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LOW-CYCLE FATIGUE STRAIN CRITERION FOR BIAXIAL NONPROPORTIONAL LOADING

The paper is devoted to development of the evaluation methods for limiting condition of metal alloys and steels under uniaxial and biaxial proportional and non-proportional low-cycle fatigue. To investigate the fatigue behavior of Type 08X18H10T stainless steels and titanium alloys BT9 strain controlled tests under 12 proportional and nonproportional loading at room temperature were carried out. A strain parameter, namely, a nonproportional strain range, is proposed to obtain a correlative dependence with lifetime. For the majority of the materials this dependence can be derived from uniaxial tests and described by a linear function. It made it possible to propose simple engineering method for lifetime prediction.

1. Introduction

Increased requirements for reliability and cost-effectiveness imposed on structural elements of modern machinery give rise to the necessity of modeling the mechanical behaviour of vital parts under loading regimes close to operating conditions. The loads occurring in service are in most cases cyclic, multiaxial and nonproportional. Under such conditions, particularly in zones of increased stress concentrations, materials can cyclically deform beyond the elastic limit and undergo considerable cyclic hardening. In the region of low-cycle fatigue there factors influence appreciably the lifetime of the material. In the case of uniaxial straining in the theory low-cycle fatigue, the dependence between the lifetime and range (amplitude) of equivalent strains is used. For nonproportional straining it is difficult to obtain such dependence taking no account of the strain

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hardening. Nonproportional loading reduces fatigue life due to the additional hardening depending on strain history or cycle path. In order to take into account the influence of the cycle path on additional hardening the nonproportional parameters have been proposed. Some of them are constructed using the stress tensor components, other using a mixed scheme, which includes both the stress and strain components. When considering the strain controlled mode of loading it is convenient to use the nonproportional parameter based on the strain components only.

The aim of this paper is analysis of experimental results of proportional and nonproportional loading a hollow cylinder specimens of Type 08X18H10T stainless steel and titanium alloy BT9 under uniaxial and biaxial strain controlled tests (12 strain paths) and to examine the modified strain criterion [1].

1. Experimental procedure

The material tested was Type 08X18H10T stainless steel and titanium alloy BT9 which have different degree strain hardening. Type 08X18H10T stainless steel is known as material which shows the large additional hardening under nonproportional loading whereas titanium alloy BT9 shows a small additional hardening. The strain controlled low-cycle fatigue tests were carried out by electromechanical machine capable of combined axial and torsional loading. The procedure and testing equipment for uniaxial and biaxial tests are described in detail in [2]. The specimen geometry is shown in Fig. 1. Table 1 lists the dimensions of specimens. The present cyclic tests performed at room temperature. For all tests the frequency was 1 cycle/min, and the strains were completely reversed. Wave form of the axial and torsional strain cycles under loading specimens of 08X18H10T steel were triangular. For titanium alloy axial and torsional strains were give as sinusoidal wave form.

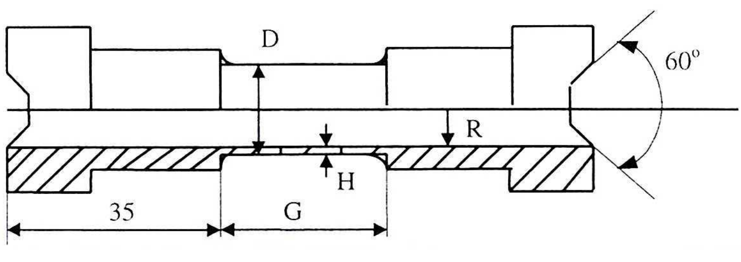


Fig. 1 Thin-wall tubular specimen

Table 1.

Dimensions of test specimens

	08X18H10T	BT9
Gale length G, mm	15	20
Outside diameter D, mm	11	11
Thickness H, mm	0.5	0.5

Figure 2 shows strain path employed for two materials. The ε and γ are the axial and shear strains. The specimens from 08X18H10T steel loaded by symmetrical cycling for different levels of total strains for proportional {1}–{5} and nonproportional {6}–{11} strain path. For titanium alloy BT9 the strain paths loading {1}, {3}, {5}, {12} were carried out.

