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The cadastre in time dimension — mathematical approach

Time is for that not everything
takes place simultaneously.

Albert Einstein

In the paper the mathematical approach of cadastre performance in time dimension is proposed. It is an amplification of the mathematical model of cadastre, which has been formulated by the Author before by means of the set theory.

INTRODUCTION

The aim of the paper is to sum up the Author's research work on the cadastre modernization. The idea of mathematical approach of cadastre, which — according to the Author's intention — enables to take advantage of the application of informatics to the possibly highest degree, has been published in [2].

The main purpose of the Author's research work was to describe the cadastre reality as a time function. He assigned the same importance to the time dimension as to other dimensions of reality being described. In other words, the time axis and the coordinate axes of applied spatial reference system are treated as equivalent. Hence, the cadastre is to provide relevant information referring not only to a given point in the cadastral space, but also to a given point on the time axis.

Therefore the solutions referring to the time aspect in proposed mathematical model of cadastre are treated by the Author as the final results of his research work on the cadastre modernization within its mathematical basis.

1. *Time in the cadastre — the traditional approach and evolutionary trends*

At present the cadastre, the real estate register and other similar institutions should be treated as spatial information systems. Their traditional purpose is to publish updated data. A great part of outdated data is also available but more as a side-effect than as a principle.

That is an anachronizm — in the presence of general development of information technologies.

In Poland the problem of time in cadastral systems has been not of importance so far — contrary to other countries. The paper of Peter van Oosterom [1] which concerns the problem of time in cadastral map can be used here as an example. Several solutions — probably very advanced — were presented there. Unfortunately these solutions have been based on traditional methods and it is difficult to find out any conception of methodological basis, which could be adequate for available information technologies and would form a complete theory.

2. *The proposed doctrine of modern cadastre in outline*

Against the background of the Author's research work results presented in [3] and [5] the following doctrine of modern cadastre was proposed in [6]:

1. The cadastre performance is continuous in time; the state in the cadastral reality in a given moment, referring to a certain point of the cadastral space, corresponds to the previous state.
2. The cadastre is submitted to permanent modernization — in the consequence of the changes of its functions in the country as well as the information systems development; this modernization is immanent and considerably consists in self-regulation.

The problem of time in functional model of cadastre — proposed in [3] and [5] — was presented graphically, not by means of relevant mathematical formulas.

3. *The essence of the proposed mathematical model of cadastre*

The mathematical model of cadastre presented by the Author in [2] has been based on the ontology theory of description. The theory is based on the following notions: *entity*, *attribute* and *relation*. Cadastral reality can be very easily described by means of this theory, most of all because the *states of things* not the *processes* belong to its domain. For a long time these three notions has been expressible by means of mathematical language (the set theory) and for decades — by means of informatics language (database).

The basic notion of model proposed is the *spatial cadastral element (PEK - przestrzenny element katastralny)* defined as a space homogenous in all aspects taken into consideration.

The database is composed of three basic sets:
G — the state of development (**G** — stan zagospodarowania)
P — the legal status (**P** — stan prawny)
M — the map modules (**M** — moduły mapy).
Relations between these sets are secured by spatial cadastral element identifier.

4. The functional model of the cadastre

4.1. Model in outline

Cadastre treated as an information system is a part of cadastre treated as an institution. As it is known, in Poland the cadastre institution is created by law; the cadastral functions referring to the country and the organ responsible for them are defined by law. On the other hand cadastre treated as an information system is created by relevant detailed regulations.

In the most general approach, there are two factors taking part in the cadastre (Fig.1):

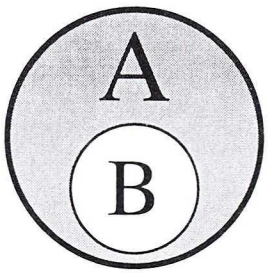


Fig. 1

A — the human factor
B — the instrumental factor.

The **A** factor has a consciousness, contrary to the **B** factor which doesn't have it. The **B** factor is quite submitted to the **A** factor. While the **A** factor acts consciously, the **B** factor Tserves only as a tool, as an instrument or — in the best case — as an automatic machine — always submitted to the **A** factor.

