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APPLICATIONS OF PLATFORM MECHANISMS WITH 6 DOF IN ACTIVE VIBRATION CONTROL SYSTEMS

The paper introduces a new design of a platform mechanism with 6 DOF. The platform is supported on three active legs, each equipped with two rotating drives. The mechanism can be used in active vibration control systems. The values of drive angular velocities are precisely controlled, so that the transmission of the base vibrations onto the platform could be minimal. The values of drive torques to be generated are determined. The mechanism was modelled using the Working Model® 3D. The effects of active vibration control are also presented.

1. Introduction

When a machine or a vehicle travels in a rough terrain, low-frequency high-amplitude vibrations are generated. These vibrations are responsible for poorer machine performance, operator's discomfort and the risk of vibration-induced illnesses. In transport of hazardous materials, vibration of the vehicle may enhance the risk of explosion and of environment pollution with toxic chemicals. In this and other similar cases, the active vibration control systems in machines and vehicles seem absolutely necessary (Fig. 1). The research work to date focused on active vibration control utilising linear hydraulic or pneumatic drives [4], [5], [7]. Models of hydraulic drives and control methods can be found in the literature on the subject. The linear drive is usually positioned in between the vibration source and the considered machine or vehicle object, thus affecting its motion within 1 DOF. In order to increase the number of DOF in which the active system should affect the object motion, another mechanism has to be added [6]. The Author suggests a platform mechanism with 6 DOF and with 6 rotation drives be used in these applications. A similar structure is considered in [9]. To analyse the operations of platform, mechanisms it is

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necessary to determine the platform position in the function of drive rotation angles. For that purpose, to describe the links positions of the mechanism, unit vectors were used [2]. Platform velocity control is effected using the Jacobian matrix of transformations of drive velocities into platform velocity components. The method of determining the Jacobian matrix is presented in [3], [8], [9], [10].

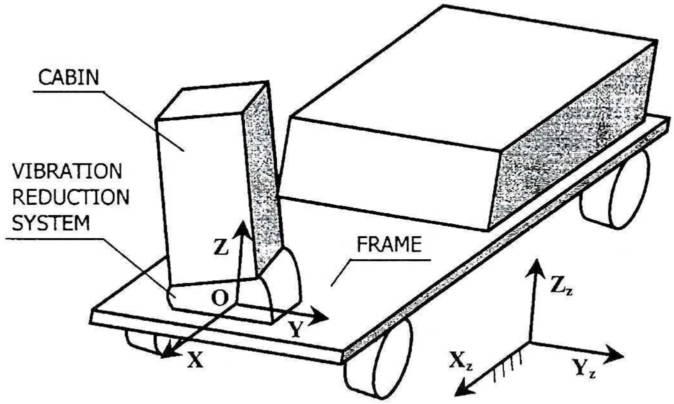

Fig. 1. Active vibration control system for an operator's cab

In general terms, the active vibration control system has three basic components:

- vehicle frame which, while moving in a rough terrain, makes complex oscillatory movements in 6 DOF, these vibrations being transmitted to the remaining parts of the vehicle,
- considered object (such as grader blade, an operator's cab or a container housing the hazardous materials) where the accelerations must not exceed the predetermined value,
- active vibration control system positioned in between the frame and the considered object, including the drives controlled in such a manner that the frame oscillatory motion is transmitted to the considered object in the minimal degree only.

The paper provides the answer to the following questions:

What values of the preset velocities of active vibro-control system drives should be fed to the controllers so that the frame vibrations should not be transmitted to the considered element of the machine?

How to generate the drive torques to implement the predetermined velocity control function?

2. Mechanism applied in active vibration control systems

Mechanisms used for active vibration control have to be rigid, furthermore a considerable motion accelerations in a relatively small working space are needed. Platform mechanisms meet those requirements. The basic components of platform mechanisms are movable legs. They lean against the base connected to the machine frame, and thus support the platform to which the considered element is attached. The supports may either passive or active – in that case they have to subtract the specified number of DOF of the platform movements with respect to the base. The structure of active supports (legs) also includes the drives. The number of drives in supporting legs ought to be equal to number of DOF in the possible platform movements where the vibrations are to be suppressed.

Let us analyse the operations of a platform mechanism with 6 DOF, supported by three active legs (Fig. 4). Each leg consists of two double - joint links, a crank and a coupler. The frame is connected to the crank by means of two revolute joints forming a Cardan joint. Rotating motion in the joints is induced by drives. Owing to the application of a bevel gear, two independent support drives can be mounted on the machine frame (Fig. 2). The crank is connected to the coupler by means of a spherical joint, while the coupler and the platform are connected with a revolute joint.

The model of a platform mechanism presented here is based on the following assumptions: there is no clearance or friction in kinematic pairs, the links are rigid and drives are not flexible.

The Author considered a similar structure of platform mechanisms in which the spherical joint is replaced with three revolute joints. During the simulations of the mechanisms operations, short-lasting though considerable acceleration and reaction force increments were reported. These are adverse effects accompanying the mechanism transformation from one configuration to another.

3. Direct position analysis

For the preset rotation angles in all six drives, the direct position analysis involves determining the passive link positions in the frame. The knowledge of passive link positions is necessary for finding the drive velocities. The positions of passive links (Fig. 2) are explicitly defined by unit vectors $\vec{n}_u$, $\vec{m}_u$, $\vec{j}_u$, $\vec{i}_u$, $\vec{k}_u$ (u = 1,2,3). The assumptions are made that the unit vectors of the drive axis $\vec{n}_u$ lie in the same plane and that unit vectors coordinates in the system xyz associated with the frame are known.