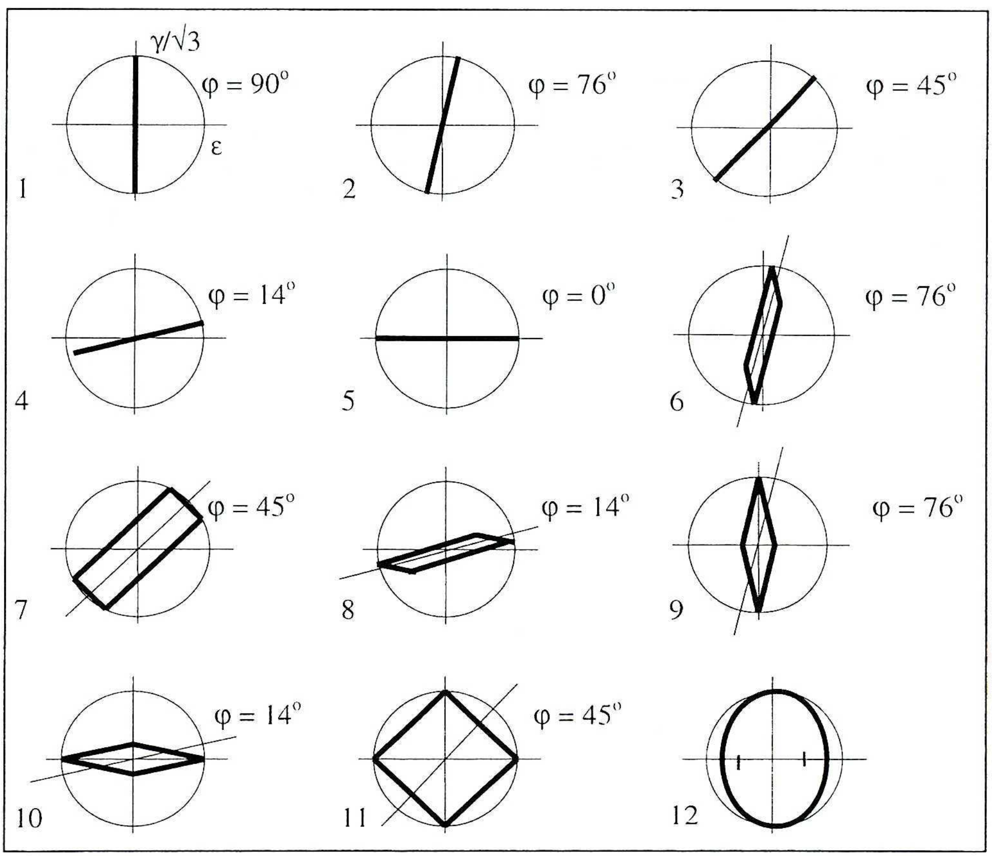


Fig. 2. Proportional and nonproportional cyclic strain path.

The failure life N was defined as the number of cycles to the formation of a crack of approximately 1.0 mm long. For each material tests were performed for three strain range levels (steel 08X18H10T: $\Delta\varepsilon$ = 0.8%, 1.0%, 1.2%; alloy BT9: $\Delta\varepsilon$ = 1.4%, 2.0%, 2.6%).

3. Experimental results

Figures 3a,b shows correlation of proportional and nonproportional low cycle fatigue lives of type 08X18H10T stainless steel and titanium alloy BT9, correspondently with the equivalent strain range $\Delta\varepsilon$ defined in ASME Code Case N-47 [3], which has been used as a design parameter for low-cycle fatigue. From this figures it can be seen that the strain path has a significant influence on the fatigue life. For the same equivalent strain range $\Delta\varepsilon$, fatigue life is longest for the case of pure torsion path {1} and shortest for pure axial straining path {5}. Analysis results also show that out-of-phase loading tends to be more damaging than in-phase loading. Reduction in nonproportional low-cycle fatigue life is connected with the degree of additional hardening.

