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A Method for Analyzing Maintenance Decisions Based on the Discrete Markov Chain

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Abstract

Production systems are stopped due to malfunctions such as rotting equipment, imbalance of rotating parts, and high vibration, which leads to loss of customers, reduction of market share and unemployment of personnel. In this research, using the absorbing Markov process, a mathematical model is formulated to analyze the maintenance policy of the production process, through which one of the four states of new, old, or failure due to deterioration or sudden failure can be allocated to the machine. It is assumed that the machine changes from one state to another with different probabilities, which are determined using a discrete Markov chain. The different maintenance policies can be analyzed to minimize the average production cost. The mathematical model is obtained using discrete Markov chain equations, and the optimal maintenance and repair policy can be analyzed by considering all types of costs, including maintenance, production, and failure costs, so that the average cost of the production process can be minimized.

Keywords

Absorbing Markov chains; Cost minimization; Maintenance and repairs; Optimal decision.

Introduction

One of the fundamental problems in the manufacturing industry is that machines deteriorate during production, leading to failure in the production system. These failures increase system costs and lead to financial losses. these losses conclude the costs of stopping the production system, the costs of repairing or replacing machines, the costs of delay in delivering the product to the customer, and even the costs of lost sales. In this regard, one of the most practical methods used in advanced industrial systems is to plan a set of systematic instructions, methods, and processes to prevent the early failure of machines and to improve the lifetime of equipment, which is known as maintenance and repair policy (Gopalakrishnan et al., 2015). In new industries, due to the automation of equipment and machines, components such as axles, bearings, and belts deteriorate over time. The failure of each of these components will stop the machine and the production system. Therefore, implementing an optimal maintenance and repair policy is mandatory to ensure the reliability of equipment and reduce the costs of deteriorated machines. Insufficient maintenance and repair decisions will be extremely costly not only because of the need to meet equipment maintenance but also because of missed opportunities. Maintenance and repairs are essential because they are a significant part of production costs, and depending on the type of industry, it covers 15–60% of total production costs (Wan et al., 2015). Due to the high impact of random factors, such as sudden equipment failure, in production systems, it is essential to determine optimal maintenance and repair policies (Angius et al., 2016). In this regard, there are various models, such as operation research models, stochastic models, and Markov models, to determine the optimal policy for maintenance and repairs, Many studies have been conducted in these fields, such as Yang et al. (2023), which proposed a time-indexed mixed-integer linear programming formulation to optimize the long-term integrated maintenance plan and maximize the total throughput. They used an algorithm that combines Benders decomposition and Lagrangian relaxation to accelerate the computational speed. Kumar et al. (2024) proposed a novel

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stochastic model and used the Markovian methodology and Chapman–Kolmogorov differential-difference equations to predict the optimal availability. In this model, computational intelligence techniques, namely, gray wolf optimization, whale optimization algorithm, and the ant lion algorithm, are used to predict the optimal availability and profit.

As one of the first research conducted in the field of maintenance optimization using Markov model relationships, Tomasevicz and Asgarpoor (2006) proposed a model to find the optimal time of preventive maintenance and repairs for a machine that consists of eight states, including three operational states, two failure states, and three states related to preventive maintenance and repairs. In this study, there is the possibility of 2 actions in each state with specific rates, and the purpose was to determine the optimal time to perform preventive maintenance and repairs to maximize the availability of the machine. Amari et al. (2006) considered equipment with deteriorating properties in six states. Unlike the study by Tomasevicz et al. (2006), where two actions are possible in each state, in this study, there is the possibility of 3 actions in each state at certain rates: performing PM, performing CM, or continuing operation. Finally, using the continuous Markov process, the optimal action in each state is determined so that equipment reliability is maximized. In another research, Andersson et al. (2022) proposed a model that determines the optimal replacement time for a multi-component system based on time-based maintenance (TBM) and condition-based maintenance (CBM) using a continuous Markov model and dynamic programing. Jin et al. (2020) determined the optimal maintenance and repair policy for a multistate deteriorating system using a continuous Markov model. Due to the uncertainty of the system change rate among the states of this model, a transition probability matrix was obtained using reversible linear integral equations, and the optimal period of preventive maintenance was finally obtained.

