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THE INVESTIGATION OF LAMELLA CLASSIFICATION PROCESS

BADANIA PROCESU KLASYFIKACJI W KLASYFIKATORZE LAMELOWYM

The paper provides a mathematical model of grain classification processes targeted at prognostication of grain size distribution on the basis of grain distribution in the feed. Predicted (theoretical) values of the cut size and separation sharpness indices (imperfection and coefficients κ_1 and κ_2 are presented. Results of grain size classification in a coal suspension in a single lamella unit are duly compiled. Measurement results are compared with predicted values. The mathematical description of the grain size distribution in the feed and final product uses Weibull's distribution (Rosin-Rammler-Bennett's distribution) as a singular case of the generalised gamma distribution. Grain sizes obtained experimentally are compared with predicted values. The experimental grade efficiency curves are approximated with several functions. The cut sizes are derived from curves approximating the experimental data set. On that basis experimental cut sizes are obtained and then compared with predicted values.

Distribution functions of grain size in classification products prognosticated on the basis of size distribution in the feed agree pretty well with the values obtained from mathematical modelling. The comparison of predicted and experimental values lead us to the conclusion that cut size obtained in experimental classification processes was nearly two times bigger than the predicted value. Separation sharpness indices obtained experimentally are similar to predicted values.

In conclusion, the results of experimental verification of the developed model are regarded as satisfactory. The mathematical model of classification processes in lamella settling tanks agrees sufficiently well with experimental results. While compared to other classification apparatuses (centrifugal separators, hydrocyclones), the lamella tank seems to provide for better separation sharpness.

Key words: lamella classification, sedimentation process, indices of classification sharpness

W artykule przedstawiono model matematyczny procesu klasyfikacji ukierunkowany na prognozowanie uziarnienia produktów klasyfikacji na podstawie uziarnienia nadawy. Przedstawiono wyniki obliczeń teoretycznych: ziarna podziałowego oraz wskaźników ostrości rozdziału: imperfekcji i wskaźników κ_1 i κ_2. Przedstawiono wyniki badań procesu klasyfikacji dla zawiesiny węglowej

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w pojedynczym elemencie klasyfikatora lamelowego. Porównano wyniki badań z wynikami obliczeń teoretycznych. Do opisu matematycznego uziarnienia nadawy i produktów klasyfikacji zastosowano rozkład Weibulla (rozkład Rosina-Rammlera-Benetta) jako szczególny przypadek uogólnionego rozkładu gamma. Doświadczalne wartości rozkładu wielkości ziaren porównano z wartościami teoretycznymi. Doświadczalne krzywe rozdziału aproksymowano za pomocą kilku funkcji. Obliczono wartości ziarna podziałowego wynikające z krzywych aproksymujących zbiór danych doświadczalnych. Na tej podstawie obliczono doświadczalne wartości wskaźników ostrości rozdziału, które porównano z wartościami teoretycznymi.

Doświadczalne dystrybuanty rozkładu wielkości ziaren produktów klasyfikacji prognozowane na podstawie rozkładu wielkości ziaren nadawy okazały się wystarczająco zgodne z wartościami obliczonymi na podstawie modelu matematycznego. Wyniki porównań wielkości teoretycznych i doświadczalnych wartości ziarna podziałowego prowadzą do wniosku, iż w doświadczalnym procesie klasyfikacji otrzymywano ziarno podziałowe około dwukrotnie większe od wartości obliczonej teoretycznie. Utworzono zależność empiryczną umożliwiającą obliczanie skorygowanej wielkości ziarna podziałowego od koncentracji zawiesiny, obciążenia powierzchniowego oraz od parametrów rozkładu. Doświadczalne wartości wskaźników ostrości rozdziału były zbliżone do wartości obliczonych teoretycznie.

W konkluzji, wyniki weryfikacji doświadczalnej opracowanego modelu oceniono pozytywnie. Model matematyczny procesu klasyfikacji w klasyfikatorze lamelowym jest wystarczająco zgodny z wynikami doświadczeń. W porównaniu do innych urządzeń stosowanych do klasyfikacji przepływowej ziaren (wirówki, hydrocyklony) klasyfikator lamelowy okazał się urządzeniem o wyższej ostrości rozdziału.

Słowa kluczowe: klasyfikacja lamelowa, proces sedymentacji, wskaźniki ostrości klasyfikacji

Nomenclature

b_0, b_1, b_2	—	empirical coefficient
d, d_x	—	grain diameter
d_0	—	scale parameter in generalized gamma distribution
d_g	—	critical grain diameter
d_{50}, d_k, d_{50c}	—	cut size, grain corrected diameter
d_N, s_N	—	parameters of normal distribution — mean value, standard deviation
$f(d)$	—	probability function of grain diameter
$2h$	—	distance between parallel plates
I	—	imperfection
l	—	length of conduit
m, σ	—	parameters of log-normal distribution function — mean value, standard deviation
n, p	—	shape parameters of generalised gamma distribution
r	—	correlation coefficient
Re	—	Reynolds number
Fr	—	Froude number
s	—	concentration of suspension
$T(d)$	—	separation curve
v	—	settling velocity of grain with the d diameter
v_g	—	settling velocity of grain with the critical d_g diameter
$\Gamma(a)$, $\Gamma(a,b)$	—	Euler gamma function

κ_1, κ_2 — separation sharpness indices
μ_0 — dynamical coefficient of fluid viscosity
θ — inclination angle of conduit
ρ, ρ_0 — density of solid phase, of fluid
φ — volume concentration rate
ψ — relative viscosity of suspension

Indices

p — fine product in overflow
w — coarse product in underflow

1. Introduction

Lamella settling tanks are in widespread use as suspension separators, classifiers and thickeners. Their structure is such that the main stream of suspension is divided into a number of inclined streams. In practice, the working space in the tank is filled with plastic packets of parallel conduits. The usual cross-section area of the lamella conduit is $2h = 0.05$ m, the length: from $l = 0.6$ to 1 m. The angle of conduit inclination is nearing 60^o. Such inclination guarantees the flow of settling grains induced by the forces of gravity. Since the sediment flows counter-current with respect to the flow of suspension, this process in known as "counter-current" sedimentation.

In physics the process of intense settling of dispersed phase particles in inclined conduits is known as Boycott's effect (1920). A.E. Boycott observed that blood in slightly inclined tubes would settle at a much faster than in tubes arranged vertically. Presently this effect is explained by an increase of the settling surface, i.e. the sum of projections of all settling surfaces onto the horizontal plane. The first industrial applications of Boycott's effect came in the 1970s as it was utilized in industrial clarification processes.

Employment of counter-current lamella packets in clarification processes leads to manifold increase of sedimentation efficiency and the financial profits thus generated are far from minor (Marciniak-Kowalska 1986, 2000a, 2000b).

Research work on possible practical applications of lamella sedimentation in the clarification processes was first undertaken in the Microfloc Company (USA). The works that afford us the best insight into the process of lamella sedimentation and its theoretical backgrounds include those by K.M. Yao (1970), H. Binder and U. Wiesmann (1983) and K.H. Nowack (1990). In Poland the studies on counter-current lamella sedimentation to intensify the clarification processes were conducted by W. Olszewski (1975), S. Bednarski (1978), J. Gęga (1979), J. Haba, J. Nosowicz, A. Pasiński (1980), W. Kowalski (1992), Z. Niedźwiedzki (2000). Recently several new publications appeared (J. Bandrowski, J. Hehlmann, H. Merta, J. Zioło). The major work is the monograph by J. Bandrowski et al. (2001) which makes an attempt to systematise and sum up the current status of theoretical studies and applications of counter-current lamella facilities in clarification of suspensions.

While analyzing the present-day status of theoretical studies, it appears there are several adequate theoretical models for predicting the sedimentation efficiency depending on the parameters of suspensions, such as probability function of grain diameters, tank design and process parameters. A clarification process is a specific case of flow-based classification where the cut size tends towards zero. In this respect the theoretical description of lamella classification is merely a generalisation of the theoretical description of clarification in inclined, thin-layered flows. In particular, the function of sedimentation efficiency of monodisperse suspensions might be interpreted as the separation function in the process of grain classification.

The main aim of the paper is to present a mathematical model of the classification process in a lamella tank targeted at prognostication of grain size distribution of the final products for Rosin-Rammler-Bennett's distribution of grain size in the feed. The results are to be compared with experimental values. Furthermore, predicted and experimental values of separation sharpness indices are to be compared, too.

2. Evaluation of classification accuracy

Lamella classifiers are in widespread use in industry, for example in hydro-classification of sands (Grzelak and Malinowski 1981; Grzelak 1992). They are reported to perform well, ensuring high efficiency and excellent sharpness parameters.