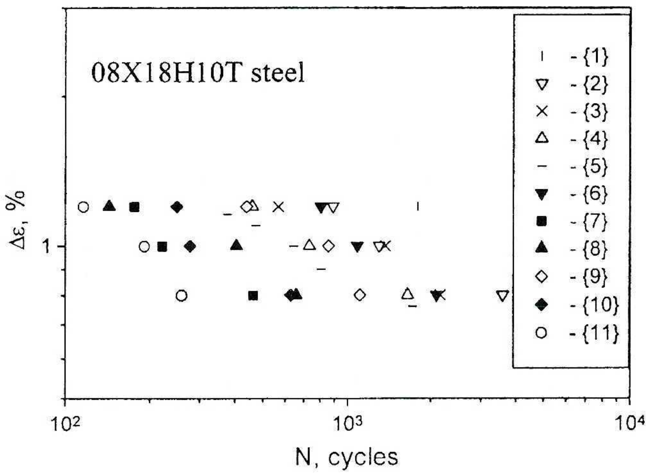
Since Type 08X18H10T stainless steel is characterized by a considerable strain hardening herefore the significant reduction in fatigue life occurs for nonproportional paths. The small reduction in low-cycle fatigue life occurs for small additional hardening and the large reduction for large additional hardening. Real materials exhibit the different properties depending on the type of the stress state. In this case even under proportional straining the life may differ by an order of magnitude depending on type of stress state.
The experimental data reported in this paper also give evidence of this fact. Thus, for Type 08X18H10T stainless steel, the lifetime under proportional straining in tension-compression is almost twice as high as that under a combined action of tension-compression and torsion. Titanium alloy shows the same trend but it features as lesser degree of hardening as a lesser difference in cyclic properties with respect to the type of the stress state.

Analysis of the experimental results leads to conclusion that the nonproportional low-cycle fatigue damage is a function of strain cyclic history and type of material. Thus, nonproportional strain parameter must take account of these two factors.

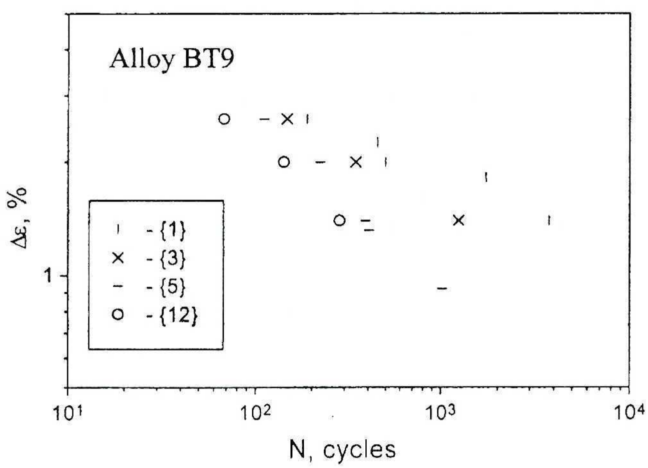
4. Nonproportional low-cycle fatigue strain parameter

We take a modified strain approach proposed in Ref. [4] as the basis for the analysis of the experimental results. According to it in [1] a new nonproportional strain range was proposed. It is determined the following expression:

$$\Delta\varepsilon_{np} = (1 + k\sin\varphi)(1 + \alpha\ \Phi)\Delta\varepsilon \qquad (1)$$



(a) Type 08X18H10T steel



(b) Titanium alloy BT9

Fig. 3. Correlation of fatigue life with ASME strain range

where α is the material constant, which defines the sensibility of the material to nonproportionality of the cycle path (according to [4] the value of α is defined as the ratio of the stress amplitude under 90 degree out-of phase loading in a steady-state regime to that under proportional loading if the effect of additional hardening is clearly pronounced under nonproportional loading); Φ is the coefficient of the cycle path nonproportionality which characterizes the geometry of cycle with respect to cyclic properties; $\Delta\varepsilon$ is the ASME maximum strain range [3]; k is the material constant which characterizes the influence of stress state on the material cyclic properties under proportional loading (for example, torsion case or axial straining case); φ is the angle of the cycle path orientation with respect to the principal axis which coincides with one of the directions whether pure torsion or tension-compression.

It's convenient to assume that the parameter of nonproportionality is equal to 1 under loading along a circular path (in this case the highest level of hardening is observed) and to 0 for any proportional path. The magnitude of this parameter for an arbitrary path lies in between the above values. The magnitude of cyclically stabilized limit stress should be calculated by interpolation within the range indicated.

Since in practice the researchers are often interested in the level of maximum stresses attained in the material at the given cyclic deformation path, the stress tensor is desired. Therefore, it may not necessarily need the expression for the nonproportionality parameter.

The hardening effects observed in the experiments with the straining along the paths of both total strains and plastic strains do not practically differ qualitatively. Keeping this in mind, it is convenient to formulate the nonproportionality parameter using a unified approach based on cycle geometry. This does not require the involvement of total or plastic strains and eliminates the transition from the total to plastic strains since the transition is not always convenient.

