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RELATIONSHIPS BETWEEN PARTICLE SIZE DISTRIBUTION AND WORK IN A GRINDING PROCESS

ZWIĄZEK POMIĘDZY SKŁADEM ZIARNOWYM A PRACĄ W PROCESIE ROZDRABNIANIA

The model presented consists of two steps. In the first step the kinematics of the grinding process is determined. It was obtained after selecting appropriate coefficients and applying the theory of the Markov processes. The obtained function is connected with the Rosin-Ramnler equation but the inclusion of the time in these equations enables the work-consumption to be determined. These formulae are somewhat similar to Rittinger's and Bond's hypotheses. Nevertheless there are some differences.

Key words: Markov processes, Rittinger and Bond Hypotheses, grinding process, grinding kinetics, work consumption

W procesach rozdrabniania ważnym zagadnieniem jest znajomość energii związanej z rozdrabnianiem minerałów. Najbardziej znane hipotezy rozdrabniania związane są z nazwiskami Kick, Rittinger, Hond (Skokołowski 1990). Stosunkowo mało znana jest hipoteza Hrocha (Sokołowski 1990). Hipotezy Rittingera i Bonda wykazują pewne podobieństwo, a mianowicie obie wiążą energię rozdrabniania ze zmianą powierzchni, dodatkowo w hipotezie Bonda uwzględnia się objętość ziarn rozdrabnianego materiału. W trakcie wyprowadzania wzorów w obu hipotezach zakłada się że rozdrabniane ziarna posiadają jednakowe wielkości i kształt. W rzeczywistych procesach rozdrabniana mamy do czynienia z krzywymi składu ziarnowego, w związku z czym w hipotezie Bonda przyjmuje się ziarno 80%, natomiast w hipotezie Rittingera średnią harmoniczną. Nasuwa się więc wniosek o konieczności uwzględnienia krzywych składu ziarnowego. W pierwszym etapie należy określić kinetykę procesu rozdrabniania. W tych wzorach powinien występować czas. Pracami tymi zajmowali się między innymi Sedlatschek i Hass (1953) oraz Pudło i Szczepańska (1976). W pracach tych, przy założeniu określonych klas ziarnowych, na podstawie bilansu masy wprowadza się odpowiednie wzory. Założenie stałości współczynników w tych równaniach prowadzi do szybkiego wzrostu ich ilości przy zwiększeniu ilości klas ziarnowych, co czyni te wzory trudne do interpretacji. Również wzory te związane są z ustalonymi klasami ziarnowymi.

Istotny postęp stanowiło rozbicie tych stałych na dwa czynniki zwane z angielska selection function i breakage function (Lynch 1977). Analizę procesu rozdrabniania przedstawia się często

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w postaci macierzowej (Broadbent i in. 1956). Ciągły opis procesu rozdrabniania prowadzi do równań różniczkowo-całkowych. Równania te dla gęstości prawdopodobieństwa występują w teorii procesów Markowa (Kazakow 1973), dla procesów rozdrabniania np. w pracach Gardner, Austin (1962) oraz Bass i in. (1954).

W niniejszej pracy określono energię rozdrabniania w dwóch etapach. W etapie pierwszym określono kinetykę procesu na podstawie teorii procesów Markowa. Wprowadzone równania związane są z teorią dyskretnych procesów Markowa z ciągłym zbiorem stanów (Kazakow 1973). Następnie określono pracę rozdrabniania. Otrzymane równania odpowiadają hipotezom Rittingera i Bonda, jednak z możliwości dowolnego wyboru charakterystycznej wielkości ziarna produktu. Prowadzi to jednocześnie do zmiany „stałej". Szczegółowy przypadek rozwiązania tego problemu przedstawiono w pracy Siwiec (1999)

Słowa kluczowe: hipotezy energetyczne rozdrabniania, procesy Markowa, kinetyka rozdrabiania, skład ziarnowy

1. Introduction

The grinding process is determined by two characteristic features:

- particle size *distribution* of ground the product being ground for a given feed,
- amount of energy consumed.

