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THE DISTRIBUTION OF MAGNETIC FIELD AND FORCES IN THE BAND SEPARATOR

ROZKŁAD POLA MAGNETYCZNEGO I SIŁ W SEPARATORZE TAŚMOWYM

The magnetic band separator is provided for the enrichment of strongly magnetic ores, as, for example, magnetite ore. The process of magnetic flocculation occurs under the influence of magnetic field. The forming particle aggregates (flocs) contain non-magnetic particles in their structure which deteriorates the separation results. In the band separator the material is subjected to several remagnetizations on its separation path during which non-magnetic particles are being liberated from the floc volumes. The separation results depend on the characteristics of the separator magnetic system and magnetic properties of the raw material.

Starting from the equations of magnetic field the author calculated the distribution of magnetic field and force in the band separator. On this basis he also determined the optimum pole pitch of the magnetic system which depends on particle sizes of the enriched raw material.

Despite the magnetic force, also mechanical forces act upon particles. The balance of forces acting upon the particle enabled the value of separation magnetic susceptibility to be calculated according to which the raw material is divided into magnetic and non-magnetic particles.

Taking into account magnetic interactions between magnetite inclusions in the particle, the dependence of particle magnetic susceptibility on the volume content of magnetite was determined and, next, theoretical indexes of magnetite ore enrichment ability were calculated.

Key words: magnetic separation, magnetic field distribution, magnetic susceptibility, band separator, indexes of enrichment ability

Separator magnetyczny taśmowy jest przeznaczony do wzbogacania rud silnie magnetycznych jak np. ruda magnetytowa. Należy on do grupy separatorów z otwartym układem magnetycznym. Pod wpływem pola magnetycznego zachodzi proces flokulacji magnetycznej. Tworzące się agregaty ziarnowe (flokuły) zawierają w swojej strukturze ziarna niemagnetyczne co powoduje pogorszenie wyników separacji. W separatorze taśmowym materiał na swej drodze separacji ulega kilkukrotnemu przemagnesowaniu w trakcie którego ziarna niemagnetyczne zostają uwalniane z objętości flokuł. Wyniki separacji są uzależnione od charakterystyki układu magnetycznego separatora oraz własności magnetycznych surowca.

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Wychodząc z równań pola magnetycznego w artykule wyliczono rozkład pola oraz siły magnetycznej w separatorze taśmowym i na tej podstawie wyznaczono optymalną podziałkę biegunów układu magnetycznego, która jest zależna od wielkości ziaren wzbogacanego surowca. Oprócz siły magnetycznej na ziarna działają siły mechaniczne. Bilans sił działających na ziarno umożliwił wyliczenie wartości podatności magnetycznej podziałowej według której następuje podział surowca na ziarna magnetyczne i niemagnetyczne.
Uwzględniając oddziaływanie magnetyczne między wpryśnięciami magnetytu w ziarnie wyznaczono zależność podatności magnetycznej ziarna od objętościowej zawartości magnetytu a następnie wyliczono teoretyczne wskaźniki wzbogacalności rudy magnetytowej.

1. Introduction

The magnetic band separator belongs to the group of separators with the so-called open magnetic system. Open magnetic systems are applied in separators provided for separating components of high magnetic susceptibility. Figure 1 shows the diagram of an open system. It is a series of pole shoes of variable polarity arranged on a plane or cylinder surface. The first one is applied in band separator, the latter in drum separators.

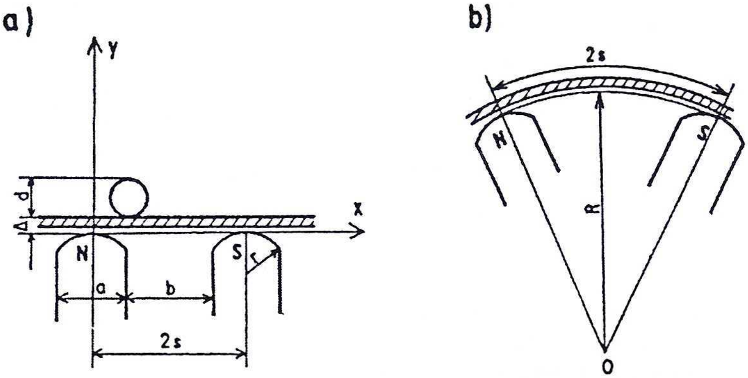

Fig. 1. Diagram of the open magnetic system: a) on the plane, b) on the cylinder surface