The theoretical description of grain classification uses several concepts from the refining processes. K.F. Tromp (1937) put forward the idea that the performance of coal refining facilities be assessed using grade efficiency curves. Tromp suggested an assessment procedure whereby the specific measures expressing the proportion of a material fraction in one of the final products to the total amount of the fraction in the feed be assigned to narrow fractions of the refined material with the known density, thus plotting a dispersion curve. In 1950 in Paris, at the Coal Processing Congress, this curve was called separation curve. It is now denoted as $T(d)$ while the relevant measures are called grade efficiency numbers.

Nevertheless, a thorough scrutiny of works published in the mid-fifties shows that the authors of major theoretical papers did not seem to be aquatinted with Tromp's concepts. The pioneer theoretical work providing the general description of grain classification processes is that by K. Sztaba (1956a, 1956b). Theoretical considerations of grain classification processes in hydraulic classifiers at various settling velocities led him to the formulation of the grain size distribution function expressed as the normal distribution function.

In later studies many Authors suggested several functions, their graphic forms resembling the experimental curves. Those most often referred to are: R.L. Plitt (1971), A.J. Lynch (1970), T.C. Erasmus (1973) and H. Trawiński (1976). L.R. Plitt suggested the following distribution curve

$$T(d) = 1 - \exp\left(-0.693 \cdot \left(\frac{d}{d_{50c}}\right)^m\right) \tag{1}$$

where d_{50c} denotes the corrected grain diameter whilst m is the measure grade efficiency.

A.J. Lynch derived the formula describing the grade efficiency curve

$$T(d) = \frac{\exp\left(\alpha \dfrac{d}{d_{50c}}\right) - 1}{\exp\left(\alpha \dfrac{d}{d_{50c}}\right) + \exp(\alpha) - 2} \tag{2}$$

where α is the measure of grade efficiency and the relationship between α and m is given as

$$\alpha = 1.54 \cdot m - 0.47 \tag{3}$$

T.C. Erasmus (1973) made use of the arc tangent function

$$T(d) = \frac{1}{2} + \frac{1}{\pi} \cdot \operatorname{arctg}\left[\frac{x-1}{x}(x+\delta)\right] \tag{4}$$

where

$$x = \frac{d}{d_{50c}}, \quad \delta > 1 \tag{5}$$

and δ — empirical coefficient.

Several approximations of the grade efficiency curve were suggested by H. Trawiński (1976). Using exponential, trigonometric and conical functions he formulated the description of grade efficiency curves in the process of classification and modified the Plitt's, Lynch's, Erasmus's equations. He applied the regression method to investigate the adequacy of thus derived formulas, checking how they would agree with experimental results.

One of the major criteria of classification efficiency is the classification accuracy. The main indicators of classification accuracy are individual numerical characteristics called the sharpness indices. Generally speaking, sharpness indicators are aimed to provide a single-number approximation of inclination of the tangent to the grade efficiency curve, passing through the inflection point understood as the point whose independent coordinate is the value of the critical grain size. Sharpness indices are closely linked to the grade efficiency curve.

Sharpness indices in most widespread use are:

- imperfection or separation inaccuracy, proposed by T. Eder (1961) and defined as follows:

$$I = \frac{d_{75} - d_{25}}{2d_{50}} \tag{6}$$

where d_{75} and d_{25} denote independent coordinates of grade efficiency curve points at which it reaches the value 0.75 and 0.25 respectively (or where grade efficiency numbers are 75% and 25%).

- coefficients κ

$$\kappa_1 = \frac{d_{75}}{d_{25}} \quad \text{and} \quad \kappa_2 = \frac{d_{65}}{d_{35}} \tag{7}$$

The method of calculating the grade efficiency curve coordinates by way of measurements of grain size distribution in the feed and in fine-grained and coarse--grained final products was developed by K. Sztaba (1956b), T.H. Eder (1961) and Z. Nowak (1982). Quite independently, the method was also suggested by J. Malewski (1980).

The common feature of mathematical descriptions of grain classification once the coordinates of the experimental grade efficiency curve are known is that various empirical functions are to be fitted to calculated results. The function best describing the grade efficiency curve being selected and its adequacy verified by way of comparison to measurement results, the sharpness indicators might now be found. Using the function approximating the grade efficiency curve and grain size distribution in the feed, the parameters of the classification process are calculated: the proportion of oversize grains in the fine-grained product and the proportion of undersize grained in the coarse product. All computations use the function approximating the grade efficiency curve. In practical applications the values of grade efficiency numbers for narrow grain classes are taken from tables.

In the study of classification of a polydisperse grain set to assess the adequacy of classification results the basic modelling tasks are:

- to find the cut size,
- to determine the grain size distributions in the final products knowing the grain distribution in the feed material,
- to determine classification accuracy.

The classification limit is the predominant parameter in classification of polydisperse grains. Another name of this important parameter is grain cut size, indicated as d_{50}, d_k, d_{50c}. It is a denominated number that denotes the grain size that divides the set of grains in two subsets: those smaller and those larger than the cut size (fine-grained and coarse-grained products).

3. Mathematical model of classification processes in a lamella classifier

A pulverised polydisperse material is separated in a lamella classifier yielding two products: fine product in overflow and coarse-grained product in underflow. The probability functions of grain diameter distribution in the products of the classification process: $f_p(d)$ in overflow and $f_w(d)$ in underflow can be derived from the formulas (Sztaba 1956b) relating then to the probability function of grain diameter distribution in the feed $f(d)$, the separation function $T(d)$ and coarse-grained product yield γ:

$$f_p(d) = \frac{1 - T(d)}{1 - \gamma} \cdot f(d) \tag{8}$$

$$f_w(d) = \frac{T(d)}{\gamma} \cdot f(d) \tag{9}$$

Assuming the distribution of suspension flow velocity in lamella classification processes is uniform throughout the whole conduit (in the system of inclined parallel plates) and that the distribution of suspension concentration in the inlet cross section is uniform too, Eqs. (8) and (9) can be rewritten as (Marciniak-Kowalska 1984)

$$f_p(d) = \frac{1 - \left(\frac{(d + d_g) - |d - d_g|}{2d_g} \right)^2}{\int_0^{d_g} f(d) \mathrm{d}d + \frac{1}{d_g^2} \int_0^{d_g} d^2 f(d) \mathrm{d}d} \cdot f(d) \tag{10}$$

$$f_w(d) = \frac{\left(\frac{(d + d_g) - |d - d_g|}{2d_g} \right)^2}{1 - \int_0^{d_g} f(d) \mathrm{d}d - \frac{1}{d_g^2} \int_0^{d_g} d^2 f(d) \mathrm{d}d} \cdot f(d) \tag{11}$$

where d_g denotes the critical grain diameter obtained from the Stokes's formula (1851). The critical grain diameter fulfilled the sedimentation theory of Hazen, i.e. the grain with diameter of d_g, which it settling velocity is equal to the quotient of fluid flow and the sum of projected surfaces on the horizontal flat plane.

$$d_g = \sqrt{\frac{18\mu_0 \cdot \psi(\varphi)}{(\rho - \rho_0)g} \cdot q} \tag{12}$$

where $\psi(\varphi)$ takes into account the suspension concentration obtained from the Kunitz's formula (Gambill 1959)

$$\psi(\varphi) = \frac{1 + 0.5\varphi}{(1 - \varphi)^4} \tag{13}$$

The unit efficiency index q present in (12) (i.e. surface loading) is equal to settling velocity of grains with the critical diameter $v(d_g)$

$$q = v(d_g) \tag{14}$$

The grade efficiency curve $T(d)$ present in the numerator of (10) and (11) might be expressed as

$$T(d) = \left(\frac{(d + d_g) - |d - d_g|}{2d_g} \right)^2 = \begin{cases} \left(\frac{d}{d_g} \right)^2 & \text{when} \quad 0 < d \le d_g \\ 1 & \text{when} \quad d_g < d < \infty \end{cases} \tag{15}$$

The values of theoretical sharpness indices: imperfection I and coefficients κ_1 and κ_2 obtained for the ideal lamella classification process (Marciniak-Kowalska 1994) are:

- imperfection I

$$I = \frac{d_{75} - d_{25}}{2d_{50}} = 0.259 \tag{16}$$

- coefficients κ_1 and κ_2

$$\kappa_1 = \frac{d_{75}}{d_{25}} = 1.732 \qquad \kappa_2 = \frac{d_{65}}{d_{35}} = 1.363 \tag{17}$$

The relationship between the critical grain diameter d_g and cut size is given as

$$d_g = \sqrt{2} \cdot d_{50} \tag{18}$$

4. Probability function of grain diameter in lamella classification

In order to obtain grain distributions in the products of lamella classification (fine products in overflow $f_p(d)$ and coarse-grained product in underflow $f_w(d)$) knowing the grain size distribution in the feed, an assumption is made that the feed grain sizes in the feed follow the Weibull's distribution (in minerals processing also known as Rosin-Rammler-Bennett's distribution RRB).

