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Discrete-time Takagi-Sugeno singular systems with unmeasurable decision variables: state and fault fuzzy observer design

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The studied problem in this paper, treat the issue of state and fault estimation using a fuzzy observer in the case of unmeasurable decision variable for Discrete-Time Takagi-Sugeno Singular Sytems (DTSSS). First, an augmented system is introduced to gather state and fault into a single vector, then on the basis of Singular Value Decomposition (SVD) approach, this observer is designed in explicit form to estimate both of state and fault of a nonlinear singular system. The exponential stability of this observer is studied using Lyapunov theory and the convergence conditions are solved with Linear Matrix Inequalities (LMIs). Finally a numerical example is simulated, and results are given to validate the offered approach.

Key words: observer design, unmeasurable decision variable, SVD approach, Lyapunov function, LMIs

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1. Introduction

Modern engineering systems are increasingly more complex which leads to sensor and actuator degradation. To overcome this catastrophic situation, the integration of faults detection and estimation (FDE) and fault tolerant control (FTC) tools [1–6], are very appreciated and required to assur the safety and stability of system against faults, and to maintain the desired performance of the whole system in various areas of applications over a long period of time.

Singular models (see [7–9] and references therein), known also as, descriptor systems or implicit systems, is a powerful and accurate way to model various industrial processes, like biological systems, chemical processes, robotics, circuit systems and so on [10]. Such systems include static and dynamic equations, are more popular than standards systems, and can interpret the behaviour of various types of processes. Recently, the issue of detecting and estimating faults for singular systems has attracted a lot of attention in different field of research see e.g. [11–13].

To build a powerful diagnosis mechanism and estimating faults, all informations about states of systems should be known and measured, but for an economics and technical constraints, is not possible for the most cases. As an alternative, the use of an observer see e.g. [14, 15], who has the ability to give the estimated state is very demanded. Moreover, designing an observer for a purely nonlinear system is still a very difficult task, instead transforming the nonlinear system into Takagi-Sugeno (T-S) models will make the design of observer more easier.

The T-S structure, is a modern approach which has been inspired from the pure fuzzy modeling [16]. Nowadays the fuzzy model of type T-S, became a very interesting model, due to the fact that it combines between the complexity of the real non linear system, as well as the simplicity and smotheness of the fuzzy representation. Thanks to the sector non linear approch (see e.g. [17–20] for more details), the non linear system translated into the T-S model, without losing informations about the original system. The T-S model is constituted of the overall T-S fuzzy system and the local system, or sub-system, each sub-system contributes in the construction of the overall T-S fuzzy system with a different degree of appartenance. The activation of these sub-system is reinforced via the intervention of the nonlinear membership function, which makes a blending of all sub-systems. In addition, the T-S models makes easier the integration of Lyapunov quadratic theory and the analysis of exponential stability conditions, which is converted to the resolution of a feasible set of linear matrix inequalities (LMIs) problems. T-S approach has won a great reputation as an effective gadget to analyze and control nonlinear systems, for the reason that such models allow finding a compromise between the good accuracy of the nonlinear behavior of the system under study, and the use of strategies that are adapted to linear systems, using the convex sum property of their nonlinear activation functions which are connected to all local sub-systems, and represents an aspect of the sector nonlinearity approach.

Various studies has investigated the issues related to state and unknown input estimation [21–23], as well as state and faults estimation [24, 25] and many approaches has been the objective of fuzzy observer design as in [21, 24] when authors use the Lipschitz constraints. In other side other approach has been applied by authors in [23, 25] based on the separation between dynamic and static relations in T-S singular model. A study has been treated in [26], aiming at the estimation of states, actuator, and sensor faults in nonlinear systems with a polytopic linear parameter varying representation. The proportional multiple integral sliding mode observer constructed in this study. A further work on the same issue has been proposed in [27]. In [28] the authors has introduced a new strategy of the trajectory tracking in T-S Lipschitz nonlinearities systems, in the basis of a proportional integral observer devoted to the control strategy, with more improvement.

This work deals with the problem of simultaneous state, actuator and sensor faults for a class of DTSSS for the case of unmeasurable decision variables. The decision variables of the singular system are not the same for the observer, and since its not mesurable like the case of the measurable decision variables, then the synthesised observer must use its own activation function, which is different from the activation function of the singular system. The decision variables which are state variables are unmeasurables and therefore unknowns, and the observer must estimate them accurately, even in the absence of measurement. Another difficulty encountered than the previous ones, is related to the passage from the theoretical quadratic Lyapunov function to the numerical resolution in the form of LMIs, and the LMIs in their first form are not linear, as the name indicates but are in the form of bilinear matrix inequalities (BMIs). These BMIs are constraints that represent more difficulty in the study, to reduce this difficulty, the BMIs are transformed into LMIs, and this passage is done by the change of variables and the Schur compliment. Feasibility and realisability is thus a problem encountered in the calculation of LMIs. Therefore, it is necessary to prepare and verify the LMIs theoretically before integrating them into existing numerical solvers. All these difficulties will be treated and verified, to improve the proposed work.

Motivated by the above, the main contribution of this paper reside in the study of a new fuzzy observer design, aiming at state and fault estimation issue resolution for a class of DTSSS in unmeasurable decision variable case. This observer are designed on the basis of SVD approach, and an augmented system aggregates state and fault as a one state vector. To prove the stability of the proposed observer, the exponential stability of Lyapunov function is studied, and the resolution of the convergence conditions are specified as a feasible set of Linear Matrix Inequalities.

