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Exact solution of flow in a composite porous channel

This article concerns fully developed laminar flow of a viscous incompressible fluid in a long composite cylindrical channel. Channel consist of three regions. Outer and inner regions are of uniform permeability and mid region is a clear region. Brinkman equation is used as a governing equation of motion in the porous region and Stokes equation is used for the clear fluid region. Analytical expressions for velocity profiles, rate of volume flow and shear stress on the boundaries surface are obtained and exhibited graphically. Effect of permeability variation parameter on the flow characteristics has been discussed.

1. Introduction

Fluid flow in a composite cylindrical channel, which is partially filled with a porous medium and partially with a clear fluid has practical applications due to its common occurrence in natural, scientific, and engineering situations. The study of porous channels flows has received tremendous attention during the last years. This is because of the significance and diversity of this research area in various applications. These applications include thermal insulation, biomedical systems, porous bearings, bio convection in porous media and crude oil extraction. The practical application of flow in annular space can be found in oil and gas wells and gas-cooled nuclear reactors. Many authors investigated problems of flow through/past porous cylindrical channel. An analytical study on the flow in a fully developed section of the three-dimensional composite channel is presented for different geometrical configurations, e.g., (i) a single porous medium of uniform porosity; (ii) two different uniform porous media (iii) three porous layers each of uniform porosity, including when assembled to produce a symmetrical situation.

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Analytical solutions for different cases are compared and the effect of the Darcy parameter is discussed by Al-Hadhrami et al. [1]. An analytical study in the fully developed section of the composite channel is presented when the channel is of constant height and composed of several layers of porous media, each of uniform porosity. In the fully developed flow regime the analytical and numerical solutions are graphically indistinguishable by Al-Hadhrami et al. [2]. Heat transfer to a fluid passing through a channel filled with porous materials is the subject which includes the derivation of the temperature solutions in channels having different cross sectional geometries. It is investigated by Haji-Sheikh and Vafai [3]. Steady flow in a composite channel bounded by two infinite parallel plates is considered, in which lower part of the channel is occupied by a clear fluid while the upper part is occupied by a fully saturated porous medium with uniform permeability. The upper plate and the porous medium are fixed while the lower plate moves with a constant velocity. Using Brinkman-Forchheimer-extended Darcy equation analytical solution is obtained by Kuznetsov [4]. A fully developed laminar forced convection inside a semi-circular channel filled with a Brinkman-Darcy porous medium and obtained analytical solutions for the flow by Wang [5]. Nield et al. [6] carried out a theoretical analysis of fully developed forced convection in a fluid-saturated porous-medium channel bounded by parallel plates, with imposed uniform heat flux or isothermal condition at the plates. They obtained an 'exact' solution of the Brinkman-Forchheimer extension of Darcy's momentum equation for flow in the channel. A calculation of the permeability of a swarm of particles is extended to closely packed particles by Brinkman [7]. Analytical study of the steady incompressible flow past a circular cylinder embedded in a constant porosity medium based on the Brinkman model and a closed form exact solution for the governing equations, which leads to an expression for the separation parameter, are reported by Pop and Cheng [8]. Fully developed forced convection inside a circular tube filled with saturated porous medium and with uniform heat flux at the wall is investigated on the basis of a Brinkman-Forchheimer model. The results for the two limiting cases of clear fluid and Darcy flow conditions are shown by Hooman and Gurgenci [9]. Exact solution for forced convection in a channel filled with porous medium is given by Vafai and Kim [10]. Laminar flow through a porous channel bounded by two parallel plates maintained at a constant and equal temperature is considered by Kaviany [11] and it is shown that the Nusselt number for fully developed fields increases with an increase in porous media shape parameter. The Brinkman-extended Darcy model (Brinkman flow) of a laminar mixed-convection flow in a annular porous region is considered by Parang and Keyhani [12], and the closed form nature of the solution is advantageous in the demonstration of the importance of the wall effect. Vadasz [13] analytically solved the fluid flow through heterogeneous porous medium in a rotating square channel having permeability variation in the vertical direction of the channel. An analytical solution is obtained for a fully developed, forced convection in a gap between two concentric cylinders. The inner cylinder is exposed to a constant heat flux and the outer outer is thermally insulated. A porous layer is attached to the inner cylinder. The effects of the permeability, thermal conductivity and the thickness of the porous material are investigated using a Brinkman-extended Darcy model by Chikh et al. [14]. An analytical solution for fully developed flow in a curved porous channel for the specific case of monotonic permeability variation according to the law $k = e^{\beta z}$ is obtained by Govender [15]. Analytical solution for a composite porous cylindrical channel is taken, in which inner and outer part of the cylindrical channel are of different permeability. Singh and Verma [16] considered two cases of permeability variation of the inner porous cylinder: linear variation and quadratic variation, and they found expressions for the velocity, rate of volume flow, average velocity and shear stress on the impermeable boundary. Verma and Datta [17] carried out the study on channel flow of a viscous incompressible fluid through a heterogeneous porous medium with linear permeability variation with permeability of the porous medium assumed to vary with the transverse distance between the plates of the channel. They obtained analytical expressions for the velocity for two cases, Poiseuille and Couette flow, and discussed the influence of various parameters on the flow. An analytical solution is obtained by using Brinkman equation for viscous, incompressible fluid in a composite cylindrical channel having inner porous cylinder of uniform permeability and outer porous cylinder of variable permeability. Two cases for outer porous cylinder are considered: (i) linear variation (ii) quadratic variation. Effect of permeability variation is discussed by Verma and Singh [18]. Exact solution for slow flow of a viscous fluid past a porous cylindrical shell is obtained by using the Brinkman equation. Flow in a clear region outside the cylindrical shell is governed by the Stokes equation. At the interface between the clear fluid and porous region, the continuity of velocity components and continuity of the stress components is assumed. Exact expression for velocity, pressure and drag on the solid cylinder has been obtained by Verma and Verma [19].