The analysis of the experimental results revealed that with the same maximum strain amplitude higher hardening is characteristic of the strain path which envelopes a larger effective area and in this case the cycle path length influence is ambiguous. Its increase leads to the increase in hardening, yet beginning with some length a reverse process occurs, i.e. a decrease in the level of maximum stresses.

As applied to plane cyclic paths in a deviatoric space of strains (biaxial loading) for specified maximum amplitude of a cycle, the nonproportionality parameter can be written as follows:

$$\Phi = \left(\left| \oint_{L'} \mathbf{e} \times d\mathbf{e} \right| \Big/ \left| \oint_{L0} \mathbf{e} \times d\mathbf{e} \right| \right)^{r}, \qquad 0 \le \Phi \le 1 \tag{2}$$

where $\mathbf{e}, d\mathbf{e}$ are the vectors of strain and strain increment, respectively; L' is the deformation path of the cycle considered or the convex equivalent path; L_0 is the circular path.

The integrals along the closed path in Eqn (2) have a simple geometrical interpretation, namely, they are doubled areas S and S_0 enveloped by the given cycle paths L' and L_0 (see Fig. 4). Then, for a circular path it is obvious that

$$\left| \oint_{L0} \mathbf{e} \times d\mathbf{e} \right| = 2S_0 = 2\pi (\varepsilon_m)^2 \tag{3}$$

Equation (1) may then be written as $\Phi = (S/S_0)^r$.

If the cycle shape is such that the integral in the numerator of eqn (1) is equal to zero, e.g. path L in Figs 4 a and b, are not convex, e.g. path L in Fig.A c (we arbitrarily call such paths non convex), then an equivalent convex path L' (Figs 4a-c) should be used in eqn (1), rather than the non convex path L considered.

The exponent r for smooth convex algebraic curve, for example circle, is equal to 1. For piecewise-broken or piecewise-smooth paths it can be determined from the following expression:

$$r = \left(1 - \left| \oint_{\mathrm{L}} \mathbf{e} \times d\mathbf{e} \right| \Big/ \left| \oint_{\mathrm{L0}} \mathbf{e} \times d\mathbf{e} \right| \right) \cdot \frac{l}{4 \Delta \varepsilon_m} \tag{4}$$

where $l = \int_{\text{cycle}} |d\mathbf{e}|$ is the length of the deformation path; $\Delta\varepsilon_m$ is the maximum strain range. For paths L in Figs 4 a and b Eqn (4) is simplified since the expression in the bracket is equal to 1:

