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Generalized inverses of cracovians – a question of paternity

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Abstract: Half a century ago two papers were published, related to generalized inverses of cracovians by two different authors, in chronological order, respectively by Jean Dommanget and by Helmut Moritz. Both independently developed papers demonstrated new theorems, however, certain similarity between them appeared. Helmut Moritz having recognized that situation, promised to mention it later in one of his published papers. This has never been done, so the author of the present paper gives some details about the situation and claims his paternity.

Keywords: Mathematical computations, least squares, cracovians

1. Historical

More than forty years ago, the author of this short communication published the results of a research on the inverse of a rectangular cracovian (Dommanget, 1963).

For the system of equations (numbered here as in the original paper)

$$\begin{array}{l} a_{11}x_1 + a_{21}x_2 + a_{31}x_3 + \ldots + a_{m1}x_m = l_1 \\ a_{12}x_1 + a_{22}x_2 + a_{32}x_3 + \ldots + a_{m2}x_m = l_2 \\ a_{13}x_1 + a_{23}x_2 + a_{33}x_3 + \ldots + a_{m3}x_m = l_3 \\ \dotfill \\ a_{1n}x_1 + a_{2n}x_2 + a_{3n}x_3 + \ldots + a_{mn}x_m = l_n \end{array} \tag{20}$$

being written in the symbolic cracovian form

$$\mathbf{X}\,\tau\mathbf{A} = \mathbf{L} \tag{21}$$

its "general" solution, *whatever the numbers n and m may be*, is given by the expressions

$$\mathbf{X} = \mathbf{L}\mathbf{A}^{-1} \quad \text{with} \quad \mathbf{A}^{-1} = (\mathbf{A}\mathbf{M})^{-1}\, \tau\mathbf{M} \tag{22}$$

where the auxiliary $n \times m$ cracovian **M** is given by

$$\mathbf{M} = \mathbf{A}\,\mathbf{P} \tag{31}$$

with **P** being $n \times n$ square cracovian of the weights on the unknowns.

On the basis of these considerations, the author of this short communication had established five theorems, two of which are as follows (original version):

Théorème IV[1]: La solution (22) du système d'équations (20) où **M** est défini par (31), est telle que le produit:

$$\mathbf{U}\,\mathbf{P}\,\mathbf{U}$$

est minimal, si les u_i composant le cracovien-colonne **U** sont les résidus des équations (20) and

Théorème V[2]: La solution (27')[3] du système d'équations (20) est telle que le produit:

$$\mathbf{X}\,\mathbf{P}\,\mathbf{X}$$

est minimal, si on choisit pour **M** l'expression:

$$\mathbf{M} = \mathbf{P}^{-1}\, \tau\mathbf{A}$$

Thirteen years after the publication (Dommanget, 1963), Helmut Moritz published in another paper on the same subject (Moritz, 1976) the following theorem:

"The set of all possible least-squares solutions of the adjustment problems (1) and (2) is given by

$$\mathbf{X} = \mathbf{L}\mathbf{A}^{-1}$$

for adjustment by parameters, model (1')[4], and

$$\mathbf{U} = \mathbf{w}\, \tau\mathbf{B}^{-1}$$

[1] ***Theorem IV***: The solution (22) of the system of equations (20) where **M** is defined by (31), is such that the product

$$\mathbf{U}\,\mathbf{P}\,\mathbf{U}$$

is minimal, if the components u_i, the elements of the column-cracovian **U**, are the residuals of the equations (20).

[2] ***Theorem V***: The solution (27') of the system of equations (20) is such that the product

$$\mathbf{X}\,\mathbf{P}\,\mathbf{X}$$

is minimum, if one chose for **M** the expression

$$\mathbf{M} = \mathbf{P}^{-1}\, \tau\mathbf{A}$$

[3] (27') is a part $\mathbf{X} = \mathbf{L}\mathbf{A}^{-1}$ of (22) in the original paper.

[4] $\mathbf{U}\,\mathbf{B} = \mathbf{w}$.

for adjustment by conditions, model (2'), provided that cracovians **A** and **B** have full rank. Here the vectors **U** and **w** consist of residuals (corrections to the observations) and misclosures, respectively". Comment of Moritz to that theorem is as follows: "Thus the least-squares adjustment solutions are simply the cracovian inverses of the observation equations (1) or the condition equations (2), respectively. *This result may look rather surprising at first sight.*"

2. Recognition

As an answer to my remark about the similarity between both demonstrations, Helmut Moritz recognized in a letter dated 28 October 1977 that[5]: "*En effet, le théorème dans mon travail publié à Cracovie s'ensuit immédiatement des théorèmes IV et V de votre publication de 1963. Je m'excuse de ne pas m'être aperçu de votre travail et je ne manquerai pas de le mentionner dans éventuels travaux futurs dans ce domaine.*"

This praiseworthy intention seems never have been put into action, the opportunities probably did not occurred.

3. Conclusion

The author of this short communication thus estimates essential to bring personally in a specific paper, the attention on the anteriority of his research on that of his colleague Helmut Moritz.

References

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Moritz H., (1976): *Cracovian inverses and least-squares estimation,* Scientific Bulletins of the Stanislaw Staszic University of Mining and Metallurgy (Cracow), No 602, Geodesy b. 45, pp. 13-19.

[5] *"Indeed, the theorem in my work that was published at Cracow, is an immediate consequence of the theorems IV and V of your publication of 1963. I am sorry that I have not seen your work and will not fail to mention it in any eventual future publications in this domain."*

Uogólnione odwrotności krakowianów – problem autorstwa

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Streszczenie

Pół wieku temu zostały opublikowane dwa artykuły dotyczące uogólnionych odwrotności krakowianów przez dwóch różnych autorów: Jean Dommangeta i Helmuta Moritza, wymienionych w kolejności chronologicznej. W obu artykułach przedstawiono niezależnie nowe teorie, które jednak zawierają pewne wspólne cechy. Helmut Moritz, po uświadomieniu sobie zaistniałej sytuacji, obiecał udzielić wyjaśnień w tej sprawie w jednej ze swoich kolejnych publikacji. Obietnica ta nigdy nie została spełniona, toteż autor niniejszej pracy podaje w niej pewne szczegóły dotyczące zaistniałej sytuacji i domaga się uznania, iż on jako pierwszy sformułował teorię uogólnionej odwrotności krakowianów.