In this paper, we considered a steady flow of a viscous, incompressible fluid in a composite cylindrical channel under constant pressure gradient. The composite channel consists of three regions. Region I is the inner cylindrical region which is porous of permeability k_1, region II is the mid region which is clear region and region III is the outer cylindrical region which is porous region of permeability k_2 as shown in Fig. 1. Brinkman equation is used as a governing equation of motion in the porous cylinders and Stokes equation is applied for the clear flow.

2. Mathematical formulation

Steady flow of a viscous incompressible fluid in a long composite porous channel has been considered in fully developed state. Inner porous cylindrical region I is of radius a, mid clear annular region II is of thickness $(b - a)$ and outer porous region III is of thickness $(c - b)$, as shown in Fig. 1. Flow in the composite channel is under applied pressure gradient $\partial p^*/\partial z^*$ and is along the axis of the cylindrical channel, which is z-axis. The governing equation of motion in

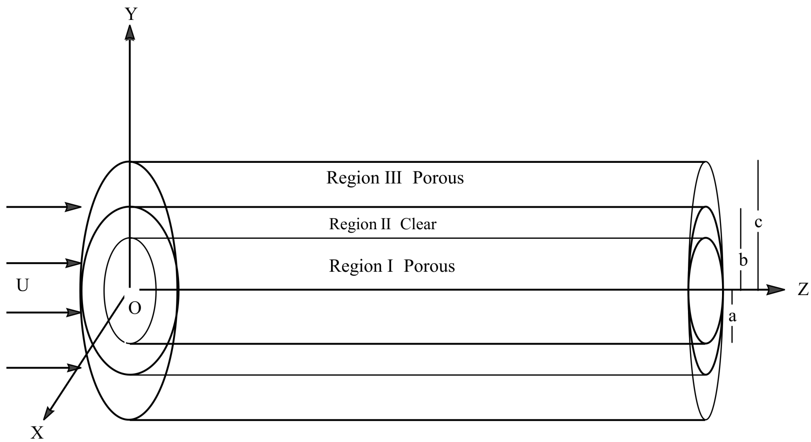

Fig. 1. Physical model of the problem

the porous region I and III is Brinkman equation [7], which for the present case in cylindrical polar coordinate (r^*, θ, z^*) is

