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Mass/heat transfer analogy method in the research of convective fluid flow through channels with a specific geometry

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Abstract The present work contains the results of the comparative analysis of the literature data and the own investigations on mass and heat transfer coefficients occurring under conditions of the convective fluid flow through channels characterised by a specific geometry. The authors focused on the available experimental investigations on mass transfer. The considered experiments were carried out using the electrochemical method named limiting current technique. Two channel geometries were discussed: the annular channel of the conventional size and the long minichannel with a square cross-section area. Taking into consideration dimensionless numbers: Schmidt and Nusselt – analogical for mass and heat transfer – the formulas describing the phenomena under consideration were included. In the case of the annular channel the laminar and turbulent range of Reynolds numbers was studied. For the square minichannel – the laminar flow is considered. The analogy between mass and heat transfer introduced by Chilton and Colburn was applied in the analysis. An equivalent boundary condition is included in considerations concerning the mass and heat transfer. It is the Dirichlet boundary condition characterised by constant temperature of the wall which corresponds to the situation of constant ion concentration at the cathode surface in the limiting current technique. The main purpose of the present work was to verify the method for the determination of heat transfer coefficients using the analogy between mass and heat transfer processes in the case of convective fluid flow through the annular and square channels. The problem discussed in the present work is impor-

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tant and actual due to the possibility of the elimination of temperature measurements in the investigations of heat transfer processes occurring in channels characterised by a specific geometry. It should be noted that sometimes temperature measurement may be difficult or even impossible. This situation also causes high uncertainty of the obtained results. Due to this problem, the presented analysis was performed also with the use of thermal results based on the analytical solution. The verification of the use of mass/heat transfer analogy method in specific cases gives the extended knowledge of correct application of the limiting current technique in heat transfer research. The main objective of the work was achieved by conducting a comparative analysis of the adequate mass and heat transfer results. The existing literature data do not provide an answer to the question about the correctness of using the limiting current technique in the case of discussed annular channels or long square minichannels. The received results make us be critical of applying the mass/heat transfer analogy method in some heat transfer cases.

Keywords: Limiting current technique; Analogy method; Annular channel; Square minichannel

Nomenclature

A	–	surface area, m^2
a	–	thermal diffusivity, m^2s^{-1}
C	–	ion concentration, $kmol \cdot m^{-3}$
D	–	diffusion coefficient, m^2s^{-1}
d	–	diameter, m
d_H	–	hydraulic diameter, m
F	–	Faraday constant, $A \cdot s \cdot kmol^{-1}$
f	–	friction factor
I	–	current, A
L	–	channel length, m
Nu	–	Nusselt number, $= \alpha \cdot d_H/\lambda$
Pr	–	Prandtl number, $= \nu/a$
Re	–	Reynolds number, $= w \cdot d_H/\nu$
Sc	–	Schmidt number, $= \nu/D$
Sh	–	Sherwood number, $= \beta \cdot d_H/D$
T	–	temperature, °C, K
t	–	time, s
w	–	mean velocity of the fluid, $m \cdot s^{-1}$

Greek symbols

α	–	heat transfer coefficient, $Wm^{-2}K^{-1}$
β	–	mass transfer coefficient, $m \cdot s^{-1}$
λ	–	thermal conductivity, $Wm^{-1}K^{-1}$
ν	–	kinematic viscosity, m^2s^{-1}

1 Introduction

Annular channels and square minichannels represent important kinds of channel geometry applied in different thermal systems. These may be: simple heat exchangers, heat sinks, elements of heating systems, minichannel heat exchangers, electronic equipment or even most complicated nuclear reactors. The problem of convective heat transfer during the fluid flow through the annular channel seems to be well recognized, however, there are selected issues that cannot be recognized as fully examined. Recent investigations indicate such problems. These include: ducts with special additional elements for enhancement of heat transfer (spiral wire inserts in cooling processes of high-temperature reactors [1], pin fins arrays manufactured by selective laser melting technology for applications in internal combustion engine components [2]), channels with special fluids transferring heat (molten salts [1, 3], new types of refrigerants [4, 5]), or channels with two-phase flow (convective boiling [4–6]). For example, the work [1] includes the results of numerical study of the flow and heat transfer of molten salt in an annular channel with a wire coil. The results were compared with those for the smooth annular channel. In turn, the paper [3] presents the results of the investigations of the heat transfer performance of molten salt steam generator used in the solar power tower plant. Various experiments on the heat transfer performance of molten salt with high pressure water/steam in a horizontal tube-in-tube have been carried out. Another topic addressed in contemporary works on heat transfer in annular channels is the problem of rotating internal cylinders. This issue is studied in works [7,8] where the authors investigated the convective heat transfer for an annulus with an inner rotating wall. Additionally, the work [8] presents the results for the finned inner pipe.

In turn, the topic of heat transfer in minichannels has also been widely researched in recent times. The comprehensive review of mathematical and numerical models applied in thermal analysis of minichannels characterized by different cross-sections is included in [9]. In the case of square minichannels the researchers studied the problem of convective nanofluid flow in minichannel heat sinks [10] or cooling performance of heat sinks for heat dissipation from electronic components [11]. Experimental investigation on laminar flow and heat transfer in square minichannels with twisted tapes is presented in [12], while the paper [13] includes the results of the numerical study of heat transfer enhancement in a square minichannel heat sink with butterfly inserts.

Another recently investigated problem related to the presented examples is mass transfer during the fluid flow through channels with an annular cross-section and square minichannels. Similar to the thermal investigations, the results of an experiment on mass transfer in an annular channel with an obstruction were reported in [14]. In turn, the special fluid nanoelectrolyte flowing through the annular channel with the inner surface working as the cathode was studied in [15]. In the case of square minichannels the works [16–18] present the results of investigations of mass transfer coefficients under conditions of the laminar and turbulent flow through the system of long square minichannels.