According to the above, most previous studies assumed a continuous Markov chain for machine-maintenance problems because the time to failure can be easily obtained using these models. The state of a system can be modeled as a discrete Markov chain that absorbs states in real problems. Also it is seen that absorbing Markov chain equations and its steady state has not been used to create a mathematical model for the analysis of maintenance and repair policies. Therefore, this research considers a production process that includes four states, two of which are absorbing. Using the absorbing Markov process, a mathematical model is formulated to analyze the maintenance policy of the production process, where one of the four states to new, old, or failure due to deterioration and sudden failure can be allocated to the machine. The optimal maintenance and repair policy is determined using discrete Markov chain equations so that the objective cost function is minimized.

Literature review

Many studies have been conducted in recent years to optimize maintenance and repair policies, and by reviewing the articles published in recent years, it can be concluded that various solution methods have been used to optimize maintenance policies. The most important methodologies are as follows:

- Operations research models;
- Stochastic models;
- Markov models;
- Analytical models;
- Simulation models;
- Bayesian networks;
- Fuzzy models;
- Multi-objective models.

Mahmud et al. (2024) proposed a model and used an analytical hierarchy process to select an appropriate maintenance strategy for cement plants. Corrective maintenance (CM), preventive maintenance, and predictive maintenance (PdM) are considered maintenance strategies.

Al-jaburi et al. (2023) proposed a model to optimize the joint selective maintenance and repairperson assignment problem when the quality of maintenance actions is uncertain. In this paper, using a robust optimization framework, the maintenance quality uncertainty is captured via non-symmetric budget uncertainty sets that enable the level of decision-maker conservatism to be controlled. In addition, the deterministic and robust problems are reformulated as mixed integer exponential conic programs that can be solved using currently available solvers.

Saini et al. (2023) proposed a model to optimize the availability of a marine power plant with two generators, one switch board, and distribution switchboards. For this purpose, a mathematical model is proposed that uses the Markov birth death process by considering the exponentially distributed failure and repair rates of all subsystems.

Dey et al. (2023) proposed a mixed integer linear programing-based optimization model to determine the optimal maintenance schedule and minimize maintenance costs.

Rasay et al. (2024) determined the optimal maintenance and repair policy for a machine that includes

3 states, good states, partial failure states, and complete failure states. The system states is determined by inspecting the system condition, and 2 action include, minor and major repairs, could be performed for the machine.

Dahia et al. (2021) proposed a quantitative approach based on a dynamic Bayesian network (DBN) to model and evaluate the maintenance of multi-state systems and their functional dependencies. According to the transition relationships between the system states modeled by the Markov process, a DBN model is established, and the objective is to evaluate the reliability and availability of the system while taking into account the impact of maintenance strategies (perfect repair and imperfect repair).

Liu and Huang (2010) proposed an optimal replacement policy based on combining the Markov model and the Universal Generating Function (UGF). They used a quasi-renewal process to evaluate the probability of system states and describe the systems behavior after imperfect maintenance.

Hongsheng et al. (2021) proposed a stochastic degradation model to simulate changes in the state of wind turbines. In this study, the average degradation trend was obtained by analyzing the properties of the stochastic degradation model, and the average degradation model was used to describe the predictive degradation model. Then, the changing trend between the actual and predicted degradation states of the wind turbine is analyzed, and based on the average degradation process, the optimal maintenance period of the wind turbine is obtained.

Alina et al. (2020) determined the optimal time for preventive maintenance and repair by using the continuous Markov model for actuators, which are one of the main parts of industrial valves. Perfect maintenance, imperfect maintenance, and machine failure are considered in the model, and the objective function of the problem is to optimize the time of preventive maintenance and repair.

Fallah Nezhad et al. (2010) considered a serial production system in which items are 100% inspected at all stages. The item has been reworked, accepted, or scrapped. As raw materials enter the production system and finally exit, a state in the Markovian model represents different conditions for the raw materials, i.e., reworking, scrapping, and accepting. In other words, an item can be in one of its three states modeled by a distinct random variable. The objective is to determine the optimal process that maximizes the expected profit per item.