$$\mu_e \left(\frac{\mathrm{d}^2 u^*}{\mathrm{d}r^{*2}} + \frac{1}{r^*} \frac{\mathrm{d}u^*}{\mathrm{d}r^*} \right) - \frac{\mu}{k} u^* = \frac{\partial p^*}{\partial z^*}, \tag{1}$$

where u^* is the fluid velocity, μ_e is the effective viscosity of porous medium, μ is the fluid viscosity, k is the permeability of porous medium and $\partial p^* / \partial z^*$ is the applied pressure gradient. For the present problem, we follow Brinkman [7] and Chikh et al. [14] and assume that $\mu_e = \mu$ (for high porosity cases). Therefore, Eq. (1) becomes

$$\frac{\mathrm{d}^2 u^*}{\mathrm{d}r^{*2}} + \frac{1}{r^*} \frac{\mathrm{d}u^*}{\mathrm{d}r^*} - \frac{u^*}{k} = \frac{1}{\mu} \frac{\partial p^*}{\partial z^*}. \tag{2}$$

Now we introduce dimensionless variables as follows

$$r = \frac{r^*}{a} \qquad \text{and} \qquad u_i = \frac{\mu u_i^*}{a^2 \left(-\partial p^* / \partial z^* \right)}.$$

The characteristic velocity is determined by $\dfrac{a^2}{\mu} \left(-\partial p^* / \partial z^* \right)$. Using these dimensionless variables in Eq. (2) and after dropping the star index for convenience, we get the following equations as obtained by Chikh et al. [14]

$$\frac{\mathrm{d}^2 u}{\mathrm{d}r^2} + \frac{1}{r} \frac{\mathrm{d}u}{\mathrm{d}r} - \frac{a^2}{k} u = -1, \tag{3}$$

where, u is the velocity in the considered region.

Thus, for inner porous cylinder of permeability k_1, the governing equation of motion is

$$\frac{\mathrm{d}^2 u}{\mathrm{d}r^2} + \frac{1}{r} \frac{\mathrm{d}u}{\mathrm{d}r} - \alpha^2 u = -1 \qquad (0 \leqslant r \leqslant 1), \tag{4}$$

where $\alpha^2 = a^2/k_1$ is called the permeability variation parameter for region I. Similarly, for outer porous cylinder of permeability k_2, the governing equation of motion is

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} - \beta^2 u = -1 \qquad (q_1 \leqslant r \leqslant q_2 = c/b), \tag{5}$$

where $\beta^2 = a^2/k_2$ is called the permeability variation parameter for region II. And, for clear region the equation of motion as mentioned by Chikh et al. [14] is

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} = -1 \qquad (1 \leqslant r \leqslant q_1 = b/a). \tag{6}$$

3. Solution and results

We have no slip condition on the surface of outer cylinder and continuity of velocity and shear stress on the interface between fluid and porous. Also, velocity is either maximum or minimum at $r = 0$ depending on the permeability of the regions. Thus, we have the following boundary conditions

$$\begin{aligned}
u_{\text{at } r=1^-} &= u_{\text{at } r=1^+}, \\
u_{\text{at } r=q_1^-} &= u_{\text{at } r=q^+}, \\
u_{\text{at } r=q_2} &= 0, \\
\left(\frac{du}{dr}\right)_{\text{at } r=0} &= 0, \\
\left(\frac{du}{dr}\right)_{\text{at } r=1^-} &= \left(\frac{du}{dr}\right)_{\text{at } r=1^+}, \\
\left(\frac{du}{dr}\right)_{\text{at } r=q_1^-} &= \left(\frac{du}{dr}\right)_{\text{at } r=q_1^+},
\end{aligned} \tag{7}$$

where, $q_1 = b/a$ and $q_2 = c/b$. The solutions of Eqs. (4), (6) and (5) as given by Chikh et al. [14] are

