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HYBRID METHOD OF STRAIN AND STRESS ANALYSIS IN FATIGUE CRACK ZONE

The paper shows the hybrid method of stress and strain distributions analysis. In the method, the results of displacement measurement were used as boundary conditions in the numerical analysis of the tested objects. The numerical analysis was performed with the use of the finite element method (FEM), whereas measurements of displacement were made by laser grating interferometry technique (moiré interferometry). Examples of tests presented in the paper show good efficiency of the method in the analysis of stress and strain distribution in the areas of their heterogeneous distribution. Mutual completion of laser grating interferometry and finite element method makes it possible to exclude their disadvantages creating broader possibilities for research impossible to achieve in separate use.

1. Introduction

A number of models used in the fatigue analysis of structural parts and criteria accepted in the evaluation of their fatigue life and strength require full information on the state of stress and strain in the areas of elements prone to cracking. Usually, in order to define it, analytical methods are used as well as the commonly available advanced numerical tools, mainly finite elements method. However, in both cases the practical possibility of their use brings about a necessity of introducing many simplifications, assumptions and limitations. It is connected mainly with the complexity and variety of the geometrical characteristics of the objects, complex character of stress and difficulties in material properties modelling.

The situation is not much different in the case of experimental method use either, in which the results of measurements are usually limited to a chosen component displacement and strain or their combinations, without making it

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possible to calculate the value of stress, especially in the case of plastic strain. Moreover, in the case of fatigue analysis, basic interest is placed on points of the highest local values and highest stress and strain gradients, located mostly at hard to inspect points for majority of measurement methods.

Both the analysis of literature as well as the experience gained during the research conducted at the Department of Machine Design of University of Technology and Agriculture in Bydgoszcz [1], [2], [3], [4] point to the fact that one of the possibilities of achieving good results in the area of stress and strain analysis is connecting the advantages of experimental and numerical methods, the effect of which are hybrid methods.

There are various configurations of methods used in the analysis of strain and stress, which can be qualified under the definition of hybrid methods. Among others, the paper [5] includes their classification from the point of view of numerical method application, detailing numerical-analytical methods, numerical-numerical and numerical-experimental (experimental-numerical).

The methodology of the hybrid analysis of strain and stress distribution is usually connected with the use of partial results of strain and displacement measurements as boundary conditions in numerical analysis [6], [7]. Thanks to this, for instance, the "lacking" components of stress and strain may be defined, i.e. with the use of non-linear material models.

The analysis of literature lets one notice numerous examples of the use in hybrid methods of the experimental laser grating interferometry and numerical analysis of finite elements [5], [8], [9], [10]. In the paper [8], the results of the measurements of displacement in the area of crack were used in the numerical analysis of strain energy release rate G_I in specimens made from titanium alloy and undergoing four-point bending.

The possibilities of using hybrid methods (based on laser grating interferometry and finite element method) in elastic and elastic-plastic fracture were also presented in the paper [9]. As examples of solutions in the area of non-linear elastic fracture, the hybrid analysis of defining the dependence of CCS (crack closure stress) on the COD (crack opening displacement) were shown, used in the analysis of energy accompanying concrete and aluminium specimen fracturing. Besides, the paper [9] also defines the value of J integral for different positions of integration contour (elastic-plastic range). The paper [10] was also devoted to results of the testing of the possibilities of using the hybrid methodology in elastic strain analysis, using the abovementioned methods. An analysis of strain was carried out for the disc under compression loading. The results of hybrid analysis were compared to the results of theoretical calculations. The authors of the paper [10] pointed to a very close compliance between the results of theoretical and hybrid analysis, showing importance of the characteristics of the mesh in the geometrical model and the types of elements used to create it.

A wide review of hybrid methods used in the analysis of issues related to fracture mechanics was presented in the paper [5]. Laser grating interferometry method and finite element method were used, among others, in the tests of the state of strain and stress at the crack tip in nuclear pressure vessel steel.

2. Hybrid, experimental-numerical method of strain and stress analysis

The basic result of the numerical analysis with the use of finite elements method are the components of strain and stress as well as the main stress/strain, intensity of the main strain/stress calculated on the basis of these values, as well as their equivalent values calculated with the use of effort criterion. These values are assigned in the mesh nodes of the tested object, whereas in other areas of the geometrical model the values of the required quantity are generated with the use of a postprocessor on the basis of the descriptions of material properties in individual finite elements.

On the other hand, the results of using experimental methods in most cases are limited to the chosen component of displacement and strain, and in some cases indirectly to stress.

Due to that fact, the possibility of "completing" the information on the state of strain and stress obtained with the use of experimental methods by using numerical methods seems to be very attractive from the point of view of the issues of experimental mechanics of solids, including specifically the issues of fatigue of material and structures.

3. Methodology of hybrid analysis of strain and stress distribution – HASS

One of the reasons why the results of experimental tests are often insufficient in the issues concerning fatigue analysis of various technical objects is the lack of direct possibilities of assigning component strain in loading conditions causing plastic strain occurrence.

On the other hand, the use of numerical analysis in such cases is not always efficient or possible. There are some reasons for it, out of which the most important ones are the difficulties in accepting the appropriate assumptions for the building of geometric and material models and the necessity of a very precise reproduction of the state of loading in the analysed object.

In many cases, the simple limitation of modelling only to the zone of the element covered by the fatigue analysis causes significant simplifications, including, among others, lack of considering the effect on influence of the rejected part of object during modelling (object decomposition effect) or lack of taking into account the changes of material properties caused by cyclic loading.

The methodology of the hybrid analysis of strain and stress in structural parts is based on the assumption of the possibility of limiting the numerical analysis of the tested object to that part of it which undergoes experimental analysis.

Fig. 1 shows a schematic presentation of the methodology of hybrid experimental-numerical method of the stress and strain analysis based on experimental laser grating interferometry technique and numerical finite elements method.

