WARSAW UNIVERSITY OF TECHNOLO	GY Index 351733	DOI: 10.24425/ace.2021.138042				
FACULTY OF CIVIL ENGINEERING		ARCHIVES OF CIVIL ENGINEERING				
COMMITTEE FOR CIVIL AND WATER ENGINEERING		AUGHIVES OF GIVIE ENGINEERING				
POLISH ACADEMY OF SCIENCES	ISSN 1230-2945	Vol. LXVII	ISSUE 3	2021		
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**Research** paper

# Numerical simulations of horizontal bearing performances of step-tapered piles

L.X. Xiong<sup>1</sup>, H.J. Chen<sup>2</sup>, Z.Y. Xu<sup>3</sup>, C.H. Yang<sup>4</sup>

Abstract: In this study, we tried to understand the horizontal bearing performances of step-tapered piles using numerical simulations. The influence of the geometric parameters, e.g. the diameter (D) and the distance (L), and the length (H) of the pile were considered, and the soil distribution imposed on the horizontal bearing capacity of the piles was simulated. Numerical results show that when the other geometrical parameters of step-tapered piles are kept unchanged: (a) the increasing diameter (D) of the enlarged upper part of step-tapered piles improves the horizontal ultimate bearing capacity of step-tapered piles; (b) reduced distance (L) improves the horizontal ultimate bearing capacity of step-tapered piles; (c) Increasing length (H) of the enlarged upper part of step-tapered piles increases the horizontal ultimate bearing capacity; (d) the reduced length (H) decreases the bending moment of the pile body. Higher soil strength surrounding the enlarged upper part of step-tapered piles can increase the horizontal ultimate bearing capacity of step-tapered piles. The change of soil strengths at the end of the step-tapered piles does not influence the horizontal ultimate bearing capacity of step-tapered piles.

Keywords: Step-tapered piles, horizontal bearing performance, numerical simulation, geometric size, soil distribution

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# **1. Introduction**

Tapered pile can be divided into two types, fully tapered and semi-tapered pile (Ghazavi & Tavasoli [1]). The tapered pile utilizes the pile-soil interaction of the wedge side part to increase the bearing capacity of the tapered pile.

In recent years, many researchers have conducted field tests, laboratory tests, theoretical analysis and numerical simulations on the vertical and horizontal bearing capacities of tapered pile and step-tapered piles. Rybnikov [2] carried out experimental investigations of bearing capacity of cast-in-place tapered pile. Jayantha & Ian [3] presented a theoretical model for the analysis of the axial response of tapered piles in cohesive ground. Sakr et al. [4] used model test to study the pile driving tapered FRP - concrete pile in dense sand. Lee et al. [5] investigated the axial load capacity of the tapered piles using cone penetration test. Zhan & Wang [6] carried out numerical analysis of load capacity behaviors of axially loading tapered pile in sand. Kong et al. [7] analyzed the load-transfer theoretically of tapered pile, and used numerical simulations to verify the developed theory. Hataf & Shafaghat [8] investigated the axial bearing capacity of tapered and their counterpart cylindrical pile in sand through numerical simulations. Nabil [9] examined the behavior of step-tapered bored piles in sand under static lateral loading through field tests. When the expanded cross-sections in an appropriate depth are observed with good soil layers, the bearing capacity of the surrounding soil layer can be sufficiently mobilized. Other results also show that a tapered pile can reduce 80% material compared with traditional piles under the same bearing capacity (Jayantha & Ian [3]; Lv et al. [10]).

The afore-mentioned results almost focused on the vertical bearing characteristics of pile with varied cross-sections, few studies concentrated on the lateral static load bearing characteristics of pile with varied cross-sections. In addition, the effect of the soil layer distribution on the bearing characteristics of the pile with varied cross-sections was rarely reported. According to the current research of pile with varied cross-section, we numerically analyzed the step-tapered piles in terms of horizontal bearing behaviors of step-tapered piles, and the influences of various geometric parameters of piles and the soil distribution on the horizontal bearing capacity of the step-tapered piles were also studied.

# 2. Calculation schemes

# 2.1. Sample preparation

The schematic diagram of pile-soil model of step-tapered piles is shown in Fig. 1 (a), and the crosssection of step-tapered piles is shown in Fig. 1 (b).