Drive axis unit vectors $\vec{m}_u$ rotate about the unit vectors $\vec{n}_u$ to which they are perpendicular. The instantaneous coordinates of the unit vectors $\vec{m}_u$ and $\vec{m}_{ou}$ in the plane determined by $\vec{n}_u$ are also known. The angle of drive rotation α_{mu} about the unit vectors $\vec{n}_u$ is found between the vectors $\vec{m}_u$ and $\vec{m}_u$. The angle of drive rotation α_{nu} about the unit vectors $\vec{m}_u$ is found between the unit vectors $\vec{n}_u$ and $\vec{j}_u$. Unit vectors $\vec{m}_u$ can be obtained from the three-vector formula, assuming that: $\vec{m}_u \circ \vec{n}_u = 0$, $\vec{m}_u \circ \vec{m}_{ou} = \cos\alpha_{nu}$ and $\vec{n}_u \circ \vec{m}_{ou} = 0$:

$$\vec{m}_u = \cos\alpha_{nu}(\vec{m}_{ou} - \vec{n}_u) + \eta_u \sin\alpha_{nu}(\vec{n}_u \times \vec{m}_{ou}), \quad (1)$$

where: η_u – sign of the configuration.

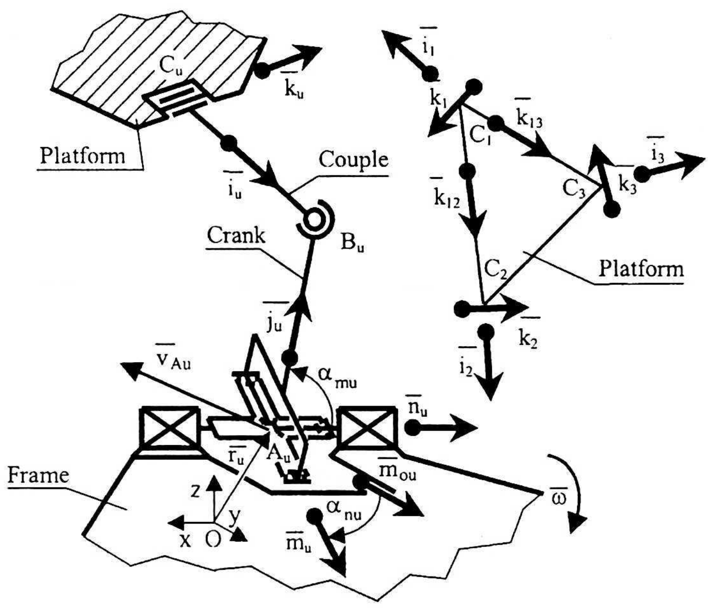

Fig. 2. Legs with the rotating drives (schematic diagram)

The unit vectors of support cranes are also determined, following an assumption that $\vec{j}_u \circ \vec{m}_u = 0$, $\vec{j}_u \circ \vec{n}_u = \cos\alpha_{mu}$, $\vec{m}_u \circ \vec{n}_u = 0$:

$$\vec{j}_u = \cos\alpha_{mu}(\vec{n}_u - \vec{m}_u) + \xi_u \sin\alpha_{mu}(\vec{m}_u \times \vec{n}_u), \quad (2)$$

where: ξ_u – sign of the configuration.

The remaining unit vectors $\vec{i}_u$ and $\vec{k}_u$ can be obtained through solving the system of eighteen algebraic equations, the coordinates of unit vectors $\vec{i}_u$ and $\vec{k}_u$ being the unknown variables. The first six equations result from the formula expressing the absolute value of unit vectors:

$$\left|\vec{i}_u\right| = 1, \qquad \left|\vec{k}_u\right| = 1, \qquad (u=1,2,3). \quad (3)$$

Assuming that unit vectors of the coupler are perpendicular to the platform unit vectors, we obtain the next three equations:

$$\vec{i}_u \circ \vec{k}_u = 0, \qquad (u=1,2,3). \quad (4)$$

Platform unit vectors are positioned at fixed and well-known angles with respect to one another, which yields the next three equations:

$$\vec{k}_1 \circ \vec{k}_2 = e_{12}, \qquad \vec{k}_1 \circ \vec{k}_3 = e_{13}, \qquad \vec{k}_2 \circ \vec{k}_3 = e_{23}. \tag{5}$$

The last six algebraic equations are formulated on the basis of projections of two close-looped vector polygons onto the axes of the coordinate system xyz (Fig. 3):

$$\overrightarrow{OA_1} + \overrightarrow{A_1B_1} + \overrightarrow{B_1C_1} + \overrightarrow{C_1C_2} = \overrightarrow{OA_2} + \overrightarrow{A_2B_2} + \overrightarrow{B_2C_2},$$

$$\overrightarrow{OA_1} + \overrightarrow{A_1B_1} + \overrightarrow{B_1C_1} + \overrightarrow{C_1C_3} = \overrightarrow{OA_3} + \overrightarrow{A_3B_3} + \overrightarrow{B_3C_3}. \tag{6}$$