The Rosin-Rammler-Bennett's distribution is actually a singular case of the triparametric (d_0 — scale parameter, p, n — shape parameters) generalised gamma distribution (Nipl 1979), the provability density function $f(d)$ being given as:

$$f(d;d_0,p,n)=\frac{n}{d_0\Gamma(p)}\left(\frac{d}{d_0}\right)^{pn-1}\cdot\exp\left(-\left(\frac{d}{d_0}\right)^n\right) \tag{19}$$

for $p = 1$.

The distribution function $F(d)$ in RRB distribution has the form

$$F(d)=\begin{cases} 1-\exp\left\{-\left(\frac{d}{d_0}\right)^n\right\} & \text{when} \quad d\geq 0 \\ 0 & \text{when} \quad d<0 \end{cases} \tag{20}$$

Basing on the assumption that probability density function of grain dimensions in the feed is given as $f(d)$, the yield of the coarse-grained product γ is derived from the formula (Camp 1946)

$$\gamma=1-\int_0^{d_g} f(d)\mathrm{d}d+\frac{1}{d_g^2}\int_0^{d_g} d^2 f(d)\mathrm{d}d=1-I_1+\frac{1}{d_g^2}I_2 \tag{21}$$

It follows from (21) that to obtain the yield γ it is sufficient that two integrals be computed

$$I_1=\int_0^{d_g} f(d)\mathrm{d}d \qquad I_2=\int_0^{d_g} d^2 f(d)\mathrm{d}d \tag{22}$$

The first of them is the distribution function (20).

In order to compute the integral I_2

$$I_2=\int_0^{d_g} d^2 f(d)\mathrm{d}d=\int_0^{d_g} d^2\frac{n}{d_0}\left(\frac{d}{d_0}\right)^{n-1}\cdot\exp\left(-\left(\frac{d}{d_0}\right)^n\right)\mathrm{d}d \tag{22}$$

the following substitution is made

$$\left(\frac{d}{d_0}\right)^n=a \qquad \text{or} \qquad d=d_0 a^{1/n} \tag{23}$$

Differentiate both sides, we get

$$\frac{n}{d_0}\left(\frac{d}{d_0}\right)^{n-1}\mathrm{d}d=\mathrm{d}a \tag{24}$$

and hence

$$dd = \frac{d_0}{n} a^{-\left(1-\frac{1}{n}\right)} da \tag{25}$$

and hence

$$\text{lower limit} \quad a_1 = 0 \qquad \text{upper limit} \quad a_2 = \left(\frac{d_g}{d_0}\right)^n \tag{26}$$

Finally, the integral I_2 is rewritten as

$$I_2 = d_0^2 \int_0^{\left(\frac{d_g}{d_0}\right)^n} a^{\left(1+\frac{2}{n}\right)-1} \cdot e^{-a} da = d_0^2 \cdot \Gamma\left(1+\frac{2}{n}, \left(\frac{d_g}{d_0}\right)^n\right) \tag{27}$$

Fig. 1 provides the plots of probability density functions of grain diameter in the feed and in the products of classification when the feed grain distribution follows the RRB pattern, the parameters in the distribution being $d_0 = 92.1$ μm, $n = 1.320$. All calculations use the formulas (10) and (11).

The parameters of the suspension assumed in the study are those characteristic of coal suspensions: ρ = 1700 kg/m^3, ρ_0 = 1000 kg/m^3, $\mu_0 = 1 \cdot 10^{-3}$ Pa·s (at the tem-

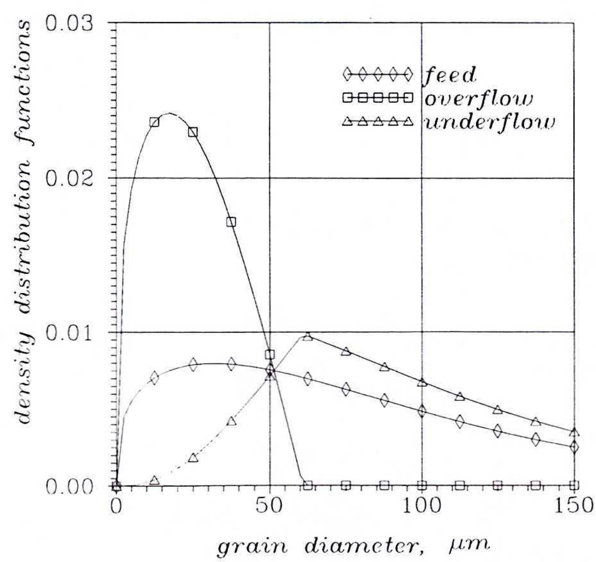

Fig. 1. Density distribution functions of grain distribution in feed and final products for RRB distribution function of feed grain diameters (d_0 = 92.1 μm and n = 1.320)

Rys. 1. Wykresy funkcji gęstości prawdopodobieństwa nadawy i produktów klasyfikacji w przypadku gdy rozkład wielkości ziaren nadawy jest rozkładem Rosina-Rammlera-Bennetta o parametrach d_0 = 92,1 μm i n = 1,320 perature of the suspension 20°C); surface loading q = 5 m/h (for the critical grain diameter d_g = 60.8 μm and cut size d_{50} = 43.0 μm).

The plotted probability function of grain diameter in fine-grained product in overflow calls for a closer scrutiny. In terms of the mathematical model of classification process this product should not contain grains that are bigger than the critical grain diameter.

To obtain the distribution functions of grain diameter in the final products (in overflow $F_p(d < d_x)$ and in underflow $F_w(d < d_x)$, it is required that formulas (10) and (11) be transformed accordingly

$$F_p(d<d_x)=\int_0^{d_x} f_p(d)\mathrm{d}d=\frac{1}{1-\gamma}\int_0^{d_x}(1-T(d))\cdot f(d)\mathrm{d}d \tag{28}$$

$$F_w(d<d_x)=\int_0^{d_x} f_w(d)\mathrm{d}d=\frac{1}{\gamma}\int_0^{d_x}T(d)\cdot f(d)\mathrm{d}d \tag{29}$$

Let us next consider the position of grain size d_x with respect to the critical grain diameter d_g; accordingly (28) and (29) can be rewritten as

$$F_p(d<d_x)=\begin{cases}\dfrac{1}{1-\gamma}\left\{\displaystyle\int_0^{d_x} f(d)\mathrm{d}d+\frac{1}{d_g}\int_0^{d_x} d^2 f(d)\mathrm{d}d\right\} & \text{when } 0<d_x<d_g\\ 1 & \text{when } d_g\le d_x<\infty\end{cases} \tag{30}$$

$$F_w(d<d_x)=\begin{cases}\dfrac{1}{\gamma}\cdot\left\{\dfrac{1}{d_g^2}\displaystyle\int_0^{d_x} d^2 f(d)\mathrm{d}d\right\} & \text{when } 0<d_x<d_g\\ \dfrac{1}{\gamma}\cdot\left\{\dfrac{1}{d_g^2}\displaystyle\int_0^{d_x} d^2 f(d)\mathrm{d}d+\int_{d_g}^{d_x} f(d)\mathrm{d}d\right\} & \text{when } d_g\le d_x<\infty\end{cases} \tag{31}$$

hence

$$F_p(d<d_x)=\begin{cases}\dfrac{1}{1-\gamma}\{I_3+I_4\} & \text{when } 0<d_x<d_g\\ 1 & \text{when } d_g\le d_x<\infty\end{cases} \tag{32}$$

$$F_w(d<d_x)=\begin{cases}\dfrac{1}{\gamma}\cdot\left\{\dfrac{1}{d_g^2}\cdot I_2\right\} & \text{when} \quad 0<d_x<d_g\\[2ex] \dfrac{1}{\gamma}\cdot\left\{\dfrac{1}{d_g^2} I_4+I_5\right\} & \text{when} \quad d_g\le d_x<\infty\end{cases} \tag{33}$$

The integral I_3 is the distribution function (20) for $d < d_x$ while I_4 is nearly identical to I_2 (27) though the upper limit of integration is different; I_5 is the difference of distribution functions

$$I_5-F_w(d<d_x)-F_w(d<d_g)=\exp\left\{-\left(\frac{d_g}{d_0}\right)^n\right\}-\exp\left\{-\left(\frac{d_x}{d_0}\right)^n\right\} \tag{34}$$

Finally, we get distribution functions of grain diameters in the products of classification

$$F_p(d_x)=\begin{cases}\dfrac{1}{1-\gamma}\left\{1-\exp\left[-\left(\dfrac{d_x}{d_0}\right)^n\right]+\left(\dfrac{d_x}{d_0}\right)^2\cdot\Gamma\left(1+\dfrac{2}{n},\left(\dfrac{d_x}{d_0}\right)^n\right)\right\} & \text{when} \quad 0<d_x<d_g\\[2ex] 1 & \text{when} \quad d_g\le d_x<\infty\end{cases} \tag{35}$$

$$F_w(d_x)=\begin{cases}\dfrac{1}{\gamma}\left\{\dfrac{1}{d_g^2}\cdot\Gamma\left(1+\dfrac{2}{n},\left(\dfrac{d_x}{d_0}\right)^n\right)\right\} & \text{when} \quad 0<d_x<d_g\\[2ex] \dfrac{1}{\gamma}\left\{\dfrac{1}{d_g^2}\cdot\Gamma\left(1+\dfrac{2}{n},\left(\dfrac{d_x}{d_0}\right)^n\right)+\exp\left\{-\left(\dfrac{d_g}{d_0}\right)^n\right\}-\exp\left\{-\left(\dfrac{d_x}{d_0}\right)^n\right\}\right\} & \text{when} \quad d_g\le d_x<\infty\end{cases} \tag{36}$$

Fig. 2 shows distribution functions of grain diameter in the feed and in the final products obtained on the basis of the probability density function in Fig. 1, for the same set of input data.