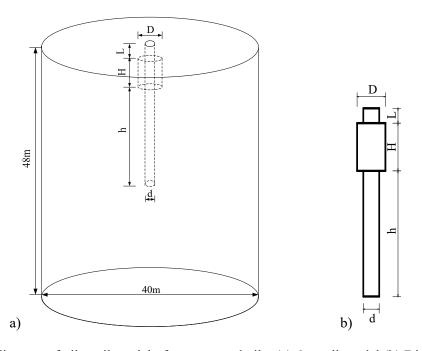


Fig. 1. Schematic diagram of pile-soil model of step-tapered pile. (a) Overall model (b) Dimension of crosssection of pile

In Fig. 1 (b), D is the diameter of the upper enlarged part of step-tapered piles, L is the distance between the upper surface of the enlarged upper part and the upper surface of step-tapered piles, H is the length of the enlarged upper part of step-tapered piles, h is the length of the enlarged lower part of step-tapered piles, d is the diameter of the enlarged lower part of step-tapered piles. Parameter  $\delta$  is defined as D / d. The numerical simulations were conducted by using the geometric dimensions of step-tapered piles and soil parameters as influence factors. The simulation parameters of different groups are in the Table 1.

Study	The parameter <i>L</i> / m	The parameter D / m	The parameter <i>d</i> / m	The parameter <i>H</i> / m	The parameter <i>h</i> / m	Soil layers surrounding the enlarged upper part	Soil layers at the pile end
1	0	1, 1.25, 1.5, 1.75, 2	1.0	4	20	saturated clay	saturated clay
2	0, 4, 8	1.5	1.0	4	20, 16, 12	saturated clay	saturated clay
3	0	1.5	1.0	2, 4, 6, 8	20	saturated clay	saturated clay
4	0	1.5	1.0	4	20	saturated clay, silty clay and sandy soil	saturated clay
5	0	1.5	1	4	20	saturated clay	saturated clay, silty clay and sandy soil

Table 1. Simulation parameters of 5 groups in this study

# 3. Finite element simulation

#### **3.1 Assumptions**

- (1) Due to horizontal force is considered to applying on the top of the step-tapered piles, threedimensional (3D) model for numerical simulation is employed. In order to avoid the influence of boundary conditions, the horizontal radius of the soil model is 20 m, and the length of the steptapered piles is 24 m. The diameter of pile with equal cross-section is 1 m, and the thickness of soil of pile-soil model is 48 m.
- (2) The soil surrounding the step-tapered piles is isotropic and homogeneous, and the pore water pressure is not considered. Mohr-Coulomb model is used when conducting numerical simulation.
- (3) The stiffness of step-tapered piles is larger than that of soils, and the step-tapered piles does not fail when the top of pile is under the vertical and lateral loads simultaneously. Thus, the perfect linear elastic model of the materials of piles is assumed in simulating the body of the piles.

# 3.2 Model establishment

#### (1) Geometry of the model

Both pile body and soil are simulated using solid deformable parts. Mohr-Coulomb friction criterion is used to analyze the contact between the pile and the soil. The thickness of the soil of the model is double of the length of the step-tapered piles, i.e. 48 m. The horizontal radius of the soil is 20 m. The diameter of the lower part of step-tapered piles is 1 m. The diameter (D) of the upper

enlarged part of step-tapered piles is set as 1 m, 1.25 m, 1.5 m, 1.75 m and 2 m, respectively. The total length of the pile body is 24 m.

The solid element of pile body if C3D8R, and soil body is C3D8R, both of them are built by gridding traverse. There are 2464 pile elements and 3000 pile nodes, and 22644 soil elements, 24238 soil nodes. The discretization is same as Xiong et al. [11]. Quadratic reduced integration units are used in this study, which promotes a high resolution in displacement solution and minimum affected by the grid distortion.

#### (2) Constitutive models and material properties of pile and soil

The concrete strength of the pile is using grade C30. The effects of the soils surrounding the enlarged upper part and the end part on the horizontal bearing capacity of the step-tapered piles will be studied. Three kinds of soils are considered in the numerical simulation, i.e., sandy soil, silty clay and saturated clay. The calculation parameters of the pile body and three kinds of soils are shown in Table 2 (Xiong et al. [11]). In this study, the stress level of normal consolidated soil is used. Therefore, the relationship between soil mechanical parameters and stress level is not considered, the dilatancy angle is fixed as 0, this parameter represents soil expansion.