Hamrol (2018) discussed methods that can be useful for more efficiently applying the power of TQM, Six Sigma, Lean manufacturing, and other strategies for process maintenance and improvement in the daily activities of companies.

Based on published articles that use Markov models for machine maintenance problem, it is concluded that the absorbing Markov chain has not been applied to machine-maintenance problems. The system state of the production process was also considered a continuous variable in most of the previous models. Also, the objective function used in most previous studies has been the maximization of reliability, availability, safety, etc.

The state of a system can be modeled as a discrete Markov chain that absorbs states in real problems. This research considers a production process that includes four states, two of which are absorbing. The optimal maintenance and repair policy is determined using discrete Markov chain equations so that the objective cost function is minimized. To the best of the authors' knowledge, the description of the production process by absorbing the Markov chain in the maintenance problem has not been addressed.

In this study, the optimal policy of maintenance and repairs is evaluated using the concepts of distinct Markov chains to minimize the average cost of the production process. Most previous studies assumed a continuous Markov chain for machine maintenance problems because the time to failure can be easily obtained using these models. In this study, a new method is developed based on a discrete Markov chain approach in which all related costs can be determined by absorbing Markov chain equations.

Notation

The notations used in this paper are as follow:

P_{ij} – the transition probability of the system from state i to state j,

C_i – expected cost in state i,

$F(C)$ – objective function of average production cost,

T – time to failure,

π_i – limiting probability of state i,

F_{ij} – probability of absorption from transient state i to absorbing state j,

P – transition probability matrix of the system states,

MC – cost of maintenance,

Q – transition probability matrix among the transient states,

R – elements related to the rows of transient states and columns of absorbing states,

S – probability of transition between the transient states of the system when the absorbing states are removed,
M – the number of transient states of the system,
N – fundamental matrix for P, which denotes the expected number of times in states before being absorbed.

Problem statement

The problem considered in this research includes a working machine with deterioration, for which there is a possibility of two types of failure: failure due to deterioration and sudden failure.

This machine includes the following four states:

State 1: ready for perfect operation (initial state).
State 2: Ready to work with lower performance than the initial state.
State 3: Failure due to the deterioration.
State 4: Sudden failure.

The four-state Markov model is shown in Fig. 1.

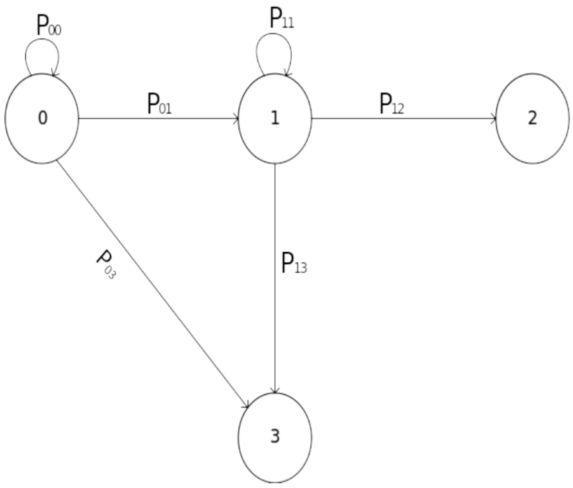

Fig. 1. State transition diagram for the system

The objective function of the model is to minimize the average cost of the production process. Two states of failure due to deterioration and sudden failure are absorbing states; thus, if the machine enters one of these states (breakdown), it is impossible to return to the other states. If the machine is in state one, in the next step, there is only the possibility of staying in its state or moving to states 2 and 4, and it is impossible to go to state 3. If the machine is in state two, in the next step, there is only the possibility of staying in its state or moving to states three and four, and it is not possible to move to state one. If the machine enters state three (the failure due to the deterioration), then it is not possible to return to the other states, and this state is one of the absorbing states of the system, in which the machine must be replaced by a new one. If the machine enters state four (sudden breakdown), it is not possible to return to other states. This state is one of the absorbing states of the system in which a new machine must be replaced with the current machine.