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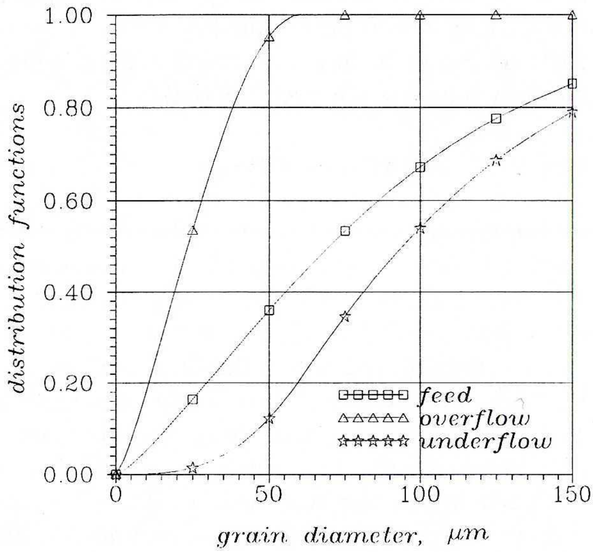

Fig. 2. Distribution function of grain diameter in the feed and in final products

Rys. 2. Wykresy dystrybuant nadawy i produktów klasyfikacji

Plots in Fig. 1 and 2 reveal that lamella classification processes are by nature not sharp. The grain fraction making up the fine product is limited and falls in the interval $(0, d_g)$, yet the grain fraction making up the coarse product contains the grains with the diameter range $(0, \infty)$ though the proportions are quite different than in the feed. As turbulence behaviours in lamella classifiers are less marked, the results of classification are perhaps better predictable than in other classifiers.

5. Investigation of lamella classification processes

Research work was carried out in the Department of Minerals Processing, Environment Protection and Waste Utilisation of the AGH-UST University in Cracow. The main component of the test rig for laboratory scale testing was a single conduit of a lamella classifier with adjustable inclination angle. It was a physical model of a real conduit (1:1). The length of the conduit was about 1 m, the dimensions of rectangular cross section area: width 0.05 m, height 0.03 m. Testing facilities also included: a feed tank complete with a mixer (agitator) and an in-built impeller pump, tanks receiving the products of classification (overflow and underflow). Underflow was removed with the use of a peristaltic pump, which also provided for flow rate control. The overflow outlet remained free.

The counter-current flow configuration was employed. The suspension flew upward from the conduit bottom section, in the direction opposite to that of sediment settling. The feed was supplied through a short pipe in the lower part of the conduit; there was also a short outlet pipe (underflow). In the upper section of the conduit, at the distance of 0.6 m from the feed supply pipe was the overflow outlet.

5.1. Methodology of laboratory tests

Prior to the tests, a homogenised suspension with the specified solid concentration, was prepared in the feed tank. At the beginning the feed was pumped into the measuring circuit for about ten minutes until the required process parameters would be achieved. During that time several portions of the suspension were introduced. As soon as the process parameters were stabilised, samples of the feed, outflow and underflow were collected to the measuring cylinders. The time period required for collecting the predetermined volume of the sample material was measured, too. Measured times required to collect the samples were then made use of in flow rate calculations. Temperature of the suspension was also measured, as it was required to calculate its viscosity. The next step involved determining the concentration of the suspension and the grain size distribution. The grain size distribution was obtained using screen analysis and sedimentation analysis methods. Another samples of the feed, underflow and overflow material were taken for chemical analysis, A single experiment, lasting about 60 minutes, involved taking several (sometimes more than ten) sets of samples.

When the geometrical parameters of the elementary conduit model, including the sedimentation area are known as well as the time required to fill the measuring cylinder with known volume, the next step involves calculating the surface loading, critical grain diameter and cut size for the specified process parameters.

5.2. Function approximating the grade efficiency curves

The grain size distribution in the feed and final products being known, one can now proceed to find the coordinates of the grade efficiency curve, employing the Nowak's method (1982) whereby the points determining the grade efficiency curve are approximated with the functions

$$T(d_x) = \frac{1}{\sqrt{2\pi} s_N} \int_{-\infty}^{d_x} \exp\left(-\frac{1}{2}\left(\frac{d - d_N}{s_N}\right)^2\right) \mathrm{d}d \tag{37}$$

$$T(d_x) = \frac{1}{\sqrt{2\pi}\sigma} \int_{0}^{d_x} \frac{1}{d} \cdot \exp\left(-\frac{1}{2}\left(\frac{\ln(d) - m}{\sigma}\right)^2\right) \mathrm{d}d \tag{38}$$

$$T(d_x) = \frac{2}{\pi}\arctan(b_0 d_x^{b_1}) \tag{39}$$

$$T(d_x) = \frac{1}{1 + b_0 d_x^{b_1}} \tag{40}$$

$$T(d_x) = \frac{1}{1 + b_0 d_x^{b_1} \cdot \exp(b_2 d_x)} \tag{41}$$

The range of function (37)–(41) is an open interval (0, 1). All these functions can be linearised. The approximating function (40) can be transformed to the form of plane equation

$$T^{-1}(d_x) = \frac{1}{s_N} \cdot d_x + \left(-\frac{d_N}{s_N}\right) \tag{42}$$

$$T^{-1}(d_x) = \frac{1}{\sigma} \cdot \ln(d_x) + \left(-\frac{m}{\sigma}\right) \tag{43}$$

where $T^{-1}(d_x)$ — function inverse to the normal distribution function.

$$\tan\left(\frac{2}{\pi}T(d_x)\right) = \ln(b_0) + b_1 \ln(d_x) \tag{44}$$

$$\ln\left(\frac{1}{T(d_x)} - 1\right) = \ln(b_0) + b_1 \ln(d_x) \tag{45}$$

$$\ln\left(\frac{1}{T(d_x)} - 1\right) = \ln(b_0) + b_1 \ln(d_x) + b_2 d_x \tag{46}$$

The critical grain diameter d_{50}, and separation sharpness indices: imperfection I and coefficients κ_1 and κ_2 were obtained. With regards to (37) and (38), equations (47) and (48) were solved numerically with respect to d_i and grain sizes corresponding to grade efficiency numbers (indices) 0.50, 0.75, 0.65, 0.35, 0.25 were derived from the formulas

$$T(d_i) = \frac{1}{\sqrt{2\pi}s_N} \int_{-\infty}^{d_i} \exp\left(-\frac{1}{2}\left(\frac{d - d_N}{s_N}\right)^2\right) \mathrm{d}d \tag{47}$$

$$T(d_i) = \frac{1}{\sqrt{2\pi}\sigma} \int_{0}^{d_i} \frac{1}{d} \cdot \exp\left(-\frac{1}{2}\left(\frac{\ln(d) - m}{\sigma}\right)^2\right) \mathrm{d}d \tag{48}$$

With regards to (39) and (40), grain sizes corresponding to grade efficiency numbers 0.50, 0.75, 0.65, 0.35, 0.25 were derived from (49) and (50):

$$d_i = \left[\frac{1}{b_0} \cdot \tan\left(\frac{1}{\pi} T(d_i) \right) \right]^{\frac{1}{b_1}} \tag{49}$$

$$d_i = \left[\frac{1}{b_0} \cdot \left(\frac{1}{T(d_i)} - 1 \right) \right]^{\frac{1}{b_1}} \tag{50}$$

With regards to the approximating function (41), nonlinear equations were solved numerically and the bisection method was applied:

$$\ln\left(\frac{1}{T(d_i)} - 1 \right) = \ln(b_0) + b_1 \ln(d_i) + b_2 d_i \tag{51}$$