The process of enrichment of magnetite ore requires the application of separators of a relatively little value of the magnetic field gradient, due to the high value of magnetic susceptibility of magnetite. The phenomenon of flocculation accompanies the separation of strongly magnetic ores. The intensity of this phenomenon depends on the magnetic field intensity and its distribution in the separator working space. Magnetic flocculation results in decreasing the quality of the obtained magnetite concentrate due to the mechanical elevating of non-magnetic particles to the magnetic product. The recovery of magnetic component in the concentrate increases with the growth of magnetic field intensity yet the content of this component in the concentrate decreases. The enriched raw material should be subjected to numerous purifying operations in order to obtain the concentrate of required quality. When applying the band separator in which the magnetic system consists of many pole shoes of variable polarity, the flocs of the magnetic product on the band change the direction of magnetization several times. During this process of remagnetization, the particle of non-magnetic component are liberated from the floc volumes where they are held by mechanical forces. Consequently, the magnetic product of lower content of non-magnetic component can be obtained.

The parameters of the separator magnetic system affect the value of magnetic force, acting upon the particle. If we want to analyze thoroughly the influence of these parameters, the distribution of magnetic field and magnetic force in the separator working space (space above the band) should be determined before.

Despite the magnetic force, the particles are subjected to the action of mechanical forces which elevate the non-magnetic particles from the stream of magnetic particles. The balance of all forces enables the separation magnetic susceptibility to be calculated and, knowing the dependence of particle magnetic susceptibility on magnetite content in the particle, it is possible to calculate theoretical separation indexes. The further part of the paper presents the derivation of the dependence of particle magnetic susceptibility on magnetite content and the calculation of theoretical separation indexes.

2. Magnetic field in band separators

Figure 2 presents the schematic diagram of band separator. The field distribution in the space above pole shoes should be determined (Fig.1a).

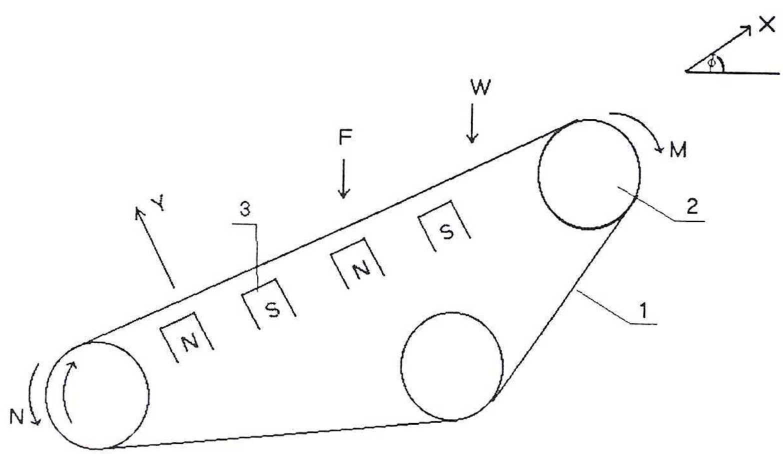

Fig. 2. Schematic diagram of the band separator: 1 – band, 2 – drum, 3 – pole shoe of magnetic system, N – reception of non-magnetic product, M – reception of magnetic product, F – place of feed intake, W – place of water spray,

Analytical methods of determining the magnetic field distribution in magnetic separators result from M a x w e l l ' s equations and field theory. The complete system of M a x - w e l l ' s equations is as follows [5]:

$$\text{rot}\,\vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t} \tag{1a}$$

$$\text{rot}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{1b}$$

$$\text{div}\,\vec{B} = 0 \tag{1c}$$

$$\text{div}\,\vec{D} = \rho, \tag{1d}$$

where: $\vec{H}$ – intensity of magnetic field, $\vec{E}$ – intensity of electric field, $\vec{B}$ – induction of magnetic field, $\vec{D}$ – induction of electric field, $\vec{j}$ – density of current, ρ – volume density of electric charge.
The vector of magnetic field intensity is sourceless in the vacuum and, moreover, for the fields constant in time, in the space without currents, magnetic and electric fields are irrotational. In such a case the equations of magnetic field are expressed as follows:

$$rot\,\vec{H} = 0 \tag{2a}$$

$$div\,\vec{H} = 0 \tag{2b}$$

There is the scalar potential $V_m(x,y,z)$ for irrotational fields, such that $\vec{H} = -\,grad\,V_m$ [11]. Substituting this dependences into equation (2b), we obtain L a p l a c e ' s equation:

$$\text{div}(\text{grad}\,V_m) \equiv \Delta V_m = 0 \tag{3}$$

where symbol Δ denotes L a p l a c e ' s operator.
There are no currents in the space above the polar shoes, magnetic field is irrotational and the field scalar potential fulfills L a p l a c e ' s equation. Due to symmetry, the problem can be considered on the plane. In the two-dimensional C a r t e s i a n system L a p - l a c e ' s equation is as follows [11]:

$$\frac{\partial^2 V_m(x,y)}{\partial x^2} + \frac{\partial^2 V_m(x,y)}{\partial y^2} = 0. \tag{4}$$