The state of the **B** factor can be treated as a cadastre development indicator. At the beginning it was represented by abacuses, paper and ink and also relevant application forms with instructions, whereas at present — by computer information systems.

As the cadastre evolves the **B** factor takes new tasks from the **A** factor. However there is a limit of this evolution, because a certain class of these tasks is not formalizable. The **B** factor evolution is determined by a degree of the data processing formalizm.

A certain class of catastral tasks is not formalizable — that means they are not feasible without the creative element used, i.e. without a human factor.

The **B** factor can be identified with the information system, which — together with the **A** factor — forms the cadastre institution. Then the following functional modules of the information system can be distinguished (Fig. 2):

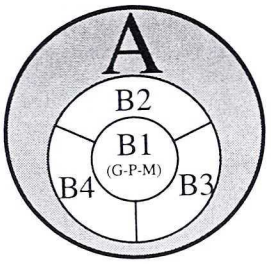


Fig. 2

B1 — storing data, e.g. as **G-P-M** files,
B2 — creating the data files for specified areas (e.g. cadastral district), updating these data files and storing the outdated data,
B3 — providing cadastre users with the access to proper data, generally in the form of values of functions processing stored data,
B4 — modifing the **B1** module (including **G-P-M** files — according to their structure changes) as well as the **B2** and **B3** modules within the system modernisation.

4.2. Data updating and storing

The changes of the cadastre description state are discrete. Irrespectively of the range of these changes, the states of **G-P-M** files on the both sides of the given time point of the change are two different states. That does not mean however that all contents of **G-P-M** files must be saved in archive (database). Storing in archive only the state of this **PEKs** which were involved by these changes is sufficient.

The change of **G-P-M** files in a given time point which consists in the archive of relevant **PEKs**, the storage of the new update state in **G-P-M** files or the replacement of these files with new **PEKs** — can be called *unit data change* (*JZD jednostkowa zmiana danych*). By the **JZD** identifier is understood the time point corresponding to it.

A change of spatial character of given **PEK** should be additionally discussed. In consideration of the fact that the analogous (spatial) change of other (adjoining) **PEKs** can be associated to it, **JZD** should also include all these adjoining **PEKs**.

4.3. Data publishing

In traditional (handmade) cadastre published data — let's call them „user data" — had most of all a form of the land register extract and the cadastral map extract.

At present the cadastral information user should be provided with data of two following forms:

1) formal document, i.e. written on paper, in native official language of given country, provided with relevant signatures and stamps,
2) data file — of standard form — saved on data medium.

At present, forms of data storied and user data (their structure, format etc.) result from contrary reasons:

— the most characteristic feature of stored data is formalizm,
— the most characteristic feature of user data is native language.

The range of information user data is determined by:

— stored data (**B1** module, including **G-P-M** files),
— functions (operations) realized on stored data (**B3** module).

4.4. Cadastre modernization

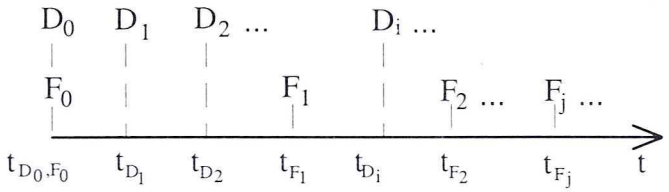
The principle of cadastre continuity can be conduisive to its conservatizm. However, stagnation of the cadastre — neither legal nor methodological nor technological — can be justified — in the presence of general development. Modernization of any system is associated sometimes with considerable costs. On the other hand any stagnation is also associated with costs which are not smaller but only later. So the problem of future permanent modernization of cadastre system can't be ignored on the cadastre planning. That is a purpose of the **B4** module.

The **B1**, **B2** and **B3** modules are — in a way — integrated by the form of data on which the functions of these modules are realized. Each of these modules can realize its functions variously, within determined structure of data storing in the **B1** module. Any change of this structure requires the modification of these modules.

Neither data structures modification nor functional modules modification can be done unconsciously i.e. without the **A** factor. Because of laborious of that modification the degree of its automatization is not the most important. However, that degree is very important for the modification of data in existing structure. A great number of these data and a magnitude of modification operations require their full automatization. That is the main purpose of the **B4** module.