The simulation part is devoted to the study of a singular fuzzy model which represent a physical system of electronic circuit, this physical system, is chosen to be modeled in the singular stucture due to the fact that this process is by nature singular, and the necessity to deal with the algebraic part which represents a constraint for the studied system imposes the choice of the singular structure, which cannot be traited with the explicit structure. The synthesised fuzzy observer will be applied on this non linear singular electronic circuit, to validate the accuracy of the proposed approach of state and fauls estimation.

The rest of this work is divided as follows, Section 2 present the mathematical formulation of DTSSS in presence of actuator and sensor fault, Section 3 debate the main result acquired on the new observer to estimate state and fault, Section 4 explicate the attainment of the coveted result in numerical application of fuzzy model, Section 5 is a epilogue of the paper.

Notation

Let the superscript $X = X^T$ be a symmetric matrix $X > 0$ and $X < 0$, $*$ designates for positive and negative definiteness and the transpose of X, I gestures the identity matrix, 0 is the zero matrix of appropriates dimensions. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ marks the n-dimensional real vectors and the set of all $n \times m$ real matrices.

2. Problem formulation of discrete-time Takagi-Sugeno singular systems

Using T-S approach, the considere DTSSS is:

$$\begin{cases} E\rho_{k+1} = \sum_{i=1}^{q} \phi_i(\gamma_k)\left(A_i\rho_k + B_i u_k + F_{ai} f_{ak}\right), \\ y_k = C\rho_k + D_a f_{ak} + F_s f_{sk}\,, \end{cases} \tag{1}$$

where $\rho_k \in \mathbb{R}^n$ is the state vector, $u_k \in \mathbb{R}^m$ is the control input, $y_k \in \mathbb{R}^p$ is the measured output vector. $f_{ak} \in \mathbb{R}^{n_a}$ is the actuator fault and $f_{sk} \in \mathbb{R}^{n_s}$ is the sensor fault. $A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$, $D_a \in \mathbb{R}^{p\times n_a}$, $E \in \mathbb{R}^{n\times n}$ where $E(\rho)$ is the singular matrix, has not full rank, such that rank$(E) < n$, $F_{ai} \in \mathbb{R}^{n\times n_a}$, $F_s \in \mathbb{R}^{p\times n_s}$, are real known constant matrices. The nonlinear system can be decomposed into the following so-called T-S sub-model:

$$\begin{cases} E\rho_{k+1} = A_i\rho_k + B_i u_k + F_{ai} f_{ak}\,, \\ y_k = C\rho_k + D_a f_{ak} + F_s f_{sk} \end{cases} \tag{2}$$

with $i = \{1, \cdots, q\}$, q is the number of sub-model. The activation functions $\phi_i(\gamma_k)$ are associated to (2) and confirm the convex sum properties:

$$0 \leqslant \phi_i(\gamma_k) \leqslant 1, \qquad \sum_{i=1}^{q} \phi_i(\gamma_k) = 1. \tag{3}$$

The system (1), the fault and matrix expression f_k, L_{ai}, G can be rewritten as:

$$\begin{cases} E\rho_{k+1} = \sum_{i=1}^{q} \phi_i(\gamma_k)\left(A_i\rho_k + B_i u_k + L_{ai} f_k\right), \\ y_k = C\rho_k + G f_k , \end{cases} \tag{4}$$

$$f_k = \begin{bmatrix} f_{ak}^T & f_{sk}^T \end{bmatrix}^T, \qquad L_{ai} = \begin{pmatrix} F_{ai} & 0 \end{pmatrix}, \qquad G = \begin{pmatrix} D_a & F_s \end{pmatrix}. \tag{5}$$

Assumption 1 *Suppose that the f_k is a constant, unknown fault signal per time interval i.e:*

$$f_{k+1} = f_k , \qquad k \in [\mathcal{T}_1\ \mathcal{T}_2], \quad \forall\, \mathcal{T}_1,\ \mathcal{T}_2 \in \mathbb{R}^+. \tag{6}$$

Using (5), the augmented DTSSS (1) with f_k and matrix expressions can be written as:

$$\begin{cases} \widetilde{E}\varpi_{k+1} = \sum_{i=1}^{q} \phi_i(\gamma_k)\left(\widetilde{A}_i\varpi_k + \widetilde{B}_i u_k\right), \\ y_k = \widetilde{C}\varpi_k ; \end{cases} \tag{7}$$

$$\begin{cases} \varpi_k = \begin{pmatrix} \rho_k^T & f_k^T \end{pmatrix}^T, \quad \widetilde{B}_i^T = \begin{pmatrix} B_i^T & 0 \end{pmatrix}, \quad \widetilde{C} = \begin{pmatrix} C & G \end{pmatrix}, \\ \widetilde{A}_i = \begin{pmatrix} A_i & L_{ai} \\ 0 & I \end{pmatrix}, \qquad \widetilde{E} = \begin{pmatrix} E & 0 \\ 0 & I \end{pmatrix}. \end{cases} \tag{8}$$

The studied problem is considered under the following conditions of regularity, impulsive observability and detectability which represents the conditions of existence of the synthsised observer:

H_1) The pair $\left(\widetilde{E},\ \widetilde{A}_i\right)$ are said to be regular, if: $i \in [1, \ldots, q]$

$$\det(s\widetilde{E} - \widetilde{A}_i) \neq 0 \qquad \forall s \in \mathbb{C}. \tag{9}$$

