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DANUTA KRZYSZTOŃ*, URSZULA SANETRA**

INVESTIGATIONS OF ROCKS UNDER TRIAXIAL COMPRESSION AT CONFINING PRESSURE FROM 0 TO 70 MPa

BADANIA TRÓJOSIOWEGO ŚCISKANIA SKAŁ PRZY WARTOŚCI CIŚNIENIA OKÓLNEGO W ZAKRESIE 0–70 MPa

The results of triaxial compression of rock samples in a stiff testing machine using a 70 MPa pressure chamber are presented. Experiments were carried out at the constant rate of longitudinal strain of a sample ($10^{-5} \cdot s^{-1}$) and at a constant confining pressure of 0, 5, 10, 15, 20, 30, 50, 70 MPa. The samples of fine-grained sandstone, siltstone and coal, at a total of 151 samples, were tested. For each rock type 3 to 5 experiments were conducted at given confining pressure.

The following rock properties: critical stress, critical strain, residual stress, residual strain and the modulus of softening were determined at different values of the confining pressure. The average values of the determined magnitudes are listed in Table 1 and presented in the form of functional dependences in the graphs (Fig. 2 to 6).

The angle of internal friction and cohesion were defined by three methods: the method of tangents to Mohr's circles, the *q–p* method and the method of two tangents to the parabolic envelope of Mohr's circles. The obtained results are listed in Table 2.

On the basis of the equation of Mohr's circles parabolic envelope, the angle of internal friction, being a function of normal stress, was determined for a compact rock in the pre-critical state and for a fractured rock in the post-critical state.

The results of triaxial tests were applied for determining the rock maximum strength according to the Hoek-Brown criterion. The obtained envelopes of maximum strength were compared with the envelopes of Mohr's circles. Then the parameters occurring in Hoek-Brown's criterion (m, s) and the corresponding magnitudes resulting from Mohr's circles envelope (φ, c) were compared.

In the conclusions, attention was paid to the increase in the modulus of softening in the low range values of confining pressure (0 to 10 MPa), and then to the decrease of this modulus with the increase of confining pressure in the range from 10 to 70 MPa.

Attention should also be paid to the determination of the angle of internal friction and cohesion by the proposed method of tangents to the parabolic envelope of Mohr's circles (Sanetra 2002). According to this method, the angle of internal friction and cohesion can be determined in the whole range of normal stress, both for compact and fractured rocks.

* WYDZIAŁ GÓRNICZY, AKADEMIA GÓRNICZO-HUTNICZA, 30-059 KRAKÓW, AL. MICKIEWICZA 30, POLAND.

** GŁÓWNY INSTYTUT GÓRNICTWA, 40-166 KATOWICE, PLAC GWARKÓW 1, POLAND.

Key words: rock stress-strain properties, angle of internal friction and cohesion, confining pressure, functional dependences, traxial strength

Przedstawiono wyniki badań trójosiowego ściskania próbek skalnych w sztywnej maszynie wytrzymałościowej przy zastosowaniu komory ciśnieniowej 70 MPa (rys.1).

Eksperyment prowadzono ze stałą prędkością odkształcenia podłużnego próbki wynoszącą $10^{-5} \cdot s^{-1}$. Stosowano 8 poziomów ciśnienia okólnego: 0, 5, 10, 15, 20, 30, 50, 70 MPa. Badano próbki piaskowca drobnoziarnistego, iłowca i węgla o łącznej liczbie 151. Dla każdego rodzaju skały przeprowadzono 3–5 eksperymentów dla zadanego ciśnienia okólnego.

Wyniki eksperymentalne uzyskiwano w postaci całkowitej charakterystyki naprężeniowo-odkształceniowej, z częścią wznoszącą i opadającą, na podstawie której określano następujące właściwości skał:

- naprężenie maksymalne zwane naprężeniem krytycznym,
- odkształcenie krytyczne odpowiadające naprężeniu krytycznemu,
- moduł odkształcenia podłużnego określany na podstawie stycznej do wznoszącej (przedkrytycznej) części charakterystyki,
- naprężenie resztkowe przedstawiające minimalną wartość naprężenia w części opadającej (pokrytycznej) charakterystyki,
- odkształcenie resztkowe odpowiadające naprężeniu resztkowemu,
- moduł osłabienia określony na podstawie nachylenia stycznej do pokrytycznej części charakterystyki, wyznaczony jako tangens kąta ostrego pomiędzy styczną a osią odkształcenia.

Średnie wartości określonych wielkości dla stosowanego ciśnienia okólnego zestawiono w tablicy 1 i przedstawiono w postaci zależności funkcyjnych na wykresach (rys. 2–6).

Wyznaczono kąt tarcia wewnętrznego i spójność trzema metodami: metodą graficzną w układzie współrzędnych (τ–σ), metodą punktową w układzie (q–p), oraz metodą dwóch stycznych do parabolicznej obwiedni kół Mohra. Uzyskane wyniki zestawiono w tablicy 2.

Mając określone równanie parabolicznej obwiedni kół Mohra wyznaczono kąt tarcia wewnętrznego jako funkcję naprężenia normalnego, zarówno dla skały zwięzłej w stanie przedkrytycznym, jak również dla skały spękanej występującej w stanie pokrytycznym (rys. 7a, b, c).

Wyniki trójosiowych badań zastosowano do wyznaczenia wytrzymałości skał zwięzłych według kryterium Hoeka-Browna (tabl. 3). Uzyskane obwiednie maksymalnej wytrzymałości (rys. 8a, b,c, d) porównano z obwiedniami kół Mohra (Sanetra, Szedel 2000; Sanetra 2002). Przeprowadzono analizę parametrów występujących w kryterium Hoeka-Browna: m, s i odpowiadających wielkości wynikających z obwiedni kół Mohra: φ, c (tabl. 4).

Na podstawie uzyskanych wyników sformułowano wnioski, które na ogół są zgodne z wynikami wcześniejszych badań (Kwaśniewski 1983, 1986; Tajduś 1990; Sanetra 1994a, b; Krzystoń i in.1998):

- Wzrost ciśnienia okólnego powoduje wzrost naprężenia krytycznego, odkształcenia krytycznego, naprężenia resztkowego, odkształcenia resztkowego oraz na ogół zmniejszanie się modułu osłabienia.
- Ilościowa zmiana poszczególnych wielkości zależy od rodzaju skały i zakresu stosowanego ciśnienia okólnego.
- Na uwagę zasługuje znacznie większy wzrost naprężenia resztkowego niż naprężenia krytycznego wraz ze wzrostem ciśnienia okólnego.
- W zakresie niskich wartości ciśnienia okólnego (5–10 MPa) występuje wzrost modułu osłabienia poprzedzający ogólną tendencję dalszego zmniejszania się modułu.
- Wzrost modułu osłabienia (odkształcenia) w zakresie niskich wartości ciśnienia okólnego (5–10 MPa) względem wartości uzyskanej w jednoosiowym stanie naprężenia można uzasadniać „pamięcią" materiału skalnego o wzmocnieniu odkształcenia występującego w przedkrytycznej części charakterystyki..
- Kąt tarcia wewnętrznego zależy od rodzaju skały i zmniejsza się wraz ze wzrostem naprężenia normalnego.
- Kąt tarcia wewnętrznego skały spękanej jest nieznacznie mniejszy niż skały zwięzłej, natomiast spójność skały spękanej jest wielokrotnie mniejsza niż skały zwięzłej.