Solving the above equation by the method of separated variables [8], the following function is obtained for the potential distribution:

$$V_m(x,y) = (C_1 e^{ky} + C_2 e^{-ky})\cos kx, \tag{5}$$

where C_1, C_2 and k are certain constants determined from boundary conditions.
Magnetic field is equal to 0 in infinity. Therefore the following condition must be fulfilled:

$$\lim_{y \to \infty} V_m(x,y) = 0. \tag{6}$$

It results from this condition that $C_1 = 0$.
As it can be seen from Figs 1 and 2, and the solution of equations (5) and (6), magnetic potential is a periodical function in relation to variable x of the period equal $2s$ where s is called a pole pitch. Consequently, the following condition must be fulfilled:

$$V_m(x,y) = V_m(x + 2s, y) \tag{7}$$

it is

$$C_2 e^{-ky} \cos kx = C_2 e^{-ky} (\cos kx \cos k2s - \sin kx \sin k2s) . \tag{8}$$

It results from equation (8) that:

$$\sin k2s = 0$$
$$\cos k2s = 1 ,$$

from which the relation between constant k and pole pitch s is obtained: $k = \frac{\pi}{s}$. Thus the distribution of magnetic potential for the open system on the plane is as follows:

$$V_m(x,y) = C_2 e^{-\frac{\pi}{s}} \cos \frac{\pi}{s} x . \tag{9}$$

The distribution of magnetic field, by components, is expressed by the formulas [14]:

$$H_x = - \frac{\partial V_m}{\partial x} = C e^{-\frac{\pi}{s} y} \sin \frac{\pi}{s} x \tag{10a}$$

$$H_y = - \frac{\partial V_m}{\partial y} = C e^{-\frac{\pi}{s} y} \cos \frac{\pi}{s} x , \tag{10b}$$

where: $C = C_2 \frac{\pi}{s}$.
The absolute value of the magnetic field intensity is:

$$H = \sqrt{H_x^2 + H_y^2} = C e^{-\frac{\pi}{s} y} . \tag{11}$$

Constant C in formula (11) is determined from the boundary condition on the surface of pole shoe of the magnetic system:

$$H(y = 0) = C = H_m \cos \frac{\pi}{s} x ,$$

where H_m is the maximum value of magnetic field intensity on the surface of the pole shoe, obtained from the measurement and depending on the magnet type, or, in case of the electro-magnetic system, on the number of coils and current intensity in the electromagnet winding.
Finally, the magnetic field distribution is expressed by the formula:

$$H = H_m e^{-\frac{\pi}{s}y} \cos \frac{\pi}{s} x . \tag{12}$$

The components of magnetic force, acting on a unit of a body volume, and the force absolute value are equal [14]:

$$\vec{f}_{mx} = \frac{1}{2} \mu_o \kappa \frac{\partial H^2}{\partial x} \vec{e}_x = (-\frac{1}{2} \mu_o \kappa H_m^2 \frac{\pi}{s} e^{-\frac{2\pi}{s}y} \sin \frac{2\pi}{s} x) \vec{e}_x \tag{13a}$$

$$\vec{f}_{my} = \frac{1}{2} \mu_o \kappa \frac{\partial H^2}{\partial y} \vec{e}_y = (-\mu_o \kappa H_m^2 \frac{\pi}{s} e^{-\frac{2\pi}{s}y} \cos^2 \frac{\pi}{s} x) \vec{e}_y \tag{13b}$$

$$f_m = \mu_o \kappa H_m^2 \frac{\pi}{s} e^{-\frac{2\pi}{s}y} \cos \frac{\pi}{s} x , \tag{14}$$

while $\vec{e}_x$, $\vec{e}_y$ are unit vectors, respectively towards axes x and y.
As it can be seen from formulas (13b) and (14), magnetic force is directed to the surface of magnetic system and its value decreases with increase of distance from this surface. The rate of decrease of the force value depends on the value of the pole pitch s. Total magnetic force should have a sufficiently large value, necessary to separate the magnetic particle from the mixture of feed particles. Therefore the pole pitch in separators for coarse-grained materials should be different than for fine-grained materials. The magnetic force in relation to s should have maximum value in order to calculate the optimum value of the pole pitch. Component F_{my} is a force attracting magnetic particles to the surface of separator band. The effectiveness of selecting magnetic particles from the feed depends on its value. Due to it, the extreme value of this component in relation to the pole pitch s is significant. Since the value of component f_{my} is changed periodically in relation to variable x, total force F_{my}, acting upon a particle of diameter d, was calculated for the average value in relation to this variable:

$$\bar{f}_{my} = \frac{1}{2} \mu_o \kappa H_m^2 \frac{\pi}{s} e^{-\frac{2\pi}{s}y} . \tag{15}$$