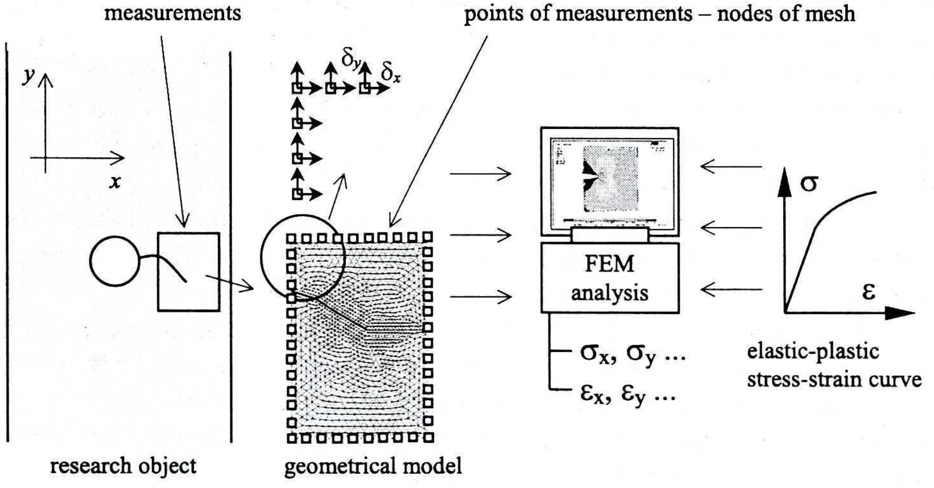


Fig. 1. Procedure of hybrid method of stress and strain distribution analysis

In the analysed limited area of the real object (i.e. in the fatigue crack zone), remaining in an arbitrary state of loading, the whole-field relative displacement measurement is performed. This area is then geometrically modelled with the help of finite element grating. Further on, at the nodes of the mesh on model boundary, kinematical forcing is introduced, equivalent to the measured δ_v and δ_u displacements.

Further numerical analysis conducted with the use of non-linear material model makes it possible to assign components of strain and stress in the tested area.

Thanks to the introduction of the state of displacement obtained in the experimental tests as the boundary conditions into the numerical model, we do not need to reproduce the full object geometry and loading conditions in the FEM analysis.

Application of the methodology described above in determination of strain and stress distribution will be presented on the example of strain and stress analysis in the area of fatigue crack zone.

In the present state of work, the analysis of the state of strain and stress is realised on two-dimensional geometric models.

4. Research objects and loading conditions

In the tests, two types of specimens with a propagating fatigue crack were used, as shown in Fig.2.

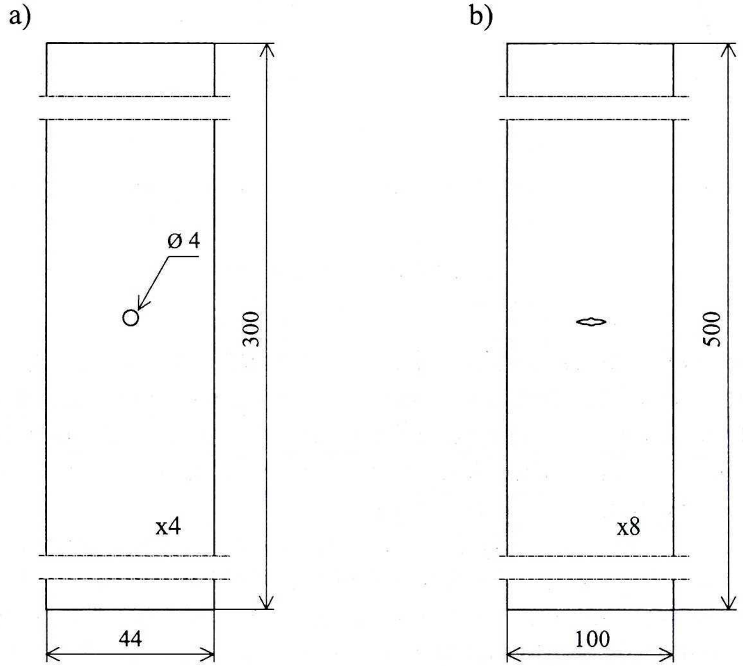


Fig. 2. Specimen: a) AL type, made of aluminium alloy D16CzATW, b) ST type, made of steel 18G2A

The specimen marked with the label AL was made of aluminium D16CzATW alloy (equivalent of 2124) with mechanical properties given in Fig. 3a. During the test, the specimen was treated with one-side axial loading with $R = 0$ and maximum nominal stress value of $S_{max} = 62.5$ MPa. Second specimen (Fig. 3b), marked ST, was made of 18G2A steel (according to Polish Standards – P355N according to EN 10028-2).

Similarly to the specimen made of aluminium alloy, during the tests of ST type specimen one-side loading was used with constant nominal stress amplitude $S_{min} = 4$MPa and $S_{max} = 64$MPa.

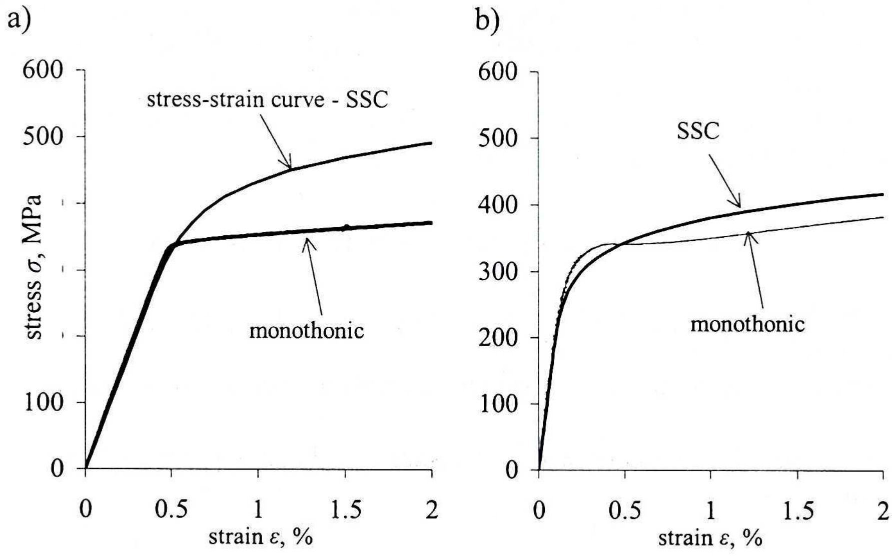


Fig. 3. Material properties of aluminium alloy D16CzATW (a) and 18G2A steel (b)

5. Experimental analysis

Experimental work conducted during the tests was aimed at two problems. The goal of the first one was the displacement measurement required for the definition of boundary conditions in numerical analysis, whereas the second one consisted of marking full strain distribution maps ε_y and ε_x in order to verify numerical tests results.

