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AMBIGUITY IN DETERMINING THE CRITICAL TEMPERATURE OF A STEEL SWAY FRAME WITH SEMI-RIGID JOINTS

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The problem of uniqueness and representativeness of steel frame fire resistance assessment is considered in this paper. The thesis, that the selection of analysis method determines the result in both qualitative and quantitative terms is given scrutiny. It is also shown, that the differences between computed values may be significant. The selection of an appropriate computational model for an analysis of this type seems to be especially important, as the possible overestimation of the fire resistance determined during computation is equivalent to an unjustified optimism of the user with respect to the safety level warranted. In the considerations presented here the critical temperature determined for the whole bearing structure is considered as the measure of sought resistance. The determined temperature is associated with the bearing structure reaching the bearing capacity limit state subject to fire conditions, treated as accidental design situation. Two alternative computational methods have been applied during calculations: the first one – classical, based on 1st order statics and using the buckling length concept for members of the considered frame, and the second one – taking account of 2nd order phenomena via simple amplification of the horizontal loads applied to the frame. Special attention has been paid to the influence exerted on the final fire resistance of the considered structure by the real joint rigidity, decreasing with increasing temperature of the structural members. The obtained results differ not only in the value of determined temperature but also in the indicated location of the weakest frame component, determining its safety.

*Keywords***:** fire resistance, steel frame, critical temperature, fully-rigid joints, semi-rigid joints, end-plate beam-to-column joints.

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1. INTRODUCTION

In the conventional procedure applied to determine the bearing capacity of a sway frame, related to the accidental design situation, when the frame besides typical external loads is subjected to additional action of fire temperature, the influence of this temperature is accounted for in general via an appropriate correction of the mechanical properties of steel used to make the bearing structural components, and in particular the characteristic yield limit and modulus of longitudinal elasticity. In such approach the flexibility of joints increasing with developing fire is disregarded, and this by itself may lead to overly optimistic estimates of fire resistance sought, and as a consequence to overestimated warranted safety level. The joint designed for persistent design conditions as nominally fully-rigid during fire usually becomes semi-rigid. This change to a large extent determines the redistribution of internal forces occurring in practice. Taking into account the actual flexibility of joints correlated with particular steel temperature and increasing with fire development significantly complicates calculations, as the flexural characteristic of the joint has to be determined a'priori for a given temperature and its change with changing temperature has to be accounted for as well. The characteristics of this type, illustrated for particular joints by appropriate relations depicted in the coordinate system of "bending moments – increase in the rotation angle" may be determined experimentally or simulated numerically [1]. It has to be underlined, however, that these characteristics would depend on the joint heating scenario assumed during analysis. This heating may be executed traditionally in the steady state heating regime, or alternatively in the transient state heating regime [2]. The research conducted so far seems to indicate, that the joint behaviour, when subjected to fire temperature, may differ substantially depending on its geometry, type and load transfer mode. It is different for end-plate beam-to-column joint [3-8], different for bolted angle beam-to-column connections [9-10] and yet different for beam-to-column shear connections [11-12]. The bolted joint behaves in a different manner than the fully welded one [13]. The comparative analysis of particular characteristics juxtaposed for various joint structures and listed for fire conditions may be found for instance in papers [14-18]. During the second stage of calculations, based on the flexural characteristics determined in advance, a substitute schematic diagram conforming to the rules of the classical component method generalized to the case of fully developed fire is developed. Then the basic parameters of the so developed formal model are calibrated [19-20]. Detailed considerations pertaining to the modelling of joint components subjected to fire conditions may be found in [21] for components located in its tension zone, and in [22] for components located in its compression zone. Paper [23] is devoted to the behaviour of column web in those conditions.

The fire resistance of steel frame subjected to the action of fire temperature is usually measured by the period of time, from fire initiation or possibly flashover in a given building compartment, during which in spite of gradual and progressing weakening, it is capable of safely resisting the loads applied to it. According to the opinion of the Authors this measure is not fully objective, as a change in the forecast fire development scenario is sufficient to obtain, after appropriate calculations, a completely different value of the resistance sought. Thus, in the presented considerations an alternative safety measure defined for the considered frame as unequivocally specified critical temperature [24] is used for formal reasoning. This temperature is defined as a certain temperature of steel, recorded in a selected component of the considered frame but representative for the whole bearing structure, after reaching which the fire resistance limit state will be achieved in the whole structural system. Of course the limit state of this type shall not be associated with immediate destruction of the structure, but only with the fact, that the probability of failure has reached an unacceptable level. In this approach the selected measure of safety does not depend on the fire development scenario considered in practice, and thus may be treated as a certain characteristic of the bearing structure under scrutiny. Let us note, that with the critical temperature identified a'priori for a bearing system, one may easily determine its safe service time under anticipated fire conditions. It is sufficient to check the fire exposure time needed to reach this temperature at the point representative for the whole structure. Unequivocal determination of the critical temperature, such that, although calibrated only locally, could be considered a measure of safety specified for the load bearing system treated as a whole, is not a trivial task, and in general is not always possible. However, due to the homogeneity of the material the measure of this type seems to be especially appealing in the fire resistance analysis of steel structures. A general assumption on the mutual proportionality of the temperature increase in individual frame components, necessary in the applied computational procedure for the forecast fire, may be considered as posing certain restrictions here. However, the analyses conducted by the Authors of this paper seem to indicate that an approximation of this type, in spite of the fact that in many practically important design scenarios seems to be hardly acceptable, in general does not result in significant quantitative errors.

The purpose of this paper may be stated as showing that the determination of the critical temperature for a given steel bearing system may be ambiguous due to formal reasons. This ambiguity, however, is not rooted in the insufficiencies of the theoretical model described above. It originates in the plethora and qualitative variety of approaches, which may be applied to perform the static analysis of the considered frame. We intend to show, that application of two, commonly treated as statically equivalent, computational algorithms: the first one – based on conventional 1st order statics with application of buckling length concept, and the second one – taking into account 2nd order phenomena by a simple amplification of the horizontal loads applied to the structure, will result, for the same frame loaded in the same manner, in two completely qualitatively and quantitatively disparate resistance forecasts. The results shown in this paper have been in parts presented in [25-27]. This paper, in the Authors intent, is a supplement to these previous works, hence completing the substantive discussion.