- Bezwymiarowe stałe m i s występujące w kryterium Hocka-Browna są w przybliżeniu analogiczne odpowiednio do kąta tarcia wewnętrznego i spójności określanych w kryterium zniszczenia Coulomba-Mohra.
- Stała materiałowa m oraz kąt tarcia wewnętrznego φ dla skał o strukturze zwięzłej przyjmują znacznie wyższe wartości dla skał mocnych (piaskowiec drobnoziarnisty) niż dla skał słabych (iłowiec, węgiel).
- Stała materiałowa m dla skał o strukturze zniszczonej (spękanej) przyjmuje zbliżone do siebie wartości zarówno dla skał mocnych, jak i słabych.
- Spójność c oraz stała materiałowa s skał spękanych znacznie spada w stosunku do tych wartości dla skał zwięzłych (Hoek, Brown 1980; Kidybiński 1982).

Analiza ilościowych zmian własności skał pod wpływem ciśnienia okólnego, wzrastającego w zakresie 0–70 MPa, pozwala na wyciągnięcie następujących ogólnych wniosków:

- spękane struktury skalne mogą spełniać warunek nośności dla określonych wartości ciśnienia okólnego i kąta tarcia wewnętrznego (Krzysztoń 2000);
- pokrytyczne właściwości skał zależą nie tylko od typu skały (piaskowiec, iłowiec, węgiel) ale również od struktury skały (piaskowiec nr 1 i nr 3);
- znajomość wartości kąta tarcia wewnętrznego i kohezji dla węgla i skał płonnych może mieć zastosowanie w określaniu wytrzymałości filarów węglowych przy uwzględnieniu warunków kontaktowych w układzie: strop–filar–spąg (Krzysztoń 2002);
- wzrost modułu osłabienia w zakresie niskich ciśnień okólnych, przejawiający „pamięć" materiału skalnego, świadczy o konieczności uwzględnienia efektów lepkich w konstytutywnym modelu pokrytycznej części charakterystyki naprężeniowo-odkształceniowej (Nawrocki, Mróz 1992).

Słowa kluczowe: właściwości naprężeniowo-odkształceniowe skał, kąt tarcia wewnętrznego i spójność, ciśnienie okólne, zależności funkcyjne, trójosiowa wytrzymałość

1. Introduction

Numerous Polish and foreign research centres carry out investigations on stress--strain properties in a triaxial state of stress. Such investigations have been conducted for many years at the Central Mining Institute, Katowice (Smołka 1994; Sanetra 1994a, b; Krzysztoń, Sanetra, Szedel 1998). Experiments were carried out in a stiff testing machine using a 30 MPa pressure chamber. These experiments were conducted at a constant rate of longitudinal strain of a sample (10^{-5} s^{-1}) and at constant confining pressures of 0, 10, 20, 30 MPa. The experimental results took the form of a complete stress-strain characteristic, with the ascending and descending parts, on the basis of which the following rock properties were determined:

- maximal stress, called the critical stress,
- critical strain corresponding to the critical stress,
- modulus of longitudinal strain determined on the basis of the slope of tangent to ascending (pre-critical) part of the stress-strain characteristic,
- residual stress presenting the minimal value of stress in the descending (post--critical) part of the stress-strain characteristic,
- residual strain, i.e. the strain corresponding to the residual stress,
- modulus of softening determined on the basis of the slope of tangent to the post-critical part of characteristic, i.e. as the tangent of acute angle between the tangent and the strain axis.

Next the influence of the confining pressure on stress-strain properties was analysed. It has been shown that the values of the investigated stress-strain parameters increased linearly with the increase of confining pressure; however, the rate of increase depends on the investigated parameter and on the type of rock. The modulus of longitudinal strain did not show uniform changes with the increase in confining pressure. However, the modulus of softening in general decreased with the increase in confining pressure.

The angle of internal friction and cohesion were determined by three methods: the method of tangents to Mohr's circles, the *q–p* method and the computational method according to the formulae resulting from Coulomb's theory (Bukowska, Sanetra, Szedel 1998).

Analysis of the results showed that in investigations of the triaxial strength of rocks, a greater number of confining pressure values should be applied to determine a more accurate picture of the critical-strength envelope as well as the residual-strength envelope. Also, more values of confining pressure in the low pressure range 0 to 10 MPa, 10 to 20 MPa should be introduced, as in these ranges experimental points were missing to obtain the correct curve to fit the stress-strain dependences as a function of the confining pressure.

This work presents the results of an investigation of triaxial compression of rock samples in a stiff testing machine using the new 70 MPa compression chamber (Krzysztoń et al. 2002). Eight levels of confining pressure: 0, 5, 10, 15, 20, 30, 50, 70 MPa were applied. Samples of fine-grained sandstone, siltstone and coal, totalling of 151 samples, were tested. The following rock properties: critical stress, critical strain, residual stress, residual strain and the modulus of softening were determined at different values of confining pressure. The results obtained, in the form of the functional dependences, are shown in the graphs.

The equation of Mohr's circles parabolic envelope was determined using the least square method; then the angle of internal friction as a function of normal stress was determined for a compact rock in the pre-critical state and for a fractured rock in the post-critical state.

The results of triaxial tests were then used to determine the maximum strength of the rock-samples according to the Hoek-Brown criterion. The resulting envelopes of maximum strength thus obtained were compared with the Mohr's circles envelopes. An analysis of the parameters occurring in Hoek-Brown's criterion (m, s) and the corresponding magnitudes resulting from Mohr's circles envelope (φ, c) was carried out.

In the conclusions, attention is focused on the increase in the modulus of softening in the low-value range of confining pressure (0 to 10 MPa), and then to the decrease of this modulus as the confining pressure in the 10 to 70 MPa range increases.