Medium	γ (kN/m <sup>3</sup> )	E (MPa)	ν	с (kPa)	<i>φ</i> (°)
Sandy soil	18.4	15	0.30	5	25
Silty clay	19.5	10.5	0.35	33.8	18.3
Saturated clay	19.2	7.6	0.35	19.5	15
Step-tapered pile	25	30000	0.20	_	-

Table 2. Parameters used for numerical analysis

#### (3) Initial conditions and boundary conditions

The bottom of the soil is the fixed, and the lateral stress boundary is sliding-supported. Mohr-Coulomb friction criterion is used to analyze the contact between the pile and the soil.

# 4. Analysis of splitting tensile test results

### **4.1. Effect of** $\delta$

In this part, the parameters H, h, L, and d for numerical analysis are 4 m, 20 m, 0 m, and 1.0 m, respectively. A horizontal force of 300 kN is applied to the top of step-tapered piles, and the

parameter  $\delta$  is taken as 1, 1.25, 1.5, 1.75 and 2, respectively. The horizontal displacement of the soil surface on the force side with different  $\delta$  is shown in Fig. 2.

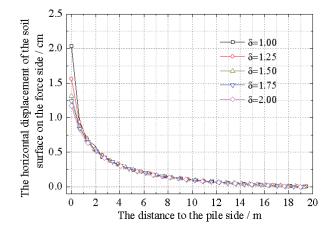


Fig. 2. The horizontal displacement of the soil surface on the force side with different  $\delta$ 

When the axis of the step-tapered piles is regarded as the center, the horizontal displacement of the soil surface on the force side decreases with the increasing radial distance to the axis of the pile. The horizontal displacement of the soil in the force direction applied is the largest. The horizontal displacement of the soil is mainly concentrated on the pile body, and the increasing diameter of the upper enlarged part of step-tapered piles significantly reduces the horizontal displacement of the soil around the pile. However, further increasing of the diameter does not significantly reduce the horizontal displacement of the soil around the pile when the parameter  $\delta$  is higher than 1.5. The horizontal displacement curve of the pile with depth of pile is shown in Fig. 3.

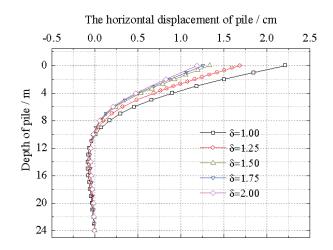


Fig. 3. Relationship curves of the horizontal displacement of pile verse depth with different  $\delta$ 

The horizontal displacement of the pile has a maximum value at the top, and decreases with the pile depth. The horizontal displacement of the pile at the same depth significantly reduces with the increasing diameter (D) of the upper enlarged part of step-tapered piles when the top of step-tapered piles is under the same horizontal force.

The horizontal displacement of the pile is shown in Fig. 4 at the D, d, H, L are 1.5 m, 1 m, 4 m and 0 m, respectively. The horizontal displacement of the pile reaches the maximum value which is same as Fig. 3.

The distribution curve of bending moment of pile with different  $\delta$  is shown in Fig. 5. The bending moment variation with the pile body is accordance with field test (Ismael [12]), which the maximum bending moment happened on pile body.

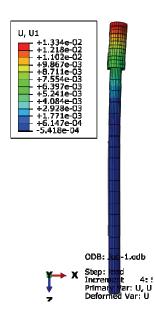


Fig. 4. Diagram of horizontal displacements of the pile

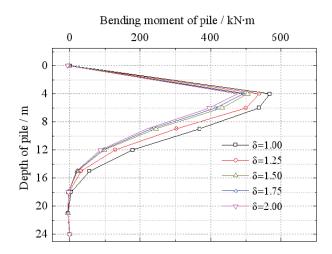


Fig. 5. Distribution curves of bending moment of pile verse pile depth with different  $\delta$ 

The increase of the diameter (D) of the upper enlarged part of step-tapered pile reduces the bending moment of the pile. Nevertheless, the increase of the diameter (D) of the upper enlarged part of step-tapered piles does not obviously reduce the bending moment of pile body when the parameter D is larger than 1.5 m.

The horizontal force-horizontal displacement curves of a single pile are shown in Fig. 6 with different  $\delta$ .