The main and important topic in heat and mass transfer considerations is the determination and validation of formulas for obtaining mass or heat transfer coefficients, taking into account certain boundary conditions.

In the present study, the comparative analysis of the results of mass/heat transfer coefficients in the case of the fluid flow through channels with annular and mini-square cross sections has been performed. Based on the available literature data and our own research, the comparison of the mass/heat transfer coefficients with the use of the analogy method has been performed. The main goal of the work is to validate the received equations in application to the considered channel geometry. The results obtained from the validation may be important in further mass/heat transfer studies.

2 Mass transfer – experimental investigation

This section discusses the results of experimental research on mass transfer during the electrolyte flow through the annular and square channels. The considered data are the results of measurements of mass transfer coefficients. The investigations have been performed using the limiting current technique.

2.1 Limiting current technique in the investigation of mass transfer in channels

The electrochemical method named the limiting current technique is a method used to determine mass transfer coefficients. The limiting current technique involves observing the controlled diffusion of ions at the electrode surface. Most often it is the cathode. Once the external voltage is applied to electrodes, which are immersed in the electrolyte, the reduction of anions occurs at the cathode and the oxidation process at the anode, by which the electric current arises in the external circuit. The current is proportional to the number of reacting ions.

Taking into account Faraday's law the current I is given by

$$I = nFAN, \tag{1}$$

where: A – surface of the cathode, n – valence charge of reacting ions, N – molar flux density, F – Faraday constant.

The process of ion transport occurs by three phenomena: convection, migration and diffusion. However, the convection element is negligibly small. In turn, the migration process can also be neglected when the additional background electrolyte is used. Taking into account only the diffusion process, according to Fick's law about molar flux density and based on the Nernst model of linear dependence of ion concentration vs the distance from the electrode surface in the diffusion layer [19], the following equation is valid

$$I = nFA\beta\left(C_b - C_w\right), \tag{2}$$

where: β – mass transfer coefficient, C_b, C_w – bulk ion concentration and ion concentration at the electrode surface, respectively.

It is impossible to calculate the mass transfer coefficient from Eq. (2) because C_w is practically non-measurable. In the limiting current technique, an increasing current is caused to flow across the electrodes by increasing the applied voltage.

If the anode surface is much bigger than the cathode surface, a further increase in the applied voltage will not lead to increased current intensity – segment AB in Fig. 1. This segment of the polarization curve is called

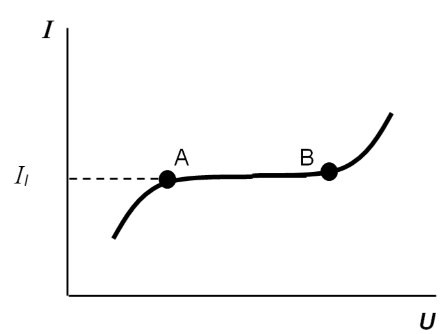


Figure 1: Current *vs* potential curve – voltammogram with the limiting current plateau region.

the limiting current plateau. Under these conditions the ion concentration at the cathode surface C_w approaches zero. Based on the measurement of the limiting current, the mass transfer coefficient can be calculated from the equation

$$\beta = \frac{I_l}{nFAC_b}\,, \tag{3}$$

where: I_l – limiting current. The experiment performed with the presented method requires measurements of the limiting current and the bulk ion concentration C_b.

Different electrochemical systems are used in the limiting current method. The ferri-ferrocyanide system is useful among others due to minimizing the reactions of deposition [20]. In this case, the oxidation-reduction reaction under controlled convective-diffusion is given by

$$[\mathrm{Fe(CN)_6}]^{-4} \xleftrightarrow{\text{oxidation} \qquad \text{reduction}} [\mathrm{Fe(CN)_6}]^{-3} + \mathrm{e}. \tag{4}$$

In order to achieve the controlled convective-diffusion process, oxygen must be removed from the electrolyte. The bubbling of the electrolyte with nitrogen satisfies this condition. Electrodes made of nickel are useful in the mentioned system. The value of the ferricyanide ions concentration C_b is measured using iodometric titration. Figure 2 presents the electrochemical system corresponding to Eq. (4).

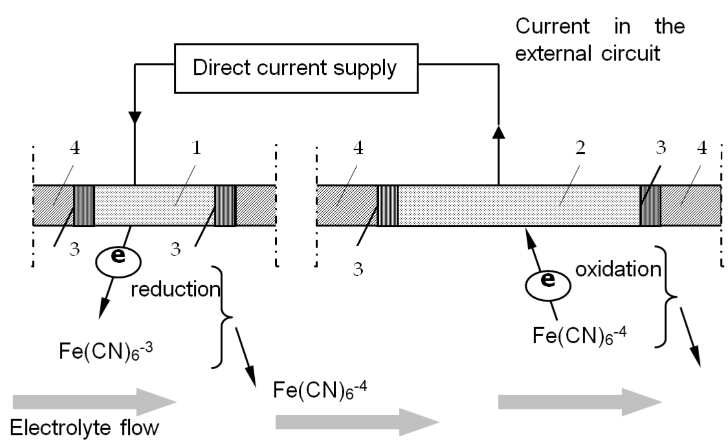


Figure 2: Scheme of the electrochemical system applied in the limiting current method; Electrolyte-aqueous solution of equimolar quantities of $K_3Fe(CN)_6$ and $K_4Fe(CN)_6$ and molar solution of sodium hydroxide NaOH: 1 – nickel cathode, 2 – nickel anode, 3 – electric insulation, 4 – construction element.

Since the ferricyanide reduction at the cathode is a limiting reaction, the cathode surface models the heat transfer surface in mass/heat transfer analogy studies.

In the electrochemical research with the use of the limiting current technique for the determination of mass transfer coefficients, the boundary condition characterised by constant ion concentration at the cathode surface takes place. This is the analogous situation to the Dirichlet condition of constant wall temperature in thermal research.