In the experimental analysis for the displacement measurement of points in the area of fatigue crack, an automated grating laser extensometer LES was used, the design and rules of functioning of which were presented, among others, in papers [11], [12], [13].

In the system for displacement measurement, a two-beam moiré interferometry method was used with the optical setup presented in Fig. 4.

The light beam emitted by the laser diode of λ = 670 nm wavelength passing through collimator lightens one-way diffraction grating of f_s = 1200 lines/mm frequency. As a result of its diffraction, two mutually coherent (coupled) A and B beams are created, which after reflecting from the controlled mirrors lighten the specimen grating applied to the object. In compliance with the rule of measurement, the beams diffracted on the grating create the image of interference fringes watched through a CCD camera.

Application of a diffraction grating as a generator of coupled A and B beams makes it possible to ensure low sensitivity to light wavelength changes.

Duplicating the optical setup shown in Fig. 4 allows us to realise the mea-

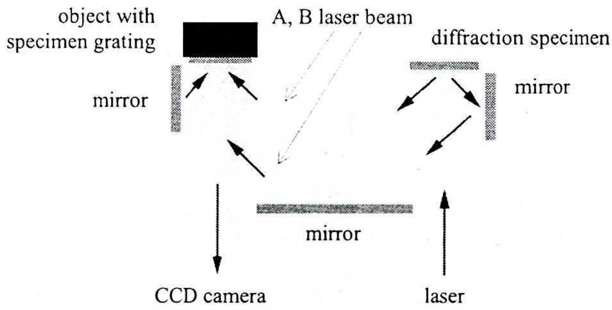


Fig. 4. Optical arrangement of laser grating extensometer LES

surement in two perpendicular directions with the use of cross-line specimen gratings.

AL and ST specimens with a measurement head applied during the tests are shown in Fig. 5.

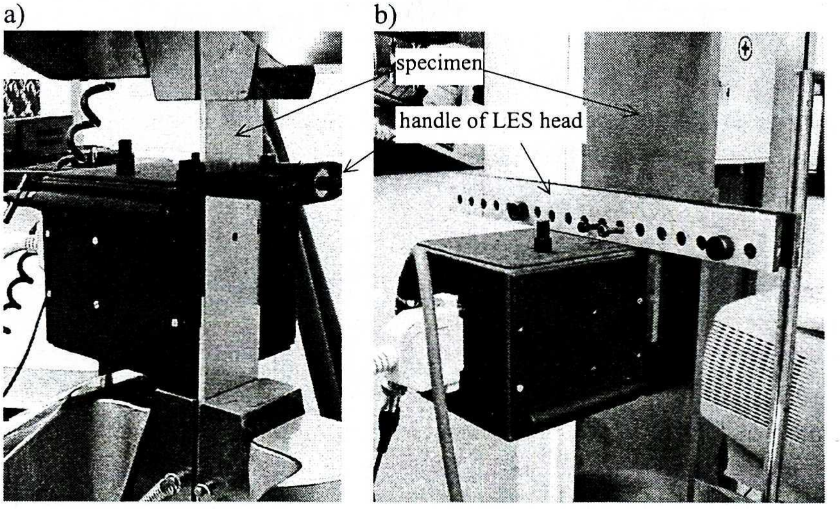


Fig. 5. Displacement measrement: a) AL type specimen, b) ST type specimen

As a result of the measurements, displacement maps in y and x directions were obtained. The displacement δ_y and δ_x in the "points" reflecting nodes on the boundary of finite elements mesh were used in the further numerical analysis, as boundary conditions.

Fig. 6 and Fig. 7 present an example of displacement distribution maps for AL and ST specimens, while in Fig. 8 and Fig. 9 their distributions in cross-sections limiting the area of numerical analysis is plotted.

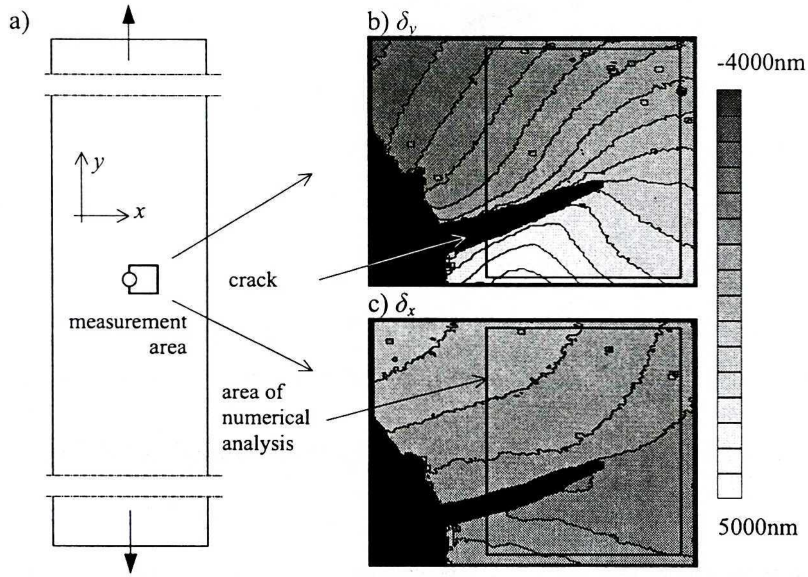


Fig. 6. Example of displacement distribution in AL type specimen for nominal stress S=62.5MPa: a) measurement area localisation, b) displacement distribution in y direction, c) displacement distribution in x direction