Thus, total force F_{my} will be equal to:

$$F_{my} = \int_{\Delta}^{\Delta+d} \bar{f}_{my}\, dy = \frac{1}{2}\mu_o \kappa H_m^2 \frac{\pi}{s} \int_{\Delta}^{\Delta+d} e^{-\frac{2\pi}{s}y}\, dy = \frac{1}{4}\mu_o \kappa H_m^2 e^{-\frac{2\pi}{s}\Delta}\left[1 - e^{-\frac{2\pi}{s}d}\right]. \quad (16)$$

The force maximum can occur for this value s for which the first derivative of the force in relation to s is equal to zero:

$$\frac{\partial F_{my}}{\partial s} = \mu_o \kappa H_m^2 \frac{2\pi}{s^2} e^{-\frac{2\pi}{s}\Delta}\left[\Delta - (\Delta + d)e^{-\frac{2\pi}{s}d}\right] = 0. \quad (17)$$

The expression for the optimum pole pitch is obtained from formula (17):

$$s = \frac{2\pi d}{\ln\dfrac{\Delta + d}{\Delta}}. \quad (18)$$

Figure 3 presents the dependence of the pole pitch on particle diameter.

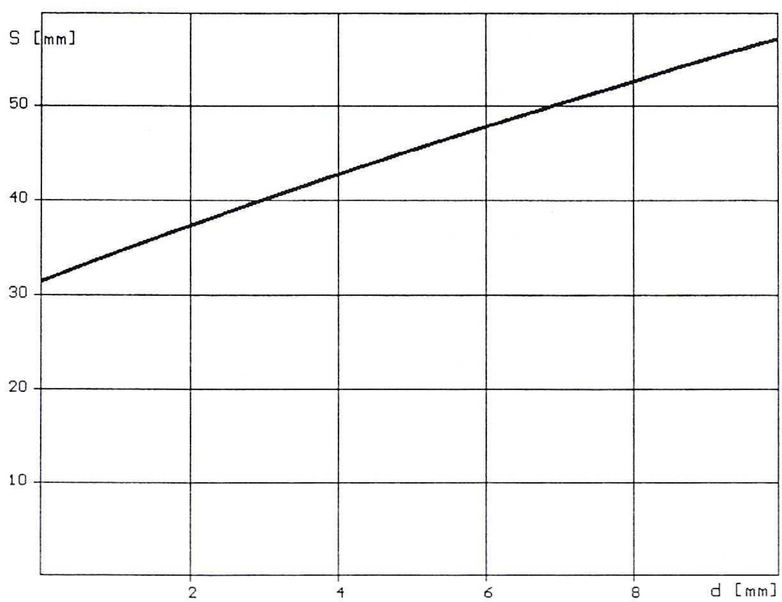

Fig. 3. Dependence of the pole pitch upon the particle size

The value of pole pitch grows with the increase of particle diameter and this growth is faster than the growth of particle diameter. This comes out from the fact that, respectively, the value of magnetic force decreases violently with the growth of distance from the pole shoe surface. In order to compensate this decrease which is manifested for large particles, the pole pitch should be enlarged. In other words, for small values of the pole pitch the magnetic stream is concentrated close to the band surface while for larger pitches the stream is dispersed in a large space.

3. Balance of acting forces and analysis of particle distribution in separation products

In the band separator the feed is continuously delivered onto the band surface. Under the effect of magnetic force the magnetic particles are subjected to flocculation, are attracted to the band surface and transported beyond the area of action of magnetic field and washed out there. Non-magnetic particles are kept in the floc volume by means of mechanical forces. During transport through the magnetic field the direction of magnetization of flocs is subject to changes during which non-magnetic particles are washed out by a stream of water from the structure of flocs. These particles are moved mechanically down the band, in the direction opposite to the band movement.

It can be seen from the above schematic diagram of the process that there are different mechanisms of dispersion of non-magnetic particles into the magnetic product (concentrate) and magnetic particles into the non-magnetic product (tailings). Non-magnetic particles are transferred to the concentrate because of flocculation while magnetic particles find their way to the tailings because of the turbulent movement of water washing down the band surface and heterogeneity of the liquid velocity field.