Introducing the defined unit vectors, the equations (6) can be given as:

$$\vec{r}_1 + l_{j1}\vec{j}_1 - l_{i1}\vec{i}_1 + l_{C_1C_2}\vec{k}_{12} = \vec{r}_2 + l_{j2}\vec{j}_2 - l_{i2}\vec{i}_2,$$

$$\vec{r}_1 + l_{j1}\vec{j}_1 - l_{i1}\vec{i}_1 + l_{C_1C_3}\vec{k}_{13} = \vec{r}_3 + l_{j3}\vec{j}_3 - l_{i3}\vec{i}_3, \tag{7}$$

where: l_{ju} – crank length in the u-th leg, l_{iu} – coupler length in the u-th leg,

$\vec{r}_u = \overrightarrow{OA_u}$,

$\vec{k}_{12}$, $\vec{k}_{13}$ – platform unit vectors in the linear relationship to $\vec{k}_1$, $\vec{k}_2$ and $\vec{k}_3$ on the basis of the three-vector formula.

The system of equations (3), (4), (5), (7) is a system of direct, complex linear equations and polynomial equations of maximally the second order. However, because of the number of equations, an explicit solution is not available. In practical solutions it is necessary to apply iterative procedures. Besides, there are theoretical methods (Grubner's base, eliminants) to bring the system of equation down to one polynomial equation of the higher order and the order of the polynomial will be equal to the theoretically obtained number of mechanism configurations. The scope of platform mechanism operations should be chosen such that it should operate in one configuration only.

4. Drive velocities

In order to determine the required instantaneous angular velocities of six drives in active legs, an assumption is made that the platform with the considered element remains immobile. That is why the linear velocities of points C_u are equal to zero ($\vec{v}_{Cu} = \vec{0}$). The linear velocity $\vec{v}_{Bu}$ of the point B_u in the u-th leg with rotating drives mounted in revolute joints of the cross-piece can be expressed starting from the linear velocities of points A_u and C_u; $\vec{v}_{Au}, \vec{v}_{Cu}$:

$$\vec{v}_{Cu} + \omega_{ku}\vec{k}_u \times \vec{i}_u\, l_{iu} = \vec{v}_{Au} + \left(\omega_{nu}\vec{n}_u + \omega_{mu}\vec{m}_u + \vec{\omega}\right) \times \vec{j}_u l_{ju}, \tag{8}$$

where: $\vec{\omega}$ - vector of frame angular velocity,

ω_{nu}, ω_{mu} – angular velocities in revolute joints of the cross-piece.

Dot-multiplying Eq (8) by $\vec{j}_u$ yields the relative angular velocity in the rotating pair C_u:

$$\omega_{ku} = \frac{\vec{j}_u \circ \vec{v}_{Au}}{\vec{j}_u \circ (\vec{k}_u \times \vec{i}_u) l_{iu}}. \tag{9}$$

Dot-multiplying Eq (8) by $\vec{n}_u$ and making use of (9) yields the relative angular velocity in the cross-piece A_u along the unit vector $\vec{m}_u$:

$$\omega_{mu} = \frac{1.}{l_{ju}\, \vec{n}_u \circ (\vec{m}_u \times \vec{j}_u)} \left[\frac{\vec{n}_u \circ (\vec{k}_u \times \vec{i}_u)}{\vec{j}_u \circ (\vec{k}_u \times \vec{i}_u)} \vec{j}_u - \vec{n}_u \right] \circ \vec{v}_{Au} + \frac{\vec{n}_u \times \vec{j}_u}{\vec{n}_u \circ (\vec{m}_u \times \vec{j}_u)} \circ \vec{\omega}. \tag{10}$$

Dot-multiplying Eq (8) by $\vec{m}_u$ and making use of (9) yields the relative angular velocity in the cross-piece A_u along the unit vector $\vec{n}_u$:

$$\omega_{nu} = \frac{1}{l_{ju}\, \vec{m}_u \circ (\vec{n}_u \times \vec{j}_u)} \left[\frac{\vec{m}_u \circ (\vec{k}_u \times \vec{i}_u)}{\vec{j}_u \circ (\vec{k}_u \times \vec{i}_u)} \vec{j}_u - \vec{m}_u \right] \circ \vec{v}_{Au} + \frac{\vec{m}_u \times \vec{j}_u}{\vec{m}_u \circ (\vec{n}_u \times \vec{j}_u)} \circ \vec{\omega}. \tag{11}$$

Thus obtained angular velocities ω_u and ω_u are the driving shaft velocities that should be generated at the given instant in order to immobilise the platform with the considered object.

5. Jacobian matrix

An important element in the analysis of platform mechanism operation is the Jacobian matrix **J** transforming the frame velocity components into the required velocities of rotating drives:

$$\mathbf{\Omega} = \mathbf{J}\mathbf{V}, \tag{12}$$

where: $\mathbf{\Omega} = [\omega_{n1}\ \omega_{m1}\ \omega_{n2}\ \omega_{m2}\ \omega_{n3}\ \omega_{m3}]^T$ – matrix of required rotating drive velocities in three legs' cross-pieces,

$\mathbf{V} = [v_{Ox}\ v_{Oy}\ v_{Oz}\ \omega_x\ \omega_y\ \omega_z]^T$ – matrix of linear velocity coordinates for the selected frame point O and of coordinates of frame angular velocity against an immobile system.