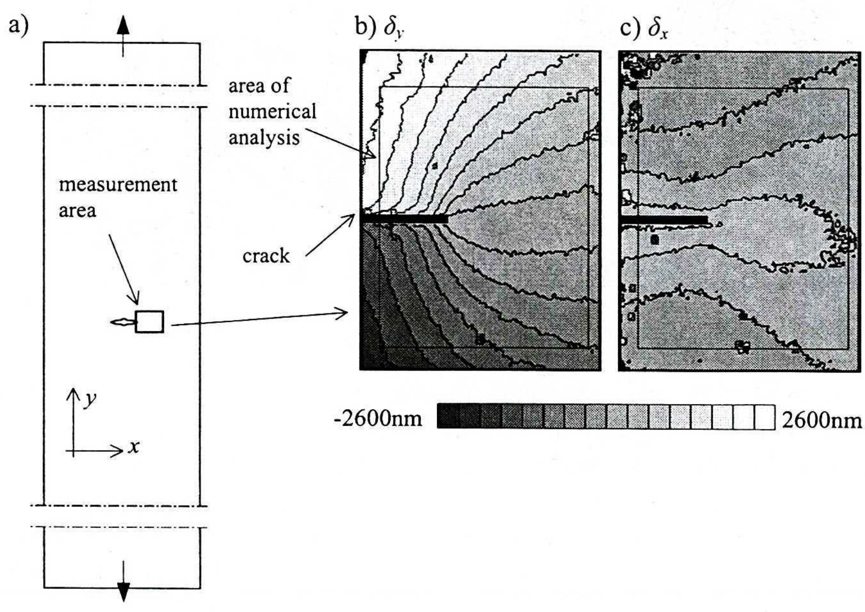


Fig. 7. Example of displacement distribution in ST type specimen for nominal stress S = 64 MPa: a) measurement area localisation, b) displacement distribution in y direction, b) displacement distribution in x direction

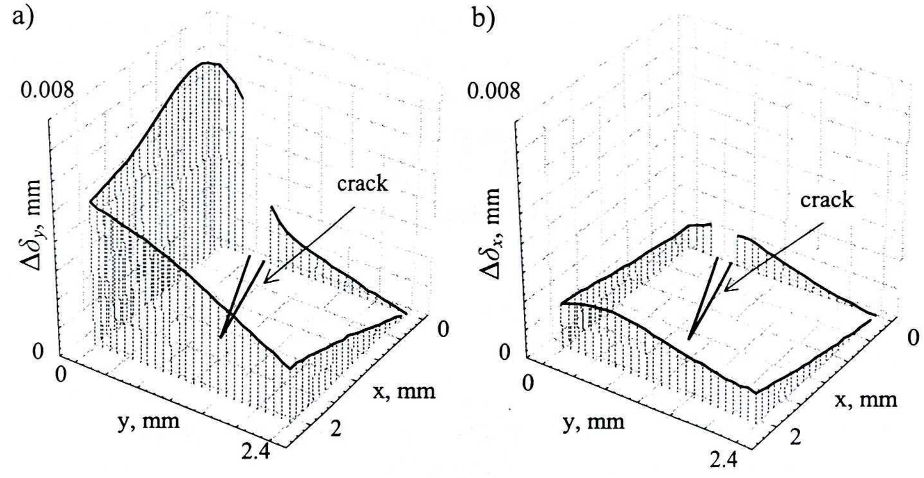


Fig. 8. Example displacement distribution for border of numerical analysis area for y (a) and x (b) direction for AL type specimen (S = 62.5 MPa)

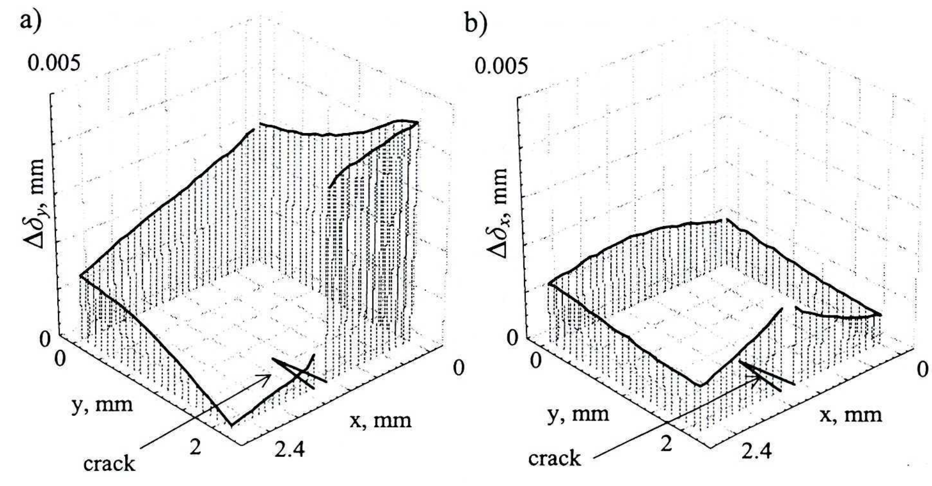


Fig. 9. Example displacement distribution for border of numerical analysis area for y (a) and x (b) direction for ST type specimen (S = 64 MPa)

During the test, the course of changes of displacements was recorded for 55 (AL specimen) and 100 (ST specimen) values of the load in several cycles of loading. However, due to the repetitive character of the conducted analyses, the paper will only present the results of tests for maximum values of nominal stress in loading cycle, e.g. for S_{max} = 62.5 MPa for AL specimen and S_{max} = 64 MPa for ST specimen.

6. Numerical analysis

The knowledge of displacement distribution in the tested areas of specimens made it possible to significantly limit the area of geometrical modelling in numerical analysis.

In the analyses conducted with the use of commercial ANSYS software, the isolated areas of specimens underwent discretisation with the help of 6-node triangular finite elements of PLANE2 type from ANSYS software library [14], [15].

In Fig. 10, the applied finite elements mesh is shown, taking into consideration the real shape of the fatigue crack for both specimens.

