ROBUST RTSS-BASED ESPRIT METHOD FOR LOW FREQUENCY OSCILLATIONS ANALYSIS

Hanjun Deng, Mouhai Liu, Minqi Yu, Dezhi Xiong

Hunan State Grid Electric Power Limited Company Power Supply Service Center (Metrology Center), Changsha, China
(✉ *dzhxiong@outlook.com*)

Abstract

Low frequency oscillations (LFOs) threaten the stability of power systems. The estimation of signal parameters via rotational invariant techniques can analyse LFOs with high accuracy only when the model order of the analysed signal is known. This paper proposes a novel model order estimation method for modal analysis of LFOs. The method first builds a singular energy spectrum to inspect whether the measurement data is being polluted by complex interferences (*e.g.*, impulsive noises). Then, a tailored Rauch–Tung–Striebel smoother is utilized to alleviate the impact of complex interferences. Afterwards, the mean value of the singular energies is adopted to determine a rough estimation of the model order of dominant modes in LFOs. Finally, the reconstruction quality indicator of the reconstructed signal is introduced for detecting and correcting overestimation and fake modes. The proposed solution is experimentally evaluated via simulations and field measurement data obtained from the phasor measurement unit installed at a generating station in North America. Results show that the method is accurate, robust, and suitable for field applications.

Keywords: Low frequency oscillations analysis, impulsive noises, model order estimation, damping ratio estimation; Rauch–Tung–Striebel smoother.

1. Introduction

The dynamic disturbances (*e.g.*, short-circuit faults) often lead to *low frequency oscillations* (LFOs) in interconnected power systems [1]. High-amplitude LFOs, which are also regarded as dominant modes, can be detrimental to the safety of power systems [2]. Fast and accurate recognition of LFOs is of great significance for reducing potential risks of power systems [3], [4]. Therefore, the monitoring of LFOs has become a critical assignment at the control centres of regional power grids. With the widespread use of *phasor measurement units* (PMUs) in modern power systems, it has become a hot issue extracting LFOs directly from PMU measurements [5,6].

To identify LFOs quickly and accurately, numerous modal analysis methods have been proposed to pick out dominant modes from synchrophasor measurements. These methods include *Discrete Fourier Transform-based* (DFT) [7–13], *Least Squares-based* (LS) methods [14–16], Prony-based methods [17,18], and *Estimation of Signal Parameters via Rotational Invariance Techniques-based* (ESPRIT) methods [19–23].

Among the methods mentioned above, the DFT-based ones are an effective frequency domain estimation tool because of their low computational burden. However, their behaviour is limited by spectrum leakage and picket-fence effects [24, 25]. Furthermore, they are restricted by spectral resolution, which is not suitable for estimating adjacent dominant modes [26]. The Prony-based methods are proposed to extract the parameters of dominant modes with high frequency resolution [17]. These methods perform accurately and can be applied for offline processing of power system data [18]. However, they are sensitive to noise and require prior knowledge of the number of dominant modes.

Recently, several solutions have been designed to resist noise interference using ESPRIT [19–22]. The main idea of ESPRIT-based methods is to estimate LFOs by separating the dominant modes from the noise subspace, which takes advantage of the nature of shift invariance in the signals. In [19], a *modified TLS-ESPRIT* method (MTLS-ESPRIT) has been proposed for parameter estimation of LFOs. The method introduces a low-pass Butterworth filter to resist coloured Gaussian noise. However, the filter deteriorates the estimation accuracy of the damping factor. In addition, its model order estimation strategy, *i.e.*, the *singular value accumulation index* (SVAI) method, does not work well under impulse noise situations. To solve the model order estimation problem, the *Extract Model Order* (EMO) method is designed to enhance the ESPRIT method [20]. In [21], a prior S-G filter is utilized to mitigate the measuring noise and *singular value accumulation percentage adjacency increment ratio* (SVAPAIR) is used to identify the number of modes in oscillation signals. Although SVAPAIR enhances the adaptability of the ESPRIT method, its performance depends on the pre-filtering stage. Different from [19, 20] and [21], the clustering technique is used in [22] to rank the dominant modes of LFOs. However, its behaviour is not clear under multiple nearly dominant modes conditions. Moreover, its multiple shift operation requires a relatively larger computation burden than traditional ESPRIT-based methods.