Equations (10) and (11) can be given in a simplified form:

$$\omega_{mu} = \vec{Q}_{mu} \circ \vec{v}_{Au} + \vec{P}_{mu} \circ \vec{\omega}, \tag{13}$$

$$\omega_{nu} = \vec{Q}_{nu} \circ \vec{v}_{Au} + \vec{P}_{nu} \circ \vec{\omega}, \qquad (u=1,2,3), \tag{14}$$

where:

$$\vec{Q}_{mu} = \frac{1}{l_{ju}\, \vec{n}_u \circ (\vec{m}_u \times \vec{j}_u)} \left[\frac{\vec{n}_u \circ (\vec{k}_u \times \vec{i}_u)}{\vec{j}_u \circ (\vec{k}_u \times \vec{i}_u)} \vec{j}_u - \vec{n}_u \right], \qquad \vec{P}_{mu} = \frac{\vec{n}_u \times \vec{j}_u}{\vec{n}_u \circ (\vec{m}_u \times \vec{j}_u)},$$

$$\vec{Q}_{nu} = \frac{1}{l_{ju}\, \vec{m}_u \circ (\vec{n}_u \times \vec{j}_u)} \left[\frac{\vec{m}_u \circ (\vec{k}_u \times \vec{i}_u)}{\vec{j}_u \circ (\vec{k}_u \times \vec{i}_u)} \vec{j}_u - \vec{m}_u \right], \qquad \vec{P}_{nu} = \frac{\vec{m}_u \times \vec{j}_u}{\vec{m}_u \circ (\vec{n}_u \times \vec{j}_u)}.$$

Let the frame point O be the point with respect to which the linear velocities $\vec{v}_{Au}$ are determined; accordingly:

$$\vec{v}_{Au} = \vec{v}_O + \vec{r}_u \times \vec{\omega}, \quad (u=1,2,3). \tag{15}$$

When Eq (15) is taken into account in (13) and (14), we get:

$$\omega_{nu} = \vec{Q}_{nu} \circ \vec{v}_O + (\vec{P}_{nu} + \vec{Q}_{nu} \times \vec{r}_u) \circ \vec{\omega}\,, \tag{16}$$

$$\omega_{mu} = \vec{Q}_{mu} \circ \vec{v}_O + (\vec{P}_{mu} + \vec{Q}_{mu} \times \vec{r}_u) \circ \vec{\omega}\,. \tag{17}$$

Taking into account (16), (17), the Jacobian matrix is given by:

$$\mathbf{J} = \begin{bmatrix} Q_{n1x} & Q_{n1y} & Q_{n1z} & P_{n1x} + (\vec{Q}_{n1} \times \vec{r}_1)_x & P_{n1y} + (\vec{Q}_{n1} \times \vec{r}_1)_y & P_{n1z} + (\vec{Q}_{n1} \times \vec{r}_1)_z \\ Q_{m1x} & Q_{m1y} & Q_{m1z} & P_{m1x} + (\vec{Q}_{m1} \times \vec{r}_1)_x & P_{m1y} + (\vec{Q}_{m1} \times \vec{r}_1)_y & P_{m1z} + (\vec{Q}_{m1} \times \vec{r}_1)_z \\ Q_{n2x} & Q_{n2y} & Q_{n2z} & P_{n2x} + (\vec{Q}_{n2} \times \vec{r}_2)_x & P_{n2y} + (\vec{Q}_{n2} \times \vec{r}_2)_y & P_{n2z} + (\vec{Q}_{n2} \times \vec{r}_2)_z \\ Q_{m2x} & Q_{m2y} & Q_{m2z} & P_{m2x} + (\vec{Q}_{m2} \times \vec{r}_2)_x & P_{m2y} + (\vec{Q}_{m2} \times \vec{r}_2)_y & P_{m2z} + (\vec{Q}_{m2} \times \vec{r}_2)_z \\ Q_{n3x} & Q_{n3y} & Q_{n3z} & P_{n3x} + (\vec{Q}_{n3} \times \vec{r}_3)_x & P_{n3y} + (\vec{Q}_{n3} \times \vec{r}_3)_y & P_{n3z} + (\vec{Q}_{n3} \times \vec{r}_3)_z \\ Q_{m3x} & Q_{m3y} & Q_{m3z} & P_{m3x} + (\vec{Q}_{m3} \times \vec{r}_3)_x & P_{m3y} + (\vec{Q}_{m3} \times \vec{r}_3)_y & P_{m3z} + (\vec{Q}_{m3} \times \vec{r}_3)_z \end{bmatrix}. \tag{18}$$

Unit vector coordinates and the length of the three cranks: l_{j1}, l_{j2}, l_{j3} are used to determine the matrix components. The unit vectors are obtained through solving the direct position problem for the platform mechanism on the basis of mechanism dimensions and the drive rotation angles. The Jacobian matrix is used to calculate the required angular velocities of the drives.

The linear velocity $\vec{v}_O$ and angular velocity $\vec{\omega}$ of the frame oscillatory motion, required for drive velocity control, can be obtained from measurements of frame accelerations in an immobile system associated with the Earth ($X_zY_zZ_z$) (Fig. 1). Unit vectors making the matrix components and the vectors of linear and angular frame velocity must be expressed in the same coordinate system. Unit vectors of the passive links in the system determined by the frame XYZ are obtained through solving the direct position problem for the mechanism. They can be expressed in the coordinate system $X_zY_zZ_z$ as long as the rotation matrix of transformations from the XYZ to $X_zY_zZ_z$, based on the measured vector of the frame inclination with respect to the reference level, is known.