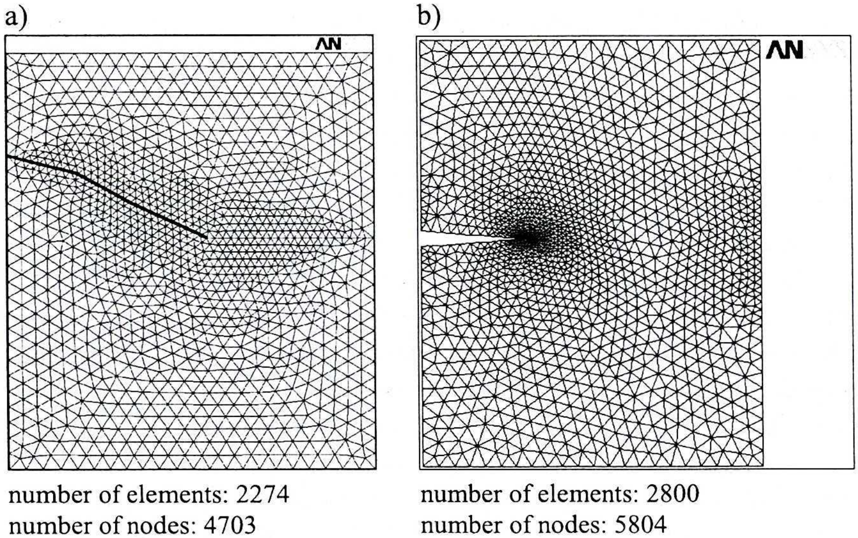
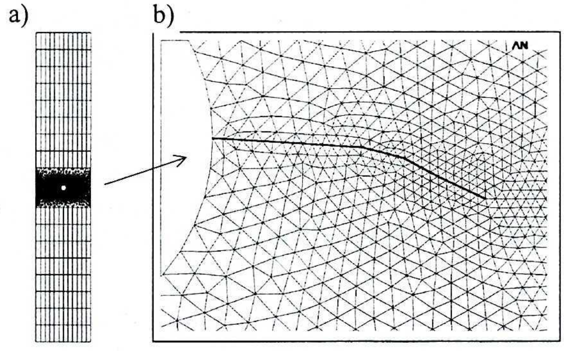


Fig. 10. Mesh of finite element analysis applied for hybrid stress and strain analysis: a) AL type specimen, b) ST type specimen

For the description of material properties, non-linear material models were used, which were obtained through discretisation of cyclic stress-strain curve shown in Fig. 3 with the help of 20 points. For comparative purposes, in the case of AL specimen, an additional numerical analysis was performed for a full, two-dimensional model of cracked specimen (Fig. 11). In this analysis, the same material model was applied and loading corresponding to loading used during experimental tests.



number of elements: 8504
number of nodes: 17008

Fig. 11. Mesh of finite element used in analysis of whole AL type specimen: a) view of whole geometrical model, b) crack zone

7. Results analysis

The conducted tests and their results allowed for carrying out two types of comparative analysis. In the case of AL specimen, a comparison of strain and stress distribution obtained by the hybrid and numerical methods was made, whereas in the case of ST specimen the comparison included the results of the analysis conducted with the use of hybrid and experimental methods.

– ***AL specimen***

During the tests, both in the case of numerical and hybrid analysis, all components of the strain and stress state in the specimen were determined for individual phases of loading cycle. For comparison purposes, the components ε_y and ε_x of strain and σ_y and σ_x of stress state were chosen. Fig. 12 shows their distributions in the zone including fatigue crack and its surrounding.

Despite significant differences in the model itself, as well as in the method of loading in numerical analysis, the results of calculations in both cases are very similar. It applies to both their qualitative and quantitative character. In both cases, the characteristic stress and strain distributions for propagating fatigue crack were obtained.

Comparison of their maximum values obtained in the hybrid and numerical analysis showed a difference of about 3% for σ_y, 30% for σ_x and about 11% for both ε_y and ε_x. Maximum stress reached higher values in the case of numerical analysis (σ_y = 372 MPa and σ_x = 222 MPa), whereas strain in hybrid analysis (ε_y = 1% and ε_x = 0.2%).