2. DESCRIPTION OF FRAME ANALYZED IN THE EXAMPLE

A two storey high two aisle steel sway frame having the dimensions depicted in Fig. 1 subjected to fire is analysed here. The distance between frames equals 6.0 m. It is assumed that the fire flashed over in both aisles on the ground floor of the considered structure. This in turn results in heating of only bottom columns and bottom girders of the bearing structure due to direct fire exposure. It is assumed, that the top columns and top girders are perfectly insulated from fire action and are not heated. The fire itself is modelled via increasing temperature of bearing components indicated above, distributed evenly over the whole length and cross section of affected members. The level of external loads applied to the structure does not change for the duration of fire.

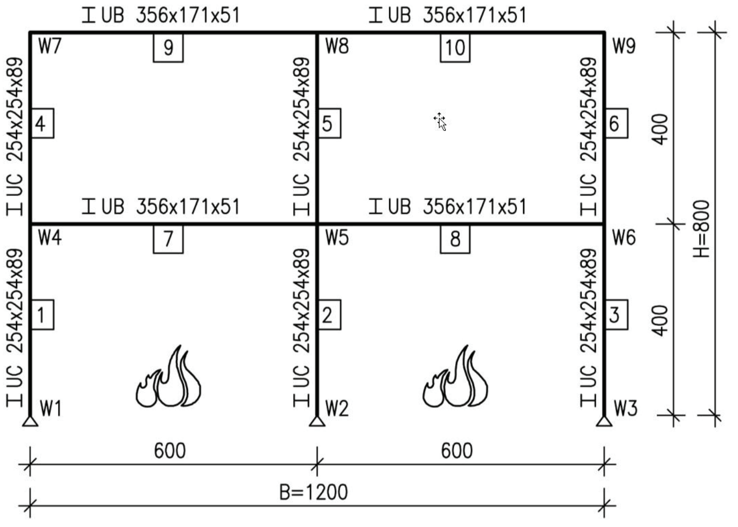

Fig. 1. The scheme of the frame considered in the example with joint numbers and location of fire indicated.

The beam-to-column joints, two sided version of which is depicted in Fig. 2, and for which the Authors had an experimentally determined flexural characteristic depending on steel temperature Θ_a presented first in [1] and depicted in Fig.3 available, have been used. Therefore it has been assumed that the girders of the considered frame are made of UB356x171x51 I-beams, while the columns are made of UC254x254x89 I-beams. Due to the same reason it has been assumed that all components of the frame are made of the steel exhibiting characteristic yield limit $f_y = 412\,\mathrm{MPa}$. The flexural characteristics of one sided joints are identical to those ascribed above to two sided joints.

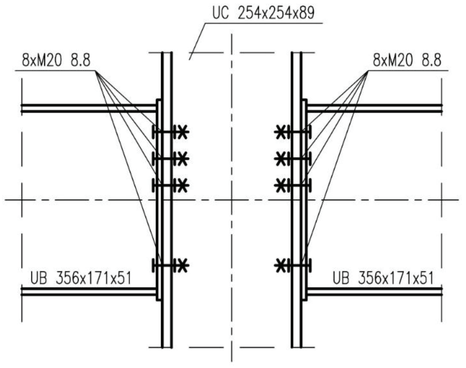

Fig. 2. Two sided end-plate beam-to-column joint used in the frame, denoted in Fig. 1 by the symbols W5 and W8. The flexural characteristics depicted in Fig. 3 pertain to this joint.

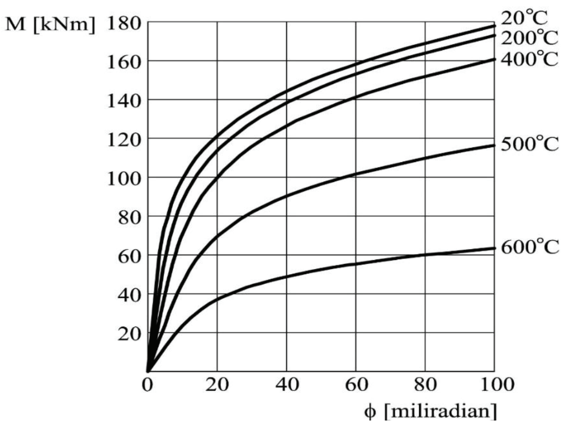

Fig. 3. Flexural characteristics of the end-plate beam-to-column joint considered in the example – based on [1] (pertain to all one sided and two sided joints).

In Fig. 4 the distribution and characteristic values of external loads applied to the frame are depicted. These values are considered independent of steel component temperature during the following analysis and thus do not change with development of simulated fire. The horizontal forces are due to the wind action and substitute loads induced by the assumed initial sway imperfection.

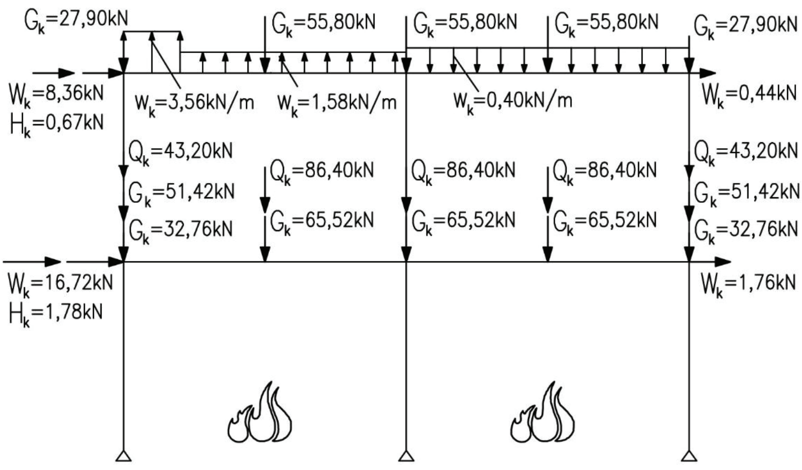

Fig. 4. External loads applied to the frame considered in the example (the level of those loads does not change during the fire).