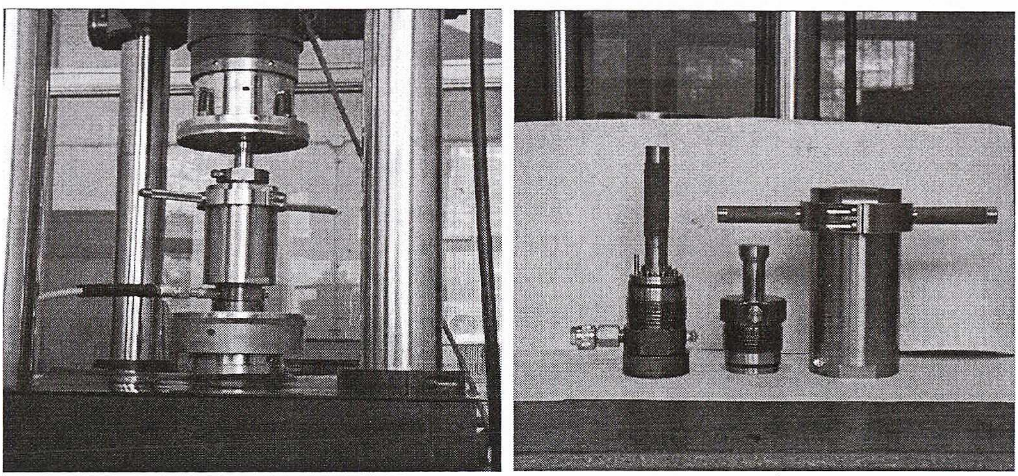
Attention is also drawn to the determination of the angle of internal friction and cohesion by the proposed method of tangents to the parabolic Mohr's circles envelope (Sanetra 2002). According to this method, the angle of internal friction and cohesion can be determined throughout the whole range of normal stress, both for compact and fractured rocks.

The complete stress-strain characteristic presents the pre-critical strain hardening as well as the post-critical strain softening of the rock subjected to a compression (Yoshinaka et al. 1996).

In this work, special attention is paid to the post-critical properties of rocks in fractured state. Determination of the behaviour of fractured rocks is of great importance in the geo-engineering practice. Although the rock surrounding the working or tunnel becomes damaged in the course of driving progress, it may provide residual support for the overburden strata. The increase in confining pressure causes the increase in critical stress and the decrease in the modulus of softening (Bieniawski 1970, Rummel and Fairhurst 1970). It also induces the increase in the residual stress as well as the extent of the fractured zone, which has an influence on the long-term stability of underground openings (Nawrocki, Mróz 1992).

2. Description of experiments

Experiments in a triaxial state of stresses were carried out in a stiff testing machine MTS-810 NEW by the conventional method: $\sigma_1 \neq \sigma_2 = \sigma_3$. The vertical stress σ_1 was set up by the axial loading of a sample by a plate of the testing machine and the horizontal stresses were induced by a hydrostatic oil pressure. The pressure chamber 70 MPa, type KTK, produced by UNIPRESS in Warsaw (Fig. 1a) and a U2 type compressor,



a b

Fig. 1. Pressure chamber 70 MPa type KTK
a — the chamber located in a stiff testing machine MTS 810-NEW;
b — preparation of samples to tests in a chamber

Rys.1. Komora ciśnieniowa 70 MPa typ KTK
a — komora umieszczona w maszynie wytrzymałościowej MTS 810-NEW;
b — przygotowanie próbek do badań w komorze

enabling constant pressure on the chosen level to be maintained during the experiment, were used.

The tests were carried out with a constant rate of longitudinal strain $\dot{\varepsilon} = 10^{-5} \cdot s^{-1}$, i.e. with the strain rate corresponding to strain rates of rocks in the vicinity of preparatory or excavation workings (Kwaśniewski 1986) at set confining pressures $p = 0, 5, 10, 15, 20, 30, 50, 70$ MPa. To prevent oil penetration into the sample, a rubber casing and a special shrinkable lag, closely adherent to the surface of the sample, were applied (Fig. 1b).

The complete stress-strain characteristic obtained in the experiment describes the pre- and post-critical states of the rock sample.

The tests were made on cylindrical samples of diameter $\phi = 30$ mm and height $h = 60$ mm, prepared according to the PN-G-04303 standard.

Three types of Carboniferous rock taken from the saddle strata of Hard Coal Mine "Polska Wirek" were investigated:

- fine-grained sandstone taken from seam 502/II roof in two places (samples no. 1 and no. 3),
- siltstone from seam 502/II roof (sample no. 2),
- coal from seam 502/II (sample no. 4).

For each rock type 3 to 5 experiments were carried out for the given pressure range. In all, 151 samples were tested, including 79 samples of fine-grained sandstone, 41 samples of siltstone and 31 samples of coal.

As a result of these experiments in uniaxial and triaxial states of stress, the following properties of the investigated rocks were determined: critical stress, critical strain, residual stress, "residual" strain and modulus of softening at different confining pressure values. The tensile strength in uniaxial state of stress was also determined. Next the values of the internal friction angle and of the cohesion were defined by different methods.

3. Results of investigations

The Carboniferous rocks of the Upper Silesian Coal Basin tested had the following values for uniaxial compressive strength R_c and tensile strength R_r:

- fine-grained sandstone (samples no. 1 and no. 3): $R_c = 116.2$ to 123.2 MPa, $R_r = 7.03$ to 7.23 MPa,
- siltstone (sample no. 2): $R_c = 60.3$ MPa, $R_r = 1.53$ MPa,
- semi-brightly-dull coal (sample no.4): $R_c = 24.4$ MPa, $R_r = 1.30$ MPa.

The average values of stress-strain parameters of the rocks investigated are listed in Table 1. The form of the obtained stress-strain characteristics for the investigated rocks (Fig. 2a, b, c) shows that the increase in confining pressure from 0 to 70 MPa (curves 1 to 8) causes an increase in the maximal (critical) stress and in the critical strain.

A considerable increase in residual stress was observed with the corresponding residual strain (Krzysztoń et al. 1998, 2002; Sanetra 2002). However, an increase

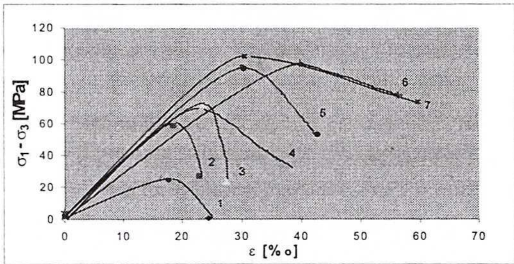
TABLE 1

Average values of stress-strain parameters for the investigated Carboniferous rocks from the Hard Coal Mine "Polska-Wirek"

TABLICA 1

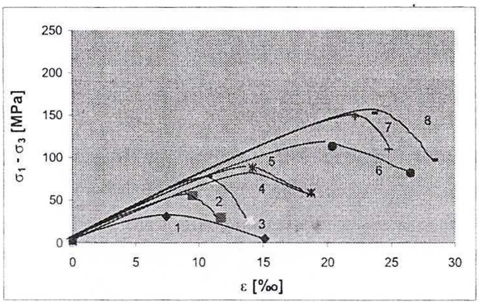
Średnie wartości parametrów naprężeniowo-odkształceniowych badanych skał karbońskich z KWK Polska-Wirek