It is also very important to know the reciprocal connection between the size of the ground-off particles *distribution* and energy consumed in that process.

Both the size *distribution* and *grinding energy* are functions of time, so this parameter should appear in the particle size distribution functions. Among the energy hypotheses of grinding the best known are (Andreew, Zwierewicz, Pierow 1961; Sokołowski 1990)

- Rittinger,
- Kick,
- Bond,
- Brach.

These hypotheses are characterized by the particle size distribution of feed and product by one parameter only, as this being a parameter of the particle size distribution different size; mean and quantiles are used. The most usual quantile is a quantile of order $p = 0.8$ (80%). It is to be expected that using different size characteristics the constants in these hypotheses should change respectively. In this paper the grinding process is described by the discrete Markov process (Bailey 1964; Fisz 1967). On assumption of formulae for the coefficients in differential equations of this process, the final function of particle-size distribution was obtained. Next the size distribution was compared with the *grinding energy*.

2. The equations of the grinding process model

Grinding products of a narrow size fraction range characterized by particle size x_1 will be considered. After grinding the fragments will be distributed in size fractions

$(x_1, x_2), (x_2, x_3) \ldots (x_{N-1}, 0)$. The probabilities for the given size fractions are $p(x_i, x_{i+1}, t)$ and are equal to the yields of size fractions $\gamma(x_i, x_{i+1}, t)$. The notations $p(x_i, x_{i+1}, \mathrm{t}) = = p_{i+1}(t)$ and $\gamma(x_i, x_{i+1}, t) = \gamma_{i+1}$ will be used.

We also have $p_1(t) = p(+x_1, t)$. According to the Markov process (Baily 196; Kazakow 1973; Fish 1967) the following equations can be written:

$$p_1(t+\Delta t) = p_1(t)(1-\lambda_1 \Delta t) \tag{1}$$

$$p_i(t+\Delta t) = p_i(t)(1-\lambda_i \Delta t) + \sum_{j=1}^{j=i-1} p_j \lambda_j \lambda_{ji} \Delta t \qquad i=2,3,\ldots$$

These equations correspond to the theorem of complete probabilities. It can be interpreted in the following manner: within time Δt the particles were not ground, if grinding occurred the particle fragments are scattered on the lower size fractions. The coefficients λ_i mean the susceptibility to grinding) and λ_{ij} are the transition coefficients. The probabilities $\lambda_i \Delta t$, λ_{ij} resemble a selection function and a breakage function described, for example in equation (Sokołowski 1990). This equation is a counterpart of the integral — differential equation of the Markov process for continuous states (Kazakow 1973). For density $w(x,t)$ it has the form:

$$\frac{\partial w(x,t)}{\partial t} = -\lambda(x,t)w(x,t) + \int_{-\infty}^{+\infty} w(x',t)\lambda(x',t)Q(x',x,t)dx' \tag{1a}$$

According to the Markov theory we have:

$$\sum_{j=i+1}^{N} \lambda_{ij} = 1 \tag{2}$$

It is an important limit on the choice ofcoefficients λ_{ij}, and the obvious relationship

$$\sum_{i=1}^{N} p_i = 1 \tag{2a}$$

The equations system (1) is a recurrence system so it can be easily solved when all coefficients are independent of time. But the number of coefficients is so numerous that it is impractical to attempt to apply these solutions. In the following considerations the coefficients λ_i, λ_{ij} are of the form.

$$\lambda_i = kx_i^n \tag{3}$$

$$\lambda_{ij} = \frac{x_{j-1}^n - x_j^n}{x_i^n}$$

The first equation states that grinding susceptibility (λ_i) is proportional to particle size at power n, the transition coefficients (λ_{ij}) are proportional to the size fraction width at power n divided by total width (x_i, 0) at power n.