The discussed method has been applied in some studies of mass transfer in channels with other geometries (in addition to those considered in this paper): short ducts with varied shapes and surface areas along the curved axes [21], short circular minichannels [22,23], long circular and rectangular microchannels [24–26], conventional circular channels [27], or short ducts consisted in cylindrical segments in different configurations [28].

Considering the convective heat transfer during the flow through the concentric annular channel, the most common case, i.e. heat transfer from the outer surface of the inner tube, is discussed. Thus, this surface works as the cathode in mass transfer investigations. In the case of square minichannel the whole inner surface of the channel was the cathode.

2.2 Experimental results — annular channel

The results of electrochemical research on mass transfer coefficients during the electrolyte flow through the annular channel were reported in [29]. The tests covered a wide range of Reynolds numbers. Both turbulent and laminar flow were studied. The work [29] includes the results of investigations of Chilton-Colburn coefficients. These results were transformed to the dimensionless mass transfer coefficient that is the Sherwood number:

$$\mathrm{Sh} = \frac{\beta d_H}{D}\,, \tag{5}$$

where: d_H – hydraulic diameter, $d_H = d_o - d_i$, d_o – inner diameter of the outer tube, d_i – outer diameter of the inner tube, D – diffusion coefficient.

The studies reported in [29] concerned two types of annular channels. The first was characterized by $L/d_H = 4.19$, and the second $L/d_H = 6.32$. The obtained Sh *vs* Re relationships in the laminar range are shown in Fig. 3. Figure 3 also includes the classical formula – Leveque correlation for tube [30]. The Leveque equation for mass transfer is given by

$$\mathrm{Sh} = 1.615\mathrm{Sc}^{1/3}\mathrm{Re}^{1/3}\left(\frac{d_H}{L}\right)^{1/3}. \tag{6}$$

Sc number was 1540 in the cited study.

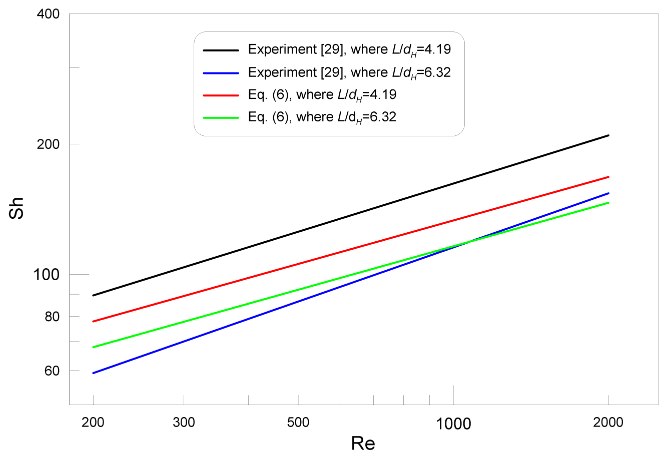


Figure 3: Sh number *vs* Re number for the laminar flow with mass transfer through the annular channel.

As can be seen from the graph in Fig. 3, Eq. (6) differs from the results of mass transfer research. The maximum deviation is about 30% for $L/d_H = 4.19$. At $L/d_H = 6.32$ (the longer channel), the deviation is smaller. It should be noted that the classical Leveque correlation was obtained from studies on the flow through the channel with a circular cross-section.

The results of mass transfer coefficients measurements in turbulent range are included in Fig. 4. The results from [14] concern the electrolyte flow

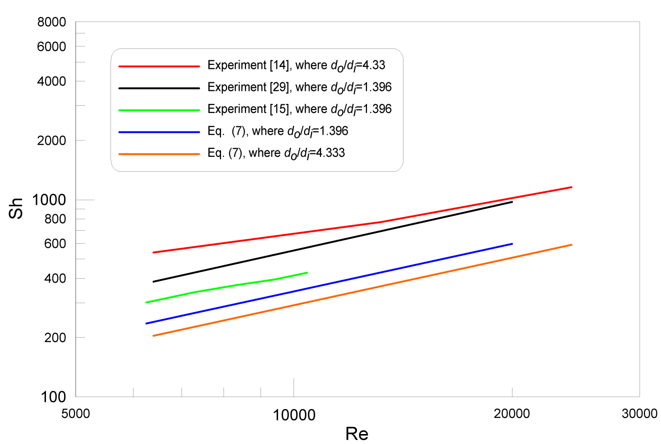


Figure 4: Sh number *vs* Re number for the turbulent flow with mass transfer through the annular channel.

through the annular duct with an obstacle. The corresponding line in Fig. 4 was received for the section of the examined channel behind the obstacle. In this case, the values of the mass transfer coefficient stabilized. In turn, the data from work [29] were averaged for the two investigated surfaces.

Figure 4 also presents the results from work [15] which were received for the same geometry of the annular channel as in [29]: $d_o/d_i = 1.396$ and $L/d_H = 4.19$. Other lines in Fig. 4 present the relationship for mass transfer during the turbulent flow in an annular channel [14]:

$$\mathrm{Sh} = 0.86 \left(\frac{d_i}{d_o}\right)^{-0.16} \mathrm{Sh}^*, \tag{7}$$

where

$$\mathrm{Sh}^* = 0.023\mathrm{Re}^{0.8}\mathrm{Sc}^{1/3}. \tag{8}$$

As can be seen in Fig. 4 the best agreement of the results from [15] with Eq. (7) for the same geometry is in the range of Re from 6000 to 10 000. The maximum obtained deviation is about 21%. It should be noted that the classical formula (8), valid for mass transfer in turbulent region, was determined using the Chilton-Colburn analogy between mass and heat transfer. Equation (8) is suitable for ducts with different cross sections. However, it was found that the channel cross-section has a significant impact on transfer processes [31]. In addition, measurement uncertainty must be taken into account. In [15] the authors reported the average relative uncertainty of ±2% for determining the mass transfer coefficient and ±8% for the Re number.