Confining pressure $\sigma_3 = \sigma_2 = p$ [MPa]	Sample no/amount of samples	Critical stress $\sigma_{cr} = \sigma_1$ [MPa]	Residual stress σ_r [MPa]	Critical strain ε_{cr} [‰]	Residual strain ε_r [‰]	Modulus of weakening M [GPa]
0	1/5	116.2	5.61	7.83	13.70	86.1
5	1a/5	200.8	60.90	14.79	17.15	104.7
10	1b/5	218.5	70.35	17.68	20.00	105.8
15	1c/5	235.8	109.72	18.05	22.70	81.7
20	1d/3	259.7	127.22	19.38	21.26	80.1
30	1e/3	327.0	161.70	23.94	28.04	79.7
50	1f/5	404.6	234.30	28.35	31.67	69.5
70	1g/4	522.8	312.62	33.03	40.40	54.0
0	2/5	30.7	4.98	7.43	15.14	17.2
5	2a/5	60.3	33.92	9.59	12.85	22.1
10	2b/4	84.8	36.99	11.09	15.92	20.9
15	2c/5	95.7	71.69	14.13	20.62	12.7
20	2d/5	107.8	78.57	14.17	17.29	15.5
30	2e/5	143.1	111.80	20.39	26.47	15.4
50	2f/5	198.3	159.46	22.18	24.80	14.8
70	2g/2	222.8	167.23	23.54	28.26	12.5
0	3/6	123.2	2.72	9.11	10.50	136.2
5	3a/5	157.0	57.79	16.36	18.55	91.1
10	3b/3	158.3	20.17	14.31	17.36	120.6
15	3c/3	193.8	98.79	19.73	21.99	82.2
20	3d/3	219.5	117.80	20.10	26.73	77.8
30	3e/4	310.0	127.72	22.67	28.61	52.1
50	3f/5	386.7	221.51	26.88	30.15	48.8
70	3g/5	482.8	339.04	32.00	53.18	47.3
0	4/7	24.4	0.71	17.70	24.50	14.1
5	4a/4	63.9	31.91	18.54	25.57	13.9
10	4b/3	81.6	33.41	23.49	27.36	15.2
15	4c/3	83.1	46.24	22.69	37.60	13.9
20	4d/3	122.2	93.51	30.46	59.63	7.1
30	4e/3	124.3	83.04	30.31	42.71	6.7
50	4f/3	147.5	127.71	39.79	56.29	6.2



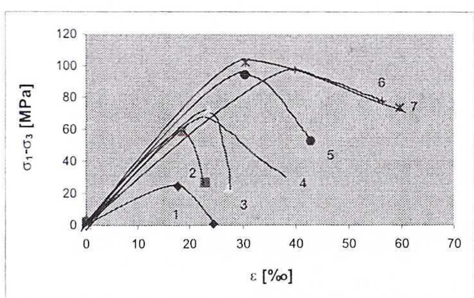
2a. Fine-grained sandstones (samples no. 1 and no. 3)

2a. Piaskowiec drobnoziarnisty (próba nr 1 i nr 3)



2b. Siltstone (sample no. 2)

2b. Iłowiec (próba nr 2)



2c. Coal (sample no. 4)

2c. Węgiel (próba nr 4)

Fig. 2. Differential stress σ_1–σ_3 as a function of axial strain ε
Curves: 1 — 0 MPa, 2 — 5 MPa, 3 — 10 MPa, 4 — 15 MPa, 5 — 20 MPa, 6 — 30 MPa, 7 — 50 MPa, 8 — 70 MPa

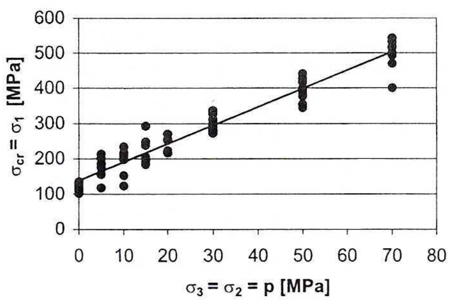
Rys. 2. Naprężenie różnicowe σ_1–σ_3 jako funkcja osiowego odkształcenia ε
Krzywe: 1 — 0 MPa, 2 — 5 MPa, 3 — 10 MPa, 4 — 15 MPa, 5 — 20 MPa, 6 — 30 MPa, 7 — 50 MPa, 8 — 70 MPa

in confining pressure results in a decrease in the modulus of softening. Similar dependences were obtained by Bieniawski (1969).

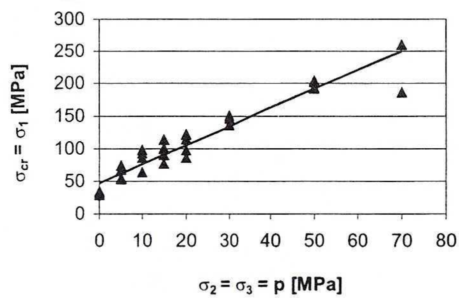
These investigations enabled the dependences between the particular parameters and the magnitude of the applied confining pressure in the range from 0 to 70 MPa to be determined, corresponding to rock deposits to depth of approximately 3000 m.

For the determination of functional dependences the following parameters were analysed as functions of the confining pressure (Fig. 3 to 6):

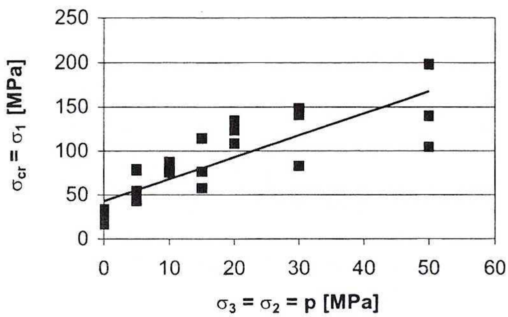
- critical stress,
- residual stress,



3a. Fine-grained sandstone
3a. Piaskowiec drobnoziarnisty
$\sigma_{cr} = 5.2045p + 138.52;\ r = 0.9695$



3b. Siltstone
3b. Iłowiec
$\sigma_{cr} = 2.8926p + 47.447;\ r = 0.9594$



3c. Coal
3c. Węgiel
$\sigma_{cr} = 2.4978p + 42.918;\ r = 0.8352$

Fig. 3. Critical stress σ_{cr} as a function of confining pressure p

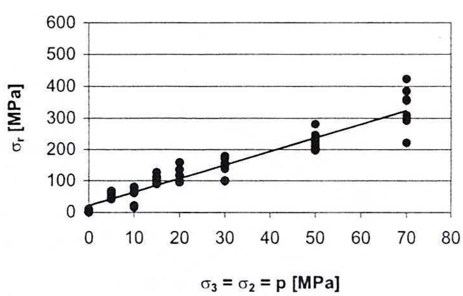
Rys. 3. Naprężenie krytyczne σ_{kr} jako funkcja ciśnienia okólnego p

- residual strain,
- modulus of softening.

