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LONGWALL OUTPUT PLAN CONSIDERED IN PROBABILITY ASPECT

PLAN WYDOBYCIA Z PRZODKA ŚCIANOWEGO W KATEGORIACH PRAWDOPODOBIEŃSTWA

During the last decades, role of the longwall production output has been considerably modified. In the eighties, and earlier decades, the production output was very often fixed by state regulations, having poor economical and technical basis. Nowadays, the longwall production planning procedures are based on economical premises, including technical conditions.

In the present study we have assumed that the longwall production output may be considered as a random variable. In order to evaluate the risk of the targeted plan collapse, including assessment of its realisation chance, a probability factor has been defined.

This procedure may be used as auxiliary tool used in planning the production output of any coal mine.

Key words: coal mining, longwall, longwall production output

Rola planu wydobycia ustalanego w kopalni dla przodka ścianowego uległa zasadniczej zmianie. W latach osiemdziesiątych i wcześniejszych poziom planu był niejednokrotnie przedmiotem uchwały o podstawach pozaekonomicznych i pozatechnicznych. Głównymi przesłankami we współczesnych procedurach ustalania planu wydobycia dla przodka ścianowego są przesłanki ekonomiczne oraz możliwości i uwarunkowania techniczne.

W niniejszej pracy założono, że wydobycie uzyskiwane z przodka ścianowego można potraktować jako zmienną losową. Wprowadzono czynnik prawdopodobieństwa w celu oszacowania ryzyka niewykonania założonego planu wydobycia lub w celu oszacowania szans jego realizacji.

Wyprowadzono wzór ogólny na obliczenie prawdopodobieństwa zdarzenia, polegającego na przekroczeniu — w warunkach danego przodka ścianowego — założonego poziomu wydobycia równego $Q_{z_{plan}}$ (wzór 14).

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Uwzględniając założenie, że wydobycie dobowe Q_d jest zmienną losową, będącą sumą zmiennych losowych ($Q_{z_1}, Q_{z_2}, ..., Q_{z_n}$), opisujących wydobycie uzyskiwane na poszczególnych zmianach, otrzymano gęstość rozkładu brzegowego zmiennej Q_d podaną wzorem (18).

W oparciu o powyższy splot funkcji, sformułowano wzór ogólny na obliczenie prawdopodobieństwa przekroczenia założonego poziomu wydobycia dobowego równego $Q_{d_{plan}}$. Wzór ten (przy założeniu dwóch zmian produkcyjnych) ma postać (22).

Wyprowadzone wzory mogą stanowić dodatkowy instrument wspomagający proces planowania działalności produkcyjnej w kopalni węgla kamiennego.

Słowa kluczowe: górnictwo węgla kamiennego, przodek ścianowy, plan wydobycia z przodka ścianowego

Longwall output plan considered in probability aspect

An outline of the calculation procedure, combining production output (per day, per shift) of given longwall and probability of its realisation, has been presented in the present study.

Diversified geological and mining conditions, including technical and organisational aspect, may cause some random fluctuations of the volume of received production output. We have assumed that the production output is a random variable, as was also assumed by Przybyła 1991, which can be described by a probability density function.

Generalized form of this function was based on commonly known relation:

$$Q_z = w_c \cdot L_c \tag{1}$$

where:

L_c — number of production cycles completed during single shift [cycle/shift],

w_c — single cycle production output, thus:

$$w_c = l \cdot h \cdot k_c \cdot \gamma \tag{2}$$

where:

l — longwall length [m],

h — longwall height [m],

k_c — single step of the production cycle, calculated according to a formula:

$$k_c = \eta_z \cdot z \tag{3}$$

η_z — average web factor [–],

z — production cycle web [m/cykl],

γ — coal bulk density [Mg/m^3].

We can assume that a single production cycle output w_c — for given longwall parameters — constitutes a constant value, however, number of production cycles completed during a single shift constitutes a random variable. We can also assume that variable L_c is described by the density function marked with a symbol f_{l_c}.

In order to determine the density f_{q_z} of random variable: $Q_z = w_c \cdot L_c$, we can use the following theorem (Krysicki et al. 1986):

If X is a continuous random variable of the density concentrated within interval (a,b), and y = g(x) is a function with derivative g'(x) ≠ 0 in this interval, whereas x = h(y) is a function inverse to y = g(x), thus the density k of continuous random variable Y = g(X) has a form:

$$k(y) = f[h(y)]\left|h'(y)\right| \tag{4}$$

for $c < y < d$, for the others y, function $k(y) = 0$

where:

$c = \min(c_1, d_1)$, $d = \max(c_1, d_1)$,

$$c_1 = \lim_{x \to a+0} g(x), \qquad d_1 = \lim_{x \to b-0} g(x)$$

based on the above relations, the probability density function of random variable Q_z can be described by the equation:

$$f_{q_z}(q_z) = \frac{1}{|w_c|} \cdot f_{l_c}\left(\frac{q_z}{w_c}\right), \qquad q_z \in R^+ \tag{5}$$