H_2) All sub-models (7) are said to be impulses observables if:

$$\operatorname{rank}\begin{bmatrix} \widetilde{E} & \widetilde{A}_i \\ 0 & \widetilde{E} \\ 0 & \widetilde{C} \end{bmatrix} = n_1 + \operatorname{rank}\widetilde{E}. \tag{10}$$

H_3) All sub-models (7) are said to be detectable if:

$$\operatorname{rank}\begin{bmatrix} s\widetilde{E} - \widetilde{A}_i \\ \widetilde{C} \end{bmatrix} = n_1, \ \forall s \in \mathbb{C}. \tag{11}$$

H_4) All sub-models (7) are equivalent to be observable, i.e:

$$\text{rank}\left(\begin{bmatrix}\widetilde{E}\\ \widetilde{C}\end{bmatrix}\right) = n_1 = n + n_a + n_s\,. \tag{12}$$

Since (12), there exist a nonsingular matrix $\begin{pmatrix}\nu_1 & \nu_2\\ \nu_3 & \nu_4\end{pmatrix}$ such that:

$$\begin{cases}\nu_1\widetilde{E} + \nu_2\widetilde{C} = I,\\ \nu_3\widetilde{E} + \nu_4\widetilde{C} = 0\end{cases} \tag{13}$$

with

$$\nu_1 \in \mathbb{R}^{n_1\times n_1}, \quad \nu_2 \in \mathbb{R}^{n_1\times p}, \quad \nu_3 \in \mathbb{R}^{p\times n_1}, \quad \nu_4 \in \mathbb{R}^{p\times p}.$$

The result obtained by the SVD approach, allows the resolution of the constant matrices, ν_1, ν_2, ν_3, ν_4, which will be incorporated in the construction phase of the fuzzy observer, in order to simplify certain complexities of synthesis, and facilitate the calculation of the gains.

3. Main result

The proposed fuzzy regular observer is presented as follows:

$$\begin{cases}\delta_{k+1} = \sum_{i=1}^{q}\phi_i(\hat{\gamma}_k)\left(M_i\delta_k + J_{1i}y_k + J_{2i}y_k + H_iu_k\right),\\ \hat{\varpi}_k = \delta_k + \nu_2 y_k + R\nu_4 y_k\,,\end{cases} \tag{14}$$

where $\hat{\varpi}_k$ denote the estimated augmented state vector. The problem of this observer is reduced to seek the matrix M_i, J_{1i}, J_{2i}, H_i and R that allow $\hat{\varpi}_k$ to converge exponentially to ϖ_k. Then let us define the state estimation error of the observer it is equal to:

$$\varepsilon_k = \varpi_k - \hat{\varpi}_k\,. \tag{15}$$

By taking into account (7), (13), gathered with (15), static and dynamic error becomes:

$$\varepsilon_k = (\nu_1 + R\nu_3)\,\widetilde{E}\varpi_k - \delta_k\,, \qquad \varepsilon_{k+1} = (\nu_1 + R\nu_3)\,\widetilde{E}\varpi_{k+1} - \delta_{k+1}\,. \tag{16}$$

According with (7), (13), (14) equation of dynamic error is represented by:

$$\begin{aligned}\varepsilon_{k+1} = &\sum_{i=1}^{q}\phi_i(\gamma_k)(\nu_1 + R\nu_3)(\widetilde{A}_i\varpi_k + \widetilde{B}_iu_k)\\ &- \sum_{i=1}^{q}\phi_i(\hat{\gamma}_k)\left(M_i\delta_k + J_{1i}y_k + J_{2i}y_k + H_iu_k\right).\end{aligned} \tag{17}$$

By applying (7), equation (17) and the expression of η_i, can be integrated as:

$$\varepsilon_{k+1} = \sum_{i=1}^{q} \phi_i(\gamma_k)(\nu_1 + R\nu_3)\left(\widetilde{A}_i\varpi_k + \widetilde{B}_i u_k\right) + \sum_{i=1}^{q} \phi_i(\hat{\gamma}_k)\left(M_i\varepsilon_k - \eta_i\varpi_k - H_i u_k\right), \tag{18}$$

$$\eta_i = M_i(\nu_1 + R\nu_3)\widetilde{E} + J_{1i}\widetilde{C} + J_{2i}\widetilde{C}. \tag{19}$$

Provided the matrices M_i, J_{1i}, J_{2i}, H_i and R satisfy:

$$\eta_i = (\nu_1 + R\nu_3)\widetilde{A}_i, \qquad H_i = (\nu_1 + R\nu_3)\widetilde{B}_i\,. \tag{20}$$

Then the above equation (18), is well defined and taking the new relation:

$$\varepsilon_{k+1} = \sum_{i=1}^{q} \phi_i(\gamma_k)(\nu_1 + R\nu_3)\left(\widetilde{A}_i\varpi_k + \widetilde{B}_i u_k\right) + \sum_{i=1}^{q} \phi_i(\hat{\gamma}_k)\left(M_i\varepsilon_k - (\nu_1 + R\nu_3)\left(\widetilde{A}_i\varpi + \widetilde{B}_i u_k\right)\right). \tag{21}$$

Moreover, from (13), (19) and (20), that yields:

$$M_i = (\nu_1 + R\nu_3)\widetilde{A}_i - J_{2i}\widetilde{C} + \left(M_i(\nu_2 + R\nu_4) - J_{1i}\right)\widetilde{C}. \tag{22}$$