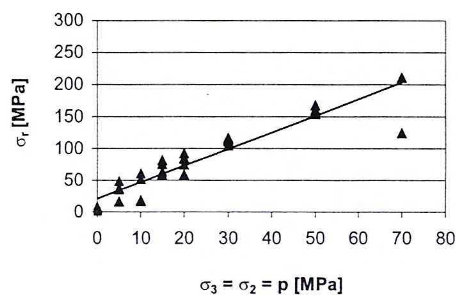
These dependences for the rocks investigated may be described as follows:

- rectilinear dependences for:
  - critical stress σ_{cr} as a function of confining pressure $\sigma_2 = \sigma_3 = p$ (Fig. 3a, b, c),
  - residual stress σ_r as a function of confining pressure $\sigma_2 = \sigma_3 = p$ (Fig. 4a, b, c).

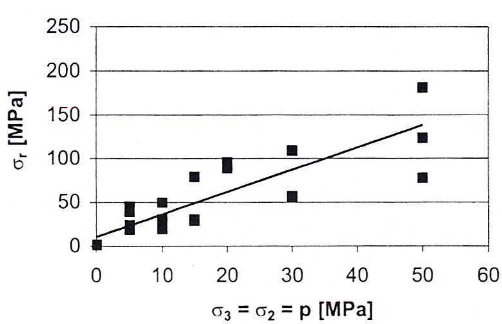
For rectilinear dependences, the correlation coefficient r changes from 0.84 to 0.97; in these cases the correlation coefficient is considerably greater than the limiting value (Volk 1973);



4a. Fine-grained sandstone
4a. Piaskowiec drobnoziarnisty
$\sigma_r = 4.3177p + 20.982$; $r = 0.9587$



4b. Siltstone
4b. Iłowiec
$\sigma_r = 2.6045p + 20.882$; $r = 0.9330$



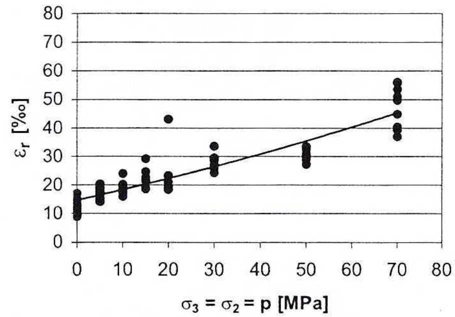
4c. Coal
4c. Węgiel
$\sigma_r = 2.5509p + 10.811$; $r = 0.8641$

Fig. 4. Residual stress σ_r as a function of confining pressure p

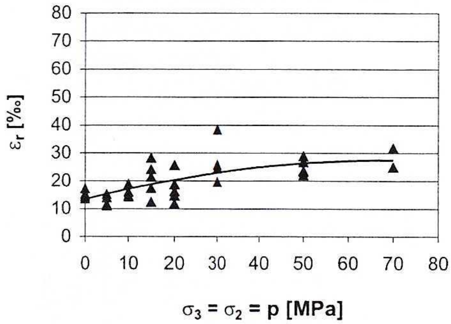
Rys. 4. Naprężenie resztkowe σ_r jako funkcja ciśnienia okólnego p

- curvilinear dependences for:
  - residual strain ε_r as a function of confining pressure $\sigma_2 = \sigma_3 = p$ (Fig. 5a, b, c)
  - modulus of softening M as a function of confining pressure $\sigma_2 = \sigma_3 = p$ (Fig. 6a, b, c).

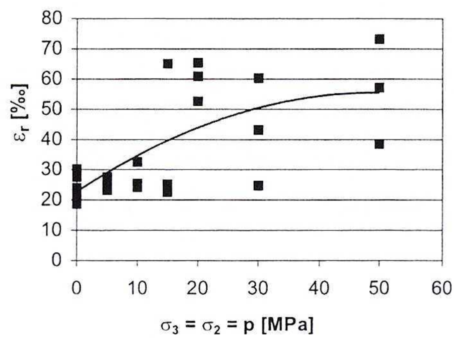
The curvilinear dependences $\varepsilon_r = f(p)$, $M = f(p)$ were expressed in the form of second order polynomials , where the correlation coefficient changed between 0.57 and 0.90. In the case of function $M = f(p)$ there is a weak dependence of the modulus of softening on the confining pressure for siltstone; in this case the correlation coefficient value is equal to 0.39 ($r_{cr} = 0.304$, Volk 1974).



5a. Fine-grained sandstone
5a. Piaskowiec drobnoziarnisty
$\varepsilon_r = 0.0011p^2 + 0.606p + 14.713$; $r = 0.9038$



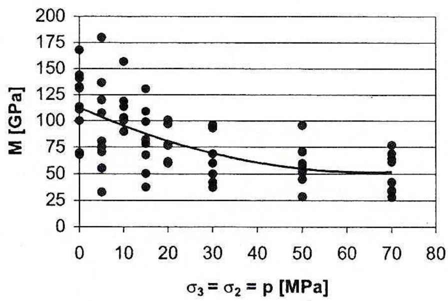
5b. Siltstone
5b. Iłowiec
$\varepsilon_r = -0.0029p^2 + 0.4062p + 13.356$; $r = 0.7070$



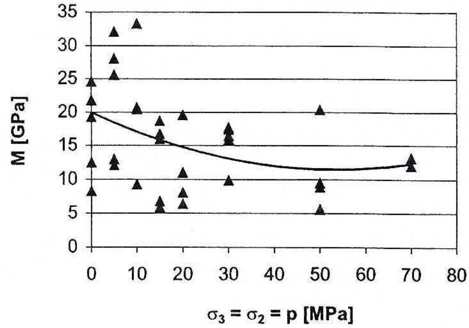
5c. Coal
5c. Węgiel
$\varepsilon_r = -0.0134p^2 + 1.3269p + 22.75$; $r = 0.6862$

Fig. 5. Residual strain ε_r as a function of confining pressure p

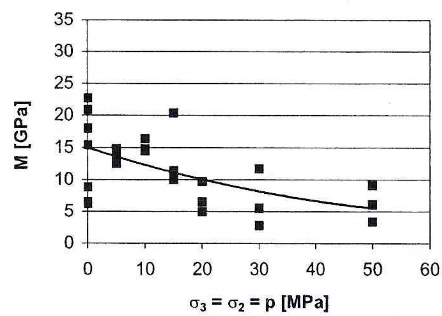
Rys. 5. Odkształcenie resztkowe ε_r jako funkcja ciśnienia okólnego p



6a. Fine-grained sandstone
6a. Piaskowiec drobnoziarnisty
$M = 0.0145p^2 - 1.8827p = 112.76$; $r = 0.6279$