4.5. The cadastre performance referring to a time axis

Because of extremely different reasons of selection of *stored data* and *user data* forms, there are two modulus assigned to common time axis: data module **D** (containing stored data) and function module **F** (containing functions which process stored data and publish user data). That is shown in Fig. 3.



D_0, D_1, D_2, D_i – *module data state (D) in particular time points resulting from data updating and from system modification*

F_0, F_1, F_2, F_j – *module function state (F) in particular time points resulting from system modification.*

Fig. 3

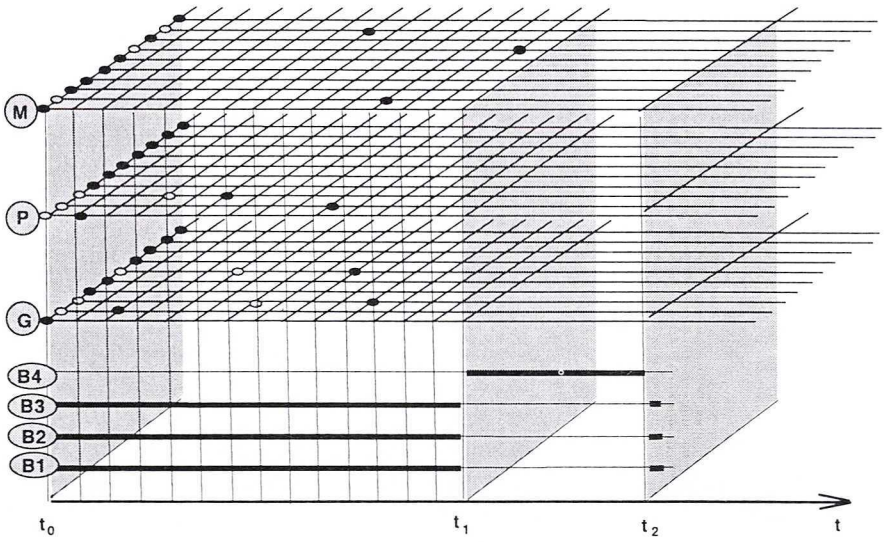


Fig. 4

Unit data change (**JZD**) includes:

1) in case of change of spatial character of given **PEK** — all **PEKs** being in spatial relation with that **PEK**,
2) in other cases — only this **PEK** in which the change takes place (relevant tuple in **G** or **P** files) or map modules in **M** file in which that **PEK** exists.

The cadastre performance is shown in Fig. 4. Particular circles in **G-P-M** files represent:

— in **G** file — single tuple of **G** relation referring to given **PEK** i.e. describing its state of developments,

— in **P** file — single tuple of **P** relation referring to given **PEK** i.e. describing its state of low,

— in **M** file — single map module, i.e. record describing geometry of relevant **PEKs** and their spatial relations.

Dark circles represent updated state of tuples and of modules of relevant **PEKs** in t_1 moment; light circles represent archival state. Points in the time interval $<t_0, t_1>$ represent time points in which updating and storing of **G-P-M** files took place.

Time interval $<t_0, t_1>$ is characterized by the constant of **B1, B2** and **B3** modules.

In time interval $<t_1, t_2>$ the modification of at least one of **B1, B2** or **B3** module takes place. The **G-P-M** files modification could also take place in that interval. In that period the purpose of **B4** module are actions involved to the modification mentioned above (as it is known — they must be formalized) and actions involved to **G-P-M** files reconstruction — because of their large-scale character.

In the t_2 moment new version of **B1-B2-B3** modules and — if necessary — new version of **G-P-M** files structure become valid.

5. *Cadastre state in time dimension*

By *the cadastral state* will be understood states of cadastral reality description included in **G-P-M** files and states of **B1, B2, B3** and **B4** modules referring to particular points on time axis.

The aim of this discussion is to refer the cadastral state to time dimension i.e. to assign it to relevant point on time axis. Let's refer this discussion *to legal status of spatial cadastral element.* This state is a problem complicated enough for expanding relevant solutions to other cadastral sets and modules.

The legal status of spatial cadastral elements can be described as the following relation [1] [2]:

$$\mathbf{P} = \{(e, o, f, u): e \in \mathbf{E}, o \in \mathbf{O}, f \in \mathbf{F}, u \in \mathbf{U},$$
spatial cadastral element e is an object of material right of a person o, f is the form of this right, u is a share$\}$.