So, taking J_{1i}, the constraint (22) leads to:

$$J_{1i} = M_i(\nu_2 + R\nu_4), \qquad M_i = (\nu_1 + R\nu_3)\widetilde{A}_i - J_{2i}\widetilde{C}. \tag{23}$$

In what follows, the dynamic of state estimation error will be treated as:

$$\varepsilon_{k+1} = \left(\sum_{i=1}^{q} \phi_i(\gamma_k) - \sum_{i=1}^{q} \phi_i(\hat{\gamma}_k)\right)\left((\nu_1 + R\nu_3)\left(\widetilde{A}_i\varpi_k + \widetilde{B}_i u_k\right)\right) + \sum_{i=1}^{q} \phi_i(\hat{\gamma}_k)(M_i\varepsilon_k) \tag{24}$$

with:

$$\begin{cases} \sum_{i=1}^{q}(\phi_i(\gamma_k) - \phi_i(\hat{\gamma}_k))\widetilde{A}_i = \sum_{i,j=1}^{q} \phi_i(\gamma_k)\phi_j(\hat{\gamma}_k)\left(\widetilde{A}_i - \widetilde{A}_j\right), \\ \sum_{i=1}^{q}(\phi_i(\gamma_k) - \phi_i(\hat{\gamma}_k))\widetilde{B}_i = \sum_{i,j=1}^{q} \phi_i(\gamma_k)\phi_j(\hat{\gamma}_k)\left(\widetilde{B}_i - \widetilde{B}_j\right). \end{cases} \tag{25}$$

Then substituting the formulation (25) in (24) produce:

$$\varepsilon_{k+1} = \sum_{i,j=1}^{q} \phi_i(\gamma_k)\phi_j(\hat{\gamma}_k)(\nu_1 + R\nu_3)\left(\Delta\widetilde{A}_{ij}\varpi_k + \Delta\widetilde{B}_{ij}u_k\right) + \sum_{i=1}^{q} \phi_i(\hat{\gamma}_k)(M_i\varepsilon_k) \tag{26}$$

with the notations:

$$\Delta\widetilde{A}_{ij} = \widetilde{A}_i - \widetilde{A}_j\,, \qquad \Delta\widetilde{B}_{ij} = \widetilde{B}_i - \widetilde{B}_j\,. \tag{27}$$

Multiplying by $\sum_{j=1}^{q} \phi_j(\hat{\gamma}_k) = 1$, the statement (26) is handled as:

$$\varepsilon_{k+1} = \sum_{i,j=1}^{q} \phi_i(\gamma_k)\phi_j(\hat{\gamma}_k)(\nu_1 + R\nu_3)\left(\Delta\widetilde{A}_{ij}\varpi_k + \Delta\widetilde{B}_{ij}u_k\right) + \sum_{i,j=1}^{q} \phi_i(\gamma_k)\phi_j(\hat{\gamma}_k)(M_j\varepsilon_k). \tag{28}$$

It has been assigned that:

$$\vartheta_{ij} = (\nu_1 + R\nu_3)\Delta\widetilde{A}_{ij}\,, \qquad \tau_{ij} = (\nu_1 + R\nu_3)\Delta\widetilde{B}_{ij}\,, \quad i,\ j \in \{1, \ldots, q\}. \tag{29}$$

So the equation (28), reduces to:

$$\varepsilon_{k+1} = \sum_{i,j=1}^{q} \phi_i(\gamma_k)\phi_j(\hat{\gamma}_k)\left(\vartheta_{ij}\varpi_k + \tau_{ij}u_k + M_j\varepsilon_k\right). \tag{30}$$

Thus, let us introduce $\widetilde{\varepsilon}_k = \begin{bmatrix} \varepsilon_k^T & \varpi_k^T \end{bmatrix}^T$, that implies:

$$\begin{cases} \Theta\widetilde{\varepsilon}_{k+1} = \sum_{i,j=1}^{q} \phi_i(\gamma_k)\phi_j(\hat{\gamma}_k)\Xi_{ij}\widetilde{\varepsilon}_k + \Lambda_{ij}u_k\,, \\ \varepsilon_k = \Gamma\widetilde{\varepsilon}_k \end{cases} \tag{31}$$

with (7) and (28) the following terms are consider:

$$\begin{cases} \Gamma = \begin{pmatrix} I & 0 \end{pmatrix}, & \Lambda_{ij}^T = \begin{pmatrix} \tau_{ij}^T & \widetilde{B}_i^T \end{pmatrix}, \\ \Theta = \begin{pmatrix} I & 0 \\ 0 & \widetilde{E} \end{pmatrix}, & \Xi_{ij} = \begin{pmatrix} M_j & \vartheta_{ij} \\ 0 & \widetilde{A}_i \end{pmatrix}. \end{cases} \tag{32}$$

Then, the convergence condition of (14), can be outlined by the following theorem.