6b. Siltstone
6b. Iłowiec
$M = 0.003p^2 - 0.3202p + 19.949$; $r = 0.3882$



6c. Coal
6c. Węgiel
$M = 0.0019p^2 - 0.2858p + 14.997$; $r = 0.5733$

Fig. 6. Modulus of softening M as a function of confining pressure p

Rys. 6. Moduł osłabienia M jako funkcja ciśnienia okólnego p

4. Determination of values for the angle of internal friction and for the cohesion

Knowledge of the values of the critical strength and residual strength in response to the applied confining pressure enables the determination of values of the angle of internal friction φ and cohesion c for both compact and fractured rocks (Sanetra 2002). The constants φ and c were determined by the following methods:

1) graphical method in the coordinate system: τ–σ,
2) the method using the co-ordinate system: q–p,

3) the method of two tangents to the parabolic envelope of Mohr's circles:
- for compact rock:
  - 3/1 — the tangent to the arc of the parabola comprising the circles for uniaxial tension and compression,
  - 3/2 — the tangent to the part of parabola comprising the circles for a triaxial state of stresses;
- for fractured rock:
  - 3/1 — the tangent to the initial part of parabola comprising the circle for uniaxial compression and the first circle for triaxial compression,
  - 3/2 — the tangent to the part of parabola comprising the circles for triaxial state of stresses.

In the method of two tangents to Mohr's circles envelope, the parabola equation using the least square method (Sanetra 2002; Pacześniowski 2002) was applied.

The values obtained of the internal friction angle and cohesion for all rocks tested are shown in Table 2.

From the table it can be seen that the values of the internal friction angle, determined by methods 1 and 2 approximate and that in both methods the values of internal friction

TABLE 2

Obtained values of internal friction angle and cohesion for compact and fractured rocks

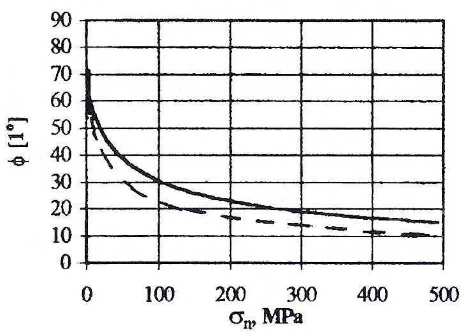
TABLICA 2

Zestawienie kątów tarcia wewnętrznego i spójności dla skał zwięzłych i spękanych

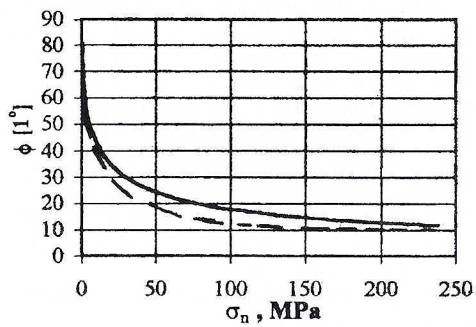
Rock	Method	Compact rock		Fractured rock	
		φ [1°]	c [MPa]	φ [1°]	c [MPa]
Fine-grained sandstone (sample no. 1)	1	41°	44.5	39°	20.0
	2	43°	32.6	37°75′	7.7
	3/1	57°23′	37.6	53°17′	12.7
	3/2	22°18′	102.9	19°07′	49.3
Siltstone (sample no. 2)	1	28°	20.5	27°	7.0
	2	28°06′	14.6	25°34′	6.6
	3/1	55°29′	9.3	47°59′	5.3
	3/2	18°19′	31.7	15°38′	20.9
Fine-grained sandstone (sample no. 3)	1	44°	50.0	40°	14.0
	2	43°16′	26.2	40°41′	1.3
	3/1	55°57′	35.2	54°34′	12.7
	3/2	21°47′	94.6	19°03′	51.5
Coal (sample no. 4)	1	34°	17.5	33°	9.0
	2	26°39′	14.0	26°79′	3.3
	3/1	56°45′	7.1	42°08′	4.5
	3/2	19°21′	22.5	14°23′	16.0

angles for compact rocks do not differ significantly from the values of the internal friction angles for fractured rock created by the residual stress. On the other hand, the cohesion of fractured rocks is usually far smaller than the cohesion of compact rocks. This is consistent with tests carried out by other investigators who reported that the cohesion might drop considerably, even as far as zero (Kidybiński 1982). Fractures influence the values of the internal friction angle to a lesser degree. In the method 3 (two tangents to the parabolic envelope of Mohr's circles) the values of the internal friction angle for fractured rocks are smaller than for compact rocks.

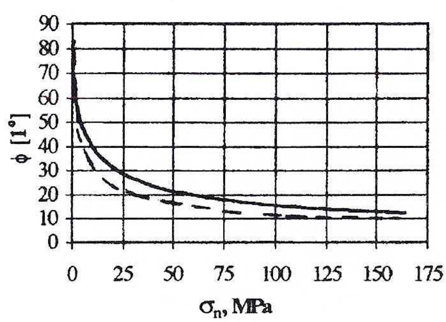
When the equation of parabolic envelope is determined for a given type of rock, then the change of the internal friction angle as a function of normal stress can be calculated. The dependence $\varphi = f(\sigma_n)$ for compact and fractured (in the post-critical state) rocks for fine-grained sandstones, siltstone and coal are given in Fig. 7a, b, c.



7a. Fine-grained sandstones
7a. Piaskowiec drobnoziarnisty



7b. Siltstone
7b. Iłowiec



7c. Coal
7c. Węgiel

Fig. 7. Angle of internal friction φ as a function of normal stress σ_n
——— compact rock, - - - - - fractured rock
Rys. 7. Kąt tarcia wewnętrznego φ jako funkcja naprężenia normalnego σ_n
——— skała zwięzła, - - - - - skała spękana

The dependences show that with an increase in normal stress the value of the internal friction angle decreases. Fine-grained sandstone has the higher internal friction angle value than siltstone and coal, both for compact and fractured rocks.

For example, at normal stress $\sigma_n = 100$ MPa the values of internal friction angles are:

- in compact rocks: $\varphi = 30°$ for sandstones
  $\varphi = 18°$ for siltstone
  $\varphi = 16°$ for coal
- in fractured rocks: $\varphi = 22°$ for sandstones
  $\varphi = 12°$ for siltstone
  $\varphi = 11°$ for coal

The investigations on the influence of confining pressures on the internal friction angle, carried out by Kwaśniewski (1983) revealed that at an increase in the confining pressure from 0 to 60 MPa the angle of internal friction decreased from 58° to 21° for fine-grained sandstones, from 60° to 16° for medium-grained sandstones and from 56° to 6° for coarse-grained sandstones.

5. Hoek-Brown's failure criterion

The empirical Hoek-Brown's criterion in its first version was mainly concerned with compact rocks, and after a certain modification fractured rocks could also be included, which considerably widened the possibility of rock-quality estimation for strong (compact) as well as for weak (fractured) rocks.