A spherical mineral particle, situated on the band surface, inclined to the horizontal direction at angle φ, is affected by a set of mechanical forces and a set of magnetic forces. The mechanical forces comprise:

a) component of gravity force (G_x):

$$G_x = \frac{\pi d^3}{6}(\rho - \rho_o)\, g \sin \varphi, \tag{19}$$

where: d – particle diameter, ρ – particle density, ρ_o – water density, g – acceleration of gravity, φ – inclination angle of band to the horizontal level,

b) hydrodynamic force (F_o), expressed by Newton – Rittinger's formula, acting along axis x (along the band) [15]:

$$F_o = \frac{\pi}{12}\rho_o d^2 u^2, \tag{20}$$

where: u – velocity of water motion on the band surface, measured in relation to the band,

c) force of friction of particles against the band surface (T_e):

$$T_e = -f_e N_e, \tag{21}$$

where: f_e – coefficient of external friction of particles against the band surface, N_e – load force,

d) force of internal friction of non-magnetic particles, contain in floc volume:

$$T_i = -f_i F_i, \tag{22}$$

where: f_i – coefficient of internal friction, F_i – force of particle interactions.
The magnetic forces comprise:
e) external force F_{my}, acting on the particle from the heterogeneous magnetic field of the separator, directed along axis y:

$$F_{my} = -\frac{\pi d^3}{6}\mu_o \kappa H_m^2 \frac{\pi}{s} e^{-\frac{2\pi}{s}y} \cos^2\frac{\pi}{s}x \tag{23a}$$

and its average value

$$\bar{F}_{my} = -\frac{\pi d^3}{12}\mu_o \kappa H_m^2 \frac{\pi}{s} e^{-\frac{2\pi}{s}y}, \tag{23b}$$

f) internal force F_i of magnetic particle interactions [2]:

$$F_i = -\frac{\pi^2 \kappa^2 d^4 H_m^2 e^{-\frac{2\pi}{s}y}}{k_1(1+\kappa N)^2 r^2}, \tag{24}$$

where: r – average distance between magnetic particles, N – demagnetization coefficient, k_1 – coefficient depending on the system of units.
The internal force of particle interactions F_i is a coulomb force as it has been observed that in the separator working space the particle magnetic interactions are of a coulomb character and not of a dipole – dipole type [4].
The force of load occurring in formula (21) is a sum of component y of the force of gravity and magnetic force:

$$N_e = -\left[\frac{\pi d^3}{6}(\rho - \rho_o) g \cos\varphi + \bar{F}_{my}\right]. \tag{25}$$

In the conditions of equilibrium, for the set motion of particles along the band, the algebraic sum of components x of external forces is equal to zero:

$$G_x + F_o - T_e = 0 \tag{26a}$$

$$\frac{\pi d^3}{6}(\rho - \rho_o) g \sin\varphi + \frac{\pi}{12}\rho_o d^2 u^2 - f_e\left[\frac{1}{2}\mu_o \kappa H_m^2 \frac{\pi}{s} e^{-\frac{2\pi}{s}y} + (\rho - \rho_o) g \cos\varphi\right]\frac{\pi d^3}{6} = 0, \tag{26b}$$

The value of particle magnetic susceptibility κ_p, can be calculated from equation (26b) which divides the feed into two components – the non-magnetic component in which magnetic susceptibility of particles fulfills the condition $\kappa < \kappa_p$ and the magnetic component for particles of susceptibility $\kappa > \kappa_p$:

$$\kappa_p = \frac{[2(\rho - \rho_o) g d \sin\varphi + \rho_o u^2 - 2 f_e (\rho - \rho_o) g d \cos\varphi] s}{\pi \mu_o f_e d\, H_m^2\, e^{-\frac{2\pi}{s} y}} . \tag{27}$$

Such a method of deriving the expression for separation magnetic susceptibility, considering only external forces acting upon the particle and neglecting particle interactions, corresponds to the situation in which each particle separately, without the presence of neighbouring particles, moves in the separator working space. The existence of forces of internal interactions between particles affects the dispersion of magnetic and non-magnetic particles into improper separation products and, particularly, magnetic particles into non-magnetic product.

It results from formula (27) that the value of magnetic susceptibility κ_p increases with the growth of the band inclination angle φ, growth of the pole pitch s, growth of velocity of water motion on the band surface u, growth of the distance from the band surface y, (which depends on separators productivity), and it also increases with the decrease of magnetic field intensity H_m. It is also larger for finer particles. Therefore all the above factors affect the separation results.

