	16	25	41	17	18	35
Autocorrelation Op	0	0	0	0	0	0
Autocorrelation Cl	240	300	540	214	307	521
Missing						
Returns Op	4	2	6	1	2	3
Returns Cl	4	1	5	3	8	11
Autocorrelation Op	0	0	0	0	0	0
Autocorrelation Cl	10	7	17	36	0	36

the information was in fact price-sensitive, alerts preceding information releases may indicate some informed dealing. In order to check whether generated alerts identify moments of possible insider trading, the alerts were compared with the list of all news releases by the company and on the company whose stock may be subject to market abuse (the list was prepared with the use of the information service of the Warsaw Stock Exchange *GPW Infostrefa*). Information is assumed to be reflected in the stock price no later than on the next trading day after the disclosure. If the alert precedes the information by five trading days or less, insider trading may be suspected. That interval is chosen in line with the findings of Dubow and Monteiro (2006) and Monteiro, Zaman, Leitterstorf (2007). Note, however, that informed dealing may take place in much greater advance, in particular in the case of mergers and acquisitions, which may be planned long time before any kind of information is released.
For intraday data on A and C stock prices, about 90% of all abnormal return alerts could be attributed to the information released around the alert. In the case of B,

this ratio amounted only to 40%, since the total number of news releases (16 in the 8 months' sample) was much smaller. More than half of alerts preceded information, which suggests a large intensity of insider trading (see tab. 3). A more detailed analysis of news releases associated with alerts revealed some interesting facts:

- A member of the executive board of company A traded its stocks in the period preceding the publication of the company's financial results. This was accompanied by abnormal returns.
- The considered period coincides with a takeover by company C – relevant news releases were almost always preceded by an alert.
- An investor systematically traded shares of B, each time crossing the 5% level which according to the law obliged him to disclose the transaction. When the purchase was disclosed, the investor sold stocks, and when the sale was disclosed, he bought the stocks. This practice was revealed later on by the stock market supervision.

In the case of open/close data, some 65% (B) to 80% (A) abnormal return alerts were attributable to the information released around the alert. 50% (C) to 65% (A and B) alerts preceded news releases which shows even a larger extent of informed dealing than the intraday data (note, however, that the sample covers a much longer time span than the sample of the intraday data). The analysis of news releases confirms major findings from the intraday data analysis.

5 Final conclusions

This paper was intended to develop the real-time market abuse detection model, which would track the market evolution. Although the model structure does not differ substantially from that of Monteiro, Zaman, Leitterstorf (2007), an attempt to extend the analysis beyond the closing prices seems, however, to add much value to the research. The utilization of intraday data is an important step towards the real-time fraud detection, while separate handling of opening and closing data, motivated by different economic characteristics of that kind of data, is particularly useful in identifying some kinds of manipulation like marking-the-close. These two approaches to data appear in some cases to be complementary, as is supported by the empirical analysis outlined in this paper.
An important innovation to the modeling strategy was to introduce the time-variation in parameters using Kalman filtering. The study of parameter trajectories indicates how the assumption of constant parameters may influence results of market abuse detection and thus increase the probability of an error. A similar distortion may be created by unrealistic assumptions on probability distributions, which advocates the application of the semi-parametric approach when estimating the model as well as

bootstrapping when identifying abnormal returns.

The contribution of this paper also consists of the generalization of the extended Kalman filter outlined in Chou, Engle, Kane (1992) to estimate a time-varying parameter model together with a bivariate GARCH effect in disturbances in the observation equation. A constant conditional correlation parametrization seems to be a plausible way to model the effect in question, which in the case of zero conditional correlation simplifies to the simultaneous estimation of two univariate GARCH processes. Additionally, this paper developed some techniques of a more accurate statistical verification of state variables' significance and a proper standardization of Kalman filter residuals, which are the basis for the identification of abnormal returns.

As evidence suggests, the proposed model is quite successful in identifying moments of potential market abuse activities. Return alerts generated by the model were attributable to subsequent information releases, while autocorrelation alerts coincided with previously released cases of market manipulation. Such alerts provide an important piece of information for the financial supervision institutions, which may look for frauds that otherwise would remain undetected. That is an approach to fraud detection that is qualitatively different from providing kind of 'statistical proofs' of frauds, which have already been under suspicion.

The framework developed in this paper may be with no particular effort applied to the analysis of real existing stock markets. The model, once estimated, may be updated on a near-continuous basis thanks to the Kalman filter and may automatically generate alerts out-of-sample (the frequency of both depends on the time interval chosen in the model development). The empirical analysis is in favour of such an approach, especially in the case of open/close data analysis, showing a satisfactory out-of-sample performance of the estimated model.

References

[1] Alexander C., (2001), *Market models*, John Wiley and Sons, Chichester.

[2] Chou R., Engle R. F., Kane A., (1992), Measuring risk aversion from excess returns on a stock index, *Journal of Econometrics* 52.

[3] Dubow B., Monteiro N., (2006), Measuring market cleanliness, Financial Securities Authority Occasional Papers 23.

[4] Efron B., (1987), *The jackknife, the bootstrap, and other resampling plans*, CBMS-NSF Regional Conference Series in Applied Mathematics 38.

[5] Emerson R., Hall S. G., Zalewska-Mitura A., (1997), Evolving market efficiency with an application to some Bulgarian shares, *Economics of Planning* 30, 75-90.

[6] Engle R. F., (2000), The econometrics of ultra-high-frequency data, *Econometrica* 68, 1-22.

[7] Engle R. F., Kroner K. F., (1995), Multivariate Simultaneous Generalized ARCH, *Econometric Theory* 11, 122–150.

[8] Hamilton J. D., (1986), A standard error for the estimated state vector of a state-space model, *Journal of Econometrics* 33, 387-397.

[9] Hamilton J. D., (1994), *Time Series Analysis*, Princeton University Press.

[10] Meulbroek L., (1992), An empirical analysis of illegal insider trading, *Journal of Finance* 47, 1661-1700.

[11] Minenna M., (2003), The detection of market abuse on financial markets: A quantitative approach, *Quaderni Di Finanza* 54.

[12] Mitchell M. L., Netter J. M., (1994), The role of financial economics in securities fraud cases: Applications at the securities and exchange commission, *The Business Lawyer* 40, 545-–590.

[13] Monteiro N., Zaman Q., Leitterstorf S., (2007), Updated measurement of market cleanliness, Financial Securities Authority Occasional Papers 25.

[14] Rockinger M., Urga G., (2000), The evolution of stock markets in transition economies, *Journal of Comparitive Economics* 28, 456-–472.

[15] Watson M. W., (1989), Recursive solution methods for dynamic linear rational expectations models, *Journal of Econometrics* 41, 65-89.

[16] Worthington A. C., Higgs H., (2003), Weak-form market efficiency in European emerging and developed stock markets, School of Economics and Finance Discussion Papers and Working Papers Series 159, School of Economics and Finance, Queensland University of Technology.

[17] Zalewska-Mitura A., Hall S. G. (1999), Examining the first stages of market performance: a test for evolving market efficiency, *Economic Letters* 64, 1-12.