Theorem 1 *Using Assumptions 1 and hypotheses H_1), H_2), H_3), H_4), the state estimation error between the DTSSS (1) and its fuzzy observer (14), converges exponentially towards zero, if given $0 < \sigma < 1$ and there exist matrices $P_1 > 0$, $P_2 > 0$, while parameter matrices are V, G_{2j}, Y, K_{2j} and S_{2j}, $j = \{1, \ldots, q\}$ such that the following LMIs are satisfied:*

$$\kappa_{ij} = \begin{pmatrix} \kappa_{11} & * & * \\ \kappa_{21} & \kappa_{22} & * \\ \kappa_{31} & \kappa_{32} & \kappa_{33} \end{pmatrix} < 0, \quad \forall (i, j) \in \{1, \ldots, q\}. \tag{33}$$

As a consequences, applying some simplifications that gives a finite numbers of checked LMIs:

$$\begin{cases} \kappa_{11} = \widetilde{A}_j^T \nu_1^T P_1 \nu_1 \widetilde{A}_j + \widetilde{A}_j^T \nu_1^T V \nu_3 \widetilde{A}_j - \widetilde{A}_j^T \nu_1^T G_{2j} \widetilde{C} \\ \qquad + \widetilde{A}_j^T \nu_3^T V^T \nu_1 \widetilde{A}_j + \widetilde{A}_j^T \nu_3^T Y \nu_3 \widetilde{A}_j - \widetilde{A}_j^T \nu_3^T S_{2j}^T \widetilde{C} \\ \qquad + \widetilde{C}^T K_{2j} \widetilde{C} - \widetilde{C}^T G_{2j}^T \nu_1 \widetilde{A}_j - \widetilde{C}^T S_{2j} \nu_3 \widetilde{A}_j - \sigma^2 P_1 , \\ \kappa_{21} = \Delta\widetilde{A}_{ij}^T \nu_1^T P_1 \nu_1 \widetilde{A}_j + \Delta\widetilde{A}_{ij}^T \nu_1^T V \nu_3 \widetilde{A}_j - \Delta\widetilde{A}_{ij}^T \nu_1^T G_{2j} \widetilde{C} \\ \qquad + \Delta\widetilde{A}_{ij}^T \nu_3^T V^T \nu_1 \widetilde{A}_j + \Delta\widetilde{A}_{ij}^T \nu_3^T Y \nu_3 \widetilde{A}_j - \Delta\widetilde{A}_{ij}^T \nu_3^T S_{2j}^T \widetilde{C}, \\ \kappa_{31} = \Delta\widetilde{B}_{ij}^T \nu_1^T P_1 \nu_1 \widetilde{A}_j + \Delta\widetilde{B}_{ij}^T \nu_1^T V \nu_3 \widetilde{A}_j + \Delta\widetilde{B}_{ij}^T \nu_3^T V^T \nu_1 \widetilde{A}_j \\ \qquad + \Delta\widetilde{B}_{ij}^T \nu_3^T Y \nu_3 \widetilde{A}_j - \Delta\widetilde{B}_{ij}^T \nu_1^T G_{2j} \widetilde{C} - \Delta\widetilde{B}_{ij}^T \nu_3^T S_{2j}^T \widetilde{C}, \\ \kappa_{22} = \Delta\widetilde{A}_{ij}^T \nu_1^T P_1 \nu_1 \Delta\widetilde{A}_{ij} + \Delta\widetilde{A}_{ij}^T \nu_1^T V \nu_3 \Delta\widetilde{A}_{ij} - \sigma^2 \widetilde{E}^T P_2 \widetilde{E} \\ \qquad + \Delta\widetilde{A}_{ij}^T \nu_3^T V^T \nu_1 \Delta\widetilde{A}_{ij} + \Delta\widetilde{A}_{ij}^T \nu_3^T Y \nu_3 \Delta\widetilde{A}_{ij} + \widetilde{A}_i^T P_2 \widetilde{A}_i , \\ \kappa_{32} = \Delta\widetilde{B}_{ij}^T \nu_1^T P_1 \nu_1 \Delta\widetilde{A}_{ij} + \Delta\widetilde{B}_{ij}^T \nu_1^T V \nu_3 \Delta\widetilde{A}_{ij} + \widetilde{B}_i^T P_2 \widetilde{A}_i \\ \qquad + \Delta\widetilde{B}_{ij}^T \nu_3^T V^T \nu_1 \Delta\widetilde{A}_{ij} + \Delta\widetilde{B}_{ij}^T \nu_3^T Y \nu_3 \Delta\widetilde{A}_{ij} \\ \kappa_{33} = \Delta\widetilde{B}_{ij}^T \nu_1^T P_1 \nu_1 \Delta\widetilde{B}_{ij} + \Delta\widetilde{B}_{ij}^T \nu_1^T V \nu_3 \Delta\widetilde{B}_{ij} + \widetilde{B}_i^T P_2 \widetilde{B}_i \\ \qquad + \Delta\widetilde{B}_{ij}^T \nu_3^T V^T \nu_1 \Delta\widetilde{B}_{ij} + \Delta\widetilde{B}_{ij}^T \nu_3^T Y \nu_3 \Delta\widetilde{B}_{ij} . \end{cases} \tag{34}$$

The currently available software packages for handling with the LMIs problems is the LMI Toolbox of Matlab, it accepts problem statements in a high level mathematical form and solves the problem with interior point methode. In fact a more computationally efficient algorithm for solving LMIs problems is the interior point method which represent an optimisation technique, and as said above, for the resolution of these LMIs, we used LMI Toolbox of Matlab. It is based on theory of interior-point polynomial-time methods described in [29–32].