It is well known that the ESPRIT-based methods can analyse LFOs with high accuracy only when the model order of the analysed signal is known [23]. However, most of the existing model order estimation methods often fail when the frequencies of dominant modes are close to each other under low SNR conditions. Due to a relatively small difference between the singular values of modes and noise in such a case, it is difficult to separate signal space from noise space. To this end, the *iterative cumulative sum of squares* (ICSS) method is presented in [27] to estimate model order, which aims to find boundary in the magnitude of singular values/eigenvalues of the autocorrelation matrix. The ICSS method performs better than the SVAI and EMO methods. In [28], the *adaptive model order estimation* (AMOE) method is proposed to further solve the model order estimation issue. However, the methods mentioned above still do not work well when complex interference, *i.e.*, strong noise, impulse noise, and mixed density modes, is present in the PMU measurements. Overall, there are three major drawbacks in the existing ESPRIT-based methods in complex interference situations: i) lack of a robust model order estimation method which will constrain the behaviour of ESPRIT-based methods under complex interference conditions; ii) lack of an effective index to quantify the performance of modal analysis results; iii) lack of an effective mechanism for fake mode detection. Such substantial shortcomings limit the ESPRIT-based method from being applied in actual working environments.

To solve the problems presented above, this paper proposed a new solution for dominant modes estimation using a singular energy spectrum, *Rauch–Tung–Striebel* (RTS) smoother, and *reconstruction quality indicator* (RQI). The proposed method is named the RTSS-ESPRIT method. It first builds a singular energy spectrum to inspect whether the analysed data is undergoing complex interference. If the analysed data is contaminated by complex interference, a specially designed RTS smoother is introduced to alleviate the effect of the complex interference. Then, an accurate rough model order can be obtained by comparing multiples of the mean value of the singular energy spectrum. Afterward, the RQI is defined to quantify the quality of the

estimated dominant modes resulting from the ESPRIT method. Finally, a procedure is designed to determine the optimally dominant modes set by avoiding fake modes and overestimation. As a result, the estimated dominant modes have the smallest reconstruction error compared with existing ESPRIT-based modal analysis methods.

2. Signal model and ESPRIT Method

In this section, the most frequently-used signal model of LFOs in power systems is first introduced, and the classical ESPRIT method is then recalled.

2.1. Low-Frequency Oscillation Model

Generally, LFOs in power systems can be modelled as the sum of damped real-valued sinusoids as [20]:

$$x(n) = \sum_{i=1}^{M} \left(A_i e^{\beta n T_i} \cos\left(2\pi n T_s f_i + \theta_i\right) \right) + \xi(n),\tag{1}$$

where $x(n)$ is the PMU measurement result, *i.e.*, the values of the reported power flow by PMUs. A_i, f_i, β_i, and θ_i refer to the amplitude, frequency, damping factor, and phase, respectively, of the ith dominant mode of $x(n)$. T_s refers to the reporting interval of PMUs and M is the number of decomposed modes. $\xi(n)$ refers to the contribution from noise and unknown disturbances. The ESPRIT-based methods are superior in parameter estimation of the dominant modes due to their super-resolution feature in the frequency domain. Therefore, they are used to extract the dominant modes from PMU measurements (*i.e.*, modal analysis). For this application, the input data is considered as PMU measurements of N consecutive points, hence, $1 \leq n \leq N$ and N is specified as odd.

2.2. Overview of the Classical ESPRIT Method

A simple recall of the ESPRIT method is provided in this subsection. First, the detrended PMU measurements are formed into a square matrix. A square Hankel structure is defined as:

$$\boldsymbol{H} = \begin{bmatrix} x(1) & x(2) & \cdots & x(L) \\ x(2) & x(3) & \cdots & x(L+1) \\ \vdots & \vdots & \vdots & \vdots \\ x(L) & x(L+1) & \cdots & x(N) \end{bmatrix}_{L \times L},\tag{2}$$

where $L = (N+1)/2$, and L should be even to ensure that singular values occur in pairs. Then the *singular value decomposition* (SVD) is applied to Hankel matrix $\boldsymbol{H}$, and one obtains [29]:

$$\boldsymbol{H} = \boldsymbol{U}\boldsymbol{S}\boldsymbol{V}^T,\tag{3}$$

where the superscript $(\bullet)^T$ is the transpose operator, $\boldsymbol{U}_{L \times L}$ and $\boldsymbol{V}_{L \times L}$ are unitary matrixes, and $\boldsymbol{S}_{L \times L}$ is a diagonal matrix, which includes all the singular values of $\boldsymbol{H}$ as:

$$\mathbf{S} = \text{diag}\left[\sigma_{1_1}, \sigma_{1_2}, \cdots, \sigma_{i_1}, \sigma_{i_2}, \cdots, \sigma_{M_1}, \sigma_{M_2}, \cdots, \sigma_{G_1}, \sigma_{G_2}\right],\tag{4}$$

where $G = L/2$, and $\sigma_{1_1} \geq \sigma_{1_2} \geq \cdots \geq \sigma_{M_1} \geq \sigma_{M_2} \geq \cdots \geq \sigma_{G_1} \geq \sigma_{G_2}$. Here, a pair of consecutive singular values, *i.e.*, σ_{i_1} and σ_{i_2}, are considered corresponding to the ith mode [30].