To evaluate the accuracy of approximation of (37)–(41), the linear correlation coefficient r and the value of F-Fisher statistics were obtained, revealing that most accurate approximation is obtained when formulas (41) and (40) are employed, while (38) and (37) are next in line. Approximation using (3)) was much less accurate. Finally, Eq. (40) was taken for further considerations. It is only slightly less

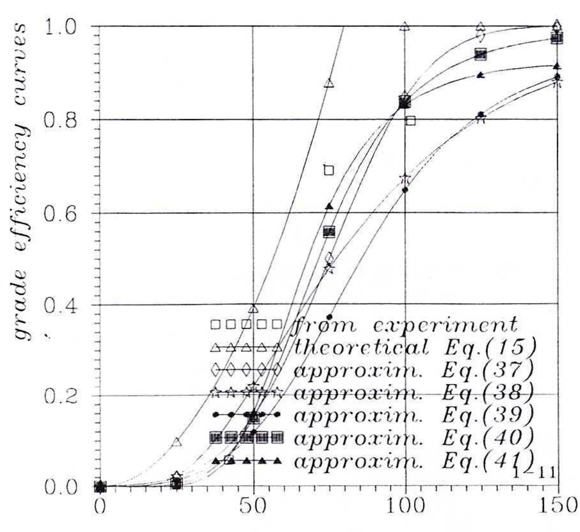

Fig. 3. Selected approximating function

Rys. 3. Przykładowe wykresy funkcji aproksymujących accurate than (41) yet computations are much less complex. The linear correlation coefficients for (40) would range from 0.965 to 1. Approximating functions are compared in Fig. 3.

The calculated values of the critical grain diameter d_{50} and sharpness indices: imperfection I and coefficients κ_1 and κ_2 were then compared with the predictions: (16) and (17). Relative errors b_{rel} was computed as the difference between the experimental and then approximated value w_{approx} and the predicted value $w_{theor.}$ related to the theoretical value $w_{theor.}$ of an indicator multiplied by 100%

$$b_{rel} = \frac{w_{approx} - w_{theor}}{w_{theor}} \cdot 100\% \qquad (52)$$

6. Discussion of results

Plots of grade efficiency curves and approximating functions derived from (40) and (41) are shown in Fig. 4–7.

Cut size d_{50} error, imperfection I error and indices κ_1 and κ_2 errors are shown in Fig. 8–11.

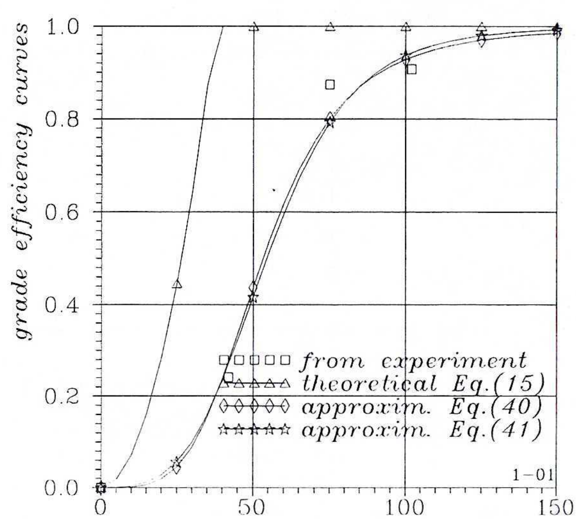

Fig. 4. Comparison of predicted and experimental grade efficiency curves (solid concentration in feed: s = 10.7 kg/m^3; overflow rate q = 2.1 m/h, feed grain composition parameters: d_0 = 48.0, n = 0.405)

Rys. 4. Porównanie teoretycznych i doświadczalnych krzywych rozdziału dla koncentracji zawiesiny s = 10,7 kg/m^3, obciążenie powierzchniowe q = 2.1 m/h, parametry rozkładu wielkości ziaren w nadawie d_0 = 48.0, n = 0.405

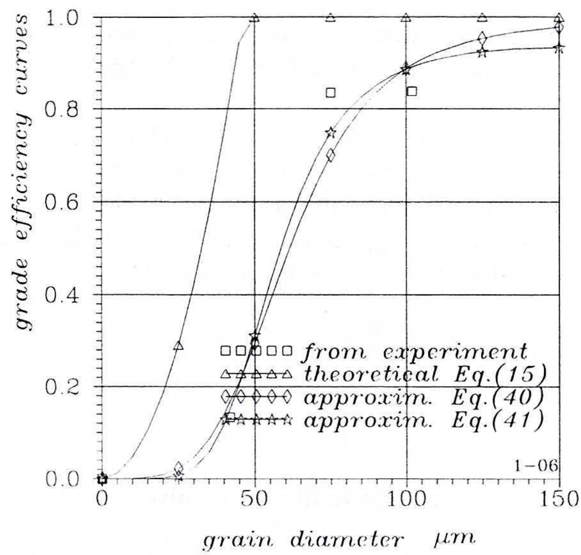

Fig. 5. Comparison of predicted and experimental grade efficiency curves (solid concentration in feed: s = 18.8 kg/m^3; surface loading q = 3.1 m/h, feed grain composition parameters: d_0 = 45.4, n = 0.392)

Rys. 5. Porównanie teoretycznych i doświadczalnych krzywych rozdziału dla koncentracji zawiesiny s = 18,8 kg/m^3, obciążenie powierzchniowe q = 3,1 m/h, parametry rozkładu wielkości ziaren w nadawie d_0 = 45,4, n = 0,392

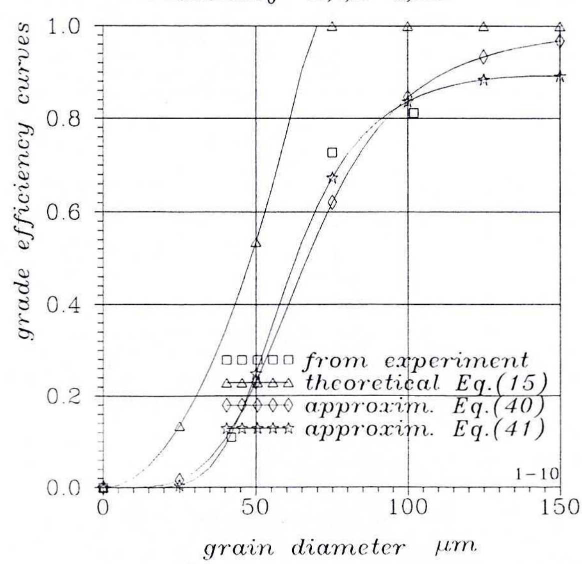

Fig. 6. Comparison of predicted and experimental grade efficiency curves (solid concentration in feed: s = 30.3 kg/m^3; overflow rate q = 6.1 m/h, feed grain composition parameters: d_0 = 48.3, n = 0.403)

Rys. 6. Porównanie teoretycznych i doświadczalnych krzywych rozdziału dla koncentracji zawiesiny s = 30.3 kg/m^3, obciążenie powierzchniowe q = 6.1 m/h, parametry rozkładu wielkości ziaren w nadawie d_0 = 48,3, n = 0,403

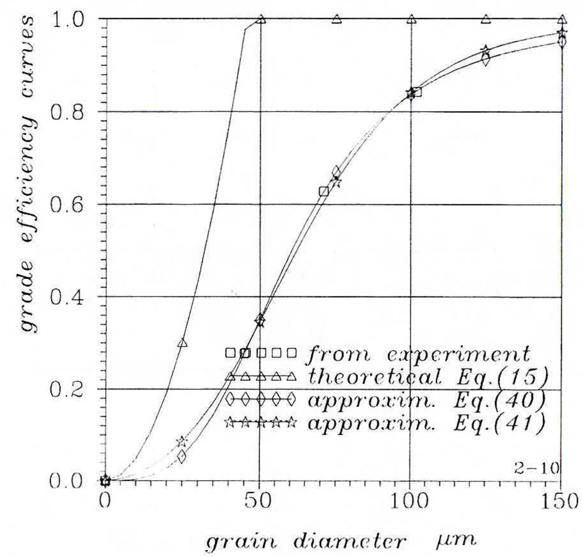

Fig. 7. Comparison of predicted and experimental grade efficiency curves (solid concentration in feed: s = 78.1 kg/m^3; overflow rate q = 2.7 m/h, feed grain composition parameters: d_0 = 18.5, n = 0.564)

Rys. 7. Porównanie teoretycznych i doświadczalnych krzywych rozdziału dla koncentracji zawiesiny s = 78,1 kg/m^3, obciążenie powierzchniowe q = 2,7 m/h, parametry rozkładu wielkości ziaren w nadawie d_0 = 18,5, n = 0,564