2.3 Experimental results — square minichannel

The results of investigations of mass transfer coefficients during the laminar flow through the square minichannel using the limiting current technique were reported in [17]. The authors experimentally investigated the system of 100 mm long, 2 mm wide minichannels. The hydraulic diameter characterizing the square channel is equal to its width. The work analysed not fully developed laminar flow region. Due to the long channel with a small hydraulic diameter that was investigated, the change of ion concentration along the channel length has been taken into account. Thus, the average bulk ion concentration in Eq. (3) has been included.

The experiment was carried out on the stand, the diagram of which is shown in Fig. 5.

Similar to the considerations concerning the mass transfer in the annular duct, the results presented in the form of Chilton-Colburn coefficients were

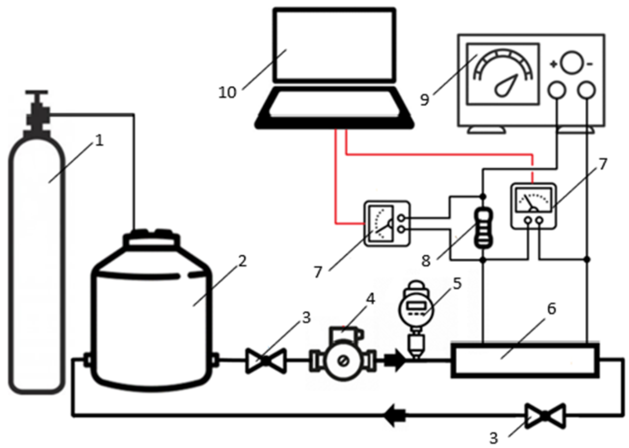


Figure 5: Scheme of the experimental system to measure the mass transfer coefficients in square minichannels [17]: 1 – cylinder with nitrogen, 2 – tank with the electrolyte, 3 – valve; 4 – pump, 5 – flowmeter, 6 – test section with the system of square minichannels, 7 – laboratory digital multimeter, 8 – high precision standard resistor, 9 – DC laboratory power supply, 10 – computer with data logger.

recalculated to the Sherwood number form. The correlation obtained as a fitting of the experimental results is given by

$$\mathrm{Sh} = 0.468\mathrm{Re}^{0.443}\mathrm{Sc}^{1/3}. \tag{9}$$

Figure 6 includes the received Sh results in the considered range of Re numbers. It should be noted that the uncertainty of mass transfer coef-

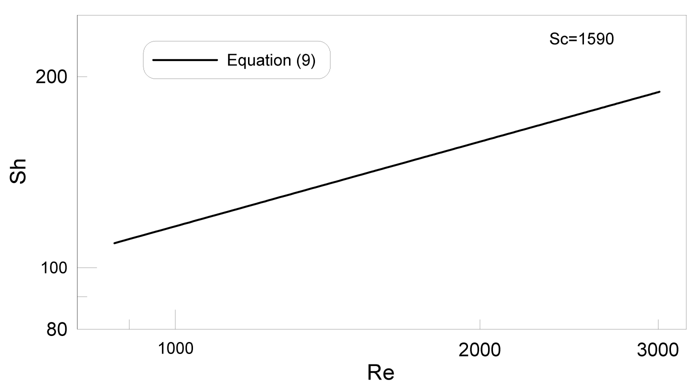


Figure 6: Mass transfer in square minichannel.

ficients determination was reported as equal to 2.5% [17]. In the case of channels characterized by small hydraulic diameters, this factor is of great importance.

3 Mass/heat transfer analogy results

3.1 Analogy between the heat and mass transfer

In the processes of mass transfer the concentration gradient of the transferred molecules or ions plays a major role. In heat transfer processes, it is a temperature gradient. The analogy between heat and mass transfer processes makes it possible to transform the results of thermal experiment into analogous mass transfer results and vice versa. This is due to the similarity of the differential equations for energy and mass transfer in the boundary conditions when the transport gradients are expressed in terms of dimensionless variables. The non-dimensional transport equations take the form

$$\frac{D\hat{C}}{Dt} = \frac{1}{\mathrm{Re}\,\mathrm{Sc}}\frac{\partial^2\hat{C}}{\partial\hat{x}^2}\,, \tag{10}$$

for mass transfer, and

$$\frac{D\hat{\Theta}}{Dt} = \frac{1}{\mathrm{Re}\,\mathrm{Pr}}\frac{\partial^2\hat{\Theta}}{\partial\hat{x}^2}\,, \tag{11}$$

for heat transfer, where: $\hat{C}$, $\hat{\Theta}$, $\hat{x}$ – dimensionless variables representing the concentration, temperature difference and the generalized coordinate. The terms $\frac{D\hat{C}}{Dt}$ and $\frac{D\hat{\Theta}}{Dt}$ are material derivatives.

The existence of the analogy between transfer processes requires that certain conditions have to be fulfilled [32]: there is no mass or energy produced in the system, there is no radiation and no viscous dissipation, the mass or heat transfer processes do not affect the fluid velocity profile, and the fluid physical properties are constant.

Dimensionless concentration and temperature profiles as solutions of Eqs. (10) and (11) are the same when the boundary conditions are equivalent for a given geometrical configuration and dimensionless numbers Sc and Pr are equal.

Under steady-state conditions, the solution to the mass transfer problem has a general form

$$\mathrm{Sh} = f(\mathrm{Re}, \mathrm{Sc}), \tag{12}$$

whereas the heat transfer has a form

$$\mathrm{Nu} = f(\mathrm{Re}, \mathrm{Pr}). \tag{13}$$

Thus, the Sh and Nu numbers are analogous terms which represent the mass transfer intensity by diffusion and the heat transfer intensity by convection.