$$u(r) = A_1 I_0(\alpha r) + A_2 K_0(\alpha r) + \frac{1}{\alpha^2} \qquad (0 \leqslant r \leqslant 1), \tag{8}$$

$$u(r) = B_1 \log r + B_2 - \frac{r^2}{4} \qquad (1 \leqslant r \leqslant q_1 = b/a) \tag{9}$$

and

$$u(r) = C_1 I_0(\beta r) + C_2 K_0(\beta r) + \frac{1}{\beta^2} \qquad (q_1 \leqslant r \leqslant q_2 = c/b), \tag{10}$$

where, I_0 and K_0 are the modified Bessel functions of zeroth order of first and second kind, respectively. Here A_1, A_2, B_1, B_2, C_1 and C_2 are the constants of integration to be determined. Using boundary conditions (7), we get the constants A_1, A_2, B_1, B_2, C_1 and C_2 as given below

$$\begin{aligned}
A_1 &= \frac{2R - 2S - T - Y}{2\Delta}, \\
A_2 &= 0, \\
B_1 &= \frac{1}{2} - \frac{\alpha I_1(\alpha)(-2R + 2S + T + Y)}{2\Delta}, \\
B_2 &= \frac{4 + \alpha^2}{4\alpha^2} - \frac{I_0(\alpha)(-2R + 2S + T + Y)}{2\Delta}, \\
C_1 &= -\frac{\alpha I_1(\alpha) K_0(\beta q_2)(-2R + 2S + T + Y)}{2\lambda\Delta} \\
&\quad - \frac{\beta q_1^2 K_0(\beta q_2) + 2q_1 K_1(\beta q_1) - \beta K_0(\beta q_2)}{2\beta\lambda}, \\
C_2 &= \frac{\alpha I_1(\alpha) I_0(\beta q_2)(-2R + 2S + T + Y)}{2\lambda\Delta} \\
&\quad - \frac{-\beta q_1^2 I_0(\beta q_2) + 2q_1 I_1(\beta q_1) + \beta I_0(\beta q_2)}{2\beta\lambda},
\end{aligned} \tag{11}$$

where

$$\begin{aligned}
\Delta &= -\beta^5 I_0(\alpha) K_0(\beta q_2) I_0(\beta q_2) K_1(\beta q_1) + I_1(\beta q_1) K_0(\beta q_2) \\
&\quad + \frac{\alpha I_1(\alpha)}{q_1} \left[-\beta^4 K_0(\beta q_2) \{ I_0(\beta q_1) K_0(\beta q_2) - I_0(\beta q_2) K_0(\beta q_1) \} \right. \\
&\quad \left. + q_1 \beta^5 \log q_1 K_0(\beta q_2) \{ I_0(\beta q_2) K_1(\beta q_1) + I_1(\beta q_1) K_0(\beta q_2) \} \right], \\
\lambda &= \beta q_1 \left[I_1(\beta q_1) K_0(\beta q_2) + I_0(\beta q_2) K_1(\beta q_1) \right], \\
R &= \beta^5 \left(-\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{q_1^2}{4} - \frac{1}{4} \right) K_0(\beta q_2) \\
&\quad - K_0(\beta q_1) \left[-I_0(\beta q_2) K_1(\beta q_1) - I_1(\beta q_1) K_0(\beta q_2) \right], \\
S &= \beta^2 \left\{ -\frac{\beta^2 q_1}{2} K_0(\beta q_2) - \beta K_1(\beta q_1) \right\} \{ I_0(\beta q_1) K_0(\beta q_2) \\
&\quad - I_0(\beta q_2) K_0(\beta q_1) \} \\
T &= \frac{\beta^2}{q_1} \left[K_0(\beta q_2) \{ I_0(\beta q_1) K_0(\beta q_2) - I_0(\beta q_2) K_0(\beta q_1) \} \right], \\
Y &= -\beta^5 \log(q_1) K_0(\beta q_2) \left\{ I_0(\beta q_2) K_1(\beta q_1) + I_1(\beta q_1) K_0(\beta q_2) \right\}.
\end{aligned} \tag{12}$$