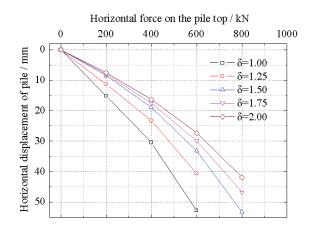


Fig. 6. The horizontal force-horizontal displacement curves of pile with different  $\delta$ 

According to the 'Building Foundation Design Code of China' (GB50007-2011), the loading can be removed if the horizontal displacement is larger than 30–40 mm. The horizontal force can be taken as the horizontal ultimate bearing capacity of a single pile if the horizontal displacement equals to 40 mm. Diameter (D) increasing of the upper enlarged part of step-tapered piles elevates the horizontal ultimate bearing capacity of the pile. However, this effect is not obvious if the parameter D is larger than 1.5 m.

### 4.2. Influence of L

In this part, the parameters D, d, and H are 1.5 m, 1 m, and 4 m, respectively, and the top of steptapered piles is applied by using a horizontal force of 300 kN. The maximum horizontal displacement of surface soil of the pile side with different L is shown in Fig. 7.

The maximum horizontal displacement of the surface soil of the pile side is observed at the pile side in the direction of force applied. The maximum horizontal displacement of the surface soil increases with the increase of L in a certain range.

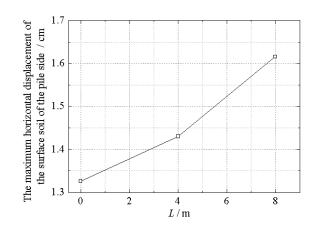


Fig. 7. The maximum horizontal displacement of the surface soil of the pile side with different L

The horizontal displacement curve of the pile with different L is shown in Fig. 8.

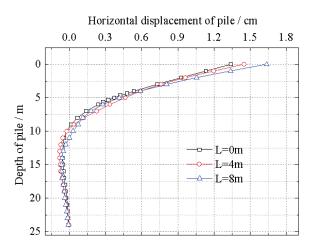


Fig. 8. Relationship curve of the horizontal displacement of pile verse pile depth with different L

When the top of pile is under the same horizontal force, the horizontal displacement of the pile at the same depth increases with increasing L. Therefore, the increase of the parameter L can not obviously reduce the horizontal displacement of the pile.

The bending moment distribution along the pile with different L is shown in Fig. 9. The maximum bending moment of the pile significantly increase with the increasing L. Therefore, the decrease of L can reduce the bending moment of the pile.

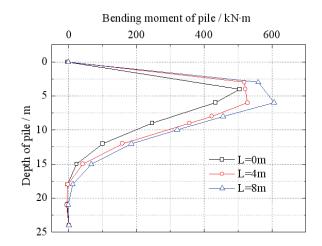


Fig. 9. Distribution curves of bending moment of pile verse pile depth with different L

The horizontal force-horizontal displacement curve of a single pile with different L is shown in Fig. 10. The decrease of L can increase the horizontal ultimate bearing capacity of the pile.

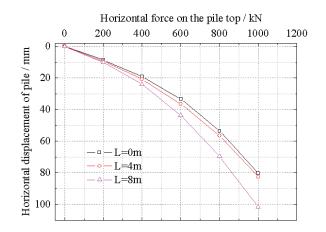


Fig. 10. The horizontal force-horizontal displacement curves of pile with different L

# 4.3. Influence of H

In this part, the parameters D, d, and L are kept as 1.5 m, 1 m, and 0 m, respectively. The top of step-tapered piles is under a horizontal force of 300 kN. The horizontal displacement of the soil surface on the force side with different H is shown in Fig. 11.

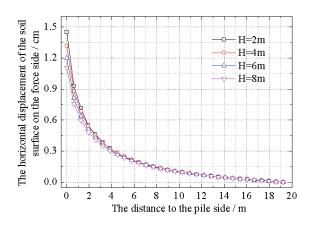


Fig. 11. The horizontal displacement of the soil surface on the force side with different H

The horizontal displacement of the soil is mainly concentrated around the pile body, and the increasing H significantly reduces the horizontal displacement of the soil around the pile. The maximum horizontal displacement of surface soil of the pile side with different H is shown in Fig. 12.

