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Research paper

Implicit neural state functions in hybrid reliability analysis of plane frame

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Abstract: The objective of the article involves presenting innovative approach to the assessment of structural reliability analysis. The primary research method was the First Order Reliability Method (FORM). The Hasofer–Lind reliability index in conjunction with transformation method in the FORM was adopted as the reliability measure. The implicit limit state functions were used in the analysis. The formulation of the random variables functions were created in the Matlab software by means of neural networks (NNs). The reliability analysis was conducted in Comrel module of Strurel computing environment. In the proposed approach, Hybrid FORM method (HF) used the concept in which NNs replaced the polynomial limit state functions obtained from FEM (Finite Elements Method) for chosen limit parameters of structure work. The module Comrel referenced Matlab files containing limit state functions. In the reliability analysis of structure, uncertain and uncorrelated parameters, such us base wind speed, characteristic snow load, elasticity modulus for steel and yield point steel are represented by random variables. The criterion of structural failure was expressed by four limit state functions – two related to the ultimate limit state and two related to the serviceability limit state. Using module Comrel values of the reliability index with the FORM method were determined. Additionally, the sensitivity of the reliability index to random variables and graph of partial safety factors were described. Replacing the FEM program by NNs significantly reduces the time needed to solve the task. Moreover, it enables the parallel formulation of many limit functions without extending the computation time.

Keywords: Hybrid FORM method, limit state functions, neural networks, partial safety factors, reliability analysis, sensitivity

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1. Introduction

Nowadays reliability evaluation of structures rely on idealized concept of limit states and their verification by the partial safety factors [1]. The role of partial safety factors is to ensure the required level of structural reliability. Probabilistic methods are natural extension of the limit state method. In practical applications, existing FEM (Finite Element Method) software is often combined with modules for reliability analysis. In this way, the estimation of the probability of failure is carried out by commercial programs such as Ansys [2] or Sofistik [3]. Ansys has two tools, namely the Ansys Probabilistic Design System and the Ansys Design Xplorer. Variational technology is offered in both tools. Rely is an add-on to the Sofistik program that performs reliability analysis, where the engineering system of interested is modeled using one of Sofistik finite element modules. The kernel of Rely is powered by the stand-alone software package Strurel [4]. Different structural reliability methods are provided: FORM (First Order Reliability Method), SORM (Second Order Reliability Method), Monte Carlo, Importance Sampling, Line Sampling, Directional Sampling, Adaptive Sampling and Subset Simulation. However, the disadvantage of this solution is the long computation time which follow from multiple calculations of the limit state function for different realizations of the random variable vector. In the paper, the authors propose alternative solution to the problem by implementing the Hybrid FORM algorithm [5, 6]. The assessment of the failure probability is calculated using the FORM method, but formulation of a limit state function is realized in Matlab by means of neural network. This solution is possible in Comrel – module of Strurel [4], one of the most complete collections of software modules for probabilistic modeling in structural engineering. There are built-in interfaces to the limit state functions cast into user defined Matlab's functions. Program offer very rich possibilities of modelling and computation. Various sensitivities can be computed by Comrel which can be as important for the user as the failure probabilities it computes.

2. Materials and methods

2.1. Reliability analysis

The formulation of the time-independent reliability analysis problem assumes that the design parameters whose values are characterized by uncertainty are represented by the n-dimensional random vector $\mathbf{X}$. The random parameters may be: the dimensions of the cross-section, material constants (Young's modulus, yield point) and load multipliers. The random vector $\mathbf{X}$ takes values in the n-dimensional space of real numbers. In the reliability analysis, we assume that the structure may be in one of two permissible states: safe state or failure state. A failure should be understood here as failure to meet a certain restriction imposed by the designer on the work of the structure. This limitation is called the limit state function. It is assumed that the parameters describing the state of the structure are treated as random variables. The limit state function $G(X_1, \ldots, X_n)$ is a function of these parameters such that $G(\mathbf{x}) \leq 0$ – failure area Ω_f and $G(\mathbf{x}) > 0$ – safe area Ω_S. Each such function is associated with a given limit state. In reliability problems of building structures, the limit state function can be linear or non-linear. If the joint density function $f(\mathbf{x})$ exists for the probability distribution, then the failure probability can be expressed in terms of the integral:

$$P_f = P\left(G(\mathbf{x}) \leq 0\right) \quad \rightarrow \quad P_f = \int_{\Omega_f} f(\mathbf{x})\, \mathrm{d}x \tag{2.1}$$

Calculating the integral is not an easy task. A good approximation is the FORM method. The main FORM principle is to approximate the reliability problem by a linear model. In the first step we have to transform the problem to one in which all random variables are described by standard normal distributions. Substituting $X_1, \ldots, X_n$ standardized normal variables $Z_1, \ldots, Z_n$ we obtain a new limit state function $G(Z_1, \ldots, Z_n)$. Now the failure probability can be described as follow:

$$P_f = \int_{\Omega_f} f_{\mathbf{X}}(\mathbf{x})\, \mathrm{d}\mathbf{x} = \int_{\Delta_f} \prod_{i=1}^{n} \varphi(z_i)\, \mathrm{d}\mathbf{z}_1\, \mathrm{d}\mathbf{z}_2 \ldots \mathrm{d}\mathbf{z}_n \tag{2.2}$$

$$\Omega_f = \{\mathbf{x} \colon g(\mathbf{x}) \leq 0\} \quad \rightarrow \quad \Delta_f = \{\mathbf{z} \colon G(\mathbf{z}) \leq 0\} \tag{2.3}$$

$$g(x) = 0 \quad \rightarrow \quad g\left[T^{-1}(z)\right] = G(z) = 0, \qquad \mathbf{Z} = T(\mathbf{X}) \tag{2.4}$$

where: $f_{\mathbf{X}}(\mathbf{x})$ – joint probability density function, $\varphi(\mathbf{z}_i)$ – standard normal joint probability density function, Ω_f – failure area in space $\mathbf{x}$, Δ_f – failure area in space $\mathbf{z}$.