In order to illustrate let's relation lets use graphic example shown in Fig. 5, taken from [2] and referring to legal status of lands (parcel is a kind of *spatial cadastral element*):

$$\mathbf{P} = \{(d, o, f, u): d \in \mathbf{D}, o \in \mathbf{O}, f \in \mathbf{F}, u \in \mathbf{U},$$
parcel d is an object of material right of the person o, f is the form of this right, u is a share$\}$.

[1] As it is known, the relation is defined as a subsset of a Cartesian product; the Cartesian product of **A** and **B** is designated by $\mathbf{A} \times \mathbf{B}$ and defined as a set of ordered (a, b) which $a \in A$ and $b \in B$.

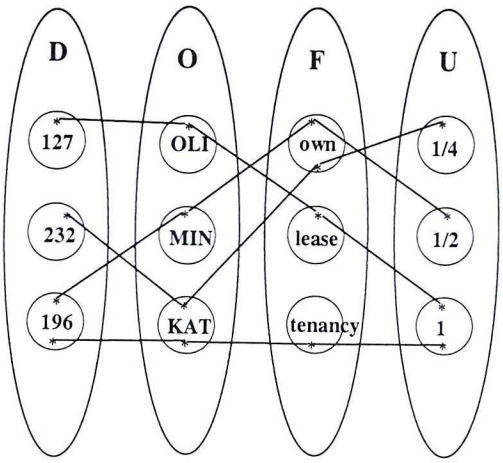


Fig. 5

The changes of legal status spatial cadastral element can be divided into two categories:

1) changes of particular *e, o, f* and *u* elements — as internal changes

2) changes of relation on **E, O, F** and **U** files.

The changes of particular *e, o, f* and *u* elements e.g. address changes, name changes, clerical errors corrections etc. also can be treated as changes of legal status — that depends on convention. Let both categories of changes mentioned above be treated as changes of legal status.

Constancy periods of particular *e, o, f* and *u* elements will be treated as time intervals describe by means of *ordered pairs*, where *predecessor* and *successor* determine the initial point and the terminal point of given interval respectively:

— in case of the *e* element — it is an interval $<t^e_p, t^e_k>$

— in case of the *o* element — it is an interval $<t^o_p, t^o_k>$

— in case of the *f* element — it is an interval $<t^f_p, t^f_k>$

— in case of the *u* element — it is an interval $<t^u_p, t^u_k>$.

Existence period files $\mathbf{T}^e$, $\mathbf{T}^o$, $\mathbf{T}^f$ and $\mathbf{T}^u$ of referring elements will be written as ordered sets of particular *constancy periods* [2] of these elements:

$$\mathbf{T}^e = \{\langle t^e_{p_1}, t^e_{k_1}\rangle, \langle t^e_{p_2}, t^e_{k_2}\rangle, \ldots\}$$

$$\mathbf{T}^o = \{\langle t^o_{p_1}, t^o_{k_1}\rangle, \langle t^o_{p_2}, t^o_{k_2}\rangle, \ldots\}$$

$$\mathbf{T}^f = \{\langle t^f_{p_1}, t^f_{k_1}\rangle, \langle t^f_{p_2}, t^f_{k_2}\rangle, \ldots\}$$

$$\mathbf{T}^u = \{\langle t^u_{p_1}, t^u_{k_1}\rangle, \langle t^u_{p_2}, t^u_{k_2}\rangle, \ldots\}$$

[2] Not always the terminal point of given constancy periods on given axis covers with initial point of next period; between these periods *interruption periods* can occur.

where: $t^z_{p_1} < t^z_{p_{i+1}}$, $t^z_{k_1} < t^z_{k_{i+1}}$; $z = e, o, f, u$; $i = 1, 2, 3,...$

That is presented on in Fig. 6, where the t axis is a time axis. On the t^e, t^o, t^f and t^u time axes the following periods are shown: *constancy periods* of *e, o, f* and *u*, elements – using bold line segments and *interruption periods* of referring elements – using thin line segments. On the t axis the following periods are shown: *common parts of constancy periods* of *e, o, f* and *u* – using bold line segments and *interruption periods* any of these elements – using thin line segments. As it is easy to notice, *common parts of constancy periods* of *e, o, f* and *u* elements are *constancy periods of legal state* **P** and *interruption periods* of particular *e, o, f* and *u* elements are *indefiniteness periods of legal state* **P**.