6. Kinetostatics of the mechanism

In order to properly select the drives, one needs the time characteristics of the expected values of torque generated motor shaft loading. As the mass of the platform together with the considered object is much greater than that of support links, an assumption is made that external loads are imposed on the platform only as the force $\vec{F}_D$ applied at point D on the platform and the moment of force $\vec{M}_D$ (Fig. 3). The components of the force $\vec{F}_D$ are: the force of gravity, inertia force of the linear drift movement and the inertia force resulting from the fact that linear and torsional frame vibrations are not completely reduced and hence are transmitted to the platform. The components of the force moment $\vec{M}_D$ are: inertia force moment of the drifting motion (when the vehicles turns) and the inertia force

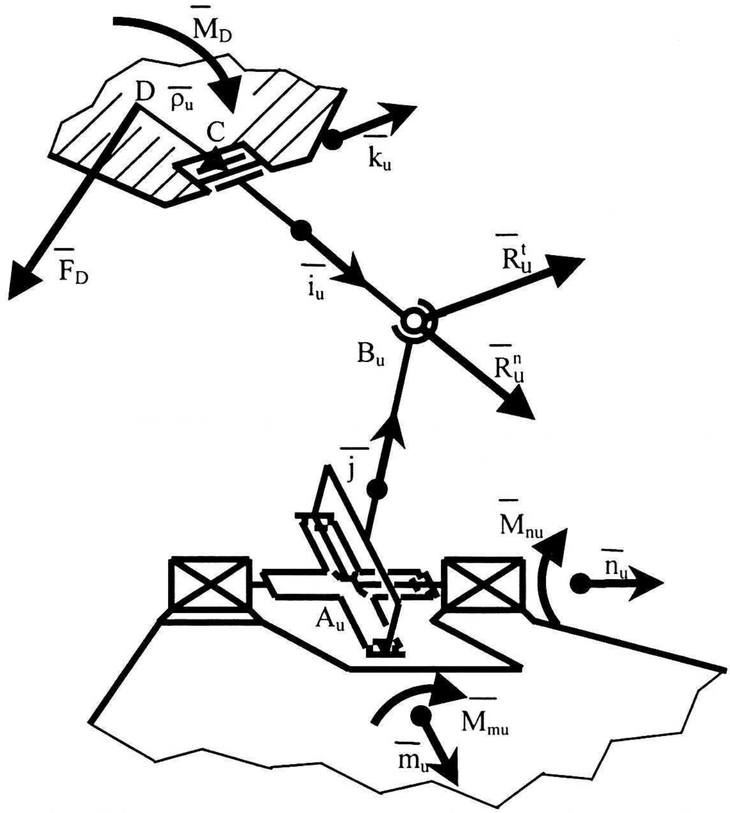

Fig. 3. Forces and moments acting upon the support (schematic diagram)

moment resulting from incomplete reduction of torsional vibrations. The force of gravity and inertia of leg links are thus neglected. Since the rotating pairs C_u do not transmit the torque in the directions $\vec{k}_u$, the reaction forces in pairs B_u (u=1,2,3) will have only two components: the normal component $\vec{R}_u^n$ in the direction coinciding with the unit vector $\vec{i}_u$ and the tangent one $\vec{R}_u^t$ in the direction coinciding with the unit vector $\vec{k}_u$. Vector equations of equilibrium for the system consisting of a platform with three couplers are:

$$\vec{F}_D + \sum_{u=1}^{3} \left(R_u^n \vec{i}_u + R_u^t \vec{k}_u \right) = \vec{0}, \tag{19}$$

$$\vec{M}_D + \sum_{u=1}^{3} \left(\vec{\rho}_u + l_{iu} \vec{i}_u \right) \times \left(R_u^n \vec{i}_u + R_u^t \vec{k}_u \right) = \vec{0}. \tag{20}$$

Equations (19) and (20) can be rewritten in the matrix form:

$$\mathbf{HR} = -\mathbf{M}, \tag{21}$$

where: $\mathbf{R} = \begin{bmatrix} R_1^n & R_1^t & R_2^n & R_2^t & R_3^n & R_3^t \end{bmatrix}^T$,

$$\mathbf{M} = \begin{bmatrix} F_{Dx} & F_{Dy} & F_{Dz} & M_{Dx} & M_{Dy} & M_{Dz} \end{bmatrix}^T,$$