The dimensionless velocity of the fluid at any point within region I, region II and region III is given by Eqs. (8), (9) and (10), respectively. Constants A_1, A_2, B_1, B_2, C_1 and C_2 are given by (11). The graphical presentation of velocity profiles for $\alpha = \beta$, $\alpha < \beta$ and $\alpha > \beta$ is given in Fig. 2. In the limiting case, when α and $\beta \to 0$ (i.e., when permeability of the porous medium is infinite in all regions) in Eqs. (8), (9) and (10), we obtain

$$\lim_{\substack{\alpha\to 0\\ \beta\to 0}} u = \frac{\left(q_2^2 - r^2\right)}{4}, \tag{13}$$

which is velocity profile for classical Hagen-Poiseuille flow of a clear fluid through a cylindrical channel of radius q_2.

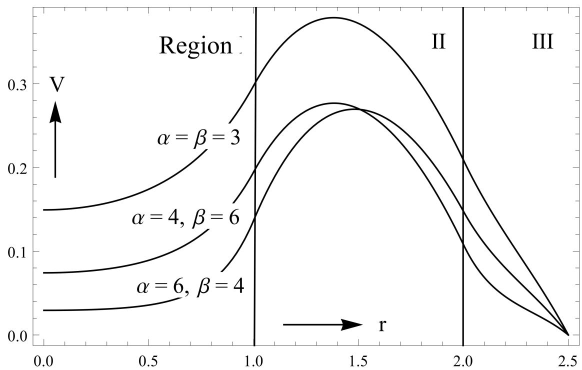

Fig. 2. Variation of velocity u with radial distance r in the composite channel for different values of α and β when $q_1 = 2$ and $q_2 = 2.5$

3.1. Rate of volume flow

The dimensionless rate of volume flow through cross-section of the inner porous cylinder is given by

$$Q_1 = 2\pi \int_0^1 u(r) r \, \mathrm{d}r. \tag{14}$$

Substituting $u(r)$ from Eq. (8) and after integration (ref. [20]), we obtain

$$Q_1 = \frac{[2\pi\alpha A_1 I_1(\alpha) + \pi]}{\alpha^2}. \tag{15}$$

Similarly, the dimensionless rate of volume flow through cross-section of the clear region II is given by

$$Q_2 = 2\pi \int_1^{q_1} u(r) r \, \mathrm{d}r. \tag{16}$$

Substituting $u(r)$ from Eq. (9) and after integration, we obtain

$$Q_2 = \frac{\pi}{8}\left[q_1^2\{8B_1 \log q_1 - 4B_1 + 8B_2\} + 4B_3 - 8B_2 - q_1^4 + 1\right] \tag{17}$$

and dimensionless rate of volume flow through cross-section of the outer porous cylinder is given by

$$Q_3 = 2\pi \int\limits_{q_1}^{q_2} v(r) r \, \mathrm{d}r. \tag{18}$$

Substituting $u(r)$ from Eq. (10) and after integration, we obtain

$$\begin{aligned} Q_3 = \frac{\pi}{\beta^2} & [q_1\{2\beta C_2 K_1(\beta q_1) - 2\beta C_1 I_1(\beta q_1)\} + q_2\{2\beta C_1 I_1(\beta q_2) \\ & - 2\beta C_2 K_1(\beta q_2) + q_2\} - q_1^2\Big], \end{aligned} \tag{19}$$

where I_1 and K_1 are the modified Bessel functions of first kind of order one and A_1, A_2, B_1, B_2, C_1 and C_2 are given by Eq. (11). In the evaluation of the above integrals the following identity (ref. [20]) has been used

$$\left(\frac{1}{z}\frac{\mathrm{d}}{\mathrm{d}z}\right)^m \left\{z^\nu \hat{A}_\nu(z)\right\} = z^{\nu-m}\hat{A}_{\nu-m}(z), \tag{20}$$

with $m = 1$ and $\nu = 1$. $\hat{A}_\nu$ denotes I_ν and $e^{\nu\pi i}K_\nu$.