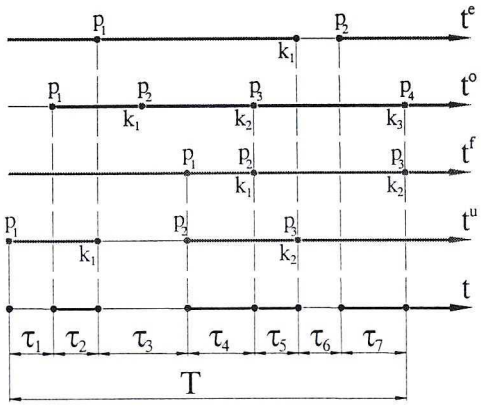


Fig. 6

Let the *indefiniteness periods of legal state* **P** be treated as one of forms of this state and let particular its *constancy periods* (including also *indefiniteness periods*) be assigned by τ. Than – in considering period **T** which is a set of *constancy periods* τ – *legal state of spatial cadastral elements* $\mathbf{P}^{\mathrm{T}}$ can be described as a relation being a subset of Cartesian product of **T**, **E**, **O**, **F** and **U** sets:

$$\mathbf{P}^{\mathrm{T}} \subset \mathbf{T} \times \mathbf{E} \times \mathbf{O} \times \mathbf{F} \times \mathbf{U}$$

defined as follows:

$$\mathbf{P}^{\mathrm{T}} = \{(\tau, e, o, f, u): \tau \in \mathbf{T}, e \in \mathbf{E}, \mathbf{o} \in \mathbf{O}, f \in \mathbf{F}, u \in \mathbf{U},$$
in constancy period τ spatial cadastral element e is an object of material right of the person o, f is the form of this right, u is the share$\}$.

As it is easy to notice, if the term of *constancy periods of legal* state **P** is extended on *constancy periods* of relation on **E**, **O**, **F** and **U** files than legal state $\mathbf{P}^{\mathrm{T}}$ will include – in time dimension – both categories of legal state changes i.e. internal changes of *e, o, f* and *u* elements as well as changes of relation on **E**, **O**, **F** and **U** files.

FINAL REMARKS

Everything should be simplified
as much as it is possible but not more.

Albert Einstein

The legal status approach mentioned above has been verified experimentally using relevant information system.

In the light of research results presented in the paper the following remarks can be formulated:

1. Taking into consideration the functionality of cadastral system – especially while it should function in time dimension – it's necessary to divide data *to user data and data stored in database.*

2. The less data stored in database are transformed the less the volume of the part of database storing them grows slower. However unlimited atomization of data not always leads to improvement of given system. That problem is more complicated. As usual in technique – also other factors of system effectiveness should be taken into consideration.

3. Sometimes problems treated as complicated can be described in unexpectedly simple way. Sometimes this is a matter of understanding its essence. Various fields of knowledge – often distant from each other – can be very helpful. According to the Author's opinion, knowledge treated as unity gives more chance for finding simply solutions.

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Kataster w wymiarze czasu – matematyczne ujęcie

Streszczenie

Rzeczywistość katastralną można opisać za pomocą następujących trzech podstawowych pojęć: stosowanego w matematyce pojęcia *relacji*, wcześniej zdefiniowanych przez autora pojęcia *przestrzennego elemenetu katastralnego* oraz pojęcia *modułu mapy*.

W rezultacie, rzeczywistość katastralną można zapisać za pomocą następujących trzech zbiorów wzajemnie powiązanych *identyfikatorami przestrzennych elementów katastralnych*: **G** – stan zagospodarowania, **P** – stan prawny, **M** – moduł mapy.

Różne są postaci *danych przechowywanych* w bazie danych i *danych użytkowych*, tj. udostępnianych użytkownikom katastru w języku naturalnym. Im mniejszym stopniem przetworzenia charakteryzują się *dane przechowywane* w bazie danych, tym w mniejszym stopniu – z upływem czasu – wzrasta objętość tej części danych, która służy do ich przechowywania.