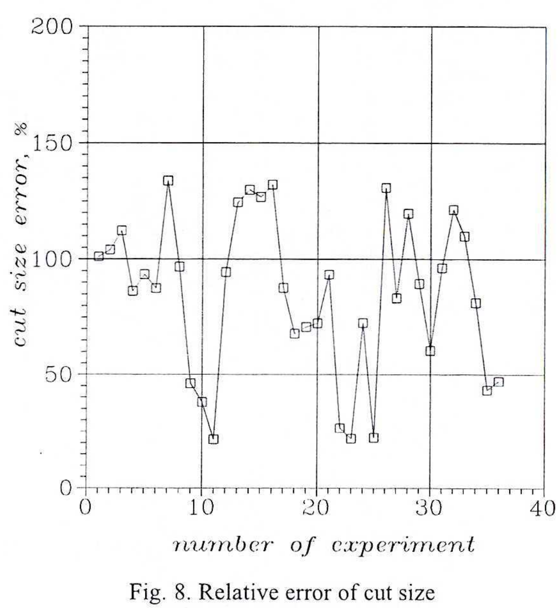

Fig. 8. Relative error of cut size

Rys. 8. Błąd względny ziarna podziałowego

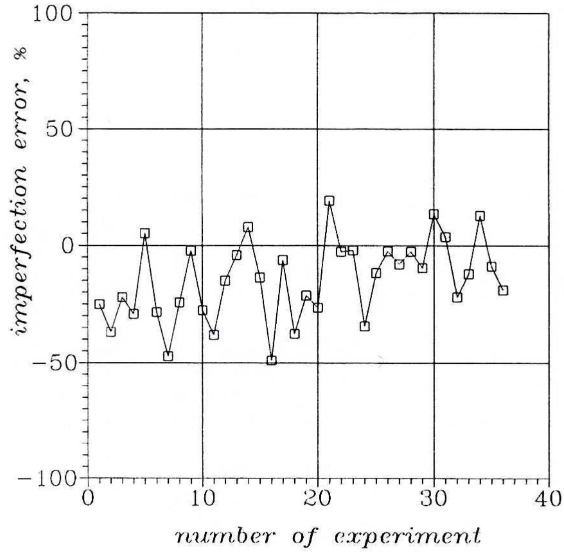

Fig. 9. Relative error of imperfection

Rys. 9. Błąd względny imperfekcji

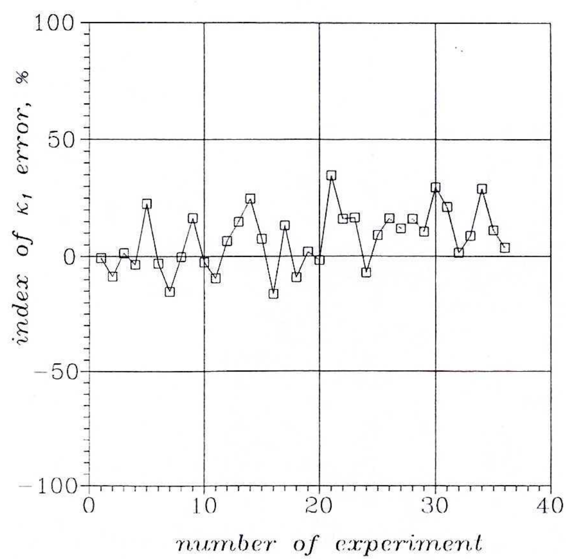

Fig. 10. Relative error of κ_1

Rys. 10. Błąd względny wskaźnika κ_1

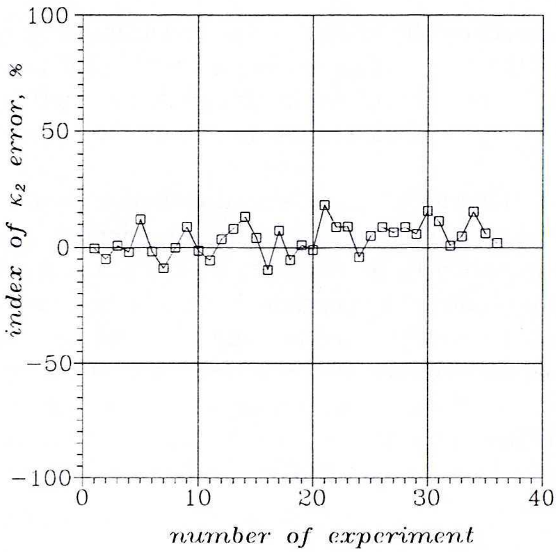

Fig. 11. Relative error of κ_2

Rys. 11. Błąd względny wskaźnika κ_2

The number of 36 experiments were analysed altogether. Solid concentration in the suspension used ranged from 10 to 70 kg/m^3 (the mean value: 29.5 kg/m^3). Surface loading would range from 1.5 to 9.2 m/h (mean value 3.7 m/h). The average linear velocity of flow varied from $3.5 \cdot 10^{-3}$ to $26 \cdot 10^{-3}$ (the mean value: $9.4 \cdot 10^{-3}$ m/s). Re number for suspension flow inside the conduit ranged from 100 to 850 (the mean value: 320). Froude's number varied from $2 \cdot 10^{-5}$ to $190 \cdot 10^{-5}$ (the mean value: $32 \cdot 10^{-5}$). The values of Reynolds and Froude's numbers indicate that suspension flows would meet the criteria for laminar (Re < 2000) and stabilised flows (Fr > 10^{-5}) (Olszewski 1975).

The analysis of cut size errors and errors of separation sharpness indices (Fig. 8–11) reveals that error involved in cut size calculation is the greatest, the stated value being always too high and ranging from about ten to nearly 100% (the mean value: 84%). Imperfection error and indices of κ coefficients are decidedly smaller — imperfection: −14.7%; κ_1: +7.5%; κ_2:+3.9%.

It appears that factors affecting the magnitude of relative error include the solid concentration, surface loading and the presence of flow disturbances (such as counter--flowing streams). In order to determine the corrected grain diameter d_{50c} the empirical formula is derived:

$$d_{50c} = \frac{4.800 \cdot n^{0.2531}}{s^{0.0572} \cdot q^{0.1669} \cdot d_0^{0.1044}} \cdot d_{50} \tag{53}$$

which allows for correcting the critical grain dimension d_{50c} [μm] in relation to solid concentration s [kg/m^3], surface loading q [m/h], RRB distribution parameters d_0 [μm] and d_{50} [μm]. The value of the linear correlation coefficient was 0.535. The applicability range of this empirical formula covers the above values of relevant variables.

Fig. 12–15 show function of grain size distribution in the feed and in final products. The good agreement shown by feed grain distribution confirms the adequacy of RRB distribution applied to its description. The grain distribution in the fine product in overflow is predicted less accurately than in the coarse-grained product in underflow. A certain asymmetry becomes apparent: experimental values of grain distribution functions for overflow are lower than theoretical ones while the values of grain distribution functions for underflow are, as a rule, higher than the predicted ones. This is best seen in Fig. 16–18, providing the comparison between theoretical grain size distribution functions and those obtained in the course of present tests.

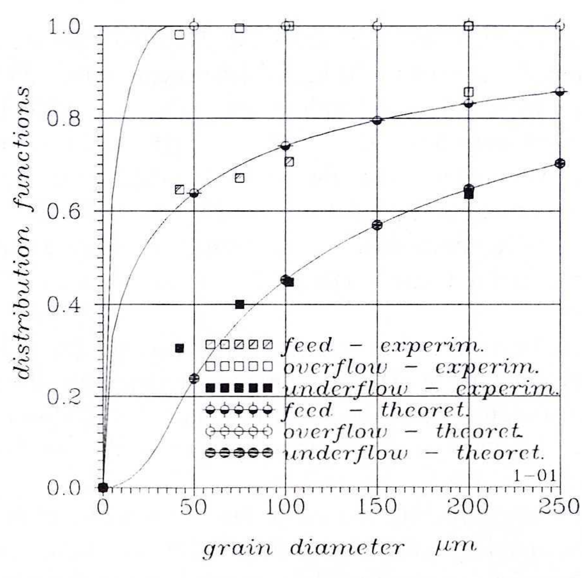

Fig. 12. Comparison of predicted and experimental grade efficiency curves (solid concentration in feed: $s = 10.7$ kg/m^3; overflow rate $q = 2.1$ m/h, feed grain composition parameters: $d_0 = 48.0$, $n = 0.405$)

Rys. 12. Porównanie teoretycznych i doświadczalnych krzywych rozdziału dla koncentracji zawiesiny $s = 10{,}7$ kg/m^3, obciążenie powierzchniowe $q = 2{,}1$ m/h, parametry rozkładu wielkości ziaren w nadawie $d_0 = 48{,}0$, $n = 0{,}405$