The dimensionless rate of volume flow through the cross-section of the composite channel is

$$Q = Q_1 + Q_2 + Q_3 \tag{21}$$

or

$$\begin{aligned} Q = \frac{\pi}{8\alpha^2\beta^2} & \Big[8\beta^2(2\alpha A_1 I_1(\alpha) + 1) + 8\alpha^2\{q_1\{2\beta C_2 K_1\,(\beta q_1) \\ & - 2\beta C_1 I_1(\beta q_1)\} + q_2\{2\beta C_1 I_1(\beta q_2) - 2\beta C_2 K_1(\beta q_2) + q_2\} - q_1^2\} \\ & + \alpha^2\beta^2\{q_1^2(8B_1 \log q_1 - 4B_1 + 8B_2) + 4B_3 - 8B_2 - q_1^4 + 1\}\Big]. \end{aligned} \tag{22}$$

The dimensionless volume flow rate Q_0 for clear fluid flow (when permeability is infinite) can be obtained by taking limit α and $\beta \to 0$ in Eq. (22). We get

$$Q_0 = \lim_{\substack{\alpha\to 0\\ \beta\to 0}} Q = \frac{\pi q_2^4}{8}, \tag{23}$$

which is classical result for Poiseuille flow.

3.2. Average velocity

The dimensionless average velocity of the flow is defined as

$$u_{avg} = \frac{Q}{\pi q^2}\,. \tag{24}$$

Substituting Q from the Eq. (22) into the above Eq. (24), we get the average velocity of the flow within the composite channel as

$$\begin{aligned} u_{avg} = \frac{1}{8q^2\alpha^2\beta^2}\Big[&8\beta^2\{2\alpha A_1 I_1(\alpha)+1\}+8\alpha^2\{q_1\{2\beta C_2 K_1(\beta q_1)\\ &-2\beta C_1 I_1(\beta q_1)\}+q_2\{2\beta C_1 I_1(\beta q_2)-2\beta C_2 K_1(\beta q_2)+q_2\}-q_1^2\}\\ &+\alpha^2\beta^2\{q_1^2(8B_1\log q_1-4B_1+8B_2)+4B_3-8B_2-q_1^4+1\}\Big], \end{aligned} \tag{25}$$

where constants A_1, B_1, B_2, C_1 and C_2 are given by Eq. (11). For clear fluid flow average velocity of the flow is obtained by taking limit α and $\beta \to 0$ in Eq. (25). We get

$$\lim_{\substack{\alpha\to 0\\ \beta\to 0}} u_{avg} = \frac{q_2^4}{8q^2}, \tag{26}$$

which is well known average velocity for classical Hagen-Poiseuilli flow.

3.3. Shearing stress on the impermeable surface of the channel

The dimensionless shearing stress at any point within inner porous cylinder is given by,

$$\tau_{rz}(r) = -\frac{\mathrm{d}u}{\mathrm{d}r}\,. \tag{27}$$

Substituting u from Eq. (8) and differentiating the modified Bessel functions $I_o(\alpha r)$ and $K_o(\alpha r)$ with the use of the identity $\frac{\mathrm{d}}{\mathrm{d}r}I_o(r) = I_1(r)$ and $\frac{\mathrm{d}}{\mathrm{d}r}K_o(r) = -K_1(r)$ (ref. [20]), we get

$$\tau_{rz}(r) = -\alpha A_1 I_1(r\alpha). \tag{28}$$

Similarly, the dimensionless shearing stress at any point within clear region II is given by,

$$\tau_{rz}(r) = -\frac{\mathrm{d}u}{\mathrm{d}r}\,. \tag{29}$$

Substituting u from Eq. (9) in above equation and after differentiation we get,

$$\tau_{rz}(r) = \frac{r^2-2B_1}{2r} \tag{30}$$

and the dimensionless shearing stress at any point within outer porous cylinder is given by,