Based on the analytical description of the transfer processes and the results of experimental research, the simple formula for determining the unknown convective transfer coefficients from the mass/heat transfer analogy takes the form

$$\frac{\mathrm{Nu}}{\mathrm{Sh}} = \left(\frac{\mathrm{Pr}}{\mathrm{Sc}}\right)^z, \tag{14}$$

where the exponent z is obtained from experiments.

Chilton and Colburn proposed that $z = 1/3$ [33]. Researchers determined z from a large database which is the result of many experiments performed. These were the experiments with the flow through circular pipes, around cylinders as well as over flat smooth plates. The Chilton-Colburn analogy has been found to be valid mainly for the fully developed turbulent flow. However, the analogy can also be used in the case of laminar flow [34]. It is valid for gases and liquids within the ranges of Sc number from 0.6 to 2500 and Pr number from 0.6 to 100 [32]. In turn, the validation of the Chilton-Colburn mass/heat transfer analogy for processes occurring in minichannels is presented in [35]. The author focused on circular and rectangular channels with a small hydraulic diameter. The conducted analysis did not give a clear answer to confirm the Chilton-Colburn analogy in the considered minichannels because both convergence and divergence of the results were noted.

3.2 Comparison of the mass and heat transfer results – the case of annular channel and square minichannel

In order to make a comparative analysis of the results of mass/heat transfer analogy, some correlations for the convective fluid flow through channels were considered.

The first formula based on the theoretical analysis relates to the laminar flow through the circular pipe [36]. It is given by

$$\mathrm{Nu} = \Bigg\{ 3.66^3 + 0.7^3 + \left[1.615 \left(\mathrm{RePr}\frac{d}{L} \right)^{\frac{1}{2}} - 0.7 \right]^3 + \left[\left(\frac{2}{1+22\mathrm{Pr}} \right)^{\frac{1}{6}} \left(\mathrm{RePr}\frac{d}{L} \right)^{\frac{1}{2}} \right]^3 \Bigg\}^{1/3}. \tag{15}$$

Equation (15) is valid for the Dirichlet boundary condition of constant wall temperature. This is the analogous condition to the constant ion concentration at the cathode surface for mass transfer phenomena.

In turbulent region of the flow through the annular channel the comprehensive formula based on the results of many experiments [37, 38] is given by

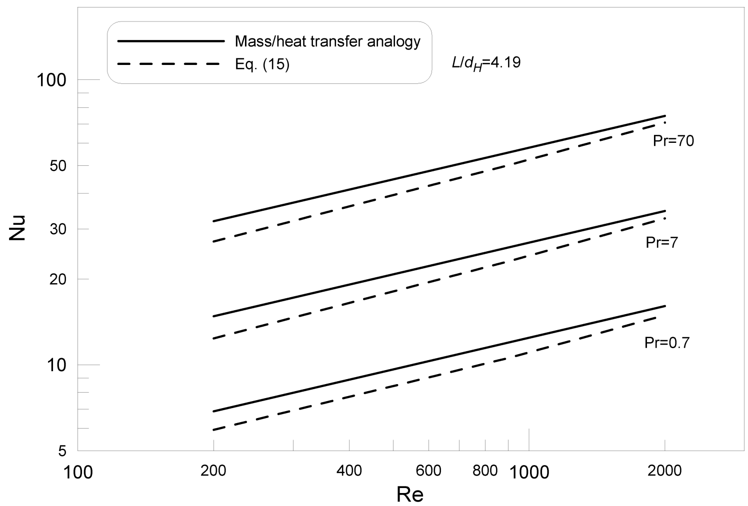
$$\mathrm{Nu} = \frac{\frac{f_{\mathrm{ann}}}{8} (\mathrm{Re}-1000)\, \mathrm{Pr}}{1 + 12.7 \left(\frac{f_{\mathrm{ann}}}{8} \right)^{1/2} \left(\mathrm{Pr}^{2/3} - 1 \right)} \left[1 + \left(\frac{d_H}{L} \right)^{2/3} \right] 0.75 \left(\frac{d_i}{d_o} \right)^{-0.17} W. \tag{16}$$

The friction factor f_{ann} in Eq. (16) depends on the diameter ratio $\frac{d_i}{d_{o_i}}$. It is given by [37]

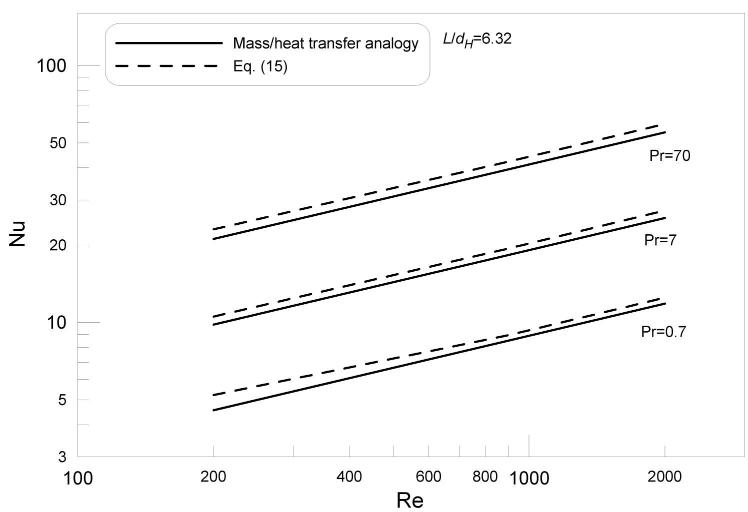
$$f_{\mathrm{ann}} = \left\langle 1.8 \log_{10} \mathrm{Re} \left\{ \frac{\left[1 + \left(\frac{d_i}{d_{o_i}} \right)^2 \right] \ln\left(\frac{d_i}{d_{o_i}} \right) + \left[1 - \left(\frac{d_i}{d_{o_i}} \right)^2 \right]}{\left[1 - \left(\frac{d_i}{d_{o_i}} \right)^2 \right] \ln\left(\frac{d_i}{d_{o_i}} \right)} \right\} - 1.5 \right\rangle^{-2}. \tag{17}$$