Next, we linearize the limit state function in the design point (Fig. 1). The coordinates of this point are calculated from the condition of minimizing the distance from the origin of the standardized coordinate system to the limit surface. This distance measures the probability of a failure and is referred to as the Hasofer–Lind reliability index. The value of the reliability index obviously depends on the correct determination of the design point. Many algorithms have been developed to calculate the design point. The first, from the works of Hasofer and Lind [7], Rackwitz and Fiessle [8], are based on gradient procedures. The slow

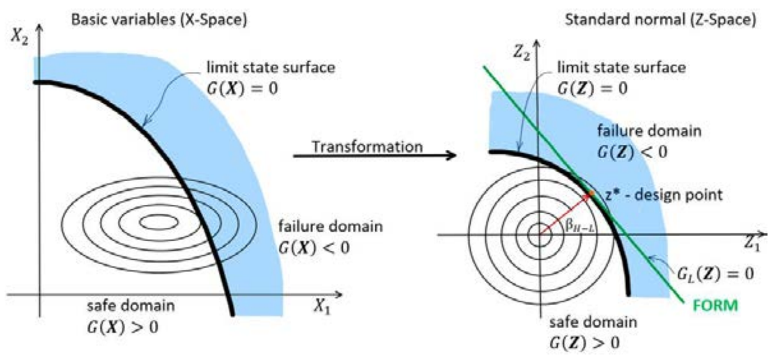


Fig. 1. Graphical interpretation of the reliability index

convergence of the Rackwitz–Fiessler algorithm was partially removed by applying the step length reduction procedure proposed by Abdo [9]. An alternative to the Abdo–Rackwitz–Fiessler procedure are Sequential Quadratic Programming (SQP) algorithms [10,11]. These algorithms use additional information about the second derivatives in the search direction determination procedure.

The accuracy of the results obtained using the Hasofer–Lind index is sufficient for practical purposes. It has gained great popularity as a measure of reliability, especially in combination with transformation methods that use full information on the distributions of primary variables **X**. The method works very well both in terms of the reliability of the element [12, 13] and the system [14–16]. An extremely valuable advantage of the FORM method is the ability to calculate sensitivity of the reliability index with respect to any parameters of the task. In the paper sensitivity of reliability index with respect to changes in the random variables is defined by vector $\boldsymbol{\alpha}$. The sensitivity of the reliability index to the coordinates of the design point z^* has the form:

$$\left.\frac{\partial \beta}{\partial Z_i}\right|_{z=z^*} = \alpha_i \quad i = 1, \ldots, n \tag{2.5}$$

The vector $\boldsymbol{\alpha}$ can be taken as a relative measure of the importance of the standardized variables. The higher the value, the greater the sensitivity of β to the variable. The negative value means that an increase in the value of the variable will result in a decrease in the reliability index β. A positive value indicates an increase in the reliability index β with an increase in the value of the variable. The coordinates of vector $\boldsymbol{\alpha}$ also correspond to the direction cosines of a hyperplane. The foregoing definition of sensitivity makes sense only if the original vector **X** is independent.

In order to better interpret and compare the obtained sensitivity values, it is convenient to introduce a normalized sensitivity measure, the so-called elasticities of the reliability index in the form:

$$E_\beta(p_i) = \frac{\partial \beta}{\partial p_i} \cdot \frac{p_i}{\beta} \tag{2.6}$$

It can be interpreted as the percentage change of β when a parameter is changed by 1%. Program Comrel provides elasticities for deterministic parameters, mean and standard deviation of a random variables. Elasticities for the standard deviation of a random variables are almost always negative because an increase in standard deviations decreases reliability.

$$E_\beta(\sigma_i) = \frac{\partial \beta}{\partial \sigma_i} \cdot \frac{\sigma_i}{\beta} \tag{2.7}$$

$$E_\beta(\mu_i) = \frac{\partial \beta}{\partial \mu_i} \cdot \frac{\mu_i}{\beta} \tag{2.8}$$

$$E_\beta(d) = \frac{\partial \beta}{\partial d} \cdot \frac{d}{\beta} \tag{2.9}$$

where: d – deterministic parameter, σ_i – standard deviation of random variable Z_i, μ_i – mean value of random variable Z_i.

Additionally in the Strurel program we can estimate partial safety factors γ. The partial safety factors are a practical way to express the design value in terms of characteristic values.

2.2. Hybrid form algorithm

Hybrid reliability analysis is a relatively new approach that uses neural networks (NN) to develop the limit state function (see e.g. [17]). In the proposed approach, HF (Hybrid FORM method) uses the concept in which NNs replace the polynomial boundary functions obtained from FEM for chosen limit parameters of structure work. The problem of function formulation falls into the class of so-called direct problems, i.e., those for which a solution can be found with a complete set of input data that define unambiguous features of the task. The following stages proposed algorithm can be distinguished below, (Fig. 2).