4. Dependence of particle magnetic susceptibility on magnetite volume content

For the sake of simplification it was assumed that magnetite inclusions are of spherical shape. Let magnetite inclusions be arranged uniformly in the particle volume. A sphere of radius r (Fig.4) was cut around the inclusion of radius a. Only one inclusion is found inside this sphere. The volume content of magnetite λ in the considered spherical space will be:

$$\lambda = \frac{a^3}{r^3} \tag{28}$$

and, consequently

$$r = \frac{a}{\sqrt[3]{\lambda}} . \tag{29}$$

The sphere of radius a was placed in homogeneous magnetic field of intensity $\vec{H}_o$, directed horizontally along axis x (Fig.4). Magnetic field intensity inside the sphere will be [10]:

$$\vec{H}_1 = \frac{3}{\mu + 2} \vec{H}_o = (1 - k) \vec{H}_o , \tag{30}$$

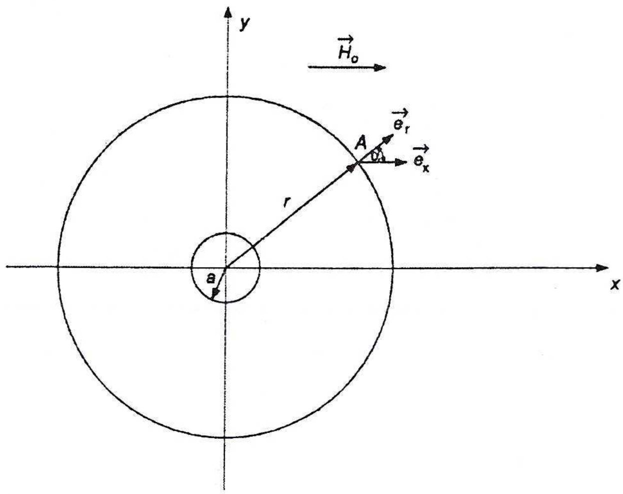

Fig. 4. Spherical inclusion of magnetite surrounded by sphere

where: $k = \dfrac{\mu - 1}{\mu + 2}$, μ – relative magnetic permeability of magnetite. Simultaneously, it was assumed that magnetic permeability of the medium in which the sphere is immersed is equal to 1. The sphere of ferromagnetic material (which is magnetite), placed in the magnetic field, is subject to magnetization and acts as a dipole of the following dipole moment [1]:

$$\vec{m} = k\, a^3\, \vec{H}_o . \tag{31}$$

Accordingly, magnetic field intensity outside the sphere in point A (Fig.4) will be:

$$\vec{H}_2 = \vec{H}_o + \vec{H}_{dip}(\vec{r}), \tag{32}$$

where $\vec{H}_{dip}$ is magnetic field intensity of a dipole and equals [10]:

$$\vec{H}_{dip} = \frac{3(\vec{m}\cdot\vec{r})\vec{r}}{r^5} - \frac{\vec{m}}{r^3}. \tag{33}$$

After taking into consideration (31), expression (33) will take the form:

$$\vec{H}_{dip} = \frac{3\, k\, H_o\, a^3}{r^3}\cos\alpha\, \vec{e}_r - \frac{k\, H_o\, a^3}{r^3}\vec{e}_x, \tag{34}$$

where $\vec{e}_r$ and $\vec{e}_x$ are units vectors, accordingly towards vector $\vec{r}$ and axis x, while α is an angle of inclination of vector $\vec{r}$ to axis x (Fig.4).
According to the above, magnetic field intensity $\vec{H}_2$ and its x-th component H_{2x} will be:

$$\vec{H}_2 = \frac{3\,k\,H_o\,a^3}{r^3}\cos\alpha\,\vec{e}_r + \left(H_o - \frac{k\,H_o\,a^3}{r^3}\right)\vec{e}_x \tag{35a}$$

$$H_{2x} = H_o - \frac{k\,H_o\,a^3}{r^3} + \frac{3\,k\,H_o\,a^3}{r^3}\cos^2\alpha. \tag{35b}$$

After taking into consideration dependence (29), the x-th field component outside the sphere will be connected with the volume content of magnetite by the following formula:

$$H_{2x} = H_o(1 - k\,\lambda + 3\,k\,\lambda\cos^2\alpha)\,. \tag{36}$$

Since the field outside the sphere is symmetrical against axis x, therefore it can be assumed that the average value of field H_{2x} in the first quarter $(0 - \pi/2)$ will an average value of field outside the sphere. Thus:

$$\bar{H}_{2x} = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} H_{2x}\,d\alpha = H_o\left(1 + \frac{1}{2}k\,\lambda\right). \tag{37}$$

It can be seen from formula (37) that the average value of the field outside the sphere is larger than the field H_o by the component $\frac{1}{2}\,k\,\lambda\,H_o$.