$$\mathbf{H} = \begin{bmatrix} i_{1x} & k_{1x} & i_{2x} & k_{2x} & i_{3x} & k_{3x} \\ i_{1y} & k_{1y} & i_{2y} & k_{2y} & i_{3y} & k_{3y} \\ i_{1z} & k_{1z} & i_{2z} & k_{2z} & i_{3z} & k_{3z} \\ (\vec{\rho}_1 \times \vec{i}_1)_x & ((\vec{\rho}_1 + l_{i1}\vec{i}_1) \times \vec{k}_1)_x & (\vec{\rho}_2 \times \vec{i}_2)_x & ((\vec{\rho}_2 + l_{i2}\vec{i}_2) \times \vec{k}_2)_x & (\vec{\rho}_3 \times \vec{i}_3)_x & ((\vec{\rho}_3 + l_{i3}\vec{i}_3) \times \vec{k}_3)_x \\ (\vec{\rho}_1 \times \vec{i}_1)_y & ((\vec{\rho}_1 + l_{i1}\vec{i}_1) \times \vec{k}_1)_y & (\vec{\rho}_2 \times \vec{i}_2)_y & ((\vec{\rho}_2 + l_{i2}\vec{i}_2) \times \vec{k}_2)_y & (\vec{\rho}_3 \times \vec{i}_3)_y & ((\vec{\rho}_3 + l_{i3}\vec{i}_3) \times \vec{k}_3)_y \\ (\vec{\rho}_1 \times \vec{i}_1)_z & ((\vec{\rho}_1 + l_{i1}\vec{i}_1) \times \vec{k}_1)_z & (\vec{\rho}_2 \times \vec{i}_2)_z & ((\vec{\rho}_2 + l_{i2}\vec{i}_2) \times \vec{k}_2)_z & (\vec{\rho}_3 \times \vec{i}_3)_z & ((\vec{\rho}_3 + l_{i3}\vec{i}_3) \times \vec{k}_3)_z \end{bmatrix}.$$

The reaction forces $-\vec{R}_u^n$ and $-\vec{R}_u^t$ acting at the points B_u upon the leg cranks give rise to moments $\vec{n}_u$ and $\vec{m}_u$ which, when projected in the driving shaft directions, are equal to torque applied to those drives:

$$M_{nu} = l_{ju}\vec{j}_u \times \left(-R_u^n \vec{i}_u - R_u^t \vec{k}_u\right) \circ \vec{n}_u, \tag{22}$$

$$M_{mu} = l_{ju}\vec{j}_u \times \left(-R_u^n \vec{i}_u - R_u^t \vec{k}_u\right) \circ \vec{m}_u. \tag{23}$$

Equations (22) and (23) can be expressed in the form of matrices:

$$\mathbf{M}_{nap} = -\mathbf{KLR}, \tag{24}$$

where: $\mathbf{M}_{nap} = \begin{bmatrix} M_{n1} & M\ _1 & M_{n2} & M\ _2 & M_{n3} & M\ _3 \end{bmatrix}^T$,

$$\mathbf{K} = \begin{bmatrix} (\vec{j}_1 \times \vec{i}_1) \circ \vec{n}_1 & (\vec{j}_1 \times \vec{k}_1) \circ \vec{n}_1 & 0 & 0 & 0 & 0 \\ (\vec{j}_1 \times \vec{i}_1) \circ \vec{m}_1 & (\vec{j}_1 \times \vec{k}_1) \circ \vec{m}_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & (\vec{j}_2 \times \vec{i}_2) \circ \vec{n}_2 & (\vec{j}_2 \times \vec{k}_2) \circ \vec{n}_2 & 0 & 0 \\ 0 & 0 & (\vec{j}_2 \times \vec{i}_2) \circ \vec{m}_2 & (\vec{j}_2 \times \vec{k}_2) \circ \vec{m}_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & (\vec{j}_3 \times \vec{i}_3) \circ \vec{n}_3 & (\vec{j}_3 \times \vec{k}_3) \circ \vec{n}_3 \\ 0 & 0 & 0 & 0 & (\vec{j}_3 \times \vec{i}_3) \circ \vec{m}_3 & (\vec{j}_3 \times \vec{k}_3) \circ \vec{m}_3 \end{bmatrix},$$

$$\mathbf{L} = \begin{bmatrix} l_{j1} & l_{j1} & 0 & 0 & 0 & 0 \\ l_{j1} & l_{j1} & 0 & 0 & 0 & 0 \\ 0 & 0 & l_{j2} & l_{j2} & 0 & 0 \\ 0 & 0 & l_{j2} & l_{j2} & 0 & 0 \\ 0 & 0 & 0 & 0 & l_{j3} & l_{j3} \\ 0 & 0 & 0 & 0 & l_{j3} & l_{j3} \end{bmatrix}.$$

Taking into account (21) and (22) the matrix of the drive torques can be finally rewritten as:

$$\mathbf{M} \quad = \mathbf{KLH}^{-1}\mathbf{M}\,. \tag{25}$$

Eq (25) is the basis for calculation of shaft loads generated in the drives during the operation of active vibration control systems, provided the external loading applied to the platform is known.

7. Results of control simulation

Because of the complexity of the mechanism structure, simulation test would be required [1]. These tests may be used to verify formulas applied to control the drives. The model for simulations (Fig. 4) was created using Working Model 3D. The geometrical parameters of the model are: $l_{iu} = 0.5$ [m]; $l_{ju} = 0.36$ [m]; (u=1,2,3), $\rho_1 = 0.288$ [m], $\rho_2 = 0.290$ [m], $\rho_3 = 0.288$ [m], $r_1 = 0.292$ [m], $r_2 = 0.294$ [m], $r_3 = 0.291$ [m]. Vectors ρ_1, ρ_2, ρ have their origin in the centre of platform gravity and vectors r_1, r , r_3 – in the centre of frame gravity.