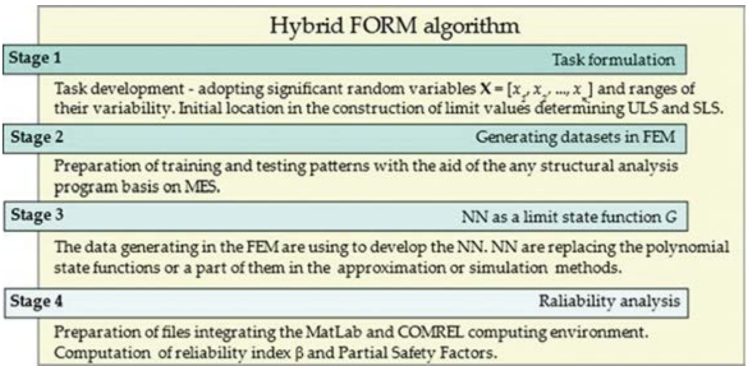


Fig. 2. The stages of the Hybrid FORM analysis

Hybrid methods in computing combine methodologies based on different algorithms. Most often it is a combination of classic methods, the so-called Hard Computing (e.g. FEM) with Soft Computing methods (e.g. NN), which have only recently been more widely used in engineering computations. While the theoretical foundations of Soft Computing methods were developed in the mid-20th century [18], it is owing to the development of computer technologies in the 21st century that they can be used effectively. Hybrid methods are often classified according to the degree and method of integration between the components. Examples include parallel, series and sequential systems [17]. In the proposed approach, the algorithm combines three methodologies – FEM, NN, FORM. The system structure is mixed, i.e. series-parallel.

The Finite Element Method (FEM) is a numerical technique that allows to simulate any physical phenomenon. The theoretical basis of this method is voluminously and well described in the literature, cf. [19–22]. In this work, the authors used FEM – Stage 2 (Fig. 2) to perform multiple numerical simulations. The set of data on the structure and the corresponding results gained in this way, then served as patterns for the formulation of neural networks – Stage 3 (Fig. 2).

Neural networks (NNs) are mathematical computational structures which are inspired by the operating scheme of a biological prototype. Currently, NN are widely used in various scientific and engineering fields. A comprehensive review of the use of NN in civil engineering was presented in literature, see e.g. [6, 17, 23–27]. The theoretical basis of NN is described in many publications, e.g. [28, 29]. One type of network dedicated to analyse regression problems was used in the article. This network is adapted to mapping an input data set to an output function, i.e. $\mathbf{X} \rightarrow \mathbf{Y}(\mathbf{X}, \mathbf{w})$, where $\mathbf{X}$, $\mathbf{Y}$ – input and output vector, $\mathbf{w}$ – vector of network parameters.

Figure 3 shows a simplified network for the analysis of regression problems. Due to the direction of the flow of signals, this network is called unidirectional. Neurons, that is computing units, are not interconnected in the layers. The presented network is a two-layer network in which the hidden layer is composed of H neurons, the output, i.e. the result of the network operation, can be any number of $\mathbf{Y}$ responses. Each network of this type must be undergo a learning process, most often on patterns, i.e. on sets of input and output data assigned to each other. The purpose of this process is the adaptive selection of network parameters. The values of the input signals are then multiplied by the computed weights $\mathbf{w}$, summed up, and transmitted to the neurons. There they are transposed by the F_h activation functions. In the further part of the work it was assumed that the activation functions of hidden neurons are bipolar sigmoids and the output layer is linear. The use of continuous functions of activation enables the objective function to be minimized with gradient methods. The effective Levenberg-Marquardt learning algorithm was adopted during the network formulation [30]. The network learning process was assessed using Mean Square Error (MSE):

$$MSE = \frac{1}{P} \sum_{p=1}^{P} \sum_{i=1}^{O} \left(y_{ip} - \hat{y}_{ip}\right)^2 \tag{2.10}$$

where: P – number of patterns, O – number of outputs, y_p – vector of network response, $\hat{y}_p$ – vector of expected values.

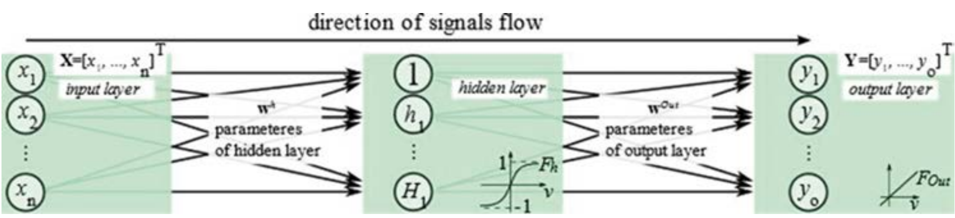


Fig. 3. Architecture of unidirectional two-layer network

Another measure of the quality of the prepared network may be the coefficient of determination, i.e. the square of the linear correlation coefficient r^2. The Deep Learning Toolbox implemented in the Matlab computing environment was used in the computations [31, 32].

3. Case study

3.1. Stage 1 – Task formulation

In the paper the reliability of plane frame was analysed. Steel frame with rigid nodes in the ridge line, column to foundation connections and beam to column connections was designed (Fig. 4). The elements are made of S235 steel with the yield point f_y = 235 MPa and elasticity modulus E = 210 GPa. Columns are designed of HEB 200 I-shaped beams and girts of IPE 200 I-shaped beams. In the reliability analysis the following uncorrelated probabilistic variables were used: V_b – base wind speed, S_k – characteristic snow load, E – elasticity modulus for steel, f_y – yield point for S235 steel. Description of random variables was shown in Table 1.