$$r = \frac{l}{4 \Delta \varepsilon_m} \tag{5}$$

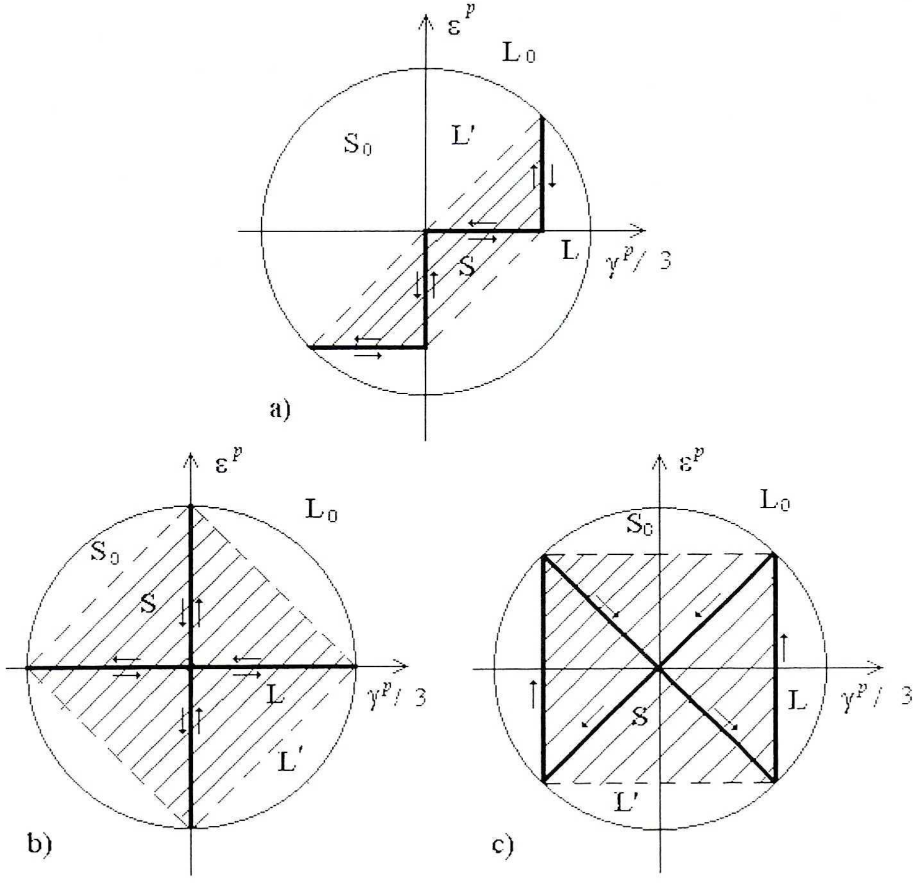


Fig 4. Cyclic paths and enveloped areas

The nonproportionality parameters for all cyclic path Fig. 2. show in Table 2.

Table 2.

The nonproportionality parameter values

Path	{1}	{2}	{3}	{4}	{5}	{6}	{7}	{8}	{9}	{10}	{11}	{12}
Φ	0	0	0	0	0	0.38	0.6828	0.38	0.45	0.45	0.89	0.95

5. Experimental determination of the model parameters

As follows from expression (1), to specify the model we have to define the constants α and k. It is obvious that to determine them we should use the experimental data from both uniaxial and biaxial low-cycle fatigue tests. We assume the existence of a linear dependence in logarithmical coordinates of the strain range $\Delta\varepsilon$ on the number of cycles to the initiation of fracture. In this case, three base experiments are enough to determine the parameter α. They are low-cycle fatigue tests under uniaxial straining for two levels of the strain range $\Delta\varepsilon_1$ and $\Delta\varepsilon_2$ and an experiment with any nonproportional straining at the strain range $\Delta\varepsilon_1$ or $\Delta\varepsilon_2$.

Following the [4], coefficient α is determined from the following expression:

$$\alpha = \frac{\sigma_{\max}^{c} - \sigma_{\max}^{t}}{\sigma_{\max}^{c}}, \quad (6)$$

where $\sigma_{\max}^{c}$, $\sigma_{\max}^{t}$ are the maximum stress amplitude under 90 degrees out-of-phase and pure torsion loading consequently. Since for 08X18H10T steel the value of maximum stress amplitude under 90 degree out-of-phase is unknown the constant α will be determined basing on an assumption of the existence of a unified law-cycle fatigue curve for proportional and nonproportional loading. Then

$$\alpha = \frac{1}{\Phi}\left[\left(\frac{\Delta\varepsilon_1}{\Delta\varepsilon_2}\right)^{\lg\frac{N_2}{\overline{N}_2}/\lg\frac{N_2}{N_1}} - 1\right] \quad (7)$$

where N_1, N_2, $\overline{N}_2$ are the lives under uniaxial and nonproportional straining, Φ is the coefficient of nonproportionality corresponding of nonproportional base test.

If we know the slope of the low-cycle fatigue line n under uniaxial deformation in advance, the expression for determining parameter α will somewhat simplified:

$$\alpha = \frac{1}{\Phi}\left[\left(\frac{\overline{N}_2}{N_2}\right)^{n} - 1\right]. \quad (8)$$

By complete analogy, we can obtain expression for determining parameter k. As in the previous case, we have to perform at least three experiments. Two experiments at different strain ranges $\Delta\varepsilon_1$ and $\Delta\varepsilon_2$ are mode along the directions which coincides with the principal one ($\varphi = 0^o$), and the third along another arbitrary direction ($\varphi = \varphi'$) with one of strain range $\Delta\varepsilon_1$ or $\Delta\varepsilon_2$.