Hoek-Brown's criterion is expressed by the following equation (Hoek, Brown 1980):

$$\sigma'_1 = \sigma'_3 + (m\sigma_c\sigma'_3 + s\sigma^2_c)^{\frac{1}{2}} \tag{1}$$

where:

σ'_1 — maximal effective principal stress,
σ'_3 — minimal effective principal stress, or confining pressure in a triaxial compression test,
σ_c — uniaxial compression strength,
$m,\ s$ — empirical constants with positive values.

The constant m is enclosed in the range from 0.001 (for very fractured rocks) to about 25 (for compact rocks). However, the values of constant s range from 0 to 1 for the fractured rock mass and compact rocks, correspondingly.

For compact rocks ($s = 1$) the shape of the empirical curve depends on constant m. Hoek and Brown have shown that the value of constant m depends on the type of rock and changes from 7 (for carbonate rocks) to 25 (for coarse-grained igneous and metamorphic rocks).

The failure criterion is given in the dimensionless form by dividing both sites of equation (1) by the uniaxial compression strength σ_c:

$$\frac{\sigma'_1}{\sigma_c} = \frac{\sigma'_3}{\sigma_c} + \left(m\frac{\sigma'_3}{\sigma_c} + s \right)^{\frac{1}{2}} \tag{2}$$

Dimensionless constants m and s are approximately analogical to the angle of internal friction and to the cohesion, correspondingly determined using the conventional Coulomb-Mohr failure criterion.

6. Application of Hoek-Brown's failure criterion to the results of rock samples in triaxial compression

The values of triaxial strength of rocks were calculated according to Hoek-Brown's criterion given in the normalised form (Hoek, Brown 1980):

$$\frac{\sigma_1}{\sigma_c} = \frac{\sigma_3}{\sigma_c} + \left(m\frac{\sigma_3}{\sigma_c} + 1.0 \right)^{\frac{1}{2}} \tag{3}$$

where:

σ_1 — maximal axial stress,
σ_3 — confining pressure,
σ_c — uniaxial compression strength,
m — material constant.

The material constant m for a compact rock ($s = 1$) is determined by the formula (Hoek 1983):

$$m = \frac{1}{\sigma_c}\left[\frac{\sum x_i y_i - \frac{\sum x_i y_i}{n}}{\sum x_i^2 - \frac{(\sum x_i^2)}{n}} \right] \tag{4}$$

where x_i and y_i are a pair of successive measurement points,

$x_i = \sigma_3$,
$y_i = (\sigma_1 - \sigma_3)^2$,
n — number of measurement points.

The value of constant s for fractured rocks is given by the equation (Hoek 1983):

$$s = \frac{1}{\sigma_c^2}\left[\frac{\Sigma y}{n} - m\sigma_c \frac{\Sigma x}{n}\right] \tag{5}$$

For a very fractured rock mass, when s is close to zero, the material constant m is calculated from the following formula:

$$m = \frac{\Sigma y}{\sigma_c \cdot \Sigma x} \tag{6}$$

The results for fine-grained sandstone — sample no. 1 are listed in Table 3.

The triaxial maximum strengths for siltstone (sample no. 2), for fine-grained sandstone (sample no. 3) and for coal (sample no. 4) were determined analogously.

The results for particular experimental series are presented graphically in the form of maximum strength envelopes (Fig. 8a, b, c, d).

The maximum strength in the triaxial state of stresses expressed by the dimensionless ratio of the maximal axial stress σ_1 to the uniaxial compression strength σ_c has comparable values for both experimental and analytical data.

The maximal strength envelopes determined on the basis of experimental and analytical data are practically the same for fine-grained sandstone (sample no. 1) and siltstone (sample no. 2). However, they differ by a certain shift along the y-axis for

TABLE 3

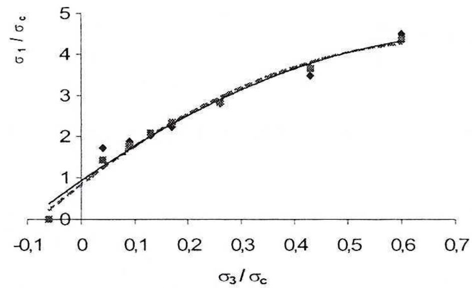
Determination of triaxial maximum strength according to Hoek-Brown's criterion

TABLICA 3

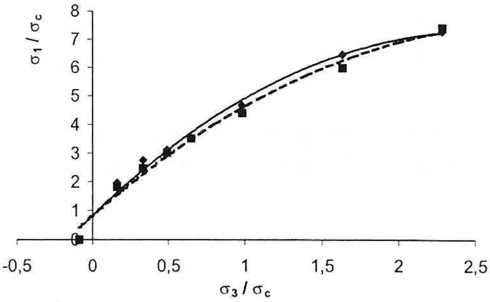
Wyznaczanie trójosiowej wytrzymałości według kryterium Hoeka-Browna

fine-grained sandstone — sample no. 1.
piaskowiec drobnoziarnisty – pr.nr 1
σ_c = 116.2 MPa
m = 21.80

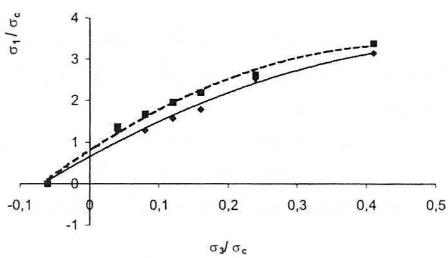
σ_1 [MPa]	σ_3 [MPa]	σ_1/σ_c	σ_3/σ_c	$\frac{\sigma_3}{\sigma_c} + \left(m\frac{\sigma_3}{\sigma_c} + 1\right)^{\frac{1}{2}}$	σ_t [MPa]
200.8	5	1.73	0.04	1.44	–5.3
218.5	10	1.83	0.09	1.78	
235.8	15	2.03	0.13	2.08	
259.7	20	2.23	0.17	2.35	
327.0	30	2.81	0.26	2.83	
404.6	50	3.48	0.43	3.65	
522.8	70	4.50	0.60	4.36	



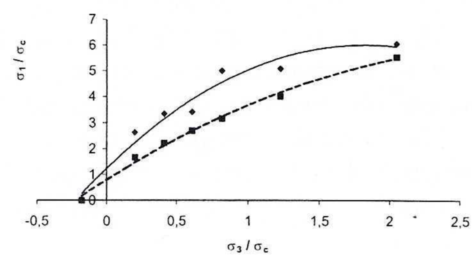
8a. Fine-grained sandstone — sample no. 1
8a. Piaskowiec drobnoziarnisty — próba nr 1



8b. Siltstone — sample no. 2
8b. Iłowiec — próba nr 2



8c. Fine-grained sandstone — sample no. 3
8c. Piaskowiec drobnoziarnisty — próba nr 3



8d. Coal — sample no. 4
8d. Węgiel — próba nr 4

Fig. 8 Envelopes of rock maximum strength according to Hoek-Brown's criterion
◆ - - - - - experimental values, ■ ——— numerical values

Rys. 8. Obwiednie maksymalnej wytrzymałości skał według kryterium Hoeka-Browna
◆ - - - - - wartości eksperymentalne, ■ ——— wartości obliczeniowe

fine-grained sandstone (sample no. 3) and coal (sample no. 4). In the case of sandstone, the analytical values are greater than the experimental values. However, in the case of coal the relation is reciprocal: the experimental strength values are higher than the analytical strength values.