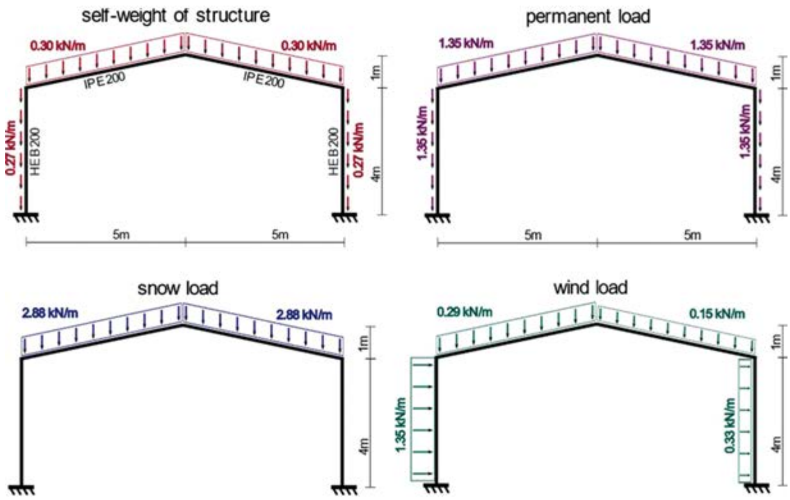


Fig. 4. Steel plane frame with the most critical load combination

Table 1. Description of the random variables

Random variable	Probability density function	Mean value (μ_X)	Standard deviation (σ_X)	Coefficient of variation (ν_X)
V_b	Normal	22 [m/s]	2.2 [m/s]	10%
S_k	Normal	1 [kN/m^2]	0.1 [kN/m^2]	10%
E	Normal	210 [GPa]	10.5 [GPa]	5%
f_y	Normal	235 [MPa]	11.75 [MPa]	5%

The four limit state functions were formulated – for Ultimate Limit State (ULS) and Serviceability Limit State (SLS). These functions have been defined in places of the maximum efforts and displacements at a given load (Fig. 5). Each of the functions, and thus each of the reliability analyses, was carried out separately – it was not a system analysis.

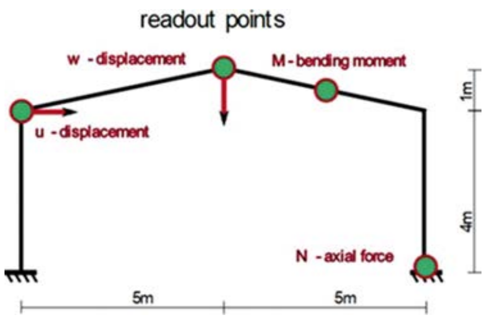


Fig. 5. The places of the maximum efforts and displacements

The first two limit state functions were related to Ultimate Limit State (ULS):

– condition of the non-exceeding of the section bearing capacity during load of bending moment:

$$G_1(\mathbf{X}) = 1 - \frac{M(\mathbf{X})}{M_{c,Rd}} \tag{3.1}$$

where: M – the maximum bending moment in the form of neural limit function of random variables grouped in vector: $\mathbf{X} = \{V_b, S_k, E, f_y\}$, $M_{c,Rd}$ – the bending resistance of the section.

– condition of the non-exceeding of the critical force in the columns:

$$G_2(\mathbf{X}) = 1 - \frac{N(\mathbf{X})}{P_{kr}} \tag{3.2}$$

where: N – the maximum axial force in the form of neural limit function of random variables grouped in vector: $\mathbf{X} = \{V_b, S_k, E, f_y\}$, P_{kr} – critical axial load that causes buckling in the column.

The next two limit state functions were related to Serviceability Limit State (SLS):

– condition of the non-exceeding of the permissible horizontal displacement:

$$G_3(\mathbf{X}) = 1 - \frac{u(\mathbf{X})}{u_{\lim}} \tag{3.3}$$

where: u – the maximum horizontal displacement in the form of neural limit function of random variables grouped in vector: $\mathbf{X} = \{V_b, S_k, E, f_y\}$, $u_{\lim}$ – the permissible horizontal displacement.

– condition of the non-exceeding of the permissible vertical displacement:

$$G_4(\mathbf{X}) = 1 - \frac{w(\mathbf{X})}{w_{\lim}} \tag{3.4}$$

where: w – the maximum vertical displacement in the form of neural limit function of random variables grouped in vector: $\mathbf{X} = \{V_b, S_k, E, f_y\}$, $w_{\lim}$ – the permissible vertical displacement.

3.2. Stage 2 – Generating datasets in FEM

In order to generate a set of training and testing patterns, the analysed structure scheme was modelled in Autodesk Robot Structural Analysis Professional (ARSAP). For further analysis four characteristic points were selected. There were read values specific for the ULS – bending moment (M), normal force (N), and for the SLS – vertical and horizontal displacements (w, u). The scheme of the location of the readout points is shown in Fig. 5.

In total, 206 simulations were performed for various sets of random variable patterns, which are presented in Table 1. At this stage, the range of variables was extended according to the 3σ rule, i.e. the values were taken from the $[\mu_X - 3\sigma_X, \mu_X + 3\sigma_X]$ ranges [33]. 125 sets of patterns were generated for training the network and 81 for testing it. Sample package of the computed sets of structure parameters in Table 2 are presented.

Table 2. Example sample sets of patterns calculated in the FEM program

Number of set	E [GPa]	V_b [m/s]	S_k [kN/m^2]	M [kNm]	N [kN]	u [cm]	w [cm]
1	190	15.4	0.63	–26.1	25.73	0.568	–2.243
14	190	22	1.035	–35.35	32.54	0.778	–3.032
43	200	25.3	0.9	–32.79	30.54	0.721	–2.649
56	210	18.7	0.63	–26.41	25.89	0.575	–2.138
82	220	18.7	0.765	–29.29	28.05	0.56	–2.168
106	230	18.7	0.63	–26.41	25.89	0.494	–1.862
117	230	25.3	0.765	–29.91	28.38	0.586	–2.093
125	230	28.6	1.17	–38.86	35.02	0.734	–2.736

3.3. Stage 3 – NN as a limit state function G(X)

For the development of implicit limit state functions, a backward propagation network with 3-H-4 structure was adopted (Fig. 6).