Production output w_c of a single cycle is always greater than zero, thus the formula takes a form:

$$f_{q_z}(q_z) = \frac{1}{w_c} \cdot f_{l_c}\left(\frac{q_z}{w_c}\right), \qquad q_z \in R^+ \tag{6}$$

where:

$f_{q_z}(q_z)$ — probability density function of random variable Q_z — shift output,

$f_{l_c}\left(\frac{q_z}{w_c}\right)$ — probability density function of random variable,

L_c — number of production cycles per shift $\left(L_c = \frac{q_z}{w_c}\right)$.

Variable L_c can be also expressed by a formula:

$$L_c = \frac{T_e}{T_c} \tag{7}$$

where:

T_e — effective working time in the longwall [min],

T_c — duration of a single production cycle [min].

It was assumed that T_e and T_c are random variables described with functions f_{t_e} and f_{t_c}, respectively.

It was also assumed that the time T_e is the longwall working time during a single shift, minus the time of stoppages related to the machinery breakdowns.

Factors effecting the time T_e can be divided into two groups, comprising both geological-mining and technical-organisational factors. Within geological-mining factors, a location of the longwall with reference to a pit shaft, as well as, a distance from the job allocation point to the longwall, play also essential role. With regard to technical--organisational factors, type of working system, time needed to reach the longwall, time needed for job allocation, and time losses related to machinery breakdowns, are very essential.

Because of the diversified character of the factors mentioned above, development of the analytic model allowing to determine the level and scope of the factors in question, including their effect upon the variable T_e and its density $f_{t_e}(t)$ — for diversified geological-mining and technical-organisational conditions — is very complicated.

In his monograph (Snopkowski 2000), the author proposed methods indicating how to define the density of function f_{t_e}, as well as, he described his model, which can be used to determine the probability density function f_{t_c} of a variable T_c (duration of single production cycle), for chosen longwall conditions.

Thus the formula (7) constitutes a quotient of two random variables of densities f_{t_e} and f_{t_c}, what means that in order to determine the density f_{l_c} of the variable L_c we can use the following theorem (Krysicki et al. 1986):

If a random variable U constitutes a quotient of random variables X and Y, that is:

$$U = \frac{X}{Y} \tag{8}$$

thus the density k_1 of the quotient of random variables X, Y can be determined by the formula

$$k_1(u) = \int_{-\infty}^{\infty} f(uy, y)|y|dy \tag{9}$$

as well as, in a case when X, Y constitute independent random variables of densities f_1, f_2 respectively, in this case:

$$k_1(u) = \int_{-\infty}^{\infty} f_1(uy) f_2(y)|y|dy \tag{10}$$

Random variables T_e and T_c are independent variables, because the effective working time in the longwall has no influence upon the duration of production cycle.

Based on the above statements, generalized form of the probability density function of the variable L_c is expressed by the formula:

$$f_{l_c}(l_c) = \int_{-\infty}^{\infty} f_{t_e}(l_c t_c) f_{t_c}(t_c)|t_c|dy_c \tag{11}$$

where:

f_{l_c} — probability density function of the random variable L_c — number of production cycles per shift,

f_{t_e} — probability density function of the random variable T_e — effective working time in longwall,

f_{t_c} — probability density function of the random variable T_c — duration of single production cycle.

Assuming that random variables L_c and T_c take their values from the positive real number assemblages, we obtain:

$$f_{l_c}(l_c) = \int_0^{\infty} f_{t_e}(l_c t_c) f_{t_c}(t_c) t_c dt_c \tag{12}$$

Substituting the calculated form of the function f_{l_c} into formula (6) we obtain:

$$f_{q_z}(q_z) = \frac{1}{w_c} \int_0^{\infty} f_{t_e}\left(\frac{q_z}{w_c} t_c\right) f_{t_c}(t_c) t_c dt_c \tag{13}$$

We can assume that the production output plan for given longwall is set on the level, which is equal to $Q_{z_{plan}}$. Thus we can ask a question:

What probability we have that the shift production output from given longwall will exceed value $Q_{z_{plan}}$*?*

In order to solve this problem, we can use formula (13), taking under consideration the probability density function properties, so:

$$P(Q_z > Q_{z_{plan}}) = 1 - \frac{1}{w_c} \int_0^{Q_{z_{plan}}} \int_0^{\infty} f_{t_e}\left(\frac{q_z}{w_c} t_c\right) f_{t_c}(t_c) t_c dt_c dq_z \tag{14}$$

Thus the probability that the production output from given longwall will not exceed the targeted value $Q_{z_{plan}}$ is equal to:

$$P(Q_z \le Q_{z_{plan}}) = \frac{1}{w_c} \int_0^{Q_{z_{plan}}} \int_0^{\infty} f_{t_e}\left(\frac{q_z}{w_c} t_c\right) f_{t_c}(t_c) t_c dt_c dq_z \tag{15}$$

A factor named "twenty four hours factor", which constitutes total output received from given longwall in time of 24 hours, is commonly used in mining industry.