$$\tau_{rz}(r) = -\frac{\mathrm{d}u}{\mathrm{d}r}\,. \tag{31}$$

Substituting u from Eq. (10) in above equation and after differentiation we get,

$$\tau_{rz}(r) = -\left[\beta C_1 I_1(r\beta) - \beta C_2 K_1(r\beta)\right], \tag{32}$$

where I_1 and K_1 are modified Bessel function of order one. Shear stress on the surface of inner and outer cylinder is obtained by putting $r = q_1$, $r = 1$ and $r = q_2$ in Eqs. (28), (30) and (32), respectively and using the appropriate sign. Thus,

$$\tau_{rz}(1) = -\alpha A_1 I_1(\alpha), \tag{33}$$

$$\tau_{rz}(q_1) = \frac{q^2 - 2B_1}{2q} \tag{34}$$

and

$$\tau_{rz}(q_2) = -\left[\beta C_1 I_1(q_2\beta) - \beta C_2 K_1(q_2\beta)\right], \tag{35}$$

where A_1, B_1, B_2, C_1 and C_2 are given by Eq. (11). Dimensionless shearing stress on the surface of outer porous cylinder for clear fluid flow (i.e., when α and $\beta = 0$) is obtained by taking limit α and $\beta \to 0$ in Eq. (35). We get

$$\lim_{\substack{\alpha\to 0\\ \beta\to 0}} \tau_{rz}(q_2) = \frac{q_2}{2}\,, \tag{36}$$

which is the classical result for Poiseuille flow in a circular pipe.

4. Discussion

Fig. 2 shows the velocity profile within the composite channel computed from Eqs. (8), (9) and (10). The velocity profile is drawn for three different cases for fixed value of $q_1 = 2$, $q_2 = 2.5$; (i) when permeability of inner porous region is equal to that of outer porous region ($\alpha = \beta$), (ii) when permeability of inner porous region is greater than that of outer region ($\alpha > \beta$), (iii) when permeability of inner porous region is smaller than that of outer region ($\alpha < \beta$). Velocity increases as with r from $r = 0$ to $r = 1$ and takes maximum in the clear region II after that it decreases in region III and becomes zero on the impermeable outer surface of the channel due to no slip condition there. Velocity profiles for $\alpha = 3, 4, 6$ and $\beta = 3, 4, 6$ are sketched. We observe that as α increases velocity in region I decreases due to a decrease in permeability of the region. Similarly, an increase in β causes a decrease in velocity in the region III.

Figs. 3 and 4 represent the variation of volume flow rate with permeability parameter α and β. It is clear that with the increase of permeability parameter volume flow rate through the channel decreases in both the porous regions because the increase in permeability parameter causes a decrease in permeability of the porous medium.

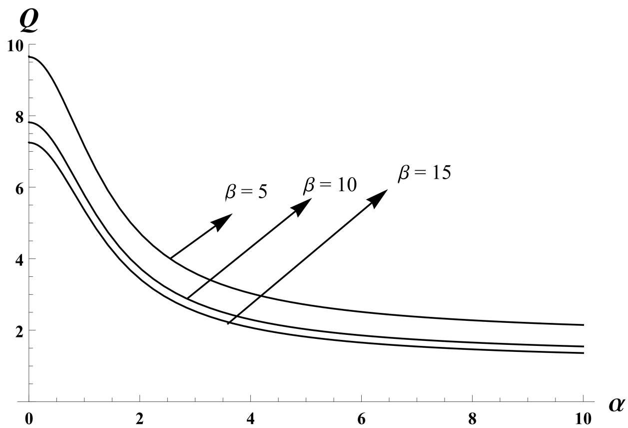

Fig. 3. Variation of volume flow rate with α for different values of β, when $q_1 = 2$ and $q_2 = 2.5$