Assuming the number of the dominant modes M is known. The right-singular vectors corresponding to the dominant modes can be expressed as:

$$V_M = \left[c_{1_1}, c_{1_2}, \cdots, c_{i_1}, c_{i_2}, \cdots, c_{M_1}, c_{M_2} \right]_{L \times 2M}, \tag{5}$$

where $c_{1_1}, c_{1_2}, \cdots, c_{i_1}, c_{i_2}, \cdots, c_{M_1}, c_{M_2}$ are the first $2M$ column vectors of matrix V^T which can be obtained from (3). By removing the last and first rows of V_M, one obtains:

$$\begin{cases} V_1 = [r_1, r_2, \cdots, r_i, \cdots, r_{L-1}]^T \\ V_2 = [r_2, r_3, \cdots, r_i, \cdots, r_L]^T \end{cases}, \tag{6}$$

where $r_1, r_2, \ldots, r_L$ are the row vectors of V_M. Then, the incidence matrix Q can be obtained by the least square method:

$$Q = \left(V_1^T V_1 \right)^{-1} V_1^T V_2, \tag{7}$$

where Q is a $2M \times 2M$ matrix whose eigenvalues can be used to estimate the frequency, damping factor, and damping ratio of each dominant mode. The related calculation formulas are:

$$\begin{cases} \hat{f}_i = \dfrac{1}{T_s} \cdot \dfrac{\mathrm{Im}\left(\log_{10}(z_i) \right)}{2\pi} & \forall i = 1, 2, \cdots, M \\[3mm] \hat{\beta}_i = \dfrac{1}{T_s} \cdot \mathrm{Re}\left(\log_{10}(z_i) \right) & \forall i = 1, 2, \cdots, M \\[3mm] \hat{\gamma}_i = -\dfrac{\hat{\beta}_i}{\sqrt{\hat{\beta}_i^2 + \left(2\pi \hat{f}_i \right)^2}} & \forall i = 1, 2, \cdots, M \end{cases}, \tag{8}$$

where $\hat{f}_i$, $\hat{\beta}_i$, and $\hat{\gamma}_i$ are the estimated frequency, damping factor, and damping ratio of the ith dominant mode. Im($\cdot$) and Re($\cdot$) return the image and real part of its argument, respectively; $z_i \in z_M = [z_1, z_1^*, \cdots, z_M, z_M^*]$, where the superscript $(\cdot)^*$ is the conjugate operator, which refers to the ith pair of successive eigenvalues of the incidence matrix Q. The eigenvalues set z_M can be easily obtained by:

$$\mathbf{z}_M = \mathrm{diag}[\mathrm{eig}(Q)]. \tag{9}$$

Once the frequencies and damping factors are obtained, the complex amplitudes can be estimated by using the LS algorithm:

$$\hat{p} = \left(\Phi^H \Phi \right)^{-1} \Phi^H x, \tag{10}$$

where the superscript $(\cdot)^H$ is the Hermitian transpose operator, $\hat{p} = [\hat{p}_1, \hat{p}_1^*, \ldots, \hat{p}_i, \hat{p}_i^*, \ldots, \hat{p}_M, \hat{p}_M^*]^T$, $x = [x(1), x(2), \ldots, x(N)]^T$, and:

$$\Phi = \begin{bmatrix} 1 & 1 & \cdots & 1 & 1 & \cdots & 1 & 1 \\ z_1 & z_1^* & \cdots & z_i & z_i^* & \cdots & z_M & z_M^* \\ \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ z_1^n & (z_1^*)^n & \cdots & z_i^n & (z_i^*)^n & \cdots & z_M^n & (z_M^*)^n \\ \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ z_1^{N-1} & (z_1^*)^{N-1} & \cdots & z_i^{N-1} & (z_i^*)^{N-1} & \cdots & z_M^{N-1} & (z_M^*)^{N-1} \end{bmatrix}. \tag{11}$$

Finally, the amplitudes and phases of all dominant modes in the LFOs signal can be obtained:

$$\begin{cases} \hat{A}_i = 2 \cdot |\hat{p}_i| \\ \hat{\theta}_i = \arctan\left(\dfrac{\mathrm{Im}(\hat{p}_i)}{\mathrm{Re}(\hat{p}_i)}\right) \end{cases}. \tag{12}$$

In fact, the ESPRIT-based methods only perform well on the premise of being able to know the true number (*i.e.*, M) of the dominant modes. Thus, estimating the value of M is one of the most difficult points in modal analysis of LFOs.