Therefore, the observer gains M_j, J_{1j}, J_{2j}, H_j, and R are respectively recovered and given by:

$$\begin{cases} M_j = (\nu_1 + R\nu_3)\widetilde{A}_j - J_{2j}\widetilde{C}, \quad R = P_1^{-1}V, \quad J_{2j} = P_1^{-1}G_{2j}\,, \\ J_{1j} = M_j(\nu_2 + R\nu_4), \qquad\qquad\qquad\qquad H_j = (\nu_1 + R\nu_3)\widetilde{B}_j\,, \end{cases} \tag{35}$$

where ν_1, ν_2, ν_3, ν_4 are such that (13) is satisfied.

Proof. (of Theorem 1) Let us consider the candidate quadratic Lyapunov function:

$$V_k = (\Theta\widetilde{\varepsilon}_k)^T P(\Theta\widetilde{\varepsilon}_k), \quad P > 0, \quad P = P^T, \quad P = \begin{pmatrix} P_1 & 0 \\ 0 & P_2 \end{pmatrix}. \tag{36}$$

$V_{k+1} - V_k$ denotes the time derivative along the trajectory of (30), it can be described as:

$$\begin{gathered} V_{k+1} - V_k < (\sigma^2 - 1)V_k\,, \qquad 0 < \sigma < 1, \\ V_{k+1} - \sigma^2 V_k = (\Theta\widetilde{\varepsilon}_{k+1})^T P(\Theta\widetilde{\varepsilon}_{k+1}) - \sigma^2(\Theta\widetilde{\varepsilon}_k)^T P(\Theta\widetilde{\varepsilon}_k) < 0. \end{gathered} \tag{37}$$

Then the property (36), is developed as:

$$\begin{aligned} V_{k+1} - \sigma^2 V_k = \sum_{i,j=1}^{q} \phi_i(\gamma_k)\phi_j(\hat{\gamma}_k) \left(\widetilde{\varepsilon}_k^T \Xi_{ij}^T + u_k^T \Lambda_{ij}^T\right) P(\Xi_{ij}\widetilde{\varepsilon}_k + \Lambda_{ij}u_k) \\ - \sigma^2 \widetilde{\varepsilon}_k^T (\Theta^T P\Theta)\widetilde{\varepsilon}_k\,. \end{aligned} \tag{38}$$

Considering (35) and (37) that inequality (33) appear under the compact form as:

$$\sum_{i,j=1}^{q} \phi_i(\gamma_k)\phi_j(\hat{\gamma}_k) \begin{pmatrix} \widetilde{\varepsilon}_k^T & u_k^T \end{pmatrix} \kappa_{ij} \begin{pmatrix} \widetilde{\varepsilon}_k \\ u_k \end{pmatrix} < 0. \tag{39}$$

With the expression of κ_{ij} is:

$$\kappa_{ij} = \begin{pmatrix} \Xi_{ij}^T P\Xi_{ij} - \sigma^2\Theta^T P\Theta & \Xi_{ij}^T P\Lambda_{ij} \\ \Lambda_{ij}^T P\Xi_{ij} & \Lambda_{ij}^T P\Lambda_{ij} \end{pmatrix}.$$

Employing (29), (32) and (38), it directly yields the desired results:

$$V = P_1 R, \quad G_{2j} = P_1 J_{2j}\,, \quad Y = R^T V, \quad S_{2j} = J_{2j}^T V, \quad K_{2j} = J_{2j}^T G_{2j}\,. \tag{40}$$

The interest is to finding the above set of LMIs, but $\kappa_{ij} < 0$ are not linear, thanks to the changes of variables V, G_{2j}, this problem can be remedy.

For end of the proof, we found the LMIs conditions (33). □

4. Simulation and results

In this section, the propounded approach of the new synthesised fuzzy observer (14) is applied on a numerical example of electronic circuit, in order to have a good and reliable estimation of state and sudden faults. This electronic circuit described by a Direct Current (DC) power source with voltage connected in series with a nonlinear capacitor, a linear inductor and a linear resistor. To understand more the structure of this model, each component of this electonic circuit is introduced:

The electrical energy, is produced and stored into a device called capacitor, the role attributed to capacitor is to store electric energy in an electric field and restore this energy to the electronic circuit when it is demanded. Resistor is considered like one of the most basic part of electrical components circuit. The principal mission of resistor in Alternating Current (AC) or even in Direct Current (DC), is to control the tansit of flow of current or voltage to other components. It resist the higher flow of electrons, that may destroy the function of other components of electrical circuit. An indutor is an electromagnet. The aim of an inductor is to store energy in a magnetic filed when a flows of electrons accros this magnetic filed, the passage of current into magnetic filed, induces an electromotive force or voltage.

In the squel, the studied DTSSS which describe the real behavior of the system, is considered to be vulnerable by sensor fault as well as actuator fault and posed as:

$$\begin{cases} E\rho_{k+1} = \sum_{i=1}^{2} \phi_i(\gamma_k)\left(A_i\rho_k + B_i u_k + F_{ai} f_{ak}\right), \\ y_k = C\rho_k + D_a f_{ak} + F_s f_{sk}\,. \end{cases} \tag{41}$$

$\rho_k = (\rho_{1k},\ \rho_{2k},\ \rho_{3k})^T$ is the state vector, with $\rho_{1k} = q(k)$ is the charge across the capacitor, $\rho_{2k} = \phi(k)_{\text{inductor}}$ is the electromagnetic field of the inductor, $\rho_{3k} = v(k)$ is the voltage of the resistor. The (DC) source of voltage is u_k, y_k is the output measurement vector, whereas f_{ak} is the actuator fault and f_{sk} is the sensor fault. The data sources of this model are from the paper [33].