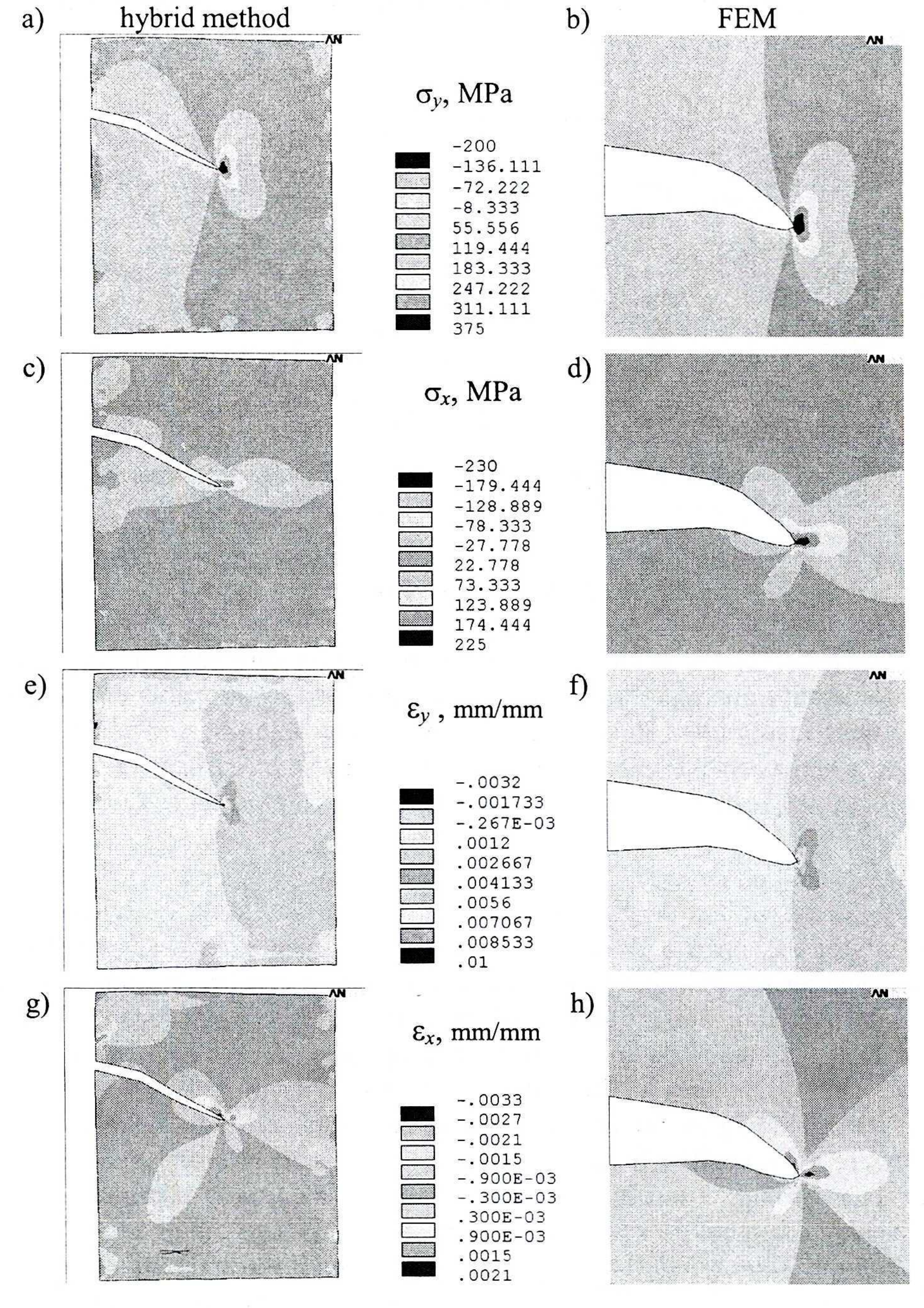


Fig. 12. Stress and strain distribution obtained by hybrid method (a, c, e, g) and numerical analysis of whole AL type specimen (b, d, f, h)

The strain and stress distributions acquired by hybrid method (Fig. 12) were determined for the specimen with certain loading history, which was not taken into account in the traditional numerical analysis. One of the main reasons for the occurrence of differences may therefore be the changes of strain state in the area of crack tip caused by accumulation of plastic strain during cyclic loading and local material properties changes, which were not taken into account in the finite element method.

Introducing the measurement results into the numerical analysis gives possibilities of considering, during calculations, the fatigue effects connected with the history of loading realised previously. However, an especially significant role plays the applied method of material properties modelling (especially in the case of one-side loading), which requires taking into consideration the influence of the cyclically changing plastic zone in the area of crack.

It also requires developing the procedures allowing for elastic unloading of the specimen with retaining the plastic effects and moving their effects to next loading cycles in the following phases of numerical analysis.

– ***ST specimen***

During the experimental part of the tests on ST specimen, beside of the measurements of displacement distribution required for the hybrid analysis, strain distributions in the area of fatigue crack were determined also. These tests made it possible to verify the strain distribution determined by the hybrid method, according to the described methodology. The example results obtained in the investigations are shown in Fig. 13.

Comparative analysis of the experimental and numerical results was conducted for strain distributions in the directions y and x determined for maximum value of nominal strain in cycle S_{max} = 64 MPa.

Images of interference fringes observed during the experimental tests and the strain distributions determined on their basis for the whole measurement field are shown in Fig. 14.

Fig. 15 shows strain distribution maps ε_y and ε_x determined by hybrid and experimental methods. The comparison of the general characteristic of the obtained strain distributions allows one to notice their similarity both in the case of ε_y and ε_x strains.

The main differences are of the local character and apply to two specimen areas: the near crack tip zone and the area along the crack line. In order to represent them in Fig. 16, strain difference maps are shown in the indicated specimen areas, while Fig. 17 compares the examples of distributions of ε_y strain in two cross-sections of the specimen in y and x directions.

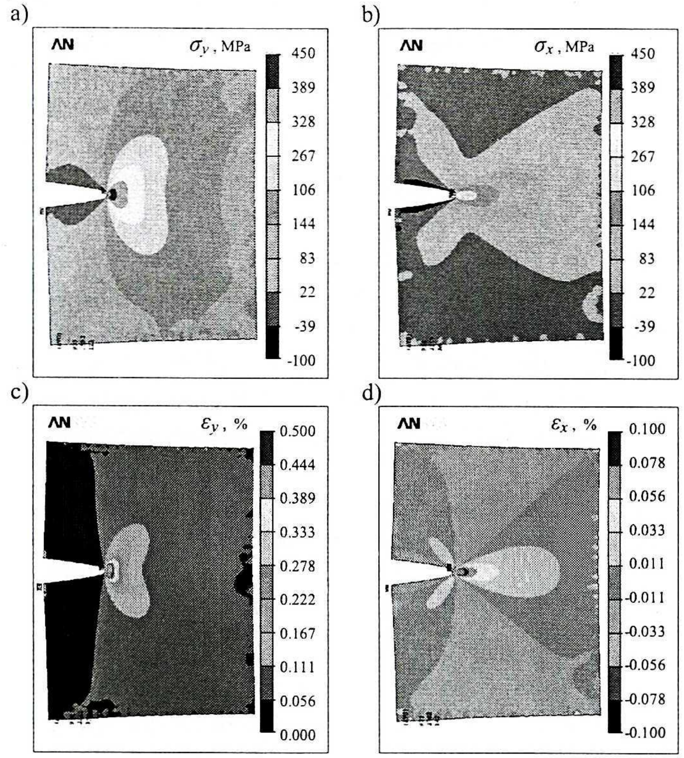


Fig. 13. Example stress (a,b) and strain (c,d) distributions determined by hybrid method for ST type specimen