3. Proposed Model Order Estimation Method

To precisely estimate the number of dominant modes (*i.e.*, the so-called model order) from the PMU measurements which are contaminated by complex interferences, a novel method is reported in this section. First, the proposed method obtains rough estimation of the model order. Then, an overestimation detection and correction procedure, based on the RQI, is used to determine the final dominant modes.

3.1. Rough Estimation of the Model Order

The rough estimation of the model order considers not only the simple interference (*i.e.*, noise with a high SNR) but also the complex interference (*e.g.*, impulse noises and strong noises). The method first takes advantage of the gap between dominant modes and the 'noise' modes in the singular energy spectrum. If complex conditions are detected, a tailored RTS smoother is used to suppress the effects of complex interferences.

3.1.1. Pre-Estimation under Simple Conditions

In relatively ideal situations, PMU measurements are mainly disturbed by white noises. This means there are only a few trivial modes in the signals being processed while the dominant modes overwhelm. The manifestation on the singular spectrum is that the singular values corresponding to the dominant mode are much larger than those of the noises. Thus, the first pre-estimation is achieved by using the mean values of the singular energy spectrum.

In the original singular spectrum within formula (4), the ith pair of consecutive singular values corresponds to the ith mode. The difference between those two singular values may be very large in some contexts (depending on parameters of the modes). Considering the robustness of model order estimation, the quadratic sum of each pair of consecutive singular values is used here to construct an improved singular spectrum called a singular energy spectrum. The new one can be expressed as:

$$s_{\mathrm{ses}} = [\sigma_1, \cdots, \sigma_i, \cdots, \sigma_M, \sigma_{M+1}, \cdots, \sigma_G], \tag{13}$$

where $\sigma_1, \cdots, \sigma_i, \cdots, \sigma_M$ correspond to the dominant modes while $\sigma_{M+1}, \cdots, \sigma_G$ are related to the contributions of trivial modes and noise. And σ_i is called energy of the ith mode and it can be calculated by the following formula:

$$\sigma_i = \sqrt{\sigma_{i_1}^2 + \sigma_{i_2}^2}. \tag{14}$$

To pick out the dominant modes from the trivial modes and 'noise' modes, a threshold η is set as follows:

$$\eta = K_{\mathrm{th}} \cdot \mathrm{Mean}\,(s_{\mathrm{ses}}), \tag{15}$$

where Mean($\cdot$) returns the mean value of s_{ses}. K_{th} represents the sensitivity coefficient which is determined by the statistical analysis of experimental results under different noise levels. K_{th} is recommended to be set to 3 and 5 for simple and complex conditions, respectively. Then the first pre-estimation M_S of the model order can be considered as the number of elements in s_{ses} with energy greater than the threshold η.

It is worth clarifying that modal analysis of LFOs only needs to distinguish dominant modes from the trivial modes and 'noise' modes. Therefore, a loose heuristic threshold based on experimental results is sufficient under relatively ideal scenarios. Moreover, model order estimation under complex disturbance conditions will be presented later.

3.1.2. Pre-Estimation under Complex Conditions

In addition to the white noise, PMU measurements are frequently affected by impulsive noises in practical scenarios, which are usually induced by improper hardware wiring, communication interferences or unavailability of the GPS time reference [31]. Unfortunately, the first pre-estimation M_S is not sufficient to deal with such complex situations.

To suppress the effects of impulse noises, a data pre-processing technique called an RTS smoother is utilized before performing the ESPRIT method. There are three stages to implement the RTS smoother: system modelling, forward filtering, and backward smoothing [32].

First, the input data sequence is hypothetically modelled as a one-dimensional linear discrete-time system as follows:

$$\begin{cases} y(n) = F_{n-1} \cdot y(n-1) + w_{n-1} \\ x(n) = H_n \cdot y(n) + v_n \end{cases}, \tag{16}$$

where $y(n)$ is the true state of the system at time n, $x(n)$ is the measurement at time n. F_{n-1} is the state-transition coefficient. H_n is the measurement coefficient. w_{n-1} and v_n are the process noise and measurement noises, which are assumed to be uncorrelated Gaussian additive noises with zero-mean and covariance Q_{n-1} and R_n, respectively.