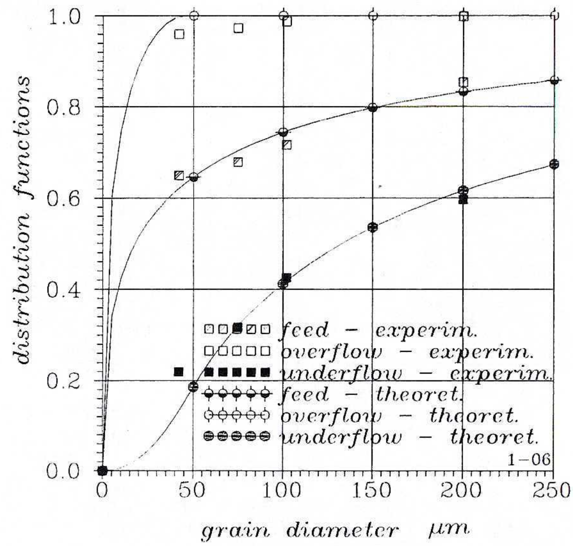

Fig. 13. Comparison of predicted and experimental grade efficiency curves (solid concentration in feed: $s = 18.8$ kg/m^3; overflow rate $q = 3.1$ m/h, feed grain composition parameters: $d_0 = 45.4$, $n = 0.392$)

Rys. 13. Porównanie teoretycznych i doświadczalnych krzywych rozdziału dla koncentracji zawiesiny $s = 18{,}8$ kg/m^3, obciążenie powierzchniowe $q = 3{,}1$ m/h, parametry rozkładu wielkości ziaren w nadawie $d_0 = 45{,}4$, $n = 0{,}392$

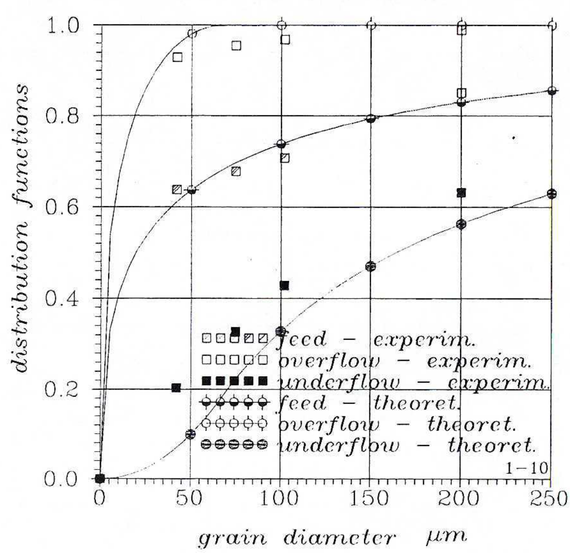

Fig. 14. Comparison of predicted and experimental grade efficiency curves (solid concentration in feed: $s = 30.3$ kg/m^3; overflow rate $q = 6.1$ m/h, feed grain composition parameters: $d_0 = 48.3$, $n = 0.403$)

Rys. 14. Porównanie teoretycznych i doświadczalnych krzywych rozdziału dla koncentracji zawiesiny $s = 30{,}3$ kg/m^3, obciążenie powierzchniowe $q = 6{,}1$ m/h, parametry rozkładu wielkości ziaren w nadawie $d_0 = 48{,}3$, $n = 0{,}403$

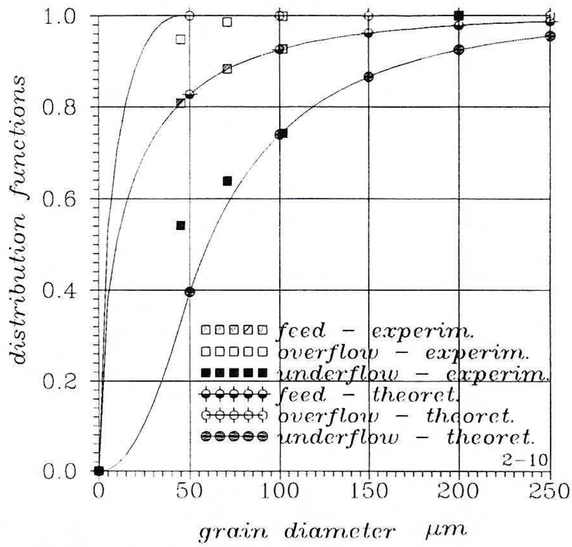

Fig. 15. Comparison of predicted and experimental grade efficiency curves (solid concentration in feed: $s = 78.1$ kg/m^3; overflow rate $q = 2.7$ m/h, feed grain composition parameters: $d_0 = 18.5$, $n = 0.564$)

Rys. 15. Porównanie teoretycznych i doświadczalnych krzywych rozdziału dla koncentracji zawiesiny $s = 78{,}1$ kg/m^3, obciążenie powierzchniowe $q = 2{,}7$ m/h, parametry rozkładu wielkości ziaren w nadawie $d_0 = 18{,}5$, $n = 0{,}564$

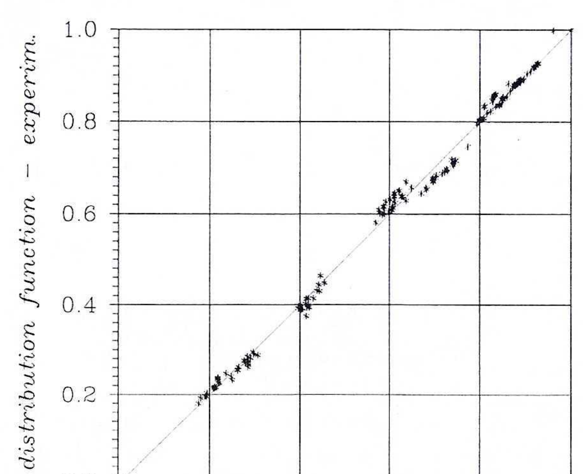

Fig. 16. Comparison between theoretical (Eq. (20)) and experimental function of grain distribution in the feed

Rys. 16. Porównanie wartości dystrybuant teoretycznych (równ. (20)) i doświadczalnych dla nadawy

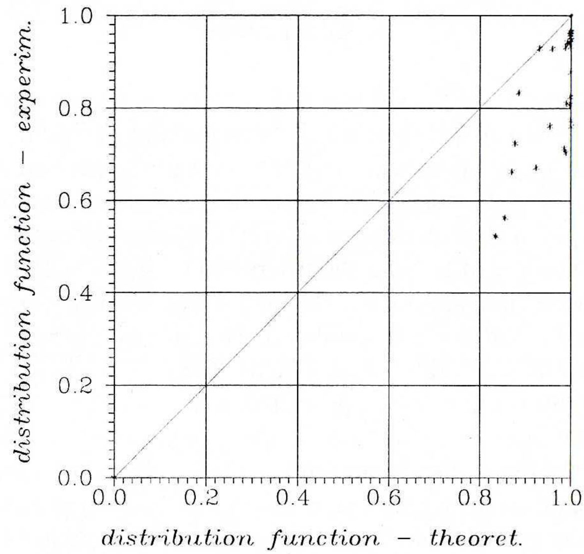

Fig. 17. Comparison between theoretical (Eq. (35)) and experimental function of grain distribution in the fine-grained product (overflow)

Rys. 17. Porównanie wartości dystrybuant teoretycznych (równ. (35)) i doświadczalnych dla produktu drobnoziarnistego (przelew)

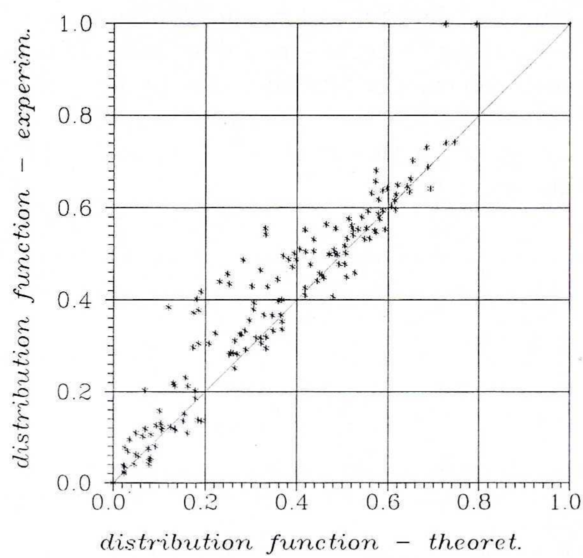

Fig. 18. Comparison between theoretical (Eq. (36)) and experimental function of grain distribution in the coarse-grained product (underflow)

Rys. 18. Porównanie wartości dystrybuant teoretycznych (równ. (36)) i doświadczalnych dla produktu gruboziarnistego (wylew)

1. Conclusions

Lamella classifiers are new facilities and theoretical descriptions of their performance are not available yet. This paper makes an attempt to fill this niche. References are made to earlier studies on lamella sedimentation processes and commonly applied classification accuracy indicators. This approach enables us to compare lamella classifiers with other types of classifiers, such as hydrocyclones and centrifugal classifiers.