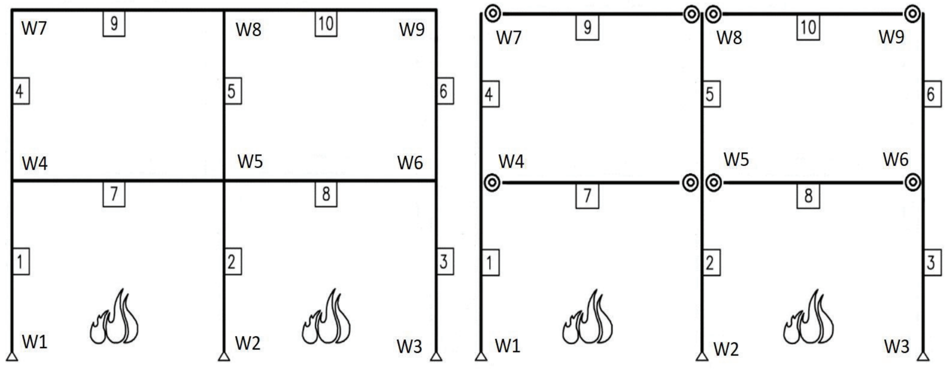

Fig. 5. Two alternative static schemes applied during the analysis related to fire situation, at left with all joints nominally fully-rigid during the whole fire, at right with semi-rigid joints, wherein the flexibility of joints W4, W5 and W6 increases with fire development, while the flexibility of joints W7, W8 and W9 remains constant during whole time of fire, as determined for the ambient temperature of $\Theta_a = 20^oC$.

As has been noted above, all the beam-to-column joints (this means both one and two sided ones) have the same structure of connections. Two variants of these joints have been analysed in the paper (Fig. 5). In the first variant these joints have been treated as nominally rigid during the whole fire. Alternatively, for comparative purposes, these joints have been treated as flexible ones, with the flexural characteristics depicted in Fig. 3. The static analysis has been performed with *Robot Structural Analysis 2010*. The dependence of yield limit and longitudinal modulus of elasticity on the temperature has been accounted for, according to the formulae: $f_{y,\Theta} = k_{y,\Theta} f_y$ and $E_{a,\Theta} = k_{E,\Theta} E_a$ where $k_{y,\Theta}$ and $k_{E,\Theta}$ are appropriate reduction coefficients listed in the code EN 1993-1-2 [28].

3. METHODS OF ANALYSIS

During the design process fire is treated as accidental design situation, this determines the application of a load combination set specific to this case. In our considerations the following combination has been accepted for calculations as representative: *characteristic dead load + characteristic permanent loads + characteristic service loads x 0.6 + characteristic wind loads x 0.2 + thermal load x 1.0*. This combination of loads is in agreement with the classical rule prescribed by the code [29], provided that the wind load is treated as the leading variable load (thus assumed at its frequent value level) while the service load is treated as the secondary load (thus assumed at its quasi-permanent value). However, one should bear in mind that during the simulated fire the hierarchy of variable loads may unexpectedly change. This assumption means also, that the horizontal substitute loads, modelling the influence of the imperfection, are determined in relation to the sum of vertical loads specified according to the combinatory rule accepted for calculations.

Two alternative computational approaches have been subjected to detailed scrutiny:

- *1st order analysis with application of the buckling length concept* – the critical load $N_{cr,y}$ is determined for the in-plane buckling case after the α_{cr} multiplier has been found (with respect to the column – for the first sway buckling mode, with respect to the girder – for the first symmetrical buckling mode); for the out-of-plane buckling the critical load $N_{cr,z}$ is determined subject to the assumption, that the out of plane buckling length of a member is equal to its theoretical length (pinned support); subsequently the relative slendernesses $\overline{\lambda}_y$

and $\overline{\lambda_z}$ are calculated followed by computation of buckling coefficients χ_y and χ_z; the lateral-torsional buckling factor χ_{LT} is determined independently,

- *simplified 2nd order analysis* – it has been checked, whether the frame is susceptible to the 2nd order phenomena ($\alpha_{cr} < 10$), next the amplification factors $\eta_{amp} = \left(1 - \frac{1}{\alpha_{cr}}\right)^{-1}$ have been determined for horizontal forces, independently at the upper and lower girder level; subsequently the values of bending moments and longitudinal forces have been found for the so amplified set of applied loads; the critical forces $N_{cr,y}$ and $N_{cr,z}$ have been computed subject to the assumption, that the member buckling lengths both in frame plane and out of that plane are equal to their theoretical lengths; the further course of action is analogous to that described above.

The ground floor column denoted as no "3" in Fig. 1, and girder no "8" supporting the ground floor ceiling are the bearing members authoritative to determine the critical temperature of the considered structure. These elements are subjected to both bending and compression, thus following the recommendations of the code EN 1993-1-2 [28], the sought critical temperature $\Theta_{a,cr}$ will be determined by satisfaction of the more stringent of two limit conditions cited below (the coefficients denoted by the Authors with additional superscript index Θ depend on the considered steel temperature):

$$\rho_1 = \rho(\Theta_{a,cr}) = \frac{N^{\Theta}_{fi,Ed}}{\chi^{\Theta}_{min,fi} A k^{\Theta}_{y,\Theta} \frac{f_y}{\gamma_{M,fi}}} + \frac{k^{\Theta}_y M^{\Theta}_{y,fi,Ed}}{W_y k^{\Theta}_{y,\Theta} \frac{f_y}{\gamma_{M,fi}}} = 1 \quad , \tag{3.1}$$

$$\rho_2 = \rho(\Theta_{a,cr}) = \frac{N^{\Theta}_{fi,Ed}}{\chi^{\Theta}_{z,fi} A k^{\Theta}_{y,\Theta} \frac{f_y}{\gamma_{M,fi}}} + \frac{k^{\Theta}_{LT} M^{\Theta}_{y,fi,Ed}}{\chi^{\Theta}_{LT,fi} W_y k^{\Theta}_{y,\Theta} \frac{f_y}{\gamma_{M,fi}}} = 1 \quad . \tag{3.2}$$