Subsequently, the two sub-systems of the studied model are given as:

$$A_1 = \begin{pmatrix} 0 & 2 & 0 \\ 0 & -2 & -1 \\ -\gamma_{k\,\min} & 0 & 1 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 0 & 2 & 0 \\ 0 & -2 & -1 \\ -\gamma_{k\,\max} & 0 & 1 \end{pmatrix}, \quad F_s = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix},$$

$$C = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad B_i = F_{ai} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad D_a = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

The activation functions $\phi_i(\hat{\gamma}_k)$, and the decision variable γ_k having for expressions:

$$\phi_1(\hat{\gamma}_k) = \frac{\gamma_k - \gamma_{k\min}}{\gamma_{k\max} - \gamma_{k\min}}, \qquad \phi_2(\hat{\gamma}_k) = 1 - \phi_1(\hat{\gamma}_k),$$

$$\gamma_k = \rho_{1k}^2 - 3\rho_{1k}q_0 + 3q_0^3, \qquad \gamma_k \in [\gamma_{k\min}, \gamma_{k\max}].$$

The proposed observer which is applied on two sub-models of the nonlinear electronic circuit system, is decomposed into two locale observers:

$$\begin{cases} \delta_{k+1} = \sum_{i=1}^{2} \phi_i(\hat{\gamma}_k)\left(M_i\delta_k + J_{1i}y_k + J_{2i}y_k + H_iu_k\right), \\ \hat{\varpi}_k = \delta_k + \nu_2 y_k + R\nu_4 y_k, \end{cases} \tag{42}$$

We test the propounded observer on a nonlinear electronic circuit system, as a result the gains of the proposed observer R, H_i, J_{2i}, J_{1i}, M_i, are generated with the resolution of LMIs (33), and numerically given as:

$$R = \begin{pmatrix} 2.2434 & -1.8253 & 0.6599 \\ 0.4417 & 0.7685 & 0.6199 \\ -2.0097 & 1.8156 & -0.8044 \\ 27.1573 & 52.9106 & 41.2931 \\ 1.9575 & -1.9620 & 1.0206 \end{pmatrix}, \quad H_1 = \begin{pmatrix} -0.0000 \\ -0.0026 \\ 0.0000 \\ -0.5115 \\ -0.0000 \end{pmatrix}, \quad H_2 = \begin{pmatrix} -0.0000 \\ -0.0026 \\ 0.0000 \\ -0.5115 \\ -0.0000 \end{pmatrix},$$

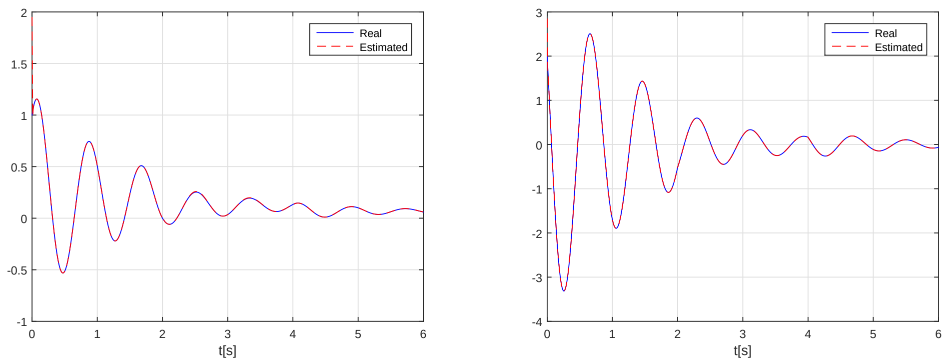
$$J_{21} = \begin{pmatrix} 0.1178 & 0.0249 & 0.1557 \\ 0.0013 & -0.2118 & -0.0079 \\ -0.0021 & -0.0174 & -0.2679 \\ 0.6146 & -49.4474 & -0.1739 \\ -0.1120 & 0.0111 & 0.4022 \end{pmatrix}, \quad J_{22} = \begin{pmatrix} 0.1178 & 0.0249 & 0.1557 \\ 0.0013 & -0.2118 & -0.0079 \\ -0.0021 & -0.0174 & -0.2679 \\ 0.6146 & -49.4474 & -0.1739 \\ -0.1120 & 0.0111 & 0.4022 \end{pmatrix},$$

$$J_{11} = \begin{pmatrix} -0.0472 & -0.0168 & -0.1966 \\ -0.0002 & -0.1960 & 0.0094 \\ 0.0624 & 0.0044 & 0.1794 \\ -0.1758 & 24.1241 & 0.2168 \\ -0.0731 & 0.0004 & -0.1880 \end{pmatrix}, \quad J_{12} = \begin{pmatrix} -0.2547 & -0.0207 & 0.0110 \\ 0.0000 & -0.1960 & 0.0092 \\ 0.2592 & 0.0081 & -0.0174 \\ 0.0331 & 24.1280 & 0.0079 \\ -0.2766 & -0.0034 & 0.0154 \end{pmatrix},$$

$$M_1 = \begin{pmatrix} 0.6287 & -0.0138 & -0.2437 & -0.0000 & 0.0756 \\ -0.0038 & -0.0478 & 0.0092 & -0.0026 & 0.0104 \\ -0.6096 & 0.0055 & 0.2418 & 0.0000 & -0.0994 \\ -0.4919 & -0.6803 & 0.0410 & 0.4885 & 0.0737 \\ 0.5973 & -0.0017 & -0.2611 & -0.0000 & 0.0907 \end{pmatrix},$$

$$M_2 = \begin{pmatrix} -0.2686 & -0.0138 & -0.2437 & -0.0000 & 0.0756 \\ -0.0028 & -0.0478 & 0.0092 & -0.0026 & 0.0104 \\ 0.2413 & 0.0055 & 0.2418 & 0.0000 & -0.0994 \\ 0.4114 & -0.6803 & 0.0410 & 0.4885 & 0.0737 \\ -0.2825 & -0.0017 & -0.2611 & -0.0000 & 0.0907 \end{pmatrix}.$$