The operating principle of lamella classifiers can be states as follows: a deep stream of suspension is divided into several thin-layered streams whose flows are laminar (low Re numbers) and stabilised (high Fr numbers). That is why the results of classification are easier to predict than in other types of classifiers. A mathematical description is proposed, followed by comparison of predicted and experimental values.

The mathematical model employed in the study is the simplest one, as many aspects of the process are idealised: distribution of flow velocity and of solid concentration in the inlet section are taken to be uniform; interactions between grains are neglected and Rosin-Rammler-Bennett's distribution is adopted in description of grain size distribution. It appears that in real-life processes many disturbing factors occur and their influence might be stronger than that of flow parameters and solid concentration, providing for additional computational complexity.

Though some aspects of the process are idealised, there is high degree of correspondence with experimental results. With regards to prognosticated grain size distributions in the final products on the basis of grain size distribution in the feed and flow characteristics, the values of distribution functions for the fine-grained product seem slightly overstated while those for coarse-grained products are underestimated. Major differences were reported in cut size prediction, due to the shifting of the separation curve towards higher values while the sufficiently high classification accuracy was retained. Accordingly, an empirical formula was proposed to provide for necessary correction of grain cut size diameter.

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REFERENCES

Boycott A.E., 1920: Sedimentation of blood corpuscles. Nature, London, 104, 952.

Bandrowski J., Merta H., Zioło J., 2001: Sedymentacja zawiesin. Zasady i projektowanie. Wyd. Politechniki Śląskiej, Gliwice, wyd. 2.

Bednarski S., 1978: Podział, systematyka i zastosowania osadników z wypełnieniem. Inż. Ap. Chem. 17, 3, 2–5.

Binder H., Wiesmann U., 1983: Sedimentation in Lamellenabscheidern. Aufbereitungs-Technik 2, 86–91; 3, 146–154.

Camp T.R., 1946: Sedimentation and the design of settling tanks. Trans. Amer. Soc. Civ. Engrs., 111, 895–958.

Culp G.L., Hansen S.P., Richardson G., 1968: High-rate sedimentation in water treatment works. J. Amer. Water Works Ass. 60, 6, 681–698.

Eder Th., 1961: Probleme der Trennschärfe. Aufbereitungs-Technik 3, 104–109; 4, 136–148; 8, 313–321; 11, 443–446; 12, 484–495.

Erasmus T.C., 1973: Plotting a smooth curve to the experimentally determined co-ordinates of a Tromp curve. Coal, Gold and Base Minerals of S.A. 6, 63–67.

Gęga J., 1979: Badania procesu płytkiej sedymentacji w celu określania głównych wymiarów elementów wielostrumieniowych w osadnikach hutniczych. Zesz. Nauk. AGH, s. Elektr. i Mech. Górn. i Hutn., 94, Prace Inst. Masz. Hutn. i Aut. 6.

Grzelak E., 1992: Klasyfikacja hydrauliczna drobnych cząstek i zawiesin. Fizykochem. Probl. Mineralurgii 26, 25–32.

Gambill W.R., 1959: How to estimate mixture viscosities. Chem. Eng. 3, 151–152.

Grzelak E., Malinowski A., 1981: Hydroklasyfikacja piasku w przemyśle kruszyw budowlanych. Mat. Sem. Nauk.-Techn. nt. „Klasyfikacja materiałów drobnouziarnionych", Kraków.

Haba J., Nosowicz J., Pasiński A., 1980: Klären und Eindicken von Suspensionen in Lamelleneindickern. Aufbereitungs-Technik 4, 198–201.

Kowalski W.P., 1992: Podstawy teoretyczne projektowania osadników z wkładami wielostrumieniowymi. Zesz. Nauk. AGH, 1500, s. Mechanika 27.

Lynch A.J., 1970: Lecture Notes on Comminution and Classification. University of Queensland, Australia.

Malewski J., 1981: Przeróbka kopalin. Zasady rozdrabniania i klasyfikacji. Wrocław, Wyd. Politechn. Wrocł.

Marciniak-Kowalska J., 1984: Skład ziarnowy produktów grawitacyjnej klasyfikacji mokrej w klasyfikatorze płytowym. Mat. XVIII Krak. Konf. Nauk.-Techn. Przer. Kopalin, Kraków-Krościenko, 133–144.

Marciniak-Kowalska J., 1986: Niektóre aspekty ekonomiczne stosowania osadników lamelowych w procesach sedymentacji mułów węglowych. Zesz. Nauk. AGH, 1104, s. Górnictwo z.128, 79–88.

Marciniak-Kowalska J., 1991: Application of enrichment indices to estimation of the process of wet grain classification in lamella classifiers. Trans. of Univ. of Min. and Met. Ostrava, Min. and Geolog. Series, Czechosłowacja, 2, v. XXXVII, 73–91.

Marciniak-Kowalska J., 1994: Some theoretical aspects of classification process in lamella classifiers. Preprints of the 12th Intern. Coal Preparation Congress, Cracow, Poland, vol. 1, 127–139.

Marciniak-Kowalska J., 1997: To the possibility of forecasting the grain-size distribution of classifying products in a lamella classifier. The Intern. Conf. "New Trends in Mineral Processing II", VSB–Technical University Ostrava, Czechy, 16–20.

Marciniak-Kowalska J., 2000a: Perspektywy zastosowań klasyfikatorów lamelowych. Zesz. Nauk. Politechniki Łódzkiej, z. 28, 137–144.

Marciniak-Kowalska J., 2000b: Funkcje rozdziału Trompa w procesie klasyfikacji lamelowej ziaren z uwzględnieniem rozwoju ruchu laminarnego. Inż. Ap. Chem. z. 3, 82–83.

Marciniak-Kowalska J., 2002a: Recovery of coal grains from the slurry in lamella classifiers. Mat. Konferencji Nauk.-Techn. nt. "Recyklace odpadu VI", Koszyce, Slovakia, 393–396.

Marciniak-Kowalska J., 2002b: Przemysłowe instalacje urządzeń sedymentacyjnych. Inż. Min., S. 2/8, 77–85.

Marciniak-Kowalska J., 2003: Analiza procesów klasyfikacji ziarnowej i wzbogacania w klasyfikatorach lamelowych. S. Rozpr. Monogr. 120, Kraków UWND AGH, 155 p.

Niedźwiedzki Z., 2000: Badania teoretyczne i eksperymentalne wypełnień osadników wielostrumieniowych. Zesz. Nauk. Politechn. Łódzkiej, z. 863.

Nipl R., 1979: O zastosowaniu uogólnionego rozkładu gamma do aproksymacji krzywych składu ziarnowego. Mat. XIII Krak. Konf. Nauk.-Techn. Przeróbki Kopalin, Kraków, 323–330.

Nowack K.H., 1990: Lamellenklärer — Eine wirtschaftliche Lösung für sauberes Wasser und hohe Eindickung. Aufbereitungs-Technik 31, 6, 304–310.

Nowak Z., 1973: Rzeczywista funkcja rozdziału zbioru ziaren w procesie klasyfikacji hydraulicznej zawiesin węglowych. Arch. Górn. t. XVIII, z. 1, 47–67.

Nowak Z., 1982: Gospodarka wodno-mułowa w zakładach przeróbki mechanicznej węgla. Katowice, Wyd. Śląsk.

Plitt L.R., 1971: The analysis of solid-solid separations in classifiers. The Can. Min. and Met. Bull. 4, 42–47.

Sztaba K., 1956a: Metoda statystyczna badania procesu klasyfikacji mokrej. Arch. Górn. t. I, 1, 33–53.

Sztaba K., 1956b: Krzywe rozdziału w procesie klasyfikacji mokrej. Arch. Górn. t. I, 2, 167–197.

Stokes G.G., 1851: On the effect of the internal friction of fluids on the motion of pendulums. Camb. Trans. v. 9.

Trawinski H., 1976: Die mathematische Formulierung der Tromp-Kurve. Aufbereitungs-Technik 5, 248–254, 9, 449–459.

Tromp K.F., 1937: Neue Wege für die Beurteitung der Aufbereitung von Steinkohlen. Glückauf 73, 125.

Tumidajski T., 1990: Wielowymiarowe oceny efektywności procesów rozdziału (podstawy teoretyczne). Zesz. Nauk. AGH, s. Górnictwo 14, 2, 77–88.

Tumidajski T., 1992: Zastosowanie metod statystycznych w analizie procesów przeróbczych. Wyd. Śląsk, Katowice.

Yao K.M., 1970: Theoretical study of high-rate sedimentation. Jour. Water Poll. Contr. Fed. 42, 2, 1970, 218–228.

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