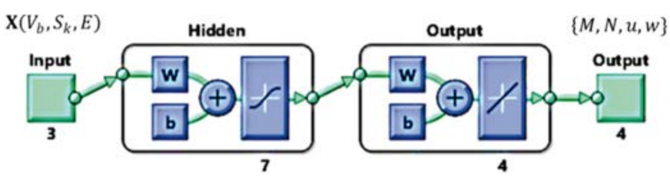


Fig. 6. Scheme of adopted NN

The three-element input vector consisted of the adopted random variables: V_b, S_k, E. The choice of the number of neurons H in the hidden layer was adaptive. That is, at the stage of preliminary computations, several versions of networks were tested, in which the number of H neurons was assumed from 4 to 16. Using $H \geq 7$, the values of the learning and testing errors (MSE) met the maximum level assumed by the authors, so the network architecture formulated in this way was adopted for further computations (Fig. 6). Table 3 details the Mean Percent Error (MPE) and MSE (Eq. (2.10)) of learning and testing for the selected network. The results show a very good mapping of the training set into the network action.

Table 3. MPE and MSE of learning and testing for the selected network

	Mean Percent Error (MPE)		Mean Square Error (MSE)	
	Learning	Testing	Learning	Testing
M	0.0077543	0.0086948	1.237e–08	1.6212e–08
N	0.0072976	0.0075214	4.9215e–09	4.8284e–09
u	0.04853	0.047326	1.9859e–07	1.7508e–07
w	0.065928	0.076951	1.2456e–08	1.5049e–08

The linear correlation coefficients for individual outputs from the network are approximately 1. Figure 7 additionally shows the absolute error histograms for learning, testing and validation of the developed network.

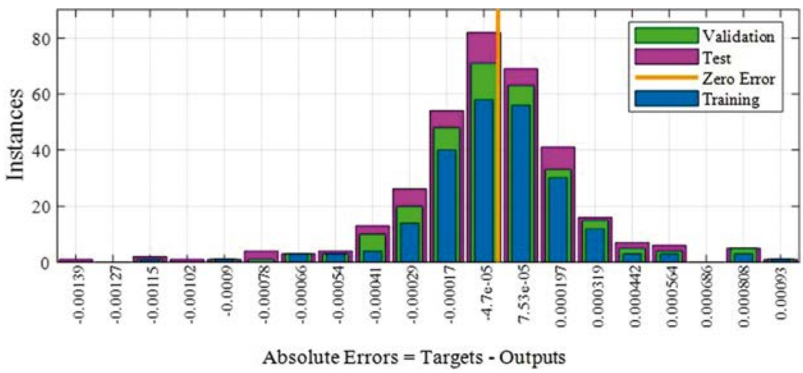


Fig. 7. Error histogram with 20 bins

Based on the error analysis, the authors concluded that the network performs the regression task very well and that it can be used in further reliability analysis without reducing its quality. The MLP:3-7-4 network (Fig. 6) preparation process after 301 epochs was completed. NN made it possible to appropriate calculate as many as four key parameters during the dimensioning (M, N for ULS) and (u, w for SLS). The neural approximation of the searched static quantities was formulated as a representation of the input vector **X** into the output vector **Y**:

$$\mathbf{X}\,(V_b, S_k, E) \;\rightarrow\; NN \;\rightarrow\; N \;\rightarrow\; \mathbf{Y}(\mathbf{X}, \mathbf{w}) \;\rightarrow\; (\mathbf{X}, \mathbf{w})\,\{M, N, u, w\} \tag{3.5}$$

where: **X**, **Y** – network input and output vectors. **w** – vector of generalized network weights.

3.4. Stage 4 – Reliability analysis

The Strurel reliability analysis program enables the use of the Matlab script as a limit state function. In this method, the stochastic model should consist of two sets of random variables R and S, where R represents the resistance of the analyzed system and S is the external load. The conditions, that must be met, in order to use the proposed HF method are:

– correct configuration of the Strurel interface to work with Matlab,
– Matlab and Strurel installed on the same system,
– preparation of Matlab script according to Strurel guidelines.

The use of the script as the limit state function allows for reliability analysis with all methods available in Strurel. The article presents using of one of the approximation methods – FORM.

4. Results

Using reliability software Comrel the value of the reliability index (β) for each limit state function with the FORM method was determined. The results are presented in Table 4.

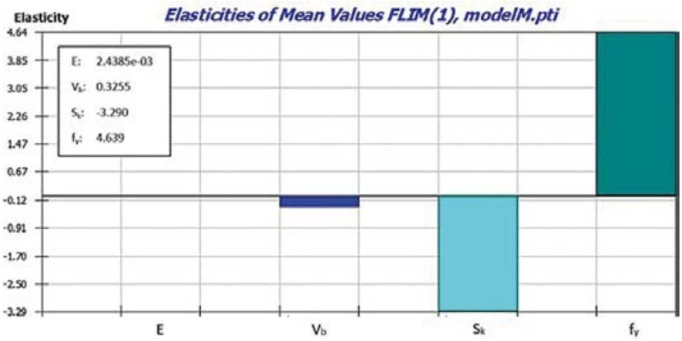
Table 4. Values of the reliability index for each limit state function