The principle aim of this work, is to synthesis a fuzzy observer to estimates simultaneously state and faults, for that, the exponential stability of the studied observer are analyzed and validated, this conducts us to study the temporal evolution of faults, which act on the output and the input of the system respectively. The time evolution of states and faults trajectories are decomposed in three time interval. For the nature and the type of the arising faults, it is a constant, unknown actuator fault f_{ak} and sensor fault f_{sk} signal per time interval. In fact, fault are characterized by unpredictable changes in the dynamics of system, which leads to undesirable behaviour of this later, then observer gives the possibility to the system to operate reliably in the presence of faults. Thus when fault occurs, that mean once it has been detected, it will be estimated. The fault estimation must specify the type of fault, its amplitude, its duration and eventually its probable evolution. Purposely to identify the sensor fault we consider that, the actuator are faultless $f_{ak} = 0$, after we found the appearance of actuator fault, then sensor becomes faultless $f_{sk} = 0$. The first fault has been applied to sensor during the time interval $[0; 2]$, $[4; 6]$, the second fault has been added to actuator during the time interval $[2; 4]$, the detection of f_{ak} and f_{sk} are very fast. A better convergence is verified with a less state estimation error, represented by the closeness of the estimated states and faults to the real ones, they demonstrates that the fuzzy observer move in the desired direction. We conclude that this observer is well synthesised and gives a good results, since both of real and estimated states as well as faults are identicals, that confirmes the achieved aim of the proposed method. The obtained results of states and faults estimation are conspicuously depicted in figures below.



(a) Charge across the capacitor (b) Electromagnetic field of the inductor

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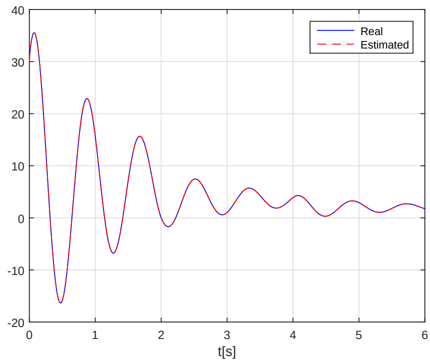
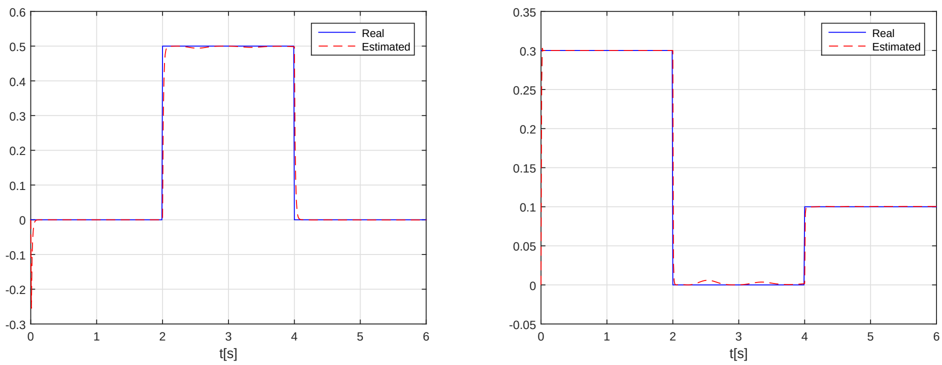


(c) Voltage of the resistor

Figure 1: States ρ_{1k}, ρ_{2k}, ρ_{3k} with their estimates



(a) Actuator fault

(b) Sensor fault

Figure 2: Faults f_{ak}, f_{sk} with their estimates

5. Conclusion

The improved strategy is suggested in the aim of designing a new fuzzy observer to estimate state and fault for DTSSS, the basis to construct this observer is the SVD approach and an augmented system. Both of exponential stability and convergence conditions of the observer which represents estimation error are studied by Lyapunov function and solved via LMIs technique. To this end, a numerical example of non linear singular electronic circuit is applicable in term of simulation to assert the competence of this method. The synthesised fuzzy observer has been applied on this non linear singular electronic circuit, to validate the accuracy of the proposed approach of state and fauls estimation. The suggested observer is synthesised in the aim of estimating the augmented state vector which includes simultaneously the state of T-S system and the occurred faults in actuator and sensor. The studied observer are well synthesised since the trajectries of the estimated states and faults are identical to the real ones. All performances of this observer are verified, in term of speed convergence with a very weak augmented state estimation error and quadratic stability validated by a feasible set of realisable LMIs. This good result will be a motivation to extend the study to a further works on the fault tolerant control (FTC) of this Takagi-Sugeno singular system, and for the uncertainty systems as well as a time delay systems.

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