Equation (16) is valid when the process of heat transfer occurs from the outer surface of the inner pipe, and the outer pipe is insulated. The parameter W in Eq. (16) characterizes the fluid properties temperature dependence. For gases, W can be written as a ratio of the bulk temperature T_b to the wall temperature T_w, raised to the specific power. In the case of liquids, the variations of fluid properties with temperature can be expressed as a ratio of Pr_b to Pr_w, raised to the specific power. Pr_b and Pr_w represent the Prandtl numbers of the fluid at the bulk and wall temperature, respectively.

Heat transfer coefficients received from the investigation on mass transfer [29] and Eq. (15) for the laminar range of Re numbers at different values of Pr number are included in Fig. 7a and 7b. Figure 7a presents the results



(a)



(b)

Figure 7: Chilton-Colburn analogy results for the annular channel in laminar region. Comparison with the theoretical solution for the circular channel: (a) $L/d_H = 4.19$, (b) $L/d_H = 6.32$.

for the shorter channel ($L/d_H = 4.19$). As can be seen, the analogy results lie above the theoretical line representing the boundary condition of constant wall temperature. The maximum deviation between the results is about 15%. For $L/d_H = 6.32$ (Fig. 7b), there is a good agreement of the mass/heat transfer analogy results with the analytical solution for a circular duct. The maximum deviation is about 8%.

The results of comparative analysis for the turbulent region of the fluid flow with mass/heat transfer through the annular channel are shown in Fig. 8. The results of the mass transfer experiment were taken from the authors' investigations [15] due to their better agreement with Eq. (7), (see Fig. 4).

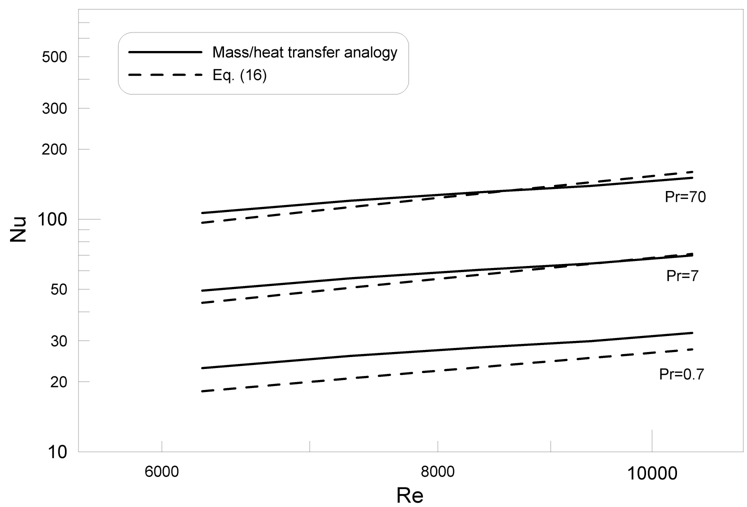


Figure 8: Chilton-Colburn analogy results for the annular channel in turbulent region – comparison with the thermal experimental results.

The analogy results were compared with the Gnielinski formula for the turbulent range of the fluid flow through the annular channel (16). Similar to the laminar region, three representative values of Pr numbers were considered. Parameter W in Eq. (16) was assumed as equal to 1. As can be seen in Fig. 8, a better compliance of the results occurs for the liquid flow at $\mathrm{Pr} = 7$ and 70. The maximum deviation in this case is about 10% while for gases ($\mathrm{Pr} = 0.7$) it is 20%.

In the comparative analysis of the case of convective fluid flow through the square minichannel, Eq. (15) valid for the laminar flow through the circular pipe was taken into account. Additionally, the analytical solution for a fully developed laminar flow in a straight minichannel with a rectangular cross-section was considered. The approximate formula of this solution has the form [39]:

$$\mathrm{Nu} = 8.235\left(1 - 2.0421K^{-1} + 3.0853K^{-2} - 2.4765K^{-3} + 1.0578K^{-4} - 0.1861K^{-5}\right), \tag{18}$$

where the parameter K is the aspect ratio of the channel cross section (the ratio of the width to the height of the channel). In the case of a square minichannel $K = 1$ and Nu number from Eq. (18) it is equal to 3.61.

Figure 9 presents the results of the comparison. Additionally, the literature data of experimental studies on heat transfer in a square minichannel have been added [10]. These are the results of research of water convective fluid flow (Pr = 7) through the system of 75 mm long, 1 mm wide minichannels.

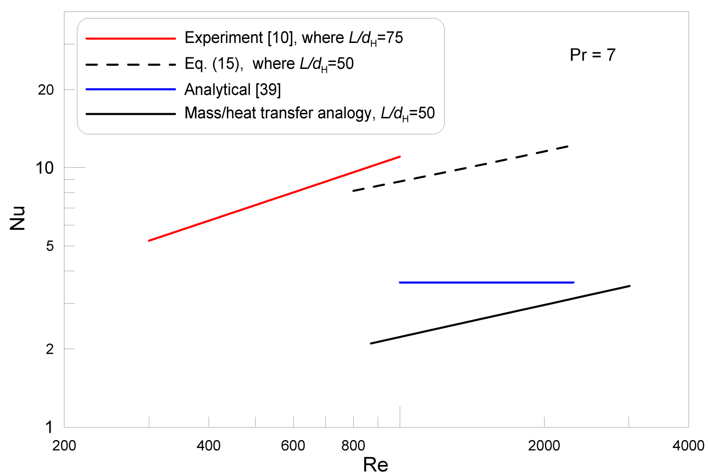


Figure 9: Chilton-Colburn analogy results for the square minichannel. Comparison with the theoretical solution for a circular channel and with heat transfer experimental results.