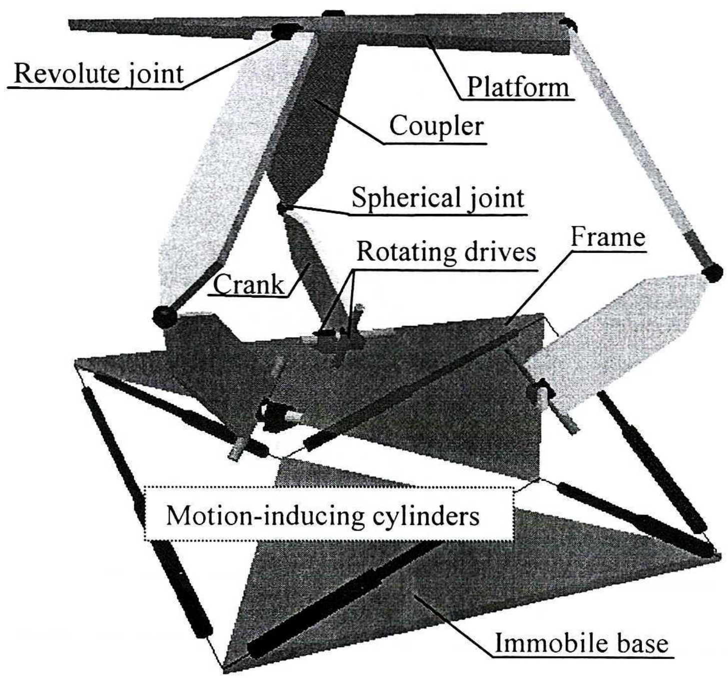

Fig. 4. Model of a platform mechanism with active supports and of the Stewart's platform mechanism inducing the frame movements

During simulations of mechanism operations, the frame is set in motion by means of six linear actuators generating harmonic disturbances: $v_1 = 0.20\cos 3t$ [m/s], $v_2 = 0.20\cos 2t$ [m/s], $v = 0.23\sin 4t$ [m/s], $v_4 = -0.22\sin 5t$ [m/s], $v_5 = 0.12\cos 5t$ [m/s], $v_6 = -0.10\sin 3t$ [m/s]. The amplitudes of acting disturbances are precisely controlled such that the extreme positions of the vibration reduction mechanism should be avoided. Rotating drives in the cross-pieces of three legs are controlled in accordance with formula (12). The platform mass is 100 kg.

The results of simulations include the comparison of coordinates of frame and platform angular acceleration vectors and coordinates of vectors of linear acceleration of frame and platform centres of gravity (Fig 6a, b). Slight deviations of platform acceleration components from the zero value are the result of numerical errors. The integration step assumed for calculations $\Delta t = 0.005$ [s]. Variations of the loading moments for the six engine shafts are also provided (Fig. 5).

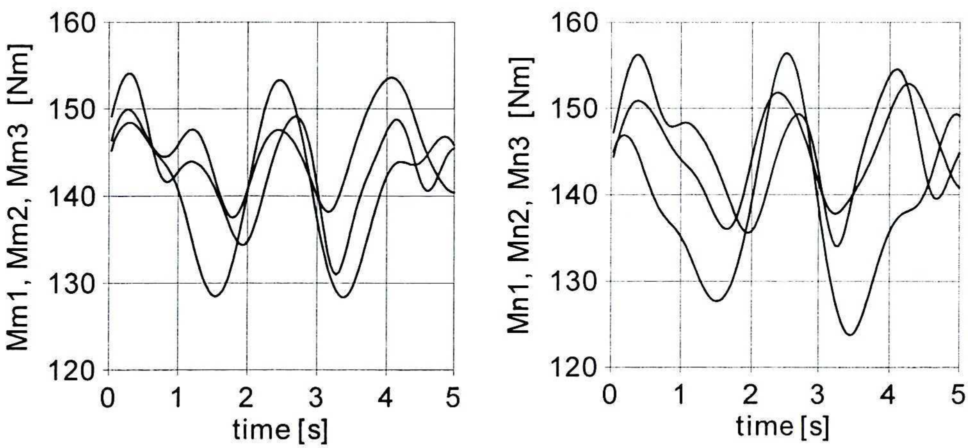

Fig. 5. Absolute values of shaft- loading torque

8. Conclusions

1. The results in the form of variability patterns of the considered machine element and frame accelerations confirm the effectiveness of the applied method of drive control.
2. The values of torques applied to the drive shaft fall in a specified interval, distant from the zero value. In order to reduce those torques and hence the drive power, it is necessary to relieve the drive, for example through the application of relief springs.