Second, the standard forward Kalman filter is performed on the PMU measurements, *i.e.*, $x(n)$, where $1 \le n \le N$. The forward recursive process can be briefly summarized as:

$$\begin{cases} P_f^-(n) = F_{n-1} \cdot P_f^+(n-1) \cdot F_{n-1} + Q_{n-1} \\ K_f(n) = \dfrac{P_f^-(n) \cdot H_n}{H_n \cdot P_f^-(n) \cdot H_n + R_n} \\ \hat{y}_f^-(n) = F_{n-1} \cdot \hat{y}_f^+(n-1) \\ \hat{y}_f^+(n) = \hat{y}_f^-(n) + K_f(n) \cdot \left(x(n) - H_n \cdot \hat{y}_f^-(n) \right) \\ P_f^+(n) = \left(1 - K_f(n) \cdot H_n \right) \cdot P_f^-(n) \end{cases} \tag{17}$$

where $K_f(n)$ is the Kalman filter gain at time n, $P_f^-(n)$ and $\hat{y}_f^-(n)$ are the predicted covariance and mean of the state $y(n)$, respectively, at time n before processing the measurement $x(n)$, $\hat{y}_f^+(n)$ and $P_f^+(n)$ are the estimated mean and covariance of the state $y(n)$, respectively, at time n after processing the measurement $x(n)$. The superscript $(\cdot)^-$ and $(\cdot)^+$ denote that the prediction and estimation are a priori and a posteriori, respectively. The subscript $(\cdot)_f$ denotes that it is a forward process. It is worth noting that the initialization parameters of the forward filter are $\hat{y}_f^+(0) = x(1)$ and $P_f^+(0) = 0$.

Finally, the backward smoothing process is performed on the posteriori estimates, *i.e.*, $\hat{y}_f^+(n)$, where $N - 1 \geq n \geq 0$. The backward recursive process can be briefly summarized as:

$$
\begin{cases}
K(n) = \dfrac{P_f^+(n) \cdot F_n}{P_f^-(n+1)} \\[2mm]
P(n) = P_f^+(n) - K(n) \cdot \left(P_f^-(n+1) - P(n+1) \right) \cdot K(n) \\[2mm]
\hat{y}(n) = \hat{y}_f^+(n) + K(n) \cdot \left(\hat{y}(n+1) - \hat{y}_f^-(n+1) \right)
\end{cases}
\tag{18}
$$

where $K(n)$ is the smoother gain at time n, $P(n)$ and $\hat{y}(n)$ are the smoother estimates for state covariance and state mean, respectively, at time n. The initialization parameters of the backward process are $\hat{y}(N) = \hat{y}_f^+(N)$ and $P(N) = P_f^+(N)$. After performing the designed RTS smoother on the original PMU measurements sequence x, one obtains a new data sequence, *i.e.*, $\hat{y} = [\hat{y}(1), \cdots, \hat{y}(n), \cdots, \hat{y}(N)]$.

3.1.3. Overall Process of the Rough Estimation Method

The flowchart of the proposed rough estimation method for the number of dominant modes is shown in Fig. 1. The overall process can be divided into four stages.
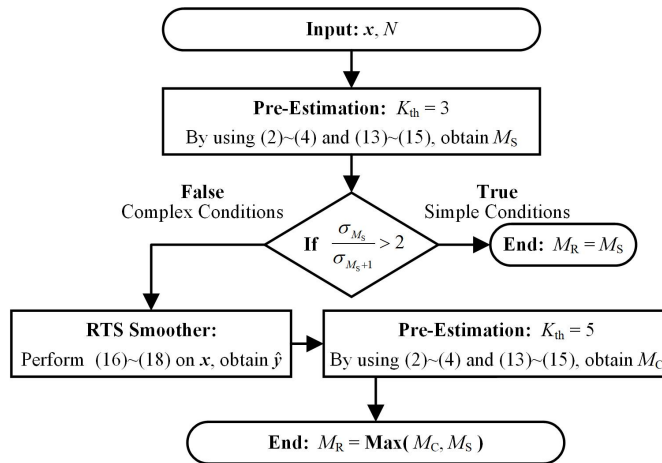

Fig. 1. The flowchart of the proposed rough estimation method for model order.

Stage 1: Assuming the detrended PMU measurements x as being in simple conditions, calculate the first pre-estimation of the model order (*i.e.*, M_S) by using (2) ~ (4) and (13) ~ (15).