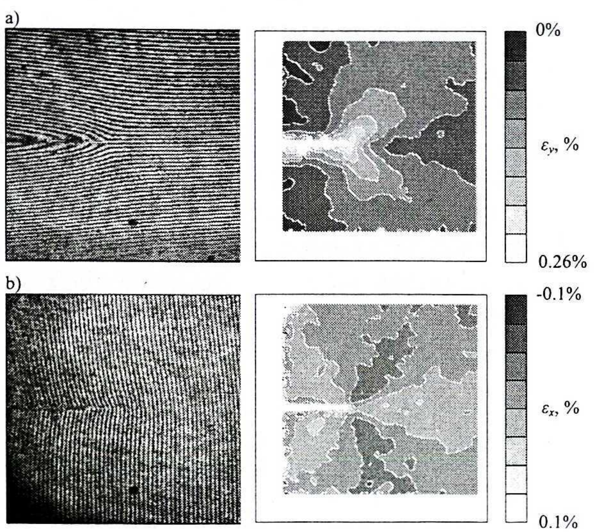


Fig. 14. Experimental strain distributions for ST type specimen: a) y direction, b) x direction

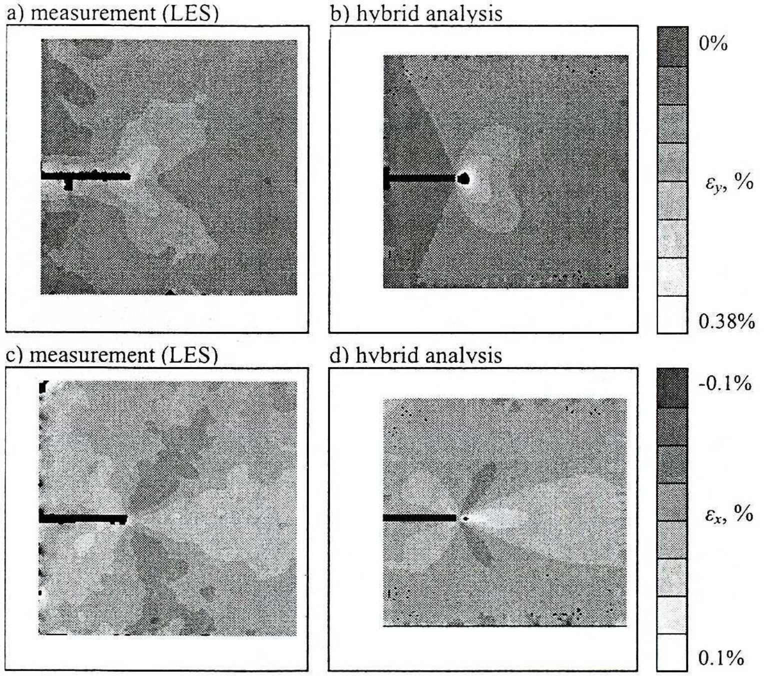


Fig. 15. Comparison of experimental (a,c) and hybrid (b,d) strain distributions

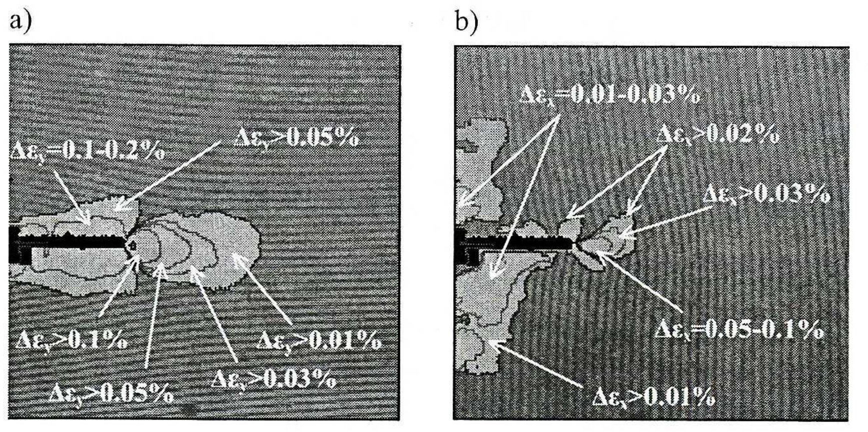


Fig. 16. Distributions of experimental and hybrid strain differences $\Delta\varepsilon$: a) y direction ($\Delta\varepsilon_y$), b) x direction ($\Delta\varepsilon_x$)