As one may easily note, the coefficient ρ_1 is related to the interaction of the in plane bending (lateral-torsional buckling is excluded) and compression with potential buckling "in the weaker direction" (this may be buckling about the y axis, due to the substantial buckling length, as well as buckling about the z axis, associated with a smaller radius of inertia). On the other hand the coefficient ρ_2 is the effect of the in plane bending, at risk of lateral-torsional buckling, interacting with compression at risk of potential buckling about the z axis. Let's pay attention to the fact that not only the global instability factors of members (χ_y, χ_z, χ_{LT}) but also the factors resulting in the nonlinear form of the interaction curve $M-N$ (in particular k_y and k_{LT}) depend on the steel temperature. In this case, however, the elastic redistribution of bending moments and other internal forces associated with, among others, changes in joint flexibility, is of fundamental importance.

4. RESULTS OBTAINED AFTER APPLICATION OF THE 1ST ORDER ANALYSIS

Stability analysis of the considered frame, related to the fire situation, was carried out separately for the design situation, where it has been assumed, that all beam-to-column joints during the whole fire exposure remain nominally fully-rigid, and for comparison, for a model taking into account their real rigidity decreasing with increasing temperature of steel. The obtained results have been juxtaposed in the Tables 1 and 2 for the column no "3" and Tables 3 and 4 for the girder no "8".

Table 1: Results referring to the column no "3", obtained after application of the 1st order analysis for a frame with nominally rigid beam-to-column joints during the whole duration of fire.

Θ_a $[^oC]$	N $[kN]$	M_y $[kNm]$	α_{cr}	$N_{cr,y}$ $[kN]$	$\overline{\lambda}_y$	χ_y	$N_{cr,z}$ $[kN]$	χ_z	χ_{LT}	ρ_1	ρ_2
20	228,2	35,7	11,18	2551,5	1,35	0,35	5842,3	0,56	0,62	0,21	0,19
100	220,4	77,0	11,15	2457,8	1,38	0,34	5842,3	0,56	0,62	0,30	0,31
200	212,8	112,0	10,07	2143,2	1,47	0,31	5258,1	0,53	0,60	0,39	0,43
300	205,2	140,9	9,00	1845,9	1,59	0,28	4673,8	0,50	0,57	0,47	0,54
400	197,7	163,3	7,93	1567,3	1,72	0,24	4089,6	0,47	0,54	0,54	0,64
500	190,8	178,3	6,86	1308, 2	1,67	0,26	3505,4	0,49	0,56	0,72	0,83
600	190,6	134,2	3,74	713,2	1,75	0,24	1811,1	0,46	0,53	1,10	1,08

Tables 1 and 3 pertain to the case of rigid joints, while Tables 2 and 4 pertain to the case of flexible joints. Those results allowed to estimate the critical temperature of the structural members at the intersection of appropriate relationships $\rho_1 = \rho_1(\Theta_a)$ and $\rho_2 = \rho_2(\Theta_a)$ with the limit level 1.0.

Those estimates are depicted in Fig. 6 and 7 (for the column no "3" and girder no "8" respectively). In those cases where this intersection occurred above the temperature of 600°C the result has been predicted by polynomial interpolation.

Table 2: Results referring to the column no "3", obtained after application of the 1st order analysis for a frame with semi-rigid beam-to-column joints, flexibility increasing with raising steel temperature.

Θ_a $[^oC]$	N $[kN]$	M_y $[kNm]$	α_{cr}	$N_{cr,y}$ $[kN]$	$\overline{\lambda}_y$	χ_y	$N_{cr,z}$ $[kN]$	χ_z	χ_{LT}	ρ_1	ρ_2
20	215,8	24,4	6,61	1426,4	1,81	0,23	5842,3	0,56	0,62	0,26	0,15
100	211,7	64,2	6,60	1397,5	1,83	0,22	5842,3	0,56	0,62	0,34	0,27
200	208,1	99,4	6,25	1301,8	1,89	0,21	5258,1	0,53	0,60	0,43	0,39
300	203,8	125,4	5,39	1098,4	2,06	0,18	4673,8	0,50	0,57	0,53	0,49
400	201,0	147,4	4,68	941,5	2,22	0,16	4089,6	0,47	0,54	0,62	0,59
500	199,8	160,0	3,48	694,4	2,29	0,15	3505,4	0,49	0,56	0,87	0,76
600	202,6	120,3	1,81	366,6	2,44	0,14	1811,1	0,46	0,53	1,48	0,99

Table 3: Results referring to the girder no "8", obtained after application of the 1st order analysis for a frame with nominally rigid beam-to-column joints during the whole duration of fire.

Θ_a $[^oC]$	N $[kN]$	M_y $[kNm]$	α_{cr}	$N_{cr,y}$ $[kN]$	$\overline{\lambda}_y$	χ_y	$N_{cr,z}$ $[kN]$	χ_z	χ_{LT}	ρ_1	ρ_2
20	-10,8	92,4	75,40	813,9	1,81	0,23	517,7	0,15	0,68	0,27	0,40
100	17,7	94,3	70,01	1240,3	1,47	0,31	517,7	0,15	0,68	0,30	0,41
200	42,9	96,6	58,65	2515,5	1,03	0,49	465,9	0,14	0,66	0,38	0,49
300	65,1	99,6	48,31	3143,1	0,92	0,54	414,1	0,13	0,64	0,46	0,57
400	83,8	103,0	39,34	3294,9	0,90	0,55	362,4	0,11	0,61	0,56	0,66
500	98,3	106,4	33,65	3308,8	0,79	0,61	310,6	0,12	0,63	0,76	0,85
600	76,8	106,4	20,00	1535,9	0,90	0,55	160,5	0,11	0,60	1,20	1,24

A comparison of the results juxtaposed in the Table 1 with corresponding results of the Table 2 leads to the conclusion, that taking into account the real flexibility of nodes resulted in substantially higher values of the coefficient ρ_1. This is a simple consequence of a longer column buckling length entered into the formulae (Fig. 8). On the other hand one should note the simultaneous decrease in the value of the coefficient ρ_2. This time it is the result of a slightly different way of bending moments redistribution.