We can assume that the 24 four hours output is a random variable, thus:

$$Q_d = \sum_{i=1}^{n} Q_{z_i} \tag{16}$$

where:

Q_d — twenty four hours output [Mg/24 h],
Q_{z_i} — shift output obtained during a shift i [Mg/shift],
n — number of production shifts per 24 hours [shift/24 h].

Probability density of the sum (coproduct) of all random variables comprises also density of the marginal distribution of variable Q_d, being a convolution of random variables function Q_{z_i}, so:

$$f_{q_d} = f_{q_{z_1}} \otimes f_{q_{z_2}} \otimes \ldots \otimes f_{q_{z_n}} \tag{17}$$

where:

f_{q_d} — probability density function of marginal distribution of the random variable Q_d (24 hours output),
$f_{q_{z_1}}$ — probability density function of marginal distribution of the random variable Q_{z_1} (first shift output),
$f_{q_{z_2}}$ — probability density function of marginal distribution of the random variable Q_{z_2} (second shift output),
$f_{q_{z_n}}$ — probability density function of the random variable Q_{z_n} (output of shift n).

According to definition of the function convolution, including independence of the random variables $Q_{z_1}, Q_{z_2}, ..., Q_{z_n}$ we can obtain:

$$f_{q_d}(q_d) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f_{q_{z_1}}(q_{z_1}) f_{q_{z_2}}(q_d - q_{z_1}) \ldots \tag{18}$$

$$\ldots f_{q_{z_n}}(q_d - q_{z_1} - \ldots - q_{z_{n-1}}) dq_{z_1} dq_{z_2} \ldots dq_{z_{n-1}}$$

For example, for $n = 2$ (two shifts per 24 hours), a marginal function f_{q_d} gets a form:

$$f_{q_d}(q_d) = \int_{-\infty}^{\infty} f_{q_{z_1}}(q_{z_1}) f_{q_{z_2}}(q_d - q_{z_1}) dq_{z_1} \tag{19}$$

symbols are the same as in the formula (17).

The 24 hours output takes its values from a positive assemblage of real numbers, thus a form of the formula for a condition of $q_d > 0$ is:

$$f_{q_d}(q_d) = \int_{0}^{\infty} f_{q_{z_1}}(q_{z_1}) f_{q_{z_2}}(q_d - q_{z_1}) dq_{z_1} \tag{20}$$

for other values q_d the function is equal to zero.

We can assume that the 24 hours output plan for this longwall is set on the level which is equal to $Q_{d_{plan}}$. Thus we can ask a question:

What probability we have that 24 hours output of given longwall will exceed the value $Q_{d_{plan}}$*?*

In order to answer the above question we can use formula (20), based on properties of the probability density function, thus:

$$P(Q_d > Q_{d_{plan}}) = 1 - \int_0^{Q_{d_{plan}}} \int_0^{\infty} f_{q_{z_1}}(q_{z_1}) f_{q_{z_2}}(q_d - q_{z_1}) dq_{z_1} dq_d \tag{21}$$

A risk that 24 hours output plan of given longwall will not be exceeded, as defined by appropriate probability, can be determined from the formula

$$P(Q_d \le Q_{d_{plan}}) = \int_0^{Q_{d_{plan}}} \int_0^{\infty} f_{q_{z_1}}(q_{z_1}) f_{q_{z_2}}(q_d - q_{z_1}) dq_{z_1} dq_d \tag{22}$$

Functions $f_{q_{z_1}}$, $f_{q_{z_2}}$ are calculated using formula (13), based on functions f_{t_e}, f_{t_c}, which are determined for individual shifts.

Usability of the probability density functions of random variable named "longwall production output", described by generalized formulas (13) and (18), means that they can also be used in solving problems, where production output level is set on the basis of assumed probability of its realisation.

Conclusions

Derived formulas, used to calculate probability of execution, or a risk of incompletion, of the targeted production plan (calculated also as the probability of such event), are of generalized character.

Functions f_{t_e} and f_{t_c} constitute source functions defined as: probability density function of the random variable T_e — effective working time in the longwall, and probability density function of the random variable T_c — duration of single production cycle. Method of identification of these functions has been described in the author's monograph (Snopkowski 2000).

In restructurized coal mines, where the production output level is often reduced, usability of the described formulas is not reasonable.

However, in locations of great output concentration (great output from one longwall), the risk assessment assuming that targeted plan would not be carried out (as considered in the probability terms), may also be used as an auxiliary tool used for mine production activity planning.

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