Using the same scheme, we obtain

$$k = \frac{1}{\sin\varphi'}\left[\left(\frac{\Delta\varepsilon_1}{\Delta\varepsilon_2}\right)^{\lg\frac{N_2}{\bar{N}_2}/\lg\frac{N_2}{N_1}} - 1\right], \tag{9}$$

and in the case when the slope of the low-cycle fatigue line n is known, we have:

$$k = \frac{1}{\sin\varphi'}\left[\left(\frac{\bar{N}_2}{N_2}\right)^{n} - 1\right]. \tag{10}$$

For Type Type 08X18H10T steel and Titanium alloy BT9 the angle $\varphi = 90^o$ corresponds to the path {1} (pure torsion), $\varphi = 0^o$ to the cycle path {5} (tension-compression) whereas angle $\varphi = 45^o$ to the path {3}.

We used two experimental point of fatigue tests at tension-compression for determination linear relationship between total strain range and fatigue life. The coefficients of linear approximation n and b are shown in Table 3. We also used expressions (8) and (10) and two additional basic test (nonproportional loading and pure tortion) for each material to determine the constants α and k according to above mentioned procedure.

Table 3.

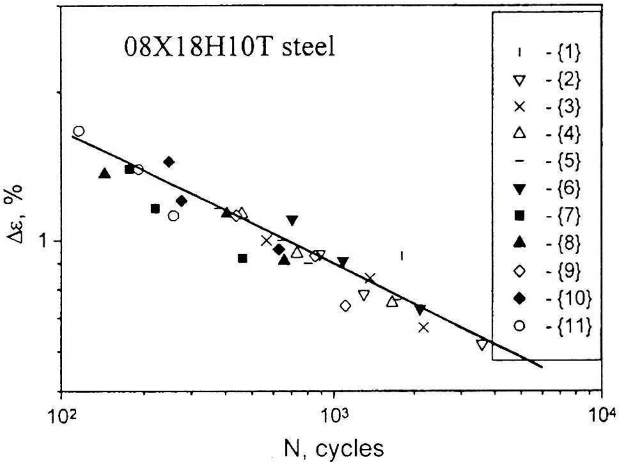
Approximation coefficients and model constants

Material	08X18H10T	BT9
N	-0.277	-0.49
B	0.778	1.43
α	0.6	0.17
K	0.25	0.26

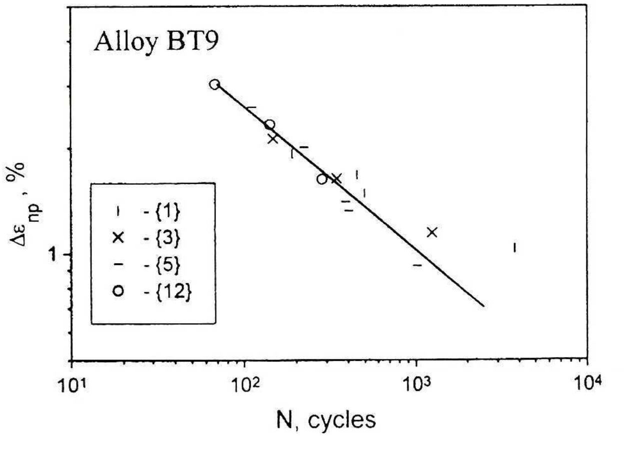
6. Analysis of the experimental results

We use expression (1) for the presentation of the experimental data on biaxial low-cycle fatigue considered in this paper. The model parameters α and k is contained in Table 3. Type 08X18H10T steel is characterized by a considerable strain hardening therefore the parameter $\alpha = 0.6$, and the difference in the cyclic properties with respect to the principal direction (path {5}) is characterized by the maximum k value equal to 0.25. Titanium alloy BT9 features a lesser degree of hardening (α = 0.17) and has similar value of k coefficient.

Figures 5a,b illustrate the correlation between the nonproportional strain range and the lifetime for the wide classes of nonproportional strain paths presented on logarithmical coordinates. As is seen, parameter (1) makes it possible to "place" the experimental points corresponding to nonproportional low-cycle loading on the relationship which can be obtained from low-cycle fatigue tests under proportional straining. Such experimental dependencies have long been obtained for many structural materials and are being used in standard documents. We remind that in our case we use uniaxial fatigue test at tension-compression. Thus, the possibility to predict the lifetime under nonproportional straining from the data on proportional low-cycle straining can become the important consequence of model (1).