Limit state function	ULS		SLS	
	$G_1(\mathbf{X})$	$G_2(\mathbf{X})$	$G_3(\mathbf{X})$	$G_4(\mathbf{X})$
Values of reliability index β	2.476	19.299	44.004	2.681

We can conclude, on the basis of the results in Table 4, that the values of the reliability index for limit state functions $G_1(\mathbf{X})$ and $G_4(\mathbf{X})$ oscillate within the limits of the standard recommendations. Therefore, these limit functions will determine the structure safety. In addition, information about a normalized sensitivity measure – so-called elasticities of the reliability index β to random variables (Fig. 8) and graphs of partial safety factors γ_i (PSF) (Fig. 9) were obtained. Due to the high value of the reliability index in the case of the limit state function $G_3(\mathbf{X})$, it was impossible to generate elasticities information and graphs of partial safety factors.

On the base of information about a normalized sensitivity measure (Fig. 8) we observe which random variable has the greatest impact on the value of the reliability index. The higher the value, the greater the sensitivity of β to the variable. According to this principle, the variable that has the greatest influence on the value of the reliability index for the limit state function $G_1(\mathbf{X})$ is yield point for S235 steel (f_y), for $G_2(\mathbf{X})$ – elasticity modulus for steel (E) and for $G_4(\mathbf{X})$ – characteristic snow load (S_k). In addition, the negative value means that an increase in the value of the variable will result in a decrease in the reliability index β. A positive value indicates an increase in the reliability index β with an increase in the value of the variable.

Reliability analysis in Comrel module also allows to obtain values of partial safety factors with the obtained level of reliability index β. Figure 9a. indicates that for condition



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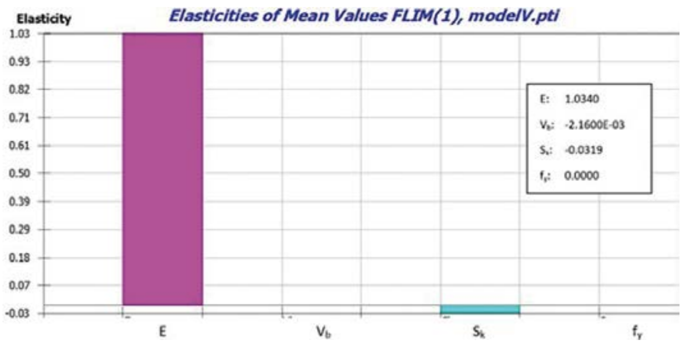

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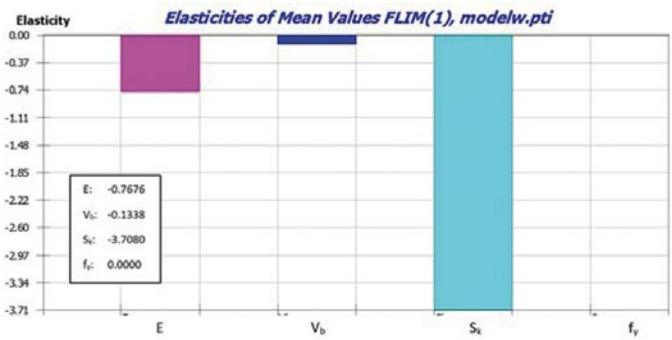
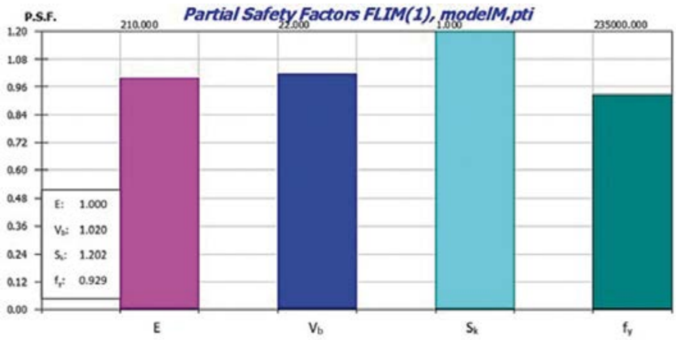


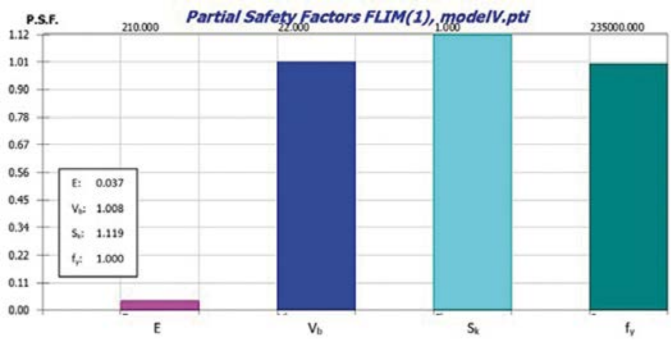
(c)

Fig. 8. Elasticity of the reliability index β to random variables for: (a) ULS – limit state function $G_1(\mathbf{X})$, (b) ULS – limit state function $G_2(\mathbf{X})$, (c) SLS – limit state function $G_4(\mathbf{X})$

of the non-exceeding of the section bearing capacity during load of bending moment ($G_1(\mathbf{X})$) and $\beta = 2.476$ the following values of partial factors were obtained: $\gamma_E = 1.000$, $\gamma_{Vb} = 1.020$, $\gamma_{Sk} = 1.202$, $\gamma_{fy} = 0.929$. Figure 9b presents that for condition of the non-exceeding of the critical force in the columns ($G_2(\mathbf{X})$) and $\beta = 19.299$ the following values