According to the above problem description, states 3 and 4 are the absorbing states of the system. The machine moves between possible states with different probabilities. The costs of the production process differ by state.

To describe the problem, the models of Wan et al. (2015) and Tomasevicz and Asgarpoor (2006) were used. In these two studies, no absorbing state was considered. In this research, the average cost of the production process, which is the objective function of the problem, is evaluated by considering two absorbing states based on separate Markov chain equations.

Problem formulation

In this problem, matrix P, which is the probability transition matrix of the system states, is as follows:

$$P = \begin{bmatrix} P_{00} & P_{01} & 0 & P_{03} \\ 0 & P_{11} & P_{12} & P_{13} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{1}$$

The transition probability matrix Q among the transient states of the system is obtained from the rows and columns of the non-absorbing states of the system in matrix P as follows:

$$Q = \begin{bmatrix} P_{00} & P_{01} \\ 0 & P_{11} \end{bmatrix}. \tag{2}$$

The matrix N (the fundamental matrix for P) that denotes the expected number of times in states before being absorbed is determined as follows:

$$N = [I - Q]^{-1}. \tag{3}$$

Parameter T is the expected number of steps before the chain is absorbed (time to failure), and it is obtained as follows (the first element of vector T'):

$$T' = N * 1. \tag{4}$$

F_{ij} denotes the probability of absorption from transient state i to absorbing state j, and it gives the following equation:

$$F = N * R. \tag{5}$$

The matrix S, which represents the probability of transition between the transient states of the system, is as follows:

$$S = \begin{bmatrix} P_{00} + P_{03} & P_{01} \\ P_{12} + P_{13} & P_{11} \end{bmatrix}. \tag{6}$$

When the system enters one of the failure states, perfect maintenance is implemented on the machine, and it returns to the new machine (state one).

π_i is the limiting probability of state i, which is obtained as follows:

$$\pi * S = \pi, \tag{7}$$

$$\sum_{i=1}^{M} \pi_i = 1. \tag{8}$$

The objective function of the problem is obtained as follows:

$$\begin{aligned} F(C) &= \frac{\text{Total failure cost}}{\text{Time to failure}} + \text{expeted operation cost} \\ &= \frac{\text{Maintenance cost} + \text{Failure cost}}{\text{Time to failure}} \\ &\quad + \text{ expeted operation cost} \\ &= \frac{MC + C_2F_{02} + C_3F_{03}}{T} + C_0\pi_0 + C_1\pi_1\,. \end{aligned} \tag{9}$$

Case study

A cooling tower, one of the main parts of the steel industry with four operational states is considered as case study. The transition probability matrix among the states without performing repairs and periodic inspection is as follows:

$$P = \begin{bmatrix} 0.8 & 0.1 & 0 & 0.1 \\ 0 & 0.7 & 0.2 & 0.1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

In addition, the transition probability among the matrix states when repairs and periodic inspections are performed is as follows:

$$P = \begin{bmatrix} 0.9 & 0.05 & 0 & 0.05 \\ 0 & 0.8 & 0.15 & 0.05 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

According to the above matrix, states 3 and 4 absorb the system.

The matrix of the expected costs is as follows:

$$C = \begin{bmatrix} 100 & 150 & 1500 & 2000 \end{bmatrix}.$$

The MC parameter, the maintenance cost, is equal to 200, which denotes the cost of periodic inspections and repairs. First, the objective function of the average cost of the production process without performing periodic inspections and repairs is determined.