(a) Type 08X18H10T steel



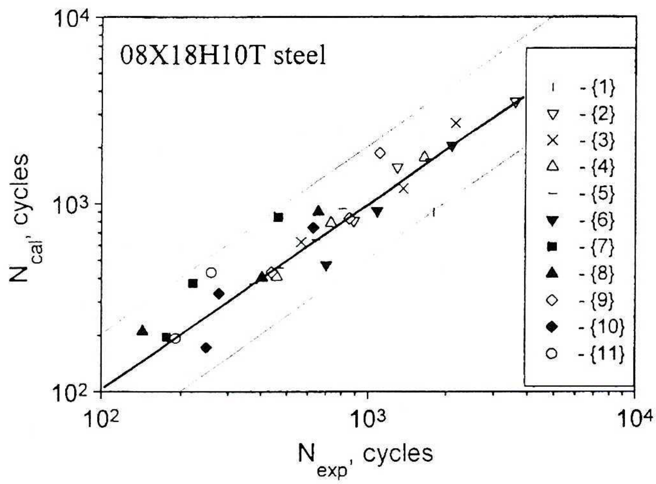
(b) Titanium alloy BT9

Fig. 5. Correlation of fatigue life with nonproportional strain range

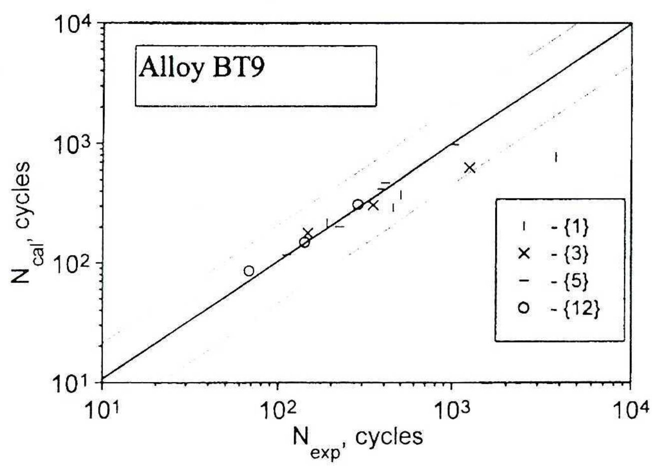
The simplest form strain criterion according to model (1) is:

$$N = (\Delta\varepsilon_{np}/10^{b})^{1/n} \qquad (11)$$

where N is fatigue life; $\Delta\varepsilon_{np}$ is the nonproportional strain range (1); n and b are the coefficients of linear regression fatigue curve for uniaxial straining. We used equation (11) to predict fatigue life. The results of this prediction are shown in Fig. 6.



(a) Type 08X18H10T steel



(b) Titanium alloy BT9

Fig. 6. Comparison of predicted lifetime and experimental

7. Conclusions

Proportional and nonproportional low-cycle fatigue tests were carried out using 12 strain path for Type 08X18H10T steel and Titanium alloy BT9 hollow cylinder specimens at room temperature.

A simple strain parameter, i.e. a nonproportional strain range, has been proposed which takes into account strain hardening and influence the type of stress state on material cyclic properties as applied to biaxial low-cycle straining.

A sufficiently good correlation between the nonproportional strain range and the lifetie, which in many cases can be described by a linear relationship, is observed for the materials considered in the paper.

The strain parameter proposed enables one to use the low-cycle fatigue relationships obtained in uniaxial tests for biaxial fatigue since in both cases the experimental points fall on the same curve. It is allows to propose simple engineering method for fatigue life prediction. The calculations performed using strain criterion exhibited good agreement with experimental data.

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Odkształceniowe kryterium nisko-cyklowego zmęczenia dla przypadków dwuosiowych obciążeń nieproporcjonalnych

S t r e s z c z e n i e

Praca jest poświęcona rozwojowi metody wyznaczenia warunków granicznych dla zmęczenia niskocyklowego stali i stopów metali pod wpływem obciążeń osiowych i obciążeń dwuosiowych proporcjonalnych i nieproporcjonalnych.

Właściwości zmęczeniowe stali nierdzewnej 08X18H10T oraz stopu tytanu BT9 badano w temperaturze pokojowej, mierząc odkształcenia dla 12 zakresów obciążeń proporcjonalnych i nieproporcjonalnych. Parametr odkształcenia, a mianowicie zakres odkształceń nieproporcjonalnych, jest przyjęty jako wielkość skorelowana z trwałością.

Dla większości materiałów zależność ta może być wyznaczona na podstawie prób osiowych i opisana funkcją liniową, co umożliwia zaproponowanie prostej inżynierskiej metody oceny trwałości.