Table 4: Results referring to the girder no "8", obtained after application of the 1st order analysis for a frame with semi-rigid beam-to-column joints, flexibility increasing with raising steel temperature.

Θ_a $[^{o}C]$	N $[kN]$	M_y $[kNm]$	α_{cr}	$N_{cr,y}$ $[kN]$	$\overline{\lambda_y}$	χ_y	$N_{cr,z}$ $[kN]$	χ_z	χ_{LT}	ρ_1	ρ_2
20	-6,2	111,6	62,20	385,6	2,63	0,12	517,7	0,15	0,68	0,32	0,46
100	19,0	111,3	60,59	1151,4	1,52	0,30	517,7	0,15	0,68	0,35	0,48
200	39,8	111,4	54,04	2149,8	1,12	0,45	465,9	0,14	0,66	0,41	0,54
300	58,3	114,5	46,82	2730,1	0,99	0,51	414,1	0,13	0,64	0,49	0,61
400	72,2	116,3	39,13	2825,0	0,97	0,52	362,4	0,11	0,61	0,56	0,68
500	79,7	119,8	30,79	2453,2	0,92	0,54	310,6	0,12	0,63	0,73	0,86
600	57,4	120,9	20,70	1187,7	1,03	0,49	160,5	0,11	0,60	1,15	1,29

Juxtaposition of Tables 3 and 4 seems to indicate a conclusion slightly different from the one drawn earlier for the column no "3". In the case of the girder no "8", after taking into account the real flexibility of nodes both coefficients increased in magnitude (i.e. coefficient ρ_1 as well as coefficient ρ_2). The change in bending moments redistribution described above resulted in the fact that a part of the bending moments previously affecting the columns was now somehow transferred to increase the load applied to the girder.

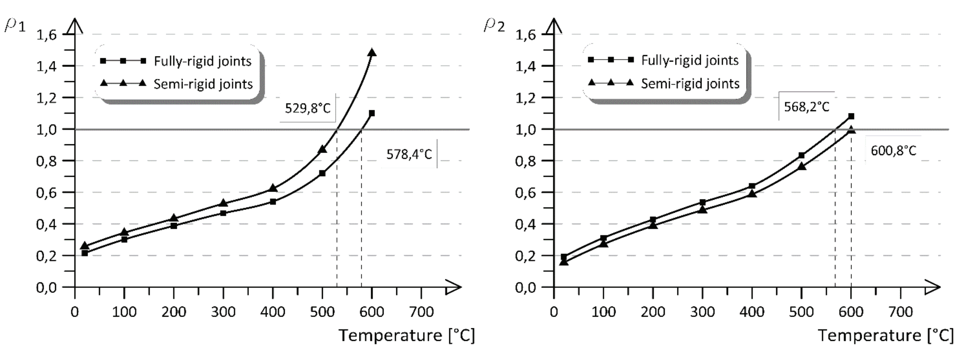

Fig. 6. Estimation of the critical temperature for the column no "3" with application of 1st order theory (at left – depending on the coefficient ρ_1, at right – depending on the coefficient ρ_2).

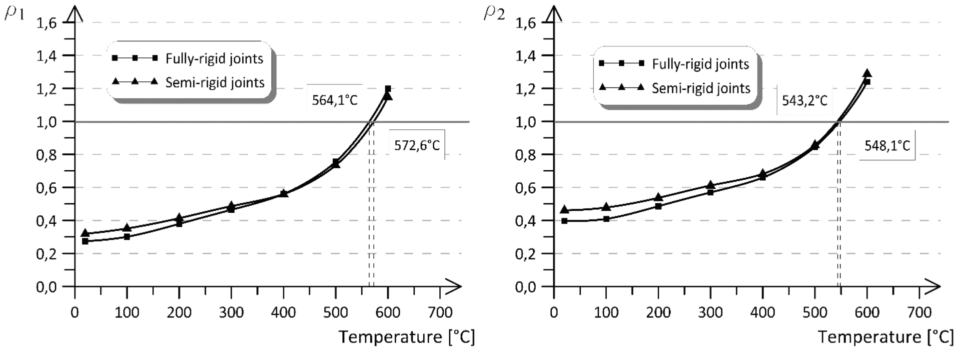

Fig. 7. Estimation of the critical temperature for the girder no "8" with application of the 1st order theory (at left – depending on the coefficient ρ_1, at right – depending on the coefficient ρ_2).

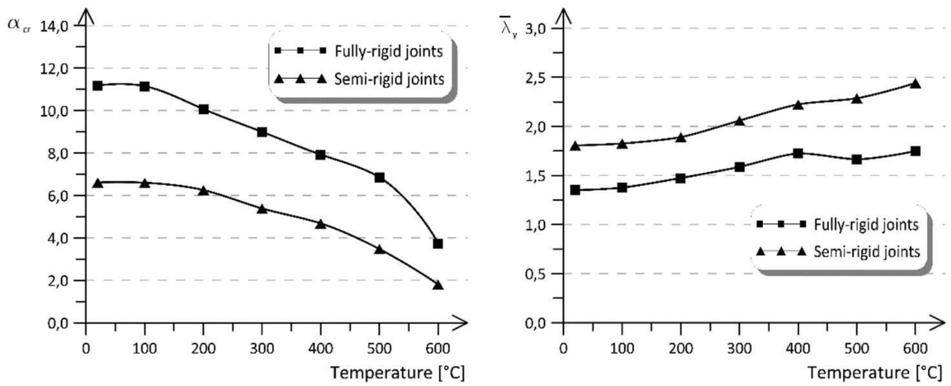

Fig. 8. Values of the α_{cr} multiplier (at left) and relative slenderness $\overline{\lambda}_y$ (at right) depending on the steel temperature Θ_a in the column no "3".