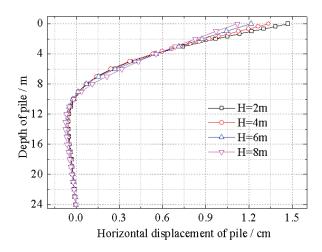


Fig. 12. Relationship curves of the horizontal displacements of pile verse pile depth with different H

When the value of parameter H is constant and the top of the pile is under the force applied, the horizontal displacement of pile decreases with the increase of pile depth. The maximum horizontal displacement of pile significantly decreases with increasing H.

The horizontal displacement at the same depth of pile decreases with increasing H. Therefore, the increasing of H can obviously reduce the horizontal displacement of the pile.

The distribution curve of bending moment of pile with different *H* is shown in Fig. 13.

The maximum bending moment of the pile significantly increases with increasing H. Therefore, the increasing H can reduce the bending moment of the pile.

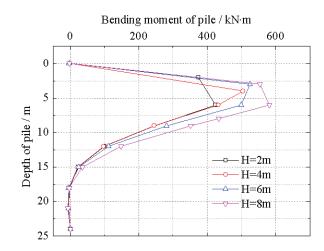


Fig. 13. Distribution curves of bending moment of pile verse pile depth with different H

The horizontal force- horizontal displacement curve of a single pile with different H is shown in Fig. 14. The increasing H increases the horizontal ultimate bearing capacity of the pile.

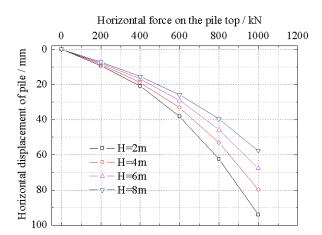


Fig. 14. The horizontal force-horizontal displacement curves of pile with different H

### 4.4. Effect of soil surrounding the enlarged upper part

In this part, the parameters D, d, H, and L are 1.5 m, 1 m, 4 m, and 0 m, and the top of step-tapered piles is under a horizontal force of 300 kN. The soil surrounding the enlarged upper part (in the range of the depth of 0–4 m from the surface of the model) is considered as saturated clay, silty clay and sandy soil and the soil in the range of the depth of 4–48 m from the surface of the model. The horizontal displacement of the soil surface on the force side with different soil surrounding the enlarged upper part is shown in Fig. 15. The horizontal displacement curve of the pile with depth of pile is shown in Fig. 16.

The increase in strength of the soil surrounding the enlarged upper part significantly reduces the horizontal displacement of the soil around the pile. The horizontal displacement of pile surrounded by sandy soil is obviously smaller than saturated clay.

The distribution curve of bending moment of pile with different soils around the enlarged upper part is shown in Fig. 17.

The maximum bending moment of the pile significantly increases with the increasing strength of the soils surrounding the enlarged upper part. Therefore, the increase of the strength of the soil surrounding the enlarged upper part can reduce the bending moment of the pile.

The horizontal force-horizontal displacement curve of single pile surrounded by different soil is shown in Fig. 18. The increase of the strength of the soils surrounding the enlarged upper part increases the horizontal ultimate bearing capacity of the pile.

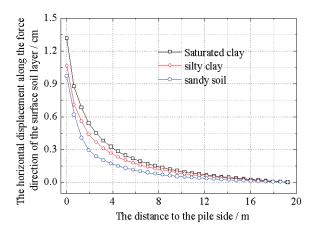


Fig. 15. The horizontal displacement of the soil surface on the force side with varied soils surrounding the enlarged upper part

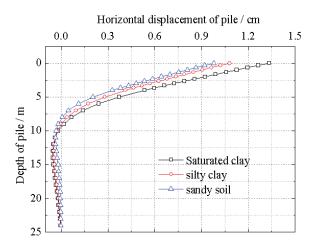


Fig. 16. The horizontal displacement curve of the pile with depth of pile with varied soils surrounding the enlarged upper part

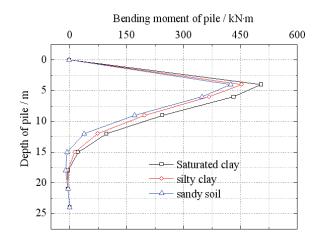


Fig. 17. Curves of bending moment of pile verse pile depth with varied soils surrounding the enlarged upper part