(a)



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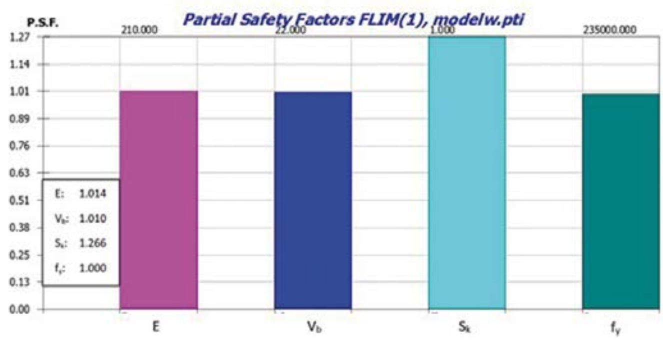

(c)

Fig. 9. Graphs of partial safety factors (PSF) for: (a) ULS – limit state function $G_1(\mathbf{X})$, (b) ULS – limit state function $G_2(\mathbf{X})$, (c) SLS – limit state function $G_4(\mathbf{X})$

of partial factors were obtained: $\gamma_E = 0.037$, $\gamma_{Vb} = 1.008$, $\gamma_{Sk} = 1.119$, $\gamma_{fy} = 1.000$. For the limit state function describing the condition of the non-exceeding of the permissible vertical displacement ($G_4(\mathbf{X})$) and $\beta = 2.681$ the following values of partial factors were obtained: $\gamma_E = 1.014$, $\gamma_{Vb} = 1.010$, $\gamma_{Sk} = 1.266$, $\gamma_{fy} = 1.000$ (Fig. 9c). Obtaining the required level of reliability index β does not involve obtaining partial safety factors at the level recommended in the standards applicable to designers, but applies to a specific design situation. In addition, we can conclude that the values of partial safety factors contained in the standard guidelines are overestimated.

5. Conclusions

The paper shows combining the advantages of the FORM and neural networks in assessing the reliability of a steel plane frame. The following conclusions may be drawn, and some recommendations are made for using the results and further investigations:

– replacing the FEM program by NNs significantly reduces the time needed to solve the task,
– Hybrid FORM algorithm enables the parallel formulation of many limit functions without extending the computation time,
– sensitivity analysis allows for the assessment of the influence of a given random variable (representing the strength features or the load of the structure) on the value of the reliability index β, and thus its impact on the structure safety,
– reliability analysis in Comrel module allows to obtain values of partial safety factors with the obtained level of reliability index β,
– obtaining a safe level of the reliability index β is not associated with obtaining the partial safety factors recommended in the Eurocode standards.

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Niejawne neuronowe funkcje stanu w hybrydowej analizie niezawodności ramy płaskiej

Słowa kluczowe: analiza niezawodności, częściowe współczynniki bezpieczeństwa, funkcja graniczna, hybrydowa metoda FORM, sieci neuronowe, wrażliwość

Streszczenie:

Obecnie ocena niezawodności konstrukcji bazuje na wyidealizowanej koncepcji stanów granicznych i ich weryfikacji poprzez zastosowanie częściowych współczynników bezpieczeństwa. Rolą częściowych współczynników bezpieczeństwa jest zapewnienie wymaganego poziomu niezawodności konstrukcji. Metody probabilistyczne są naturalnym rozszerzeniem metody stanów granicznych. W praktycznych zastosowaniach istniejące oprogramowanie MES (Metoda Elementów Skończonych) jest często łączone z modułami do analizy niezawodności. Wadą takiego podejścia jest jednak długi czas obliczeń wynikający z wielokrotnych wywołań funkcji stanu granicznego dla różnych realizacji wektora zmiennych losowych. W artykule autorzy proponują alternatywne rozwiązanie problemu poprzez implementację hybrydowej metody FORM. Ocenę prawdopodobieństwa awarii, a tym samym obliczenie wskaźnika niezawodności Hasofera-Linda, uzyskujemy poprzez zastosowanie metody aproksymacyjnej FORM. W analizie wykorzystano niejawne funkcje stanów granicznych. Formułowanie funkcji granicznych dla pewnego zakresu zmiennych losowych zostało utworzone w programie Matlab za pomocą sieci neuronowych (SN). Analiza niezawodności została przeprowadzona w module Comrel środowiska obliczeniowego Strurel. Pakiet ten zawiera wbudowane interfejsy do definiowania funkcji stanów granicznych umożliwiając odwołanie do plików Matlaba zawierających zdefiniowane przez użytkownika funkcje stanu. Proponowana przez autorów hybrydowa metoda FORM łączy trzy algorytmy obliczeniowe – metodę elementów skończonych, sieci neuronowe oraz metodę aproksymacyjną probabilistycznej oceny niezawodności konstrukcji – FORM. Struktura algorytmu systemu jest więc mieszana, tzn. szeregowo-równoległa. Analiza zadania przebiega w czterech etapach. W etapie 1 (sformułowanie zadania) przedstawiono rodzaj analizowanej konstrukcji – układ nośny w postaci ramy płaskiej. Ustrój zaprojektowano ze stali S235 o charakterystykach $f_y = 235$ MPa i $E = 210$ GPa. Połączenia słupów i rygli, słupów z fundamentem oraz rygli w kalenicy zdefiniowano jako węzły sztywne. Słupy zaprojektowano z profilu HEB200 a rygle IPE200. W analizie niezawodności konstrukcji zastosowano nieskorelowane zmienne losowe: V_b – bazowa prędkość wiatru, S_k – charakterystyczne obciążenie śniegiem gruntu, E – moduł sprężystości stali i f_y – granica plastyczności stali. Kryterium zniszczenia konstrukcji wyrażono czterema funkcjami stanu granicznego – dwie związane ze stanem granicznym nośności – SGN ($G_1(\mathbf{X})$, $G_2(\mathbf{X})$) oraz dwie związane ze stanem granicznym użytkowalności – SGU ($G_3(\mathbf{X})$, $G_4(\mathbf{X})$). Etap 2 (wygenerowanie zbiorów danych w programie MES) polegał na wygenerowaniu zestawu wzorców uczących i testujących niezbędnych do opracowania sieci neuronowej. W tym celu analizowany schemat konstrukcji zamodelowano w programie Autodesk Robot Structural Analysis Professional. Do dalszej analizy wybrano cztery punkty charakterystyczne, w których odczytywano wartości specyficzne dla SGN ($M,\ N$) oraz dla SGU ($w,\ u$). Łącznie wykonano 206 symulacji dla różnych zestawów wzorców zmiennych losowych. W tym etapie poszerzono zakres zmiennych wg zasady 3σ. Do uczenia sieci wybrano sekwencyjnie 125 zestawów wzorców a 81 zestawów przeznaczono do jej testowania. W etapie 3 (formułowanie sieci neuronowej jako funkcji stanu granicznego $G_i(\mathbf{X})$) opracowano niejawne funkcje graniczne. W tym celu zastosowano sieć neuronową o strukturze 3-H-4 ze wsteczną propagacją błędu. Trójelementowy wektor wejściowy składał się z przyjętych zmiennych losowych: V_b, S_k, E. Dobór liczby neuronów w warstwie ukrytej odbywał się w sposób