As can be seen in Fig. 9, the results of analytical study [39] are closest to the mass/heat transfer analogy results. In turn, these are lower than those from the conventional theory (Eq. (15)) and from thermal experimental results. The differences are large – even 75%. The reason for this discrepancy should be considered. The main problem is the channel geometry. A square cross-section with mini dimensions can have a major impact on the intensity of mass transfer processes. If the diffusion layer thickness is comparable with the channel hydraulic diameter, the obtained mass transfer results may be underestimated [40]. In turn, in thermal investigations, both higher or lower Nu numbers for convective fluid flow through minichannels in comparison to conventional cases can be found [41–43]. It should be noted that in the thermal investigations of the flows in small-sized channels, it is very difficult to measure the temperature, which may result in high uncertainty of the results. Thus, it can be concluded that further research is needed.

4 Conclusions

In the present study the authors have tried to carry out a comparative analysis of the Chilton-Colburn analogy results in the case of fluid flow with mass/heat transfer through the annular channel and square minichannel. The data of mass transfer were the results of measurements carried out with the use of the limiting current technique. Based on the received results, some conclusions can be drawn, such as those listed below.

The analogy between mass and heat transfer proposed by Chilton and Colburn is a simple and very useful method in heat transfer research. The analogy method may be applied when the heat experiment may be difficult to carry out. This is important when it is difficult to measure the temperature. Then, the method makes the determination of the unknown convective transfer coefficients become possible.

In the considerations of convective fluid flow through the annular channel, it is advisable to use the analogy method proposed by Chilton and Colburn. The discussed method can be particularly useful for the liquid laminar flow – in this case, the best agreement of the analogy results was obtained. The considerations made in the present study can be a base for applying the analogy method also for the annular ducts characterized by various geometrical configurations.

In the case of convective fluid flow through a long square minichannel, there is a large discrepancy between the mass and thermal experiment results. One of the possible reasons for this could be that the boundary conditions are not compatible. The Dirichlet condition characterizes the limiting current technique while the Newman condition takes place in most thermal experiments - also in the case of the experiment, the results of which were included in the comparative analysis. In the analysis of the case of annular channel, this agreement was maintained.

The second issue that may have an impact on the discrepancy of the results is the geometry of the investigated channel – mini dimensions and the cross-section shape of the channel.

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References

[1] Yang Y., Li M., Zou Y., Chen J.: *Numerical study on heat transfer characteristics of molten salt in annular channel with wire coil.* Appl. Therm. Eng. **199**(2021), 117520.

[2] Lorenzon A., Vaglio E., Casarsa L., Sortino M., Totis G., Sarago G., Lendormy E., Raukola J.: *Heat transfer and pressure loss performances for additively manufactured pin fin in annular channels.* Appl. Therm. Eng. **202**(2022), 117851.

[3] Dong X., Bi Q., Yao F.: *Experimental investigation on the heat transfer performance of molten salt flowing in an annular tube.* Exp. Therm. Fluid Sci. **102**(2019), 113–12.

[4] Zhang J., Wang J., Li W., Liu Z., Kabelac S., Tao Z., Ma L., Tang W., Sherif S.: *R410A flow boiling in horizontal annular channels of enhanced tubes. Part I: Pressure drop.* Int. J. Refrig. **137**(2022), 70–79.

[5] Zhang J., Wang J., He Y., Liu L., Li W., Tang W., Abbas A., Ayub Z.: *R410A boiling coefficient in horizontal annular channels of enhanced tubes. Part II: Heat transfer.* Int. J. Refrig. **137**(2022), 43–50.

[6] Seo J., Lee S., Yang S., Hassan Y.: *Experimental investigation of the annular flow caused by convective boiling in a heated annular channel.* Nucl. Eng. Des. **376**(2021), 111088.

[7] Fenot M., Dorignac E., Giret A., Lalizel G.: *Convective heat transfer in the entry region of an annular channel with slotted rotating inner cylinder.* Appl. Therm. Eng. **54**(2013), 345–358.

[8] Yassin M., Abd El-Hameed H.M., Shedid M.H., Helali A.H.B.: *Heat transfer Enhancement through Annular Flow using Rotating Finned Pipe.* Aerospace Sciences & Aviation Technology, ASAT-17 (2017).

[9] Soudagar M.E.M., Kalam M.A., Sajid M.U., Afzal A., Banapurmath N.R., Akram N., Mane S.D., Saleel C.A.: *Thermal analyses of minichannels and use of mathematical and numerical models.* Numer. Heat Tr. A-Appl. **77**(2020), 5, 497–537.

[10] Arshad W., Ali H.M.: *Experimental investigation of heat transfer and pressure drop in a straight minichannel heat sing using TiO_2 nanofluid.* Int. J. Heat Mass Tran. **110**(2017), 248–256.

[11] Ghasemi S.E., Ranjbar A.A., Hosseini S.M.J.: *Cooling performance analysis of water-cooled heat sinks with circular and rectangular minichannels using finite volume method.* Iran. J. Chem. Eng. **37**(2018), 2, 231–239.

[12] Feng Z., Ai X., Wu P., Lin Q., Huang Z.: *Experimental investigation of laminar flow and heat transfer characteristics in square minichannels with twisted tapes.* Int. J. Heat Mass Tran. **158**(2020), 119947.

[13] Sudheer A.P., Madanan U.: *Numerical investigation into heat transfer augmentation in square minichannel heat sink using butterfly inserts*. Therm. Sci. Eng. Prog. **36**(2022), 101522.