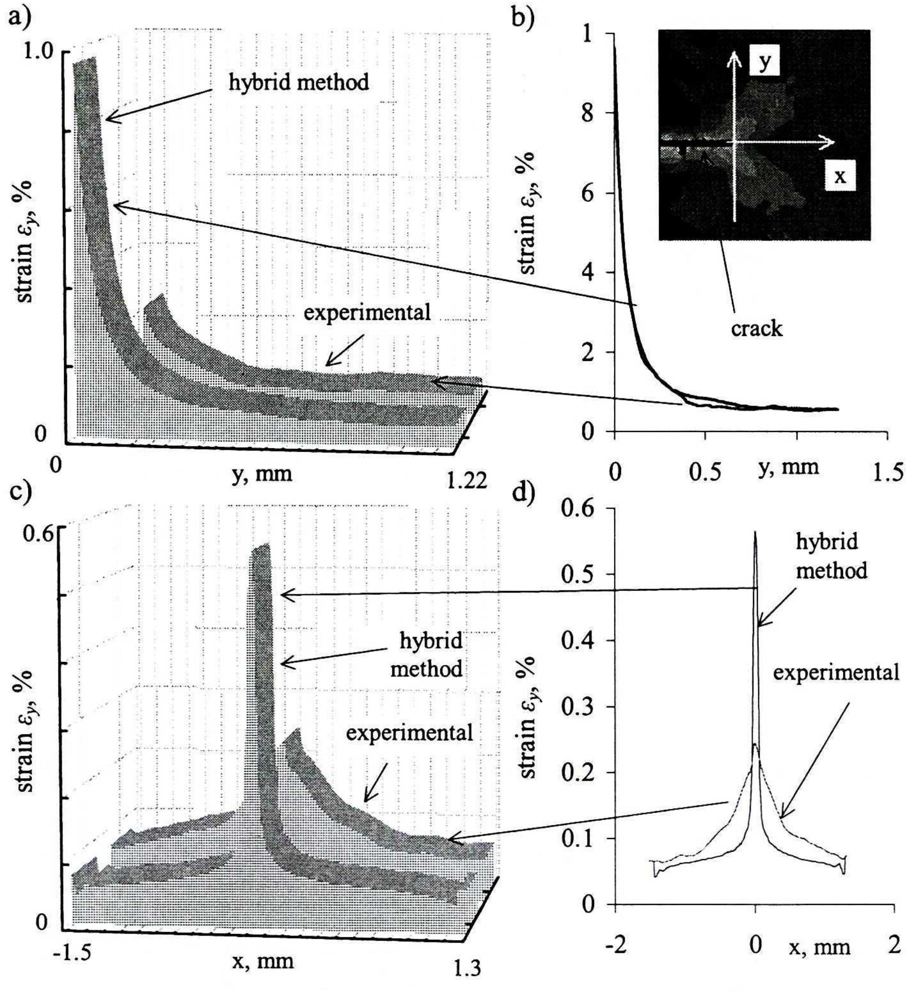


Fig. 17. Comparison of experimental and hybrid strain distributions in cross-sections y (a,b) and x (c,d)

The sources of the differences, which appeared in the comparison of the results of hybrid and experimental strain analysis, can be found in both methods.

The differences of the maximum values of strain determined in the area of crack tip in the case of the laser grating interferometry method result mainly from the difficulties of defining their value around the edge of the crack, which, taking into consideration a very high strain gradient, results in an apparent lowering of maximum values. The high strain gradient may also cause exceeding of the measurement range (excessive concentration or excessive reduction of interference fringes), especially in the case of the measurement of the full loading cycle, when the wide range of loading changes makes the full measurement impossible (for high and low loading values) for single optical setup properties.

In the case of finite elements method, the possible overestimating of the strain value on crack tip may be caused by assumptions accepted in crack modelling as well as due to neglecting the loading history in calculations.

Since the values of nominal stress did not exceed $0.2R_e$ (R_e – yield strength), the determined strain values should be less influenced by the plastic zone occurring at the crack tip. This may be confirmed by the fact that any noticeable cyclic plastic zone did not remain in the specimen after unloading.

The second area in the crack zone, which exhibits differences of the determined strain values, is the zone occurring behind crack tip. The result of the measurement has not confirmed the effect of specimen unloading on the edges of crack resulting from FEM analysis. Similarly as in the previous case, it may result from neglecting the changes of the material state in front of crack tip, appearing as a result of cyclic loading. It may also be the effect of the assumption of complete separation of the specimen along crack line accepted in the FEM analysis.

Certain possibilities of correction and verification of such assumptions in the case of hybrid method could be created by introducing the measurement data from the area of crack edge into the numerical model.

A more comprehensive verification of the suggested experimental-numerical strain and stress analysis methodology in the area of fatigue fracture requires conducting a number of further tests, i.e. in different loading ranges (with plastic strain), for different specimen geometry, for areas showing diversification of cyclic material properties.

8. Conclusions

The results of the conducted tests and their analysis allow for a positive evaluation of the research methodology used, which is influenced by several factors.

The use in fatigue investigations (and not only) of laser grating interferometry method, as well as most of the methods of whole-field strain analysis, allows for direct measurement only of the displacement occurring on the surface of the analysed objects. There is, at times, an insufficient amount of information to obtain the accompanying stress distribution, especially in the case of the occurrence of plastic strain. This limitation may be partly annulled by complementing the experimental method by numerical analysis, which leads to a significant widening of strain and stress distributions analysis possibilities.

On the other hand, from the point of view of numerical methods, the possibility of obtaining additional information on the state of object loading with the experimental method significantly simplified the numerical analysis itself, especially by avoiding the necessity of modelling full geometry of the tested object as well as its real loading.

Significant reduction of the number of finite elements in geometric model leads to a significant shortening of the analysis period with simultaneous increase of the method's accuracy, which is due to lowering of the dimensions of a single finite element.

The application of results of displacement measurement as boundary conditions in the numerical analysis makes it possible to perform the analysis in cases when real loading conditions are impossible or difficult to define. It also releases analysis from the effects of loading redistribution in relation to primary loading, e.g. the effects connected with the change of material properties of the analysed object.

The mutual complement of the laser grating interferometry and finite element method enables exclusion of its disadvantages and creates broader possibilities of testing, impossible to achieve in individual use.

One of the limitations of the presented method of stress and strain investigations is the lack of possibilities to apply it in cases of full analysis of three-dimensional objects because of insufficient range of displacement measurement results.

Both the positive results of the tests conducted with the use of hybrid strain and stress analysis method and the experience gained during their realisation enable us to indicate the main directions of further work on the proposed methodology:

- the adaptation of the method for three dimensional analysis of complex, spatial objects,
- the development of numerical analysis modules (mainly finite element method) facilitating direct use of measurement results obtained with the use of the LES system,
- investigation of the possibility of additional combination of experimental analyses results with boundary element method, which should bring a new quality in the case of its use in the analysis of objects with deep notches and cracks.

The above-mentioned investigations should result in new possibilities of analysing the problems in the area of experimental mechanics of solids, including the problems of fatigue and fracture of material and structures.

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Hybrydowa metoda analizy odkształceń i naprężeń w strefie zmęczeniowego pękania

Streszczenie

W pracy przedstawiono hybrydową metodę analizy rozkładów odkształceń i naprężeń. W metodzie, wyniki pomiarów przemieszczeń wykorzystano jako warunki brzegowe w analizie numerycznej badanych obiektów. Analizę numeryczną prowadzono z zastosowaniem metody elementów skończonych, zaś pomiary przemieszczeń realizowano metodą laserowej interferometrii siatkowej. Przykłady badań przedstawione w pracy pozwalają zauważyć dobrą skuteczność metody w analizie rozkładów odkształceń i naprężeń w obszarach o ich niejednorodnych rozkładach.

Wzajemne uzupełnianie się metody laserowej interferometrii siatkowej i metody elementów skończonych pozwala wykluczyć ich niedogodności, stwarzając szersze możliwości prowadzenia badań, niemożliwe do osiągnięcia przy ich samodzielnym zastosowaniu.