W najogólniejszym ujęciu, w funkcjonowaniu katastru uczestniczą następujące dwa czynniki:

A - czynnik ludzki,

B - czynnik instrumentalny.

Czynnik **A**, w przeciwieństwie do czynnika **B**, wyposażony jest w świadomość. Funkcjonowanie **B** jest całkowicie podporządkowane **A**. Czynnik **A** funkcjonuje na zasadzie świadomego działania, **B** – na zasadzie narzędzia, instrumentu, w najlepszym przypadku – automatu, jednak zawsze podporządkowanego czynnikowi **A**.

Utożsamiając czynnik **B** z systemem informacyjnym, który w połączeniu z czynnikiem **A** tworzy instytucję katastru, można wyodrębnić następujące moduły funkcjonalne tego systemu:

B1 – przechowywanie danych, np. w postaci zbiorów **G-P-M**,

B2 – tworzenie dla określonych obszarów (np. obrębów) zbiorów danych oraz ich aktualizowanie i archiwizowanie danych zdezaktualizowanych,

B3 – udostępnianie odpowiednich danych użytkownikom katastru, generalnie jako wartości "funkcji" realizowanych na przechowywanych danych,

B4 – modyfikacja modułów **B1** (w tym zbiorów **G-P-M** w związku ze zmianą ich struktury), **B2** i **B3** w ramach modernizacji systemu.

Z upływem czasu – wszystkie te elemnenty (moduły B1, B2, B3 i B4 oraz zbiory G-P- M) ulegają zmianom. Stany poszczególnych modułów B1, B2, B3 i B4 oraz stany elementów zbiorów G-P-M są przyporządkowane konkretnym przedziałom czasu na jednej wspólnej osi czasu. W rezultacie, zapewniona jest dostępność do informacji dotyczących dowolnego punktu czasoprzestrzeni katastralnej.

Кароль Шелига

Кадастр в измерении времени — математический подход

Р е з ю м е

Кадастровую действительность можно описать с помощью следующих трех основных понятий: применяемого в математике понятия *реляции* (связи), ранее сформулированных автором понятия *пространсмвенного кадастрового элемента* и понятия *модуля карты.*

В результате кадастровую действительность можно записать с помощью следующих трёх наборов взаимосвязанных *идентификаторами пространственных кадастровых элементов:* **G** — уровень блягоустройства, **P** — юридическое положение, **M** — модул карты.

Различными являются виды *данных, хранимых* в базе данных, и *потребительских данных,* т.е. представляемых потребителям кадастра. Чем меньшей степенью обработки характеризуются данные, хранимые в базе данных, тем в меньшей степени — с течени времени — возрастает объём той части базы данных, которая служит для их хранения.

В наиболее общем понятии, в функционировании кадастра участвуют следующие два фактора:

A — человеческий фактор,

B — инструментальный фактор

Фактор **A**, в противоположность к фактору **B**, оснащён сознанием. Функционирование **B** целиком нодчинено **A.** Фактор **A** функционирует по принципу сознательной деятельности, **B** — по принципу орудия, инструмента, в лучшем случае — автомата, но всегда однако подчинённого фактору **A**.

Отожествляя фактор **B** с унформационной системой, которая в соединении с фактором **A**, образует институт кадастра, можно выделить следующие функциональные модули этой системы:

B1 — хранение данных, например, в виде фондов **G — P — M,**

B2 — создание для определённых территорий (участков) фондов данных, а также их обновление и архивохранение устаревших данных,

B3 — предоставление соответствующих данных потребителям кадастра, главным образом как величин «функции», реализованных на хранимых данных,

B4 — модификация моделей **B1** (в том фондов **G — P — M** в связи с изменением их структуры), **B2** и **B3** в рамах модернизации системы.

С течением времени — все эти элементы (модули B1, B2, B3 и B4, а также фонды G — P — M) подвергаются изменениям. Состояние отдельных модулей B1, B2, B3 и B4 и состояние (уровень) элементов фондов G — P — M соответствует конкретным промежуткам времени на одной совместной оси времени. В результате обеспечена доступность к информации, относящейся к любому пункту кадастровому пространству-времени.