It can be checked that we have

$$\sum_{j=i+1}^{N} \lambda_{ij} = \sum_{j=i+1}^{N} \frac{x_{j-1}^{n} - x_{j}^{n}}{x_{i}^{n}} = 1 \tag{3}$$

On substitution (3) into equations (1) and after transformation we have:

$$\begin{aligned} \frac{dp_1(t)}{dt} &= -kx_1^n p_1 \\ \frac{dp_2(t)}{dt} &= -kx_2^n p_2 + k(x_1^n - x_2^n)p_1 \qquad k = 2, 3, \ldots \\ &\vdots \\ \frac{dp_k}{dt} &= -kx_k^n p_k + \sum_{i=1}^{i=k-1} k(x_{i-1}^n - x_i^n)p_i \end{aligned} \tag{4}$$

The solution of these equations is as follows (Siwiec, Nowakowski 1991):

$$\begin{aligned} p_1 &= e^{-kx_1^n t} \\ &\vdots \qquad\qquad i = 2, 3, \ldots \\ p_i &= e^{-kx_i^n t} - e^{-kx_{i-1}{}^n t} \end{aligned} \tag{5}$$

From these equations the following relationship can be obtained:

$$\sum_{i=1}^{j} p_i(t) = e^{-kx_j^n t} \tag{6}$$

which shows the probability (yield) of the material of particle-size greater than x_j. Division of the ground material into size fractions is arbitrary so as a consequence of (5) and (6) it can be written:

$$F(x,t) = 1 - e^{-kx^n t} \tag{7}$$

It is a distribution of particle size of ground material (the yield), of material passing through apertures of size x.

Function $\Phi = e^{-kx^n t}$ shows the yield of material retaing on the screen of apertures x. Functions (7) is the distributions of Rosin-Rammler (Andreew et al. 1961) $(1 - e^{bx^n})$ where the constant b has a form $b = kt$. For characterization of size distribution, quantiles of the order p are sometimes used. The quantile x_p (position parameter) indicates the proportion of material (p) passing through the screen apertures x_p.

So we have:

$$\ln(1 - p) = -kx_p^n t \tag{8}$$

The quantile of order p = 0.8 (80%) is frequently used and we have:

$$\ln 0.2 = -kx_{0.8}^{n} t \tag{9}$$

The formula (9) means that the product of grinding susceptibility (k), quantile (x_p) and grinding time (t) is constant. It changes when the order of the quantile is changed. The density function of the distribution (7) has a form:

$$f(x,t) = F_x(x,t) = nktx^{n-1}e^{-ktx^n} \tag{10}$$

For a special case when n = 1 we have:

$$f(x,t) = kte^{-ktx} \tag{11}$$

Additionally in this case, the following relations exist:

$$\bar{x} = x_m = x_{0.63} = \frac{1}{kt} \tag{12}$$

where:

$\bar{x}$ — mean value,
x_m — value of x where $f(x,t)$ has a maximum as a function of parameter t (maximum value of $f(x,t)$ for a given narrow size fraction),
$x_{0.63}$ — quantile of an order p = 0.63.

For n >1 a maximum value of density function occurs at point:

$$x_m = \sqrt[n]{\frac{n-1}{n}\frac{1}{kt}} \tag{12a}$$

At the point $x = \frac{1}{\sqrt[n]{kt}}$ the value of the distribution equals 0.63, and at the same point the maximum of the density function occurs as a function of parameter t.

3. Work done in the grinding process

The grinding process, especially the milling process is strictly connected with time. It can be assumed (8) that the work done in this process is proportional to time ($E \sim t$). In this paper the following relationship is assumed.

$$E = Mt$$

Now the formulae (8) has a form:

$$M \cdot \ln(1-p) = -kx_p^n E \tag{13}$$

when feed and ground product are characterized by D, d respectively (mean value, quantile x_p) the formulae (13) acquires a form:

$$\Delta E = M\frac{\ln(1-p)}{k}\left(\frac{1}{D^n}-\frac{1}{d^n}\right) \tag{14}$$

the only condition for exponent n is $n > 0$.
It can be assumed that the coefficient k has a form (dimension analysis):

$$k = \frac{1}{t_0}\cdot\frac{1}{d_0^n} \tag{15}$$

Now the formulae (14) has a form:

$$\Delta E = Mt_0 d_0^n \ln(1-p)\left(\frac{1}{D^n}-\frac{1}{d^n}\right) \tag{16}$$

or (because $Mt_0 = E_0$)

$$\Delta E = E_0 d_0^n \ln(1-p)\left(\frac{1}{D^n}-\frac{1}{d^n}\right) \tag{17}$$

the coefficient k is determined from the particle size distribution of the ground product.