Formula (37) can be generalized and it can be stated that it represents an average value of the field outside the sphere inside of which the volume content of magnetite is λ.
There are many inclusions of magnetite in the particle which interact mutually. The calculations considered the field resulting from interactions of two closest neighbours (inclusions of magnetite) on the line connecting their centers. In such a case additional fields (second component in formula (37)) sum up and the resultant field equals [9]:

$$\bar{H}_{2x} = H_o(1 + k\,\lambda)\,. \tag{38}$$

In order to determine the particle magnetic susceptibility a sphere of radius $R >> r$ is cut out of the particle volume and it is placed in the homogeneous field H_o with magnetic permeability of the medium equal to 1. Analogically to formula (30), magnetic field intensity inside this sphere will be:

$$H_w = H_o(1 - \bar{k})\,, \tag{39}$$

where: $\bar{k} = \frac{\bar{\mu} - 1}{\bar{\mu} + 2}$, while $\bar{\mu}$ is relative magnetic permeability of particle.

On other hand, however, the field inside the sphere can be expressed by the formula:

$$H_w = H_1\,\lambda + \bar{H}_{2x}(1 - \lambda)\,. \tag{40}$$

Substituting H_1 and $\bar{H}_{2x}$ from formulas (30) and (38) we obtain:

$$H_w = H_o(1 - k\,\lambda^2)\,. \tag{41}$$

Comparing (39) and (41) we can obtain an expression for dependence of particle magnetic permeability upon magnetite volume content:

$$\bar{k} = k\,\lambda^2\,. \tag{42}$$

Taking into consideration that $\mu = \kappa_{sm} + 1$ and $\bar{\mu} = \kappa_s + 1$ where κ_{sm} is the volume susceptibility of magnetite while κ_s the volume susceptibility of particle material, formula (42) assumes the form:

$$\frac{\kappa_s}{1 + \frac{1}{3}\,\kappa_s} = \frac{\kappa_{sm}}{1 + \frac{1}{3}\,\kappa_{sm}}\,\lambda^2 \tag{43a}$$

$$\kappa = \kappa_m\,\lambda^2\,, \tag{43b}$$

where: $\kappa = \dfrac{\kappa_s}{1 + \frac{1}{3}\,\kappa_s}$ and $\kappa_m = \dfrac{\kappa_m}{1 + \frac{1}{3}\,\kappa_{sm}}$ denote, respectively, magnetic susceptibility of a spherical particle and susceptibility of magnetite spherical inclusion for which the coefficient of demagnetizing is 1/3. This formula can be generalized for any shapes of particles and inclusions yet in place of factor 1/3 the coefficient of demagnetizing for a given shape of particle and inclusion will appear while κ will be the volume magnetic susceptibility of particles of a given shape.

5. Theoretical indexes of the process of enrichment

When the dependence of particle magnetic susceptibility on magnetite content and separation magnetic susceptibility is known, it is possible to calculate theoretical enrichment indexes of the raw material, namely the theoretical content of the magnetic component in the concentrate (β_t) and tailings (ϑ_t), enrichment ability rate (K) and recovery of magnetic component in the concentrate (ε_t). According to definition of these values, they are expressed by the following formulas [13]:

$$\vartheta_t = \int_0^{\lambda_p} \lambda\, f(\lambda)\, d\lambda \tag{44}$$

$$\beta_t = \frac{\alpha - \vartheta_t \int_0^{\kappa_p} f(\kappa)\, d\kappa}{1 - \int_0^{\kappa_p} f(\kappa)\, d\kappa} \tag{45}$$

$$K = \frac{\beta_t}{\alpha} \tag{46}$$

$$\varepsilon_t = \left(1 - \int_0^{\kappa_p} f(\kappa)\, d\kappa\right)\frac{\beta_t}{\alpha}, \tag{47}$$

where $f(\kappa)$ presents the frequency function of magnetic susceptibility of particles in the feed, α is the average content of the magnetic component in the feed, $f(\lambda)$ is the frequency function of the content of magnetic component in the feed while $\lambda_p = \sqrt{\frac{\kappa_p}{\kappa_m}}$.

The frequency function of magnetic susceptibility, occurring in the above formulas, can be obtained from the dispersive model of particle [3]. In this model the continuous phase (waste rock) is the matrix in which the magnetite inclusions are suspended, forming a dispersed phase. When we assume monodispersivity of the particle size distribution function of a sample and dispersed phase as well as Poisson's distribution of the number of magnetite inclusions in the feed particles, an expression can be derived for a particle magnetic susceptibility distribution function which can be generalized into any dispersed (polydispersive) phase.

A general form of the distribution function is as follows [3]:

$$F(\kappa) = c_1\, I\left[K_1; 1 + K_2(\kappa - \kappa_o)^{\frac{1}{2}}\right] + c_2, \tag{48}$$

where: $I(x)$ – incomplete gamma function, c_1 and c_2 – scale parameters, κ_o – magnetic susceptibility of the continuous phase whereas constants K_1 and K_2 are expressed by physical properties of the dispersive system (κ_m, κ_o, size distribution function of magnetite inclusions).