5. RESULTS OBTAINED AFTER APPLICATION OF 2ND ORDER ANALYSIS WITH AMPLIFICATION OF HORIZONTAL FORCES

Should one opt for strict adherence to code provisions, in static analysis the sensitivity of the considered frame to 2nd order phenomena should be taken into account, and should this be confirmed at least the amplification of horizontal forces should be considered. With such approach the buckling lengths of frame members are not determined, it is assumed without additional calculations that those lengths are equal to their theoretical lengths. Verification of such sensitivity has been conducted in the Table 5.

Table 5: Sensitivity coefficients to the 2nd order phenomena and amplification coefficients for the frame considered in the example.

Θ_a $\left[{}^oC\right]$	Joints fully-rigid during whole duration of fire				Joints semi-rigid with rigidity decreasing with fire development			
	α_{cr}		η_{amp}		α_{cr}		η_{amp}	
	Top floor	Bottom floor	Top floor	Bottom floor	Top floor	Bottom floor	Top floor	Bottom floor
20	45,28	11,88	1,02	1,09	13,77	6,53	1,08	1,18
100	45,38	11,88	1,02	1,09	13,77	6,53	1,08	1,18
200	42,64	10,77	1,02	1,10	13,51	6,21	1,08	1,19
300	39,57	9,65	1,03	1,12	11,04	5,28	1,10	1,23
400	36,30	8,54	1,03	1,13	9,49	4,60	1,12	1,28
500	33,27	7,40	1,03	1,16	6,28	3,41	1,19	1,42
600	22,30	4,09	1,05	1,32	3,31	1,85	1,43	2,18

As may be seen, the frame considered in this example, should one opt for disregarding the real flexibility of nodes, becomes sensitive to 2nd order phenomena ($\alpha_{cr} < 10$) only after the temperature slightly lower than 300ºC has been reached. However, this is an overly optimistic conclusion, as the values of α_{cr} coefficient juxtaposed in the right hand side of the Table 5 unequivocally indicate that the sensitivity of this type is exhibited already at the room temperature. Because of this fact, the amplification coefficients are applied in the following calculations regardless of steel temperature. The detailed results obtained after application of the simplified 2nd order analysis, conducted with simple amplification of horizontal forces are juxtaposed in the Tables 6 and 7 with respect to the column no "3" and Tables 8 and 9 with respect to the girder no "8".

A comparison of the results juxtaposed above (in the Tables 6 and 7) indicates, that this time, after application of the simplified 2nd order analysis, the introduction of real joint flexibility depending on steel temperature results in slightly lower values of the coefficients ρ_1 as well as ρ_2 . This phenomenon may be undoubtedly attributed to the changes in the redistribution of bending moments, as described in the preceding chapter. Simply speaking, a part of the bending moment attributed to the column when joints are nominally rigid has become an additional load acting on the girder when the joints are treated as semi-rigid. Let us note, that in such calculations the column buckling length is not specified. Thus a factor causing an additional variation of results observed when conventional 1st order analysis is applied is missing.

Table 6: Results related to the column no “3”, obtained after application of simplified 2nd order analysis for a frame with nominally rigid beam-to-column joints during the whole duration of fire.

Θ_a $[^oC]$	N $[kN]$	M_y $[kNm]$	$N_{cr,y}$ $[kN]$	χ_y	$N_{cr,z}$ $[kN]$	χ_z	χ_{LT}	ρ_1	ρ_2
20	226,9	44,8	17164,8	0,76	5842,3	0,56	0,62	0,17	0,22
100	220,7	77,8	17164,8	0,76	5842,3	0,56	0,62	0,24	0,31
200	213,1	112,9	15448,3	0,74	5258,1	0,53	0,60	0,31	0,43
300	205,5	141,9	13731,8	0,73	4673,8	0,50	0,57	0,36	0,54
400	198,0	164,4	12015,3	0,70	4089,6	0,47	0,54	0,41	0,64
500	191,1	179,6	10298,9	0,72	3505,4	0,49	0,56	0,55	0,84
600	191,3	136,8	5321,1	0,69	1811,1	0,46	0,53	0,76	1,10

Table 7: Results related to the column no “3”, obtained after application of simplified 2nd order analysis for a frame with semi-rigid beam-to-column joints, flexibility increasing with raising steel temperature.

Θ_a $[^oC]$	N $[kN]$	M_y $[kNm]$	$N_{cr,y}$ $[kN]$	χ_y	$N_{cr,z}$ $[kN]$	χ_z	χ_{LT}	ρ_1	ρ_2
20	215,7	22,2	17164,8	0,76	5842,3	0,56	0,62	0,13	0,15
100	211,7	64,2	17164,8	0,76	5842,3	0,56	0,62	0,21	0,27
200	208,1	99,5	15448,3	0,74	5258,1	0,53	0,60	0,28	0,39
300	204,4	127,5	13731,8	0,73	4673,8	0,50	0,57	0,34	0,49
400	201,8	149,9	12015,3	0,70	4089,6	0,47	0,54	0,39	0,60
500	201,0	163,7	10298,9	0,72	3505,4	0,49	0,56	0,52	0,78
600	205,6	130,1	5321,1	0,69	1811,1	0,46	0,53	0,74	1,05

In this juxtaposition (Tables 8 and 9) both coefficients, i.e. ρ_1 as well as ρ_2, after taking into account the real flexibility of nodes become significantly higher than the corresponding coefficients computed subject to the assumption of nominal joint rigidity during the whole course of fire. As has been indicated earlier, this is the result of changes in the static scheme when due to the redistribution a part of the bending moment acting on the columns becomes an additional load applied to girders.

Table 8: Results related to the girder no "8", obtained after application of simplified 2nd order analysis for a frame with nominally rigid beam-to-column joints during the whole duration of fire.

Θ_a $[^oC]$	N $[kN]$	M_y $[kNm]$	$N_{cr,y}$ $[kN]$	χ_y	$N_{cr,z}$ $[kN]$	χ_z	χ_{LT}	ρ_1	ρ_2
20	-4,9	92,2	7559,3	0,72	517,7	0,15	0,68	0,26	0,38
100	17,9	93,7	7559,3	0,72	517,7	0,15	0,68	0,30	0,40
200	43,1	96,0	6803,4	0,70	465,9	0,14	0,66	0,37	0,47
300	65,3	99,0	6047,4	0,68	414,1	0,13	0,64	0,46	0,54
400	84,0	102,3	5291,5	0,66	362,4	0,11	0,61	0,55	0,62
500	98,6	105,6	4535,6	0,67	310,6	0,12	0,63	0,75	0,76
600	77,4	105,1	2343,4	0,65	160,5	0,11	0,60	1,18	1,06

Table 9: Results related to the girder no "8", obtained after application of simplified 2nd order analysis for a frame with semi-rigid beam-to-column joints, flexibility increasing with raising steel temperature.

Θ_a $[^oC]$	N $[kN]$	M_y $[kNm]$	$N_{cr,y}$ $[kN]$	χ_y	$N_{cr,z}$ $[kN]$	χ_z	χ_{LT}	ρ_1	ρ_2
20	-4,2	110,8	7559,3	0,72	517,7	0,15	0,68	0,31	0,45
100	19,8	111,3	7559,3	0,72	517,7	0,15	0,68	0,35	0,47
200	41,1	111,3	6803,4	0,70	465,9	0,14	0,66	0,41	0,52
300	58,7	113,6	6047,4	0,68	414,1	0,13	0,64	0,48	0,58
400	72,7	115,2	5291,5	0,66	362,4	0,11	0,61	0,55	0,64
500	80,3	118,3	4535,6	0,67	310,6	0,12	0,63	0,72	0,78
600	59,1	117,4	2343,4	0,65	160,5	0,11	0,60	1,11	1,12

The way the critical temperature is determined during such calculations has been shown in detail in Fig. 9 (with respect to the column no "3") and in Fig. 10 (with respect to the girder no "8").

6. CONCLUDING REMARKS

The analysis performed seems to corroborate the thesis formulated at the introduction to this paper, that the representative value of the critical temperature may not be unambiguously estimated for a fairly complex bearing structure (other than a simple beam or a single column isolated from the whole bearing system). The restriction of this type is, in our opinion, an unavoidable consequence of the choice facing the designer, as he may choose one of many different courses of action, approved for use by appropriate legal regulations. However, one should be aware that these courses of action (or computational algorithms) are not formally equivalent, as they are based on different assumptions and simplifications of various types.

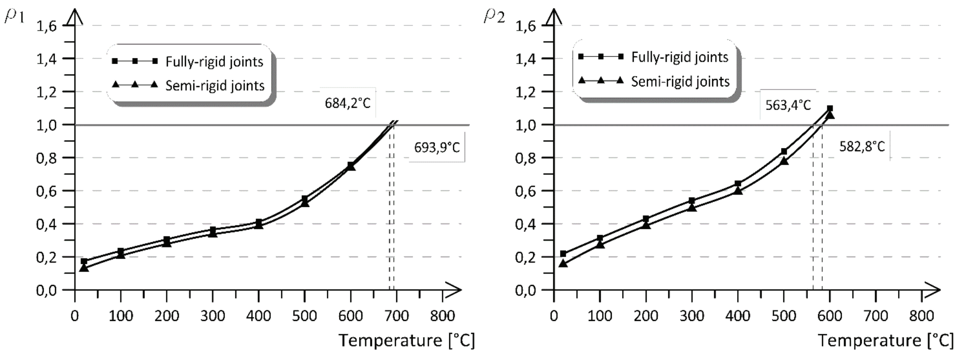

Fig. 9. Critical temperature estimation for column no “3” with application of the simplified 2nd order theory (at left – depending on the coefficient ρ_1, at right – depending on the coefficient ρ_2).

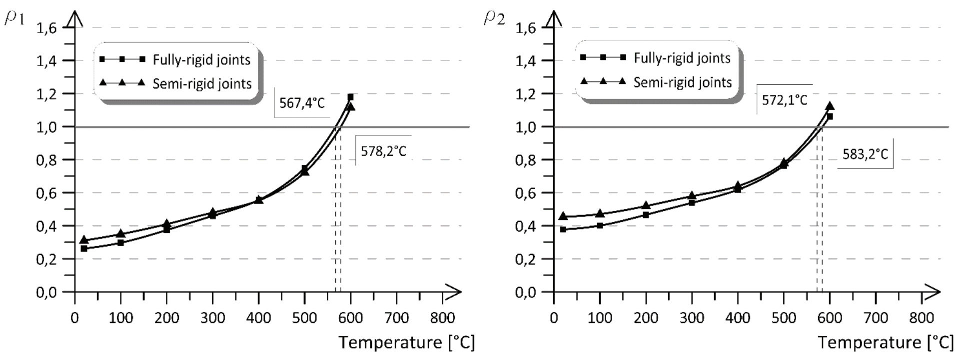

Fig. 10. Critical temperature estimation for girder no “8” with application of the simplified 2nd order theory (at left – depending on the coefficient ρ_1, at right – depending on the coefficient ρ_2).