For $n = 1$ the Rittinger hypothesis applies. (Including Brach's hypothesis). For $n = \frac{1}{2}$ the Bond hypothesis applies. In this case the sizes d, D are the quantiles of order $p = 0.8$ for product and feed.

In the special case $n = 1$ formula (17) has the form:

$$\Delta E = E_0 d_0 \ln(1-p)\left(\frac{1}{D}-\frac{1}{d}\right) \tag{17a}$$

The mean value is equal $\bar{x} = \frac{1}{kt}$, so $t_2 - t_1 = \frac{1}{k}\left(\frac{1}{\bar{x}_2}-\frac{1}{\bar{x}_1}\right)$ and energy consumption is

$$\Delta E = E_0 d_0\left(\frac{1}{\bar{x}_2}-\frac{1}{\bar{x}_1}\right) \tag{17b}$$

The formulae (17a) and (17b) are equivalent because $\bar{x} = x_{0.63}$ and $|\ln(1 - p) = = \ln 0.37 = -1|$.

The Kick's hypothesis cannot be obtained from the of applied Markov processes for grinding processes.

From formulae (7) for different time t_1, t_2 and the same value of $x = d$ is obtained.

$$\ln(1-p_1) = -kt_1 d^n \qquad t_1 > t_2$$
$$\ln(1-p_2) = -kt_2^n d^n$$

After transformations according to (17) the following relationship emerges:

$$\Delta E = \ln\frac{1-p_1}{1-p_2} E_0 \cdot d_0^n \frac{1}{d^n} \tag{17c}$$

It allows the energy consumption to be calculated in a different way.

4. Conclusions

1. It is assumed that the coefficients in the discrete Markov process have the form:

$$\lambda_i = kx_i^n$$
$$\lambda_{ij} = \frac{x_{j-1}^n - x_j^n}{x_i^n}$$

which leads to the distribution of particle size of ground product of a form:

$$F(x,t) = 1 - e^{-ktx^n}$$
$$0 < x < \infty$$

which is the Rosin-Rammler distribution when $kt = b$.

2. The value of λ_i, λ_{ij} are connected with particle size (power n) and range of particle size (power n).

3. The transition to a continuous value of x leads to the assumption that x lies in interval $0 < x < \infty$.

4. The exponent in the Markov equation has to be $n > 0$.

5. For the $x = \sqrt[n]{kt}$ the distribution $F(x, t)$ has a value 0.63.

6. For a specific value n, $n = 1$ we have $\bar{x} = x_m = x_{0.63}$.

7. When considering the grinding process of a narrow size fraction the maximum density function occurs at point $x = \sqrt[n]{kt}$. This value corresponds to the quantile x_p for $p = 0.63$.

8. For the quantile of order p we have following relationship $\ln(1-p) = -ktx_p^n$ which implies that the product of grinding susceptibility (k), grinding time (t) and quantile x_p is constant.

9. On assumption that $E = Mt$ the following theorem is obtained:

$$\Delta E = M\frac{\ln(1-p)}{k}\left(\frac{1}{D^n} - \frac{1}{d^n}\right)$$

or

$$\Delta E = E_0 d_0^n \ln(1-p)\left(\frac{1}{D^n} - \frac{1}{d^n}\right)$$

10. The value of $A\frac{\ln(1-p)}{k}$ depends on the order of quantile and grinding susceptibility (k) of material.

11. For $n = 1$ the Rittinger's hypothesis is obtained. For $n = \frac{1}{2}$ the Bond's theorem is obtained.

12. Kick's theorem can not be obtained on this way. The distribution of a given size fraction assumes the form of the Goudin-Schuman (sometime the name Andreyev is added) equation.

$$F(x) = \left(\frac{x}{d}\right)^n$$

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