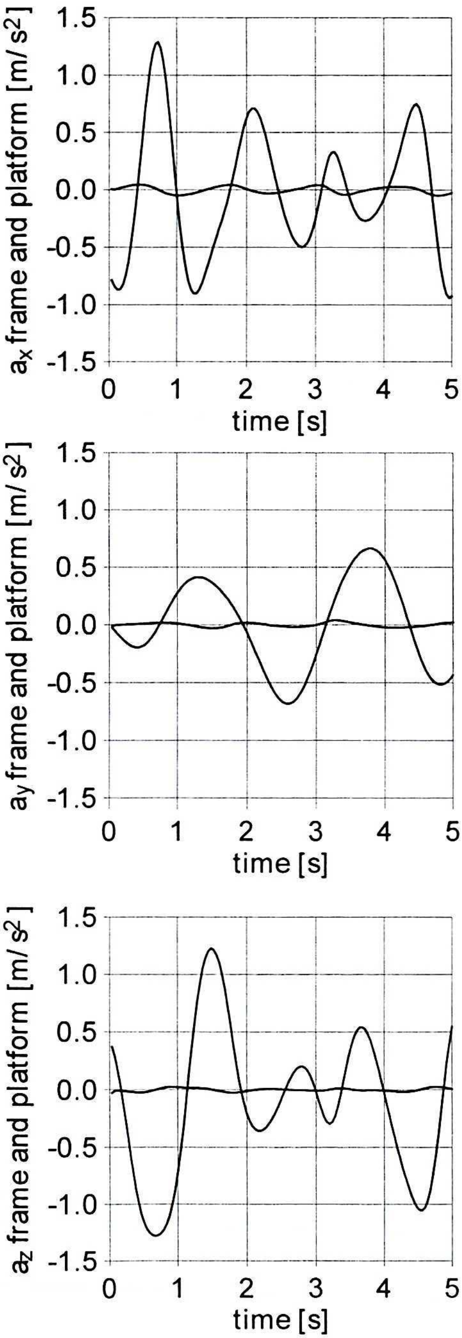

Fig. 6a. Frame and platform centre of gravity acceleration components

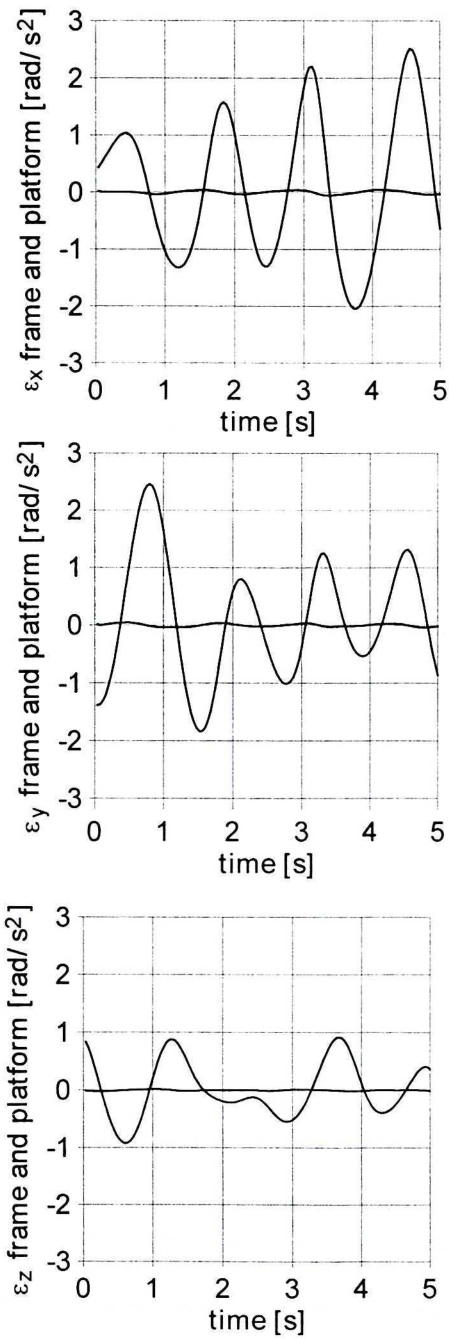

Fig. 6b. Frame and platform angular acceleration components

3. The reasons for incomplete vibration reduction are the delays in system operation, because time is needed for calculations, also due to the time constants of measuring circuit components, converters, controllers and drives. The effectiveness of the vibration reduction is improved with an increase in the frequency of acting disturbances. Furthermore, incomplete reduction of vibration may be the result of compliance of drives, joints, leg links and of excessive engine loading.
4. Geometrical dimensions of links in the platform mechanism restrict the amplitudes of the oscillating motion, otherwise the platform takes an extreme position when further movements in the direction required to leave that position become impossible. In such a situation the return motion should be possible so that the platform could resume its central position where the velocity is predetermined arbitrarily and should increase as the platform gets nearer to the extreme positions.

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Zastosowanie mechanizmu platformowego o 6 stopniach ruchliwości w układzie aktywnej redukcji drgań

Streszczenie

Artykuł zawiera propozycję nowej struktury mechanizmu platformowego o 6 stopniach swobody. Platforma tego mechanizmu jest podparta na ruchomej ramie trzema aktywnymi podporami, każda zawierająca po dwa napędy obrotowe. Mechanizm ten zastosowano do układu aktywnej redukcji drgań. Wykorzystując współrzędne wersorów związanych z ogniwami mechanizmu sformułowano układ osiemnastu równań algebraicznych położenia mechanizmu. Wyprowadzono macierz jakobianową przekształcenia prędkości ramy w prędkości obrotowe napędów oraz macierz przekształcającą obciążenia przyłożone do platformy w momenty napędów. Wyznaczono wartości prędkości kątowych oraz momentów, które powinny być realizowane przez napędy, aby zminimalizować przenoszenie drgań podstawy na platformę. Wykonano model mechanizmu w programie Working Model 3D. Do wywołania drgań ramy zastosowano mechanizm platformowy Stewarta, w którym liniowe prędkości siłowników zmieniają się harmonicznie. Przedstawiono wyniki pracy układu aktywnej redukcji drgań w postaci wykresów przyspieszeń ramy i platformy oraz przedstawiono przebiegi momentów napędów.