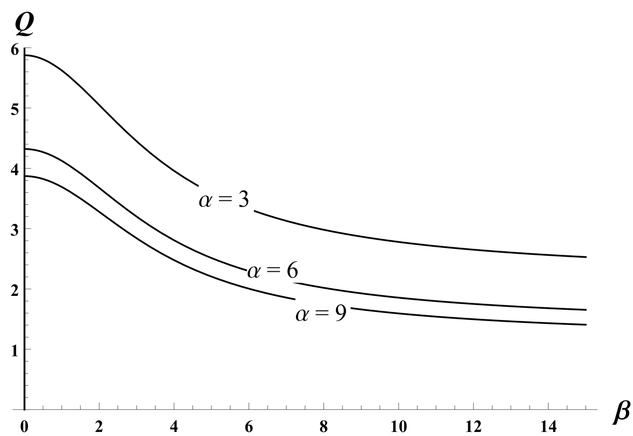

Fig. 4. Variation of volume flow rate with β for different values of α, when $q_1 = 2$ and $q_2 = 2.5$

Fig. 5 represents the variation of shear stress on the impermeable surface with the permeability parameter α for different values of permeability parameter β and fixed value of $q_2 = 2.5$, $q_1 = 2$. We observe that stress decreases with an increase in the value of α i.e., a decrease in the permeability on inner region.

Fig. 6 represents the variation of shear stress on the impermeable surface with the permeability parameter β for different values of permeability parameter α and fixed value of $q_1 = 2$, $q_2 = 2.5$. We observe that stress decreases with an increase in the value of β i.e., a decrease in the permeability on outer region.

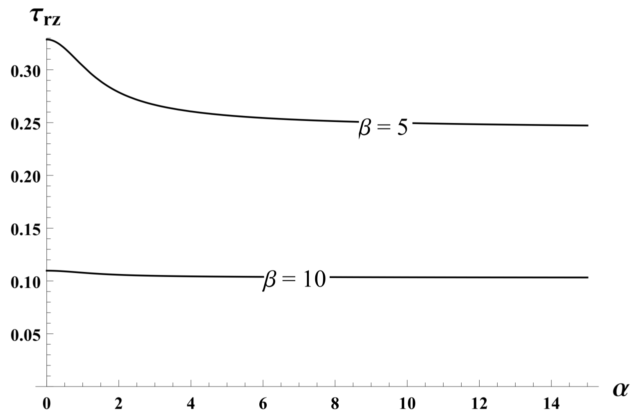

Fig. 5. Variation of shear stress with permeability parameter α for different values of β, when $q_1 = 2$ and $q_2 = 2.5$

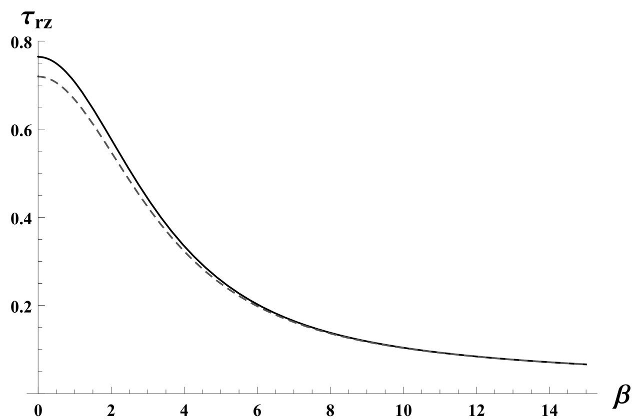

Fig. 6. Variation of shear stress on outer cylinder with permeability parameter β for different values of $\alpha = 5$ and $\alpha = 10$ (dash curve), when $q_1 = 2$ and $q_2 = 2.5$

5. Conclusions

The flow of a viscous incompressible fluid within a composite porous cylindrical channel has been investigated using the Brinkman and Stokes equation. The inner porous region and the outer porous region of the cylinder are of different permeability. An analytical solution of the governing equations for the flow within the channel has been obtained. The exact expressions for velocity, volume flow rate and sheer stress on the cylinder are obtained. In the limiting case when permeability of the porous regions tend to infinite, the obtained results reduce to the classical results of clear fluid in the cylindrical channel. We found that variation of permeability and gap parameter have a remarkable effect on the flow. The results obtained are very useful in the application of extraction of oil and filtration industries.

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