The angle of internal friction φ and cohesion *c* determined by the method of Mohr's circle tangents and the material constants (*s, m*) calculated for Hoek-Brown's failure criterion are listed in Table 4.

It can be seen from the comparison of the constants appearing in both failure criteria that the dimensionless constants *m* and *s* of Hoek-Brown's criterion are almost analogical to the angle of internal friction and cohesion, respectively determined from Coulomb-Mohr's failure criterion (Sanetra, Szedel 2000).

TABLE 4

Material constants for Coulomb-Mohr's and Hoek-Brown's failure criteria

TABLICA 4

Stałe materiałowe dla kryteriów zniszczenia Coulomba-Mohra i Hoeka-Browna

Type of rock	Coulomb-Mohr's criterion				Hoek-Brown's criterion			
	compact rock		fractured rock		compact rock		fractured rock	
	φ [1°]	c [MPa]	φ [1°]	c [MPa]	m	s	m	s
Fine-grained sandstone — sample no. 1	41	44.5	39	20.0	21.80	1	5.91	$4.3 \cdot 10^{-3}$
Siltstone — sample no. 2	28	20.5	27	14.0	11.00	1	5.92	$1.4 \cdot 10^{-3}$
Fine-grained sandstone — sample no. 3	44	50.0	40	7.0	19.16	1	5.31	$1.4 \cdot 10^{-4}$
Coal — sample no. 4	34	17.5	33	9.0	5.46	1	5.20	$2.7 \cdot 10^{-3}$

7. Conclusions

The results of triaxial compression tests of Carboniferous rocks carried out at a constant value of longitudinal strain rate $10^{-5} \cdot s^{-1}$ and at confining pressure values from 0 to 70 MPa, presented in tables 1 to 4, prompt the following conclusions:

- The increase in confining pressure from 0 to 70 MPa causes a change of the values of the parameters investigated (critical stress, residual stress, residual strain and modulus of softening) for all Carboniferous rocks tested (sandstones, siltstone, coal), which is in conformity with the results of other investigators (Tajduś 1990; Sanetra 1994a, b; Krzysztoń et al.1998).
- The critical stress of the tested rocks increases with the increase in the confining pressure range from 0 to 70 MPa: for sandstones it increases from 4 to 4.5 times in relation to the value of uniaxial critical stress. For siltstone and coal greater rates of increase of critical stresses are observed. Their critical stresses reach values equal to 6 to 7 times of the uniaxial critical stress values at confining pressure of 70 MPa.
- The confining pressure increase has its greatest influence on the residual stress. For brittle rocks as sandstones, the residual stress increases 55 to 124 times in relation to the value obtained in the uniaxial state of stress ($p = 0$ MPa). A similar increase in the residual stress appears for coal where the residual stress at 50 MPa confining pressure reaches a value of 180 times the uniaxial value. However, the residual stress for siltstone at 70 MPa confining pressure increased 33 times in comparison with the value obtained at $p = 0$ MPa.

- The residual stress at the maximal confining pressure has a large value in comparison with the vertical stress which amount to: for sandstones from 0.6 to 0.7 σ_{cr}, for siltstone 0.75 σ_{cr} and for coal 0.86 σ_{cr}.
- The increase in confining pressure also causes an increase in residual strain: in weak rocks (siltstone, coal) the residual strains increased 1.9 to 2.3 times, whereas in strong rocks (sandstones) 3 to 5 times in relation to the values obtained in uniaxial compression.
- The modulus of softening in the lower range of confining pressures (5 to 10 MPa) shows an increase to about 20% of the initial value obtained in the uniaxial state of stress ($p = 0$). For higher values of confining pressure, in the 10 to 70 MPa range, the modulus of softening decreases with the increase in confining pressure. The decrease in modulus value in relation to the value obtained in the uniaxial state of stress is different for different types of rocks: for sandstones by 35% to 63%, for siltstone 73%, for coal 44%.
- An increase in the modulus of (strain) softening in the low value range of confining pressure (5 to10 MPa) may be motivated by the "memory"of a rock material about the strain hardening which took place in the pre-critical part of the stress-strain characteristic.
- The proposed method for the determination of the internal friction angle and cohesion enables determination of the values of these parameters from the parabolic envelope of Mohr's circles for different values of normal stresses in the case of both compact rocks in the pre-critical state and for fractured rocks appearing in the post-critical state.
- The dimensionless constants m and s of Hoek-Brown's criterion are almost analogical to the angle of internal friction and cohesion, respectively, determined from Coulomb-Mohr's failure criterion.
- The material constant m and the angle of internal friction φ for rocks with compact structure assume considerably higher values for strong rocks (fine-grained sandstone R_c = 116.2 to 123.2 MPa) than for weak rocks (siltstone R_c = 30.7 MPa, coal R_c = 24.4 MPa).
- The material constant m for rocks with a fractured structure assumes the close values for strong rocks (sandstones) as well as for weak rocks (siltstone, coal).
- The value of internal friction angle for fractured rocks appearing in the post--critical phase is not significantly smaller than this value for compact rocks in the pre-critical state.
- The cohesion c and the material constant s for fractured rocks drop considerably in relation to these values for compact rocks (Hoek, Brown 1980; Kidybiński 1982).

The following general conclusions can be drawn from the analysis of quantitative changes of rock properties caused by the increase in the confining pressure in the range 0 to 70 MPa:

- structures of fractured rock may fulfil the condition of load capacity for the determined values of confining pressure and angle of internal friction (Krzysztoń 2002);

- post-critical properties of rocks depend not only on the rock type (sandstone, siltstone, coal) but also on the rock structure (sandstones no. 1 and no. 3);
- knowledge of the values of angle of internal friction and cohesion for coal and waste rocks may be used in the determination of coal pillar strength with regard to contact conditions in the set: roof-pillar-floor (Krzysztoń 2002);
- the increase in the modulus of softening in low range of confining pressure, evincing a "memory" of rock material gives evidence of viscous effects which should be taken into consideration in a constitutive model of the post-critical part of stress-strain characteristic (Nawrocki, Mróz 1992).

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REVIEW BY: PROF. DR HAB. INŻ. ANTONI KIDYBIŃSKI, KRAKÓW

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