Stage 2: Determining whether the PMU measurements x is in simple conditions. If the smallest of singular energies corresponding to the dominant modes is twice the largest of singular energies corresponding to the trivial or 'noise' modes (*i.e.*, $\sigma_{M_S} > 2 \cdot \sigma_{M_S+1}$), the data sequence x should be judged to be disturbed by simple interference. In this case, set the final rough estimation of the model order M_R to M_S.

Stage 3: If $\sigma_{M_S} \leq 2 \cdot \sigma_{M_S+1}$, the PMU measurements x could be considered disturbed by complex interference. In such a case, the RTS smoother is performed on the data sequence x to suppress the effects of complex interference. The smoothed data sequence is denoted as $\hat{y}$.

Stage 4: Performing (2) ~ (4) and (13) ~ (15) on $\hat{y}$ to obtain the second pre-estimation of the model order (*i.e.*, M_C). Finally, set the final rough estimation of the model order M_R to the maximum between M_C and M_S.

3.2. Correction and Final Determination

Although the proposed rough model order method is well equipped to deal with the challenges arising from impulsive noises and strong noises, it still has three shortcomings when dealing with field PMU data. Firstly, the sensitivity coefficient K_{th} adopted to determine the model order is a relatively loose empirical value. This means the number of dominant modes may be overestimated at some point. Secondly, in rare cases, there may be a fake mode with zero frequency if M_R is used directly with the ESPRIT. Thirdly, there is no metric to prove whether the dominant modes decomposed according to M_R are the most appropriate choice. To solve the above problems, a *reconstruction quality indicator* (RQI) is first introduced to quantify the reconstruction error of the estimated dominant modes. A procedure, which is used to determine the optimally dominant modes set, is then designed based on the minimum reconstruction error rule.

3.2.1. Reconstruction Quality Indicator

To quantify the quality of the estimated dominant modes resulting from the ESPRIT method, the RQI is defined as:

$$\text{RQI}_M = 10 \cdot \log_{10} \left(\frac{\| \hat{x} - x \|}{\| x \|} \right), \tag{19}$$

where $|| \cdot ||$ is the 2-norm operator. x and $\hat{x}$ are the detrended PMU measurements and the reconstructed data sequence, respectively. RQI_M represents the reconstruction quality when the number of the dominant modes is estimated as M. According to the definition of RQI, it can be seen that RQI denotes the error of the reconstructed sequence of the estimated dominant modes concerning the input sequence. Therefore, the smaller the value of RQI, the higher the quality of the estimated dominant modes. The mathematical formula for entries in $\hat{x}$ is given as:

$$\hat{x}(n) = \sum_{i=1}^{M} \hat{A}_i e^{\hat{\beta}_i n T_s} \cos \left(2\pi n T_s \hat{f}_i + \hat{\theta}_i \right) \quad \forall n = 1, 2, \cdots, N, \tag{20}$$

where T_s refers to reporting interval of PMUs; $\hat{A}_i$, $\hat{\beta}_i$, $\hat{f}_i$ and $\hat{\theta}_i$ are estimated parameters (*i.e.*, amplitude, damping factor, frequency, and phase) of ith dominant mode from (8) and (12).

3.2.2. Proposed Procedure for Final Determination

It is easy to infer that a more accurate M leads to a smaller reconstruction error (*i.e.*, a smaller RQI$_M$) from (19). Moreover, a loose K_{th} may lead to overestimation with a low probability. For these two reasons, it is reasonable to assume that the optimal determination can be achieved by searching for the smallest RQI value when M is less than or equal to M_R. The pseudocode of the proposed method is reported in Table 1.

As shown in Table 1, the proposed method is dedicated to finding the modal decomposition solution corresponding to the minimum reconstruction error. The whole search process is to check whether a higher RQI exists when the model order is less than M_R. This is also to avoid overestimation. In fact, the rough estimation M_R is already a relatively accurate choice. Therefore, the whole process requires no more than three iterations and the proposed method does not impose a heavy computational burden. Moreover, a potential fake mode with zero frequency will be detected and discarded, which is rarely considered by other ESPRIT-based modal analysis solutions.

Table 1. Pseudocode for the Final Determination Process.