[14] Weyns G., Nelissen G., Pembery J., Maciel P., Deconinck J., Deconinck H., Patrick M., Wragg A.: *Turbulent fluid flow and electrochemical mass transfer in an annular duct with an obstruction*. J. Appl. Electrochem. **39**(2009), 2453–2459.

[15] Wilk J., Grosicki S.: *Experimental study of electrochemical mass transfer in an annular duct with the electrolyte nanofluid*. Int. J. Therm. Sci. **129**(2018), 280–289.

[16] Wilk J., Grosicki S., Kiedrzyński K.: *Preliminary research on mass/heat transfer in mini heat exchanger*. E3S Web Conf. **70**(2018), 02016.

[17] Wilk J., Grosicki S., Smusz R.: *Mass/heat transfer analogy method in the research on convective fluid flow through a system of long square mini-channels*. Materials **15**(2022), 4617.

[18] Kumar K.A., Sarma G.V.S., Ramesh K.V.: *Mass transfer at the confining wall of a minichannel*. Mater. Today-Proc. **27**(2020), 534–538.

[19] Bard A.J., Faulkner L.R.: *Electrochemical Methods. Fundamentals and Applications*. Wiley, New York 2005.

[20] Szánto D.A., Cleghorn S., Ponce-de-León C., Walsh F.C.: *The limiting current for reduction of ferricyanide ion at nickel: the importance of experimental conditions*. AIChE J. **54**(2008), 3, 802–810.

[21] Bieniasz B: *Convectional mass/heat transfer in a rotary regenerator rotor consisted of the corrugated sheets*. Int. J. Heat Mass Tran. **53**(2010), 3166–3174.

[22] Wilk J.: *Experimental investigation of convective mass/heat transfer in short minichannel at low Reynolds numbers*. Exp. Therm. Fluid Sci. **33**(2009), 267–272.

[23] Wilk J.: *Convective mass/heat transfer in the entrance region of the short circular minichannel*. Exp. Therm. Fluid Sci. **38**(2012), 107–114.

[24] Sara O.N., Ergu Ő.B., Arzutug M.E., YapıcıS.: *Experimental study of laminar forced convective mass transfer and pressure drop in microtubes*. Int. J. Therm. Sci. **48**(2009), 1894–1900.

[25] Ergu Ő.B., Sara O.N., YapıcıS., Arzutug M.E.: *Pressure drop and point mass transfer in a rectangular microchannel*. Int. Commun. Heat Mass **36**(2009), 618–623.

[26] Acosta R.E., Mulle R.H., Tobiasz C.W.: *Transport processes in narrow (capillary) channels*. AIChE J. **31**(1985), 473-482.

[27] Beiki H., Esfahany M.N., Etesami N.: *Laminar forced convective mass transfer of γ-Al_2O_3/electrolyte nanofluid in a circular tube*. Int. J. Therm. Sci. **64**(2013), 251-256.

[28] Kiedrzyński K.: *Increasing of efficiency of the cooling duct by modification of its geometry*. Arch. Thermodyn. **41**(2020), 3, 103-123.

[29] Bieniasz B.: *The use of electrolysis and the analogy between heat and mass transfer in the design of forced convection heat exchanger elements*. WUPR, Rzeszów 1980 (in Polish).

[30] Martin H.: *The generalized Leveque equation and its practical use for the prediction of heat and mass transfer rates from pressure drop*. Chem. Eng. Sci. **57**(2002), 16, 3217–3223.

[31] Sash R.K.: *Laminar flow friction and forced convection heat transfer in ducts of arbitrary geometry*. Int. J. Heat and Mass Tran. **18**(1975), 849–862.

[32] Welty J., Wicks C., Wilson R., Rorrer G.: *Fundamentals of Momentum. Heat Mass Transfer*. Wiley, Oregon 2007.

[33] Chilton T.H., Colburn A.P.: *Mass transfer (absorption) coefficients prediction from data on heat transfer and fluid friction*. Ind. Eng. Chem. **26**(1934), 11, 1183–1187.

[34] Iwaniszyn M., Jaroszyński M., Ochońska J., Łojewska J., Kołodziej A.: *Heat and mass transfer analogy for the laminar flow: discussion of the problem*. Prace Naukowe IICh PAN **15**(2011), 37–46.

[35] Wilk J.: *Heat/mass transfer analogy in the case of convective fluid flow through minichannels*. Int. J. Therm. Sci. **156**(2020), 106467.

[36] Gnielinski V.: *On heat transfer in tubes*. Int. J. Heat Mass Tran. **63**(2013), 134–140.

[37] Gnielinski V.: *Heat transfer coefficients for turbulent flow in concentric annular ducts*. Heat Transfer Eng. **30**(2009), 6, 431–436.

[38] Gnielinski V.: *Turbulent heat transfer in annular spaces – a new comprehensive correlation*. Heat Transfer Eng. **36**(2015), 9, 787–789.

[39] Rybiński W., Mikielewicz J.: *Analytical solution of heat transfer for laminar flow in rectangular channels*. Arch. Thermodyn. **35**(2014), 4, 29–42.

[40] Wilk J.: *A review of measurement of the mass transfer in minichannels using the limiting current technique*. Exp. Therm. Fluid Sci. **57**(2014), 242–249.

[41] Hetsroni G., Mosyak A., Pogrebnyak E., Yarin L.P.: *Heat transfer in micro-channels: Comparison of experiments with theory and numerical results*. Int. J. Heat Mass Tran. **48**(2005), 25-26, 5580–5601.

[42] Celata G.P., Cumo, M., Marconi V., McPhail S.J., Zummo G.: *Microtube liquid single-phase heat transfer in laminar flow*. Int. J. Heat Mass Tran. **49**(2006), 19-20, 3538–3546.

[43] Kandlikar S.G., Garimella S., Li D., Colin S., King M.R.: *Heat Transfer and Fluid Flow in Minichannels and Microchannels*. Elsevier, Kidlington Oxford 2006.