The transient state matrix of the system is as follows:

$$Q = \begin{bmatrix} 0.8 & 0.1 \\ 0 & 0.7 \end{bmatrix}.$$

The matrix N, which represents the average number of stages until the absorption occurs, is equal to

$$N = [I - Q]^{-1} = \begin{bmatrix} 0.2 & -0.1 \\ 0 & 0.3 \end{bmatrix}^{-1} = \begin{bmatrix} 5 & 1.67 \\ 0 & 3.34 \end{bmatrix}.$$

The F_{ij} values, which are the absorption probabilities of the system, are obtained as follows:

$$\begin{aligned} F = N * R &= \begin{bmatrix} 5 & 1.67 \\ 0 & 3.34 \end{bmatrix} * \begin{bmatrix} 0 & 0.1 \\ 0.2 & 0.1 \end{bmatrix} \\ &= \begin{bmatrix} 0.334 & 0.667 \\ 0.668 & 0.334 \end{bmatrix}. \end{aligned}$$

The π_i values, which are the limiting probabilities of the transient states of the system, are equal to

$$\pi * S = \pi = \begin{bmatrix} \pi_0 & \pi_1 \end{bmatrix} * \begin{bmatrix} 0.9 & 0.1 \\ 0.3 & 0.7 \end{bmatrix} = \begin{bmatrix} \pi_0 & \pi_1 \end{bmatrix},$$

$$\pi_0 + \pi_1 = 1.$$

The objective function of the problem without maintenance is expressed as follows:

$$\begin{aligned} F(C) &= \frac{MC + C_2F_{02} + C_3F_{03}}{T} + C_0\pi_0 + C_1\pi_1 \\ &= \frac{0 + (1500 * 0.334) + (2000 * 0.667)}{6.67} \\ &\quad + (100 * 0.75) + (150 * 0.25) = 387.6. \end{aligned}$$

Now the average cost of the production process, considering the maintenance decision is computed. The objective function is obtained as follows:

$$F(C) = \frac{MC + C_2F_{02} + C_3F_{03}}{T} + C_0\pi_0 + C_1\pi_1 = \frac{200 + (1500 * 0.375) + (2000 * 0.625)}{12.5} + (100 * 0.8) + (150 * 0.20) = 271.$$

According to the above results, it is clear that the average cost of the production process can be reduced from 387.6 units to 271 units by carrying out periodic inspections and repairs; therefore, the optimal policy is to conduct periodic inspections and repairs. Thus, the proposed model can be used to analyze any maintenance policy, and the optimal policy can be determined.

It is assumed that the maintenance cost is equal to X, then the cost of the production process with maintenance decision is equaled to the cost of the production process without the maintenance decision. Thus following is obtained:

$$\frac{X + 1812.5}{12.5} + 110 = 387.6 \rightarrow X = 1657.5.$$

Thus, it can be concluded that if the maintenance cost will be less than $X = 1657.5$ it will be better to perform periodic inspections and repairs. In addition, experts can determine transition probabilities in any maintenance and repair policy, and the cost of each maintenance and repair strategy can be evaluated and select the strategy with the lowest cost. In addition, the transition probabilities for each maintenance policy can be estimated based on the historical data of previously implemented strategies.

Conclusions

Since the components of machinery are worn out and the failure of any of these parts stops the machine or production line, and since maintenance and repair costs account for a significant portion of production costs, implementing an optimal maintenance and repair policy is essential for ensuring equipment reliability and reducing downtime costs. In this research, using the absorbing Markov process, a mathematical model was developed to analyze the maintenance policy of the production process. The different maintenance policies can be analyzed to minimize the average cost of the production process. The results show the applicability of the proposed methodology in determining the optimal maintenance policy and can be used to determine the optimal maintenance and repair policies for critical equipment in various industries, including the steel industry, where the production system is continuous, and the performance and efficiency of each equipment have a significant impact on the production rate and profit. At the end of the article, a case study for one of the critical equipment of the steel industry (cooling tower) is presented, and the obtained results show that the average cost of the production process can be reduced by carrying out periodic inspections and repairs for this equipment; therefore, the optimal maintenance and repair policy for this equipment is to conduct periodic inspections. In future research, assumptions such as considering several machines instead of a single machine and the effect of a single machine's failure on the performance of other machines can be added to the problem. The optimal maintenance and repair policy can then be calculated under different conditions. It is also possible to model the problem and determine the optimal policy by considering the reservation machine. In addition to cost minimization, machine reliability maximization can be considered in the problem. The optimal maintenance and repair policy can then be calculated for the multi-objective problem.

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