Input: V, M_R, x, N, and T_s. Initialize: $\text{RQI}_{\text{final}} = +\infty$
1. for $M = M_R \rightarrow 1$
2. estimate $\hat{f}_i$ and $\hat{\beta}_i$ by using (5) ~ (9)
3. if zero frequency occurrence; go to line 1
4. estimate $\hat{A}_i$ and $\hat{\theta}_i$ by using (10) ~ (12)
5. reconstruct $\hat{x}$ by using (20)
6. calculate RQI_M by using (19)
7. if $\text{RQI}_M < \text{RQI}_{\text{final}}$
8. $\text{RQI}_{\text{final}} = \text{RQI}_M$, $M_{\text{final}} = M$
9. else
10. break
11. end if
12. end for
Output: M_{final}, $\hat{f}_i$, $\hat{\beta}_i$, $\hat{A}_i$, and $\hat{\theta}_i$ related to $\text{RQI}_{\text{final}}$.

4. Proposed Model Order Estimation Method

The performance of the proposed method is assessed using a few numerical tests in MATLAB R2019b. These tests focus on two main aspects of performance under different noise levels: 1) success rate of model order estimation; and 2) estimation accuracy of parameters of the dominant modes. To demonstrate the generality of the method, two synthetic signals were considered. The first is the classical LFOs signal, which contains four dominant modes. The frequencies of the modes distribute between 0.1 and 2.5 Hz with relatively large intervals in the frequency domain. The second includes four nearby dominant modes in the frequency domain, which makes it difficult to determine the model order under strong or impulse noise conditions. The expression of the synthetic signals is:

$$x_{1,2}(n) = \sum_{i=1}^{4} \left(A_i e^{\beta_i n T_s} \cos(2\pi n T_s f_i + \theta_i) \right), \tag{21}$$

where the amplitudes A_i are random values between 1 and 2 p.u., the phases θ_i are also random values between $-\pi$ and π. Frequencies f_i and damping factors β_i for these two synthetic signals are listed in Table 2. The sampling interval T_s is set as 1/30 s which corresponds to the most frequently used PMU reporting rate 30 Hz/s, and the sampling length is 10 s.

Table 2. Frequencies and Damping Factors of Signals of $x_1(t)$, $x_2(t)$.

Signals	$f_{1,2,3,4}$ (Hz)				$\beta_{1,2,3,4}$			
$x_1(t)$	0.21	0.79	1.61	2.33	–0.17	–0.12	–0.13	–0.07
$x_2(t)$	0.21	0.34	0.72	0.85	–0.17	–0.12	–0.13	–0.07

4.1. Success Rate of Model Order Estimation

Success rates for model order estimation are conducted to validate the robustness of the proposed method. To better assess the behaviour of the proposed method, both synthetic signals are superimposed with Gaussian white noise with zero mean. The noise levels range from 0 to 30 dB at an increment of 1 dB. The behaviour of the method is compared with those of the AMOE method [28], the SVAI method [19], and the ICSS method [27]. It is worth noting that the ICSS method outperforms the EMO method [20], the RD method [33], and the TTM method [34] according to the results reported in [27]. All reported results were obtained from 2000 independent randomized tests. The success rate for model order estimation is defined as $\left(N_{\hat{M}=4}/2000\right) \times 100\%$, where $N_{\hat{M}=4}$ refers to the number of times the number of dominant modes is estimated to be equal to 4.

The success rates regarding the model order estimation of dominant modes are reported in Fig. 2 and Fig. 3. It can be observed that for test signals containing dominant modes with evenly distributed frequencies as in $x_1(t)$, the proposed method offers a success rate of over 80% when SNR = 0 dB while the other three methods are all below 40%. The most challenging conditions for ESPRIT-based methods usually mean that test signals contain nearby dominant modes in the frequency domain as in $x_2(t)$. In such situations, a portion of the singular values corresponding to dominant modes become smaller due to the frequencies of modes being close to each other. Therefore, it is difficult to separate dominant modes from 'noise' modes when SNR is low. The proposed method benefits from the RTS smoother, as it can suppress the impact of strong noise while preserving the trend of the original signal. As shown in Fig. 3, the proposed method outperforms the other three methods when the test signal contains multiple nearby modes under low SNR conditions. This indicates that the proposed model order estimation method has better robustness and can be adapted to more complex field situations.
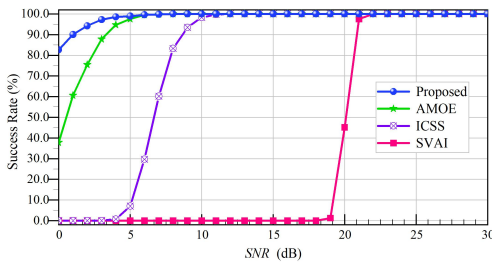

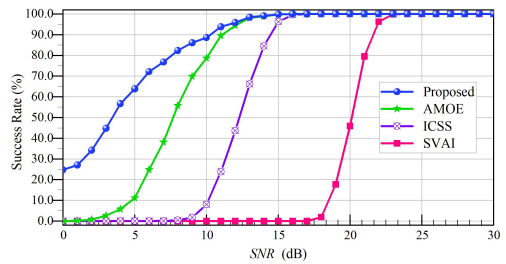Fig. 2. Success rates of dominant model order estimation for $x_1(t)$.