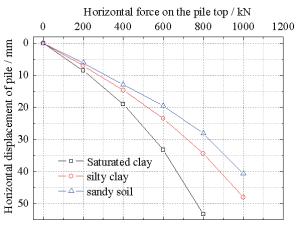


Fig. 18. The horizontal force-horizontal displacement curves with different soils surrounding the enlarged

### upper part

### 4.5. Effect of soil at the pile end

In this part, the parameters D, d, H, and L are considered as 1.5 m, 1 m, 4 m, and 0 m, and the top of step-tapered piles is applied by a horizontal force of 300 kN. The soil surrounding the enlarged upper part is considered as saturated clay, and the soil below the pile end (in the range of the depth of 24–48 m from the surface of the model) is considered as saturated clay, silty clay and sandy soil. The horizontal displacement of the soil surface on the force side is shown in Fig. 19. The increase in the soil strength below the pile end has minor effect on the horizontal displacement of the soil around the pile.

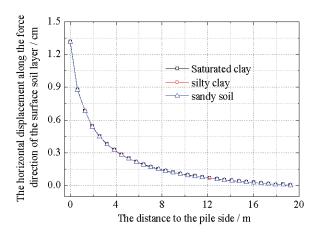


Fig. 19. The horizontal displacement of the soil surface on the force side with varied soils below the end of pile

The horizontal displacement of the pile verse pile depth is shown in Fig. 20. There is no obvious difference in the horizontal displacement of pile between saturated clay, silty clay and sandy soil below the pile end.

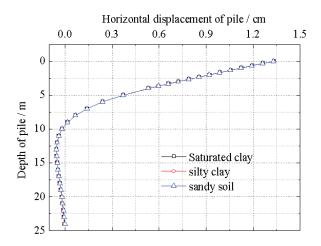


Fig. 20. The horizontal displacement curve of the pile with depth of pile with varied soils below the end of pile

The distribution curves of pile bending moment verse pile depth with varied soils below the pile end are shown in Fig. 21. The increase of the strength of soil below the end of pile is almost ineffective in reducing the bending moment of the pile under the horizontal load.

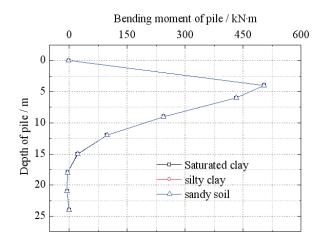


Fig. 21. Distribution curves of pile bending moment verse pile depth with varied soils below the pile end

The horizontal force-horizontal displacement curve of a single pile with different soils below the pile end are shown in Fig. 22. The increasing strengths of soils below the end of pile are almost ineffective in reducing the horizontal ultimate bearing capacity of the pile.

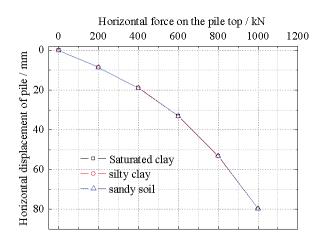


Fig. 22. The horizontal force-horizontal displacement curves with different soils below the pile end

# 5. Conclusions

1. When the soil layer of the model is homogeneous saturated clay and the other geometrical parameters (*H*, *h*, *L*, and *d*) of step-tapered piles keep unchanged, the increasing of the diameter (*D*) of the enlarged upper part of step-tapered piles reduces the horizontal displacements of the pile body and the soil surface, and improves the horizontal ultimate bearing capacity of the step-tapered piles. However, the increasing diameter (*D*) of the upper enlarged part of step-tapered

piles does not obviously improve the horizontal ultimate bearing capacity of step-tapered piles when the parameter  $\delta (= D/d)$  is larger than 1.5.