adaptacyjny, tzn. na etapie wstępnych obliczeń testowano kilka wersji, w których przyjmowano liczbę neuronów H od 4 do 16. Stosując $H \geq 7$, błędy uczenia i testowania spełniły założony przez autorów poziom maksymalny, więc do dalszych obliczeń przyjęto tak sformułowaną architekturę sieci. Etap 4 (analiza niezawodności konstrukcji) realizowany jest za pomocą modułu Comrel. W środowisku niezawodnościowym definiowane są zmienne losowe wraz z ich charakterystykami (wartością średnią, odchyleniem standardowym oraz rozkładem gęstości prawdopodobieństwa). Odpowiednio sformułowane odwołanie do pliku Matlaba zawiera zdefiniowaną przez użytkownika funkcję stanu granicznego. W ten sposób wyznaczono wartości wskaźnika niezawodności Hasofera-Linda (β) metodą FORM dla każdej z czterech funkcji stanu (dla $G_1(\mathbf{X})$ – β = 2,476, dla $G_2(\mathbf{X})$– β = 19,299, dla $G_3(\mathbf{X})$ – β = 44,004, dla $G_4(\mathbf{X})$ – β = 2,681). Dodatkowo uzyskano wykresy dotyczące wrażliwość wskaźnika niezawodności na poszczególne zmienne losowe. Na podstawie informacji o znormalizowanej mierze wrażliwości (tzw. elastyczności) obserwujemy, która zmienna losowa ma największy wpływ na wartość wskaźnika β. Im wyższa wartość, tym większa wrażliwość wskaźnika na daną zmienną. Zgodnie z tą zasadą zmienną, która ma największy wpływ na wartość wskaźnika niezawodności dla funkcji stanu granicznego $G_1(\mathbf{X})$ jest granica plastyczności stali S235 (f_y), dla $G_2(\mathbf{X})$ – moduł sprężystości stali (E) oraz dla $G_4(\mathbf{X})$ – charakterystyczne obciążenie śniegiem gruntu (S_k). Dodatkowo wartość ujemna na wykresie oznacza, że wzrost wartości zmiennej spowoduje spadek wskaźnika niezawodności β, natomiast wartość dodatnia wskazuje na wzrost wskaźnika niezawodności wraz ze wzrostem wartości zmiennej. Ciekawym elementem analizy jest możliwość uzyskania wartości cząstkowych współczynników bezpieczeństwa dla zmiennych losowych przy otrzymanym poziomie niezawodności. Na podstawie analizy wykresów otrzymano następujące dane: dla funkcji $G_1(\mathbf{X})$ i β = 2,476 – γ_E = 1,000, γ_{Vb} = 1,020, γ_{Sk} = 1,202, γ_{fy} = 0,929; dla funkcji $G_2(\mathbf{X})$ i β = 19,299 – γ_E = 0,037, γ_{Vb} = 1,008, γ_{Sk} = 1,119, γ_{fy} = 1,000, zaś dla $G_4(\mathbf{X})$ i β = 2,681 – γ_E = 1,014, γ_{Vb} = 1,010, γ_{Sk} = 1,266, γ_{fy} = 1,000. Na podstawie wykonanej analizy można wyciągnąć następujące wnioski:

– zastąpienie programu MES przez NN znacznie skraca czas potrzebny na rozwiązanie zadania,
– hybrydowa metoda FORM umożliwia równoległe formułowanie wielu funkcji granicznych bez wydłużania czasu obliczeń,
– analiza wrażliwości pozwala na ocenę wpływu danej zmiennej losowej (reprezentującej cechy wytrzymałościowe lub obciążenie konstrukcji) na wartość wskaźnika niezawodności β, a tym samym określa jej wpływ na bezpieczeństwo ustroju,
– analiza niezawodności w module Comrel pozwala na uzyskanie wartości cząstkowych współczynników bezpieczeństwa przy uzyskanym poziomie wskaźnika niezawodności β.

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