Fig. 3. Success rates of dominant model order estimation for $x_2(t)$.

4.2. Estimation Accuracy of Parameters of Dominant Modes

Simulations are implemented to evaluate the accuracy of the proposed method under noise conditions. To demonstrate the performance of the method, it has been compared with the AMOE-ESPRIT [28] and MTLS-ESPRIT [19] methods. For the MTLS-ESPRIT method, the model order of the synthetic signal is known since the SVAI method does not work well. The test signals are simulated under varying SNRs which range from 10 to 70 dB at an increment of 1 dB. All results are obtained from the statistics for 2000 independent tests and are reported as the *mean square error* (MSE). It is worth stating that results in the case of error order estimation had been removed.

The MSEs of the estimated frequencies and damping factors, for both $x_1(t)$ and $x_2(t)$, are reported in Fig. 4, Fig. 5, Fig. 6, and Fig. 7, respectively. The curves corresponding to the proposed method and AMOE-ESPRIT overlap, indicating that both methods behave in the same way. This is because the ESPRIT-based methods provide the same performance once the proper model order is known. The estimation accuracy of the proposed method is proportional to the SNR while the accuracy of the MTLS-ESPRIT method does not improve as the noise level decreases. This is due to the five-order low pass Butterworth filter adopted in the MTLS-ESPRIT method deteriorating the behaviour of the method. Comparing the MSEs of estimated parameters for $x_1(t)$ and $x_2(t)$, the estimation accuracy of the proposed method is decreased when signals contain dominant modes with multiple nearby frequencies.
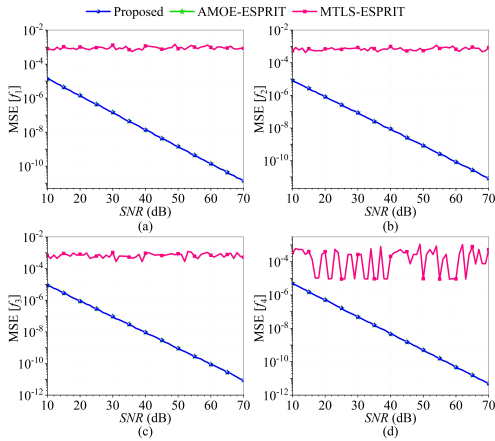

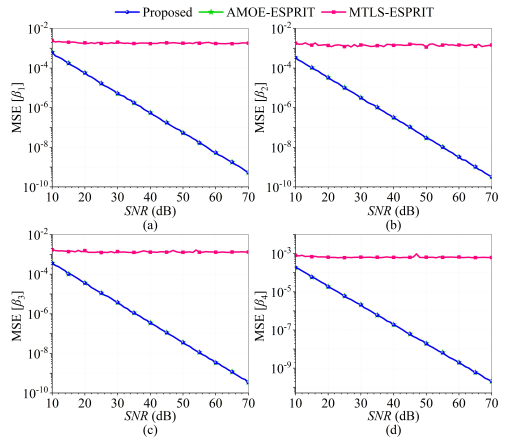Fig. 4. MSEs of the estimated frequencies versus SNR for $x_1(t)$.

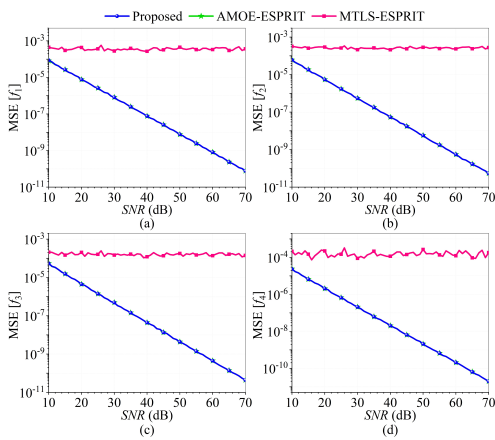Fig. 5. MSEs of the estimated damping factors versus SNR for $x_1(t)$.