An advantage of expression (48) is that all its constants can be interpreted physically. Its disadvantage is the need of use of special tables of the incomplete gamma function in calculations [12].

The frequency function of magnetic susceptibility can be also expressed by means of W e i b u l l ' s distribution [6]. This is a purely empirical approach which cannot be grounded by models, in spite of the fact that W e i b u l l ' s distribution belongs to the family of gamma distributions. The form of this frequency function and its cumulative distribution function are as follows:

$$f(\kappa) = m\,r\,\kappa^{r-1}\exp(-m\,\kappa^{r}) \tag{49a}$$

$$F(\kappa) = 1 - \exp(-m\,\kappa^{r}), \tag{49b}$$

where: m, r – distribution parameters.

Parameter m of the distribution can be expressed by an average value of magnetic susceptibility of a sample and parameter r. The average value of W e i b u l l ' s distribution is [6]:

$$\bar{\kappa} = \left(\frac{1}{m}\right)^{\frac{1}{r}} \Gamma\left(\frac{1}{r} + 1\right) \tag{50}$$

hence:

$$m = \left[\frac{\Gamma\left(\frac{1}{r} + 1\right)}{\bar{\kappa}}\right]^{r}. \tag{51}$$

Γ denotes the gamma function in the above formulas.

The frequency function of magnetite content in formula (44) can be obtained applying the theorem concerning the distribution of variable random functions [6]:

$$f(\lambda) = f[\kappa(\lambda)]|\kappa'(\lambda)|, \tag{52}$$

where $\kappa'(\lambda)$ is a derivative of function (43b) against λ.

Taking into account expressions (49a) and (43b) we can obtain the frequency function of magnetite content:

$$f(\lambda) = u\,s\,\lambda^{u-1}\exp(-s\,\lambda^{u}), \tag{53}$$

where: $u = 2r$, $s = m\,\kappa_m^r$ – distribution parameters.

Thus that is also W e i b u l l ' s distribution. The value of parameter s, analogically as for the distribution of magnetic susceptibility, can be connected with the average content of magnetite in the sample α_m and parameter $u = 2r$ by means of the following formula:

$$s = \left[\frac{\Gamma\left(\frac{1}{2r} + 1\right)}{\alpha_m}\right]^{2r}. \tag{54}$$

Taking into account distribution (53), a theoretical content of magnetic component in tailings will be:

$$\vartheta_t = s\,u \int_0^{\lambda_p} \lambda^u \exp(-s\,\lambda^u)\, d\lambda . \tag{55}$$

After substituting $s\,\lambda^u = t$ we obtain:

$$\vartheta_t = \frac{1}{s^{\frac{1}{u}}} \int_0^{s\lambda_p^u} t^{\frac{1}{u}} \exp(-t)\, dt . \tag{56}$$

The above integer constitutes an incomplete gamma function $\gamma\left(1 + \frac{1}{u}; s\,\lambda_p^u\right)$ [7].

Therefore, taking into consideration expression (54) and the relation $u = 2r$, the theoretical content of the magnetic component in tailings will be:

$$\vartheta_t = \frac{\alpha_m}{\Gamma\left(1 + \frac{1}{2r}\right)}\, \gamma\left(1 + \frac{1}{2r}; s\,\lambda_p^{2r}\right) . \tag{57}$$

On the other hand, the theoretical recovery of the magnetic component in the concentrate will be expressed by the formula:

$$\varepsilon_t = 1 - \frac{\vartheta_t}{\alpha_m} F(\kappa_p) = 1 - \frac{\gamma\left(1 + \frac{1}{2r}; s\,\lambda_p^{2r}\right)}{\Gamma\left(1 + \frac{1}{2r}\right)} [1 - \exp(-m\kappa_p^r)] . \tag{58}$$

6. Conclusions

1. Determining the distribution of magnetic field in the separators working space enables the value of magnetic force, acting upon the particle, to be calculated and, consequently, the optimization of the pole pitch of the magnetic system.
2. From the balance of all forces – magnetic and mechanical, specific for a given separator, it is possible to calculate the value of separation magnetic susceptibility, dividing the enriched material into the part characteristic for the tailings and the part characteristic for the concentrate and depending on separation conditions.
3. The distribution of magnetic susceptibility of feed particles together with the dependence of susceptibility upon the magnetite volume content contributes to

calculating the theoretical indexes of raw material enrichment ability in the function of separation magnetic susceptibility and, accordingly, separation conditions.

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