An application of the computational model with joints nominally rigid during the whole course of fire, with 1st order statics theory resulted in an estimate of the critical temperature at the level of 548.1ºC (Fig. 7). The girder no “8” proved to be authoritative in this case. Taking into account the real flexibility of joints resulted in a completely different conclusion. In this case the critical temperature has been determined as 529.8ºC, and this estimate has been obtained for the column no "3" (Fig. 6). This level of temperature would be reached in the anticipated fire much earlier. This means that the fire resistance of the considered frame, estimated previously without analyzing joint flexibility, and thus the predetermined level of safety have been overestimated. An application for analysis of the simplified 2nd order theory, based on the amplification of horizontal forces acting on the structure, usually yields estimates of the critical temperature less restrictive than those obtained by application of the classical 1st order statics. In the considered example these estimates are 563.4ºC (Fig. 9) in the case of nominally rigid joints and 572.1ºC (Fig.10) when joint flexibility is accounted for. The first of these values pertains to the column no "3" while the second to the girder no "8", i.e. quite the opposite to what has happened after application of the 1st order theory. Moreover, introduction of the real joint flexibility into the analysis resulted in slightly more optimistic estimates of the critical temperature for the considered frame. This would suggest that when joints are modeled as nominally rigid the real safety level remains underestimated. The conclusion of this type is once again a complete reversal of the conclusion drawn based on the results of 1st order analysis. Thus the differences in the obtained results are of qualitative as well as quantitative character. Their origins may be traced to not only the specification of buckling lengths for frame members in the 1st order analysis (absent from the 2nd order analysis), but also a slightly different character of bending moment redistribution under fire conditions. Should one take into account the real and temperature Θ_a dependent joint flexibility, then the bending moment acting on the girders increases gradually at the expense of the bending moment acting on columns, if compared with the model with nominally rigid joints. Which of the values listed above is then the most accurate and credible estimate of the critical temperature for the frame considered in the example? In Authors' opinion the one obtained by the 2nd order analysis applied to the model with real joint flexibility changing during fire, i.e. 572.1ºC. However, is our conclusion about choosing the least conservative of all theoretically possible estimates unambiguous and formally objective? There exist many other, more or less advanced, frame analysis procedures alternative to those shown in this paper. Application of any of those procedures would yield yet another estimate of the sought critical temperature.

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Tab. 1. Results referring to the column no “3”, obtained after application of the 1st order analysis for a frame with nominally rigid beam-to-column joints during the whole duration of fire.

Tab. 1. Wyniki odniesione do słupa numer „3”, uzyskane po zastosowaniu analizy pierwszego rzędu dla ramy z beam-to-column joints nominalnie sztywnymi przez cały czas pożaru.

Tab. 2. Results referring to the column no “3”, obtained after application of the 1st order analysis for a frame with semi-rigid beam-to-column joints, flexibility increasing with raising steel temperature.

Tab. 2. Wyniki odniesione do słupa numer „3”, uzyskane po zastosowaniu analizy pierwszego rzędu dla ramy z semi-rigid beam-to-column joints, przy podatności rosnącej ze wzrostem temperatury stali.

Tab. 3. Results referring to the girder no “8”, obtained after application of the 1st order analysis for a frame with nominally rigid beam-to-column joints during the whole duration of fire.

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Tab. 4. Results referring to the girder no “8”, obtained after application of the 1st order analysis for a frame with semi-rigid beam-to-column joints, flexibility increasing with raising steel temperature.

Tab. 4. Wyniki odniesione do rygla numer „8”, uzyskane po zastosowaniu analizy pierwszego rzędu dla ramy z semi-rigid beam-to-column joint, przy podatności rosnącej ze wzrostem temperatury stali.

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Tab. 7. Results related to the column no “3”, obtained after application of simplified 2nd order analysis for a frame with semi-rigid beam-to-column joints, flexibility increasing with raising steel temperature.

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NIEJEDNOZNACZNOŚĆ W WYZNACZANIU TEMPERATURY KRYTYCZNEJ STALOWEJ RAMY PRZECHYŁOWEJ Z PODATNYMI WĘZŁAMI

Słowa kluczowe*: odporność ogniowa, rama stalowa, temperatura krytyczna, węzły o pełnej sztywności, węzły o niepełnej sztywności, węzły doczołowe belka - słup.*

STRESZCZENIE

W pracy rozważa się problem jednoznaczności i reprezentatywności oszacowania odporności ogniowej ramy stalowej. Weryfikacji poddano tezę, że wybór metody analizy determinuje uzyskany wynik zarówno pod względem ilościowym jak i jakościowym, a różnice pomiędzy wyliczonymi wartościami mogą okazać się znaczące. Dobór miarodajnego modelu obliczeniowego w tego typu analizie wydaje się być szczególnie ważny, bowiem ewentualne przeszacowanie wyznaczonej z obliczeń odporności jest równoznaczne z nieuzasadnionym optymizmem użytkownika budynku co do gwarantowanego mu poziomu bezpieczeństwa. W prezentowanych rozważaniach miarą poszukiwanej odporności jest temperatura krytyczna specyfikowana dla całego ustroju nośnego. Nie zależy ona od prognozowanego scenariusza rozwoju pożaru i z tego względu może zostać uznana za pewnego rodzaju charakterystykę samej konstrukcji. Wyznaczana temperatura kojarzona jest z osiągnięciem przez ustrój nośny stanu granicznego nośności w warunkach pożaru traktowanego jako wyjątkowa sytuacja projektowa. Nie oznacza to jednak natychmiastowej katastrofy badanej konstrukcji ale jedynie sytuację, gdy prawdopodobieństwo tego rodzaju zdarzenia staje się już na tyle duże że nie może być dalej akceptowane. Do szczegółowej analizy wykorzystano dwie alternatywne procedury obliczeń: pierwszą – opartą o klasyczną statykę pierwszego rzędu, z wykorzystaniem koncepcji długości wyboczeniowej elementów badanej ramy, i drugą - uwzględniającą efekty drugiego rzędu przez prostą amplifikację przyłożonego do tej ramy obciążenia poziomego. Szczególną uwagę zwrócono na ocenę wpływu jaki na wynikową odporność ogniową badanego ustroju ma uwzględnienie w obliczeniach rzeczywistej sztywności węzłów, malejącej ze wzrostem

temperatury elementów. Otrzymane wyniki różnią się między sobą nie tylko wartością wyznaczonej temperatury ale i wskazaniem lokalizacji najsłabszego elementu ramy, decydującego o jej bezpieczeństwie. Różnice w oszacowaniach uzyskanych przez autorów dochodzą do $42,3^{o}C$. Ich źródłem jest nie tylko fakt specyfikacji w analizie pierwszego rzędu długości wyboczeniowej prętów ramy (czego nie ma w analizie rzędu drugiego) ale i nieco odmienny charakter realizacji w warunkach pożaru redystrybucji momentów zginających. Jeżeli uwzględnić realną i zależną od temperatury Θ_a podatność węzłów to w stosunku do modelu z węzłami w pełni sztywnymi przez cały czas pożaru zwiększa się moment zginający rygle, a to dzieje się kosztem równoczesnego zmniejszenia momentu zginającego słupy.

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