- 2. When the soil layer of the model is homogeneous saturated clay and the other geometrical parameters (*H*, *h*, *L*, and *d*) of step-tapered piles are unchanged, the increasing distance (*L*) between the upper surface of the enlarged upper part and the upper surface of step-tapered piles increases the horizontal displacement of the surface of soil and the pile body; whilst the reduced distance (*L*) reduces the bending moment of the pile body, while improves the horizontal ultimate bearing capacity of the step-tapered piles.
- 3. When the soil layer of the model is homogeneous saturated clay and the other geometrical parameters (*L*, *h*, *D*, and *d*) of step-tapered piles remain unchanged, the increasing length (*H*) of the enlarged upper part of step-tapered piles significantly reduces the horizontal displacements of the step-tapered piles and the soil around the step-tapered piles, while increases the horizontal ultimate bearing capacity of step-tapered piles. However, the reducing length (*H*) can reduce the bending moment of the pile body.
- 4. When the geometrical parameters (L, H, h, D, and d) of step-tapered piles are unchanged and the value of parameter L is kept as 0, the soils surrounding the enlarged upper part (in the range of the depth of 0–H from the surface of the model) are set as saturated clay, silty clay and sandy soil, and the soil in the range of below the depth of H from the surface of the model is saturated clay, the increasing strength of soils surrounding the enlarged upper part of step-tapered piles reduces the horizontal displacements of the pile body and the soil around the step-tapered piles, and reduces the bending moment of the pile body, but can improve the horizontal ultimate bearing capacity of step-tapered piles.
- 5. When the geometrical parameters (*L*, *H*, *h*, *D*, and *d*) of step-tapered piles are unchanged and the value of parameter *L* is kept as 0, the soil surrounding the enlarged upper part (in the range of the depth of 0 H + h from the surface of the model) is saturated clay, the soils below the pile end are saturated clay, silty clay and sandy soil, the changing soil strengths below the pile end have slight effect on reducing the horizontal displacements of the pile body and the soil around the pile.

### Acknowledgments:

This work was supported by the Systematic Project of Guangxi Key Laboratory of Disaster Prevention and Engineering Safety (Grant No. 2019ZDK051), and the Open Research Fund of State

Key Laboratory of Simulation and Regulation of Water Cycle in River Basin(China Institute of Water Resources and Hydropower Research, Grant NO. IWHR-SKL-201708).

### References

- M. Ghazavi, O. Tavasoli, "Characteristics of non-uniform cross-section piles in drivability", Soil Dynamics and Earthquake Engineering 43: pp. 287–299, 2012.
- [2] A.M. Rybnikov, "Experimental investigations of bearing capacity of bored-cast-in-place tapered piles", Foundation Engineering 43: pp. 48–52, 1990.
- [3] K.K. Jayantha, D.M. Ian, "Axial response of tapered piles in cohesive frictional ground", Journal of Geotechnical and Geoenvironmental Engineering 119: pp. 675–693, 1993.
- [4] M. Sakr, M.H. El Naggar, M. Nehdi, "Wave equation analyses of tapered FRP-concrete piles in dense sand", Soil Dynamics and Earthquake Engineering 27: pp. 166–182, 2007.
- [5] J.H. Lee, K.H. Paik, D.H. Kim, S.H. Hwang, "Estimation of axial load capacity for bored tapered piles using CPT results in sand", Journal of Geotechnical and Geoenvironmental Engineering 135: pp. 1284–1294, 2009.
- [6] Y.G. Zhan, H. Wang, "Numerical study on load capacity behavior of tapered pile foundations", Journal of Geotechnical and Geoenvironmental Engineering 17: pp. 1969–1980, 2012.
- [7] G.Q. Kong, H. Zhou, H.L. Liu, X.M. Ding, R. Liang, "A simplified approach for negative skin friction calculation of special-shaped pile considering pile-soil interaction under surcharge", Journal of Central South University of Technology, 21: pp. 3648–3655, 2014.
- [8] N. Hataf, A. Shafaghat, "Optimizing the bearing capacity of tapered piles in realistic scale using 3D finite element method", Geotech Geol Eng 33: pp. 1465–1473, 2015.
- [9] F.I. Nabil, "Behavior of step tapered bored piles in sand under static lateral loading", Journal of Geotechnical and Geoenvironmental Engineering 136: pp. 669–676, 2010.
- [10] Y.R. Lv, H.L. Liu, X.M. Ding, G.Q. Kong, "Field tests on bearing characteristics of x-section pile composite foundation", Journal of Performance of Constructed Facilities 26: pp. 180–189, 2012.
- [11] L.X. Xiong, H.J. Chen, "A numerical study and simulation of vertical bearing performance of step-tapered pile under vertical and horizontal loads", Indian Geotech J 50: pp. 383–409, 2020.
- [12] N.F. Ismael, "A behavior of laterally loaded bored piles in cemented sands", Journal of Geotechnical Engineering 116: pp. 1678–1699, 1990.

Received: 2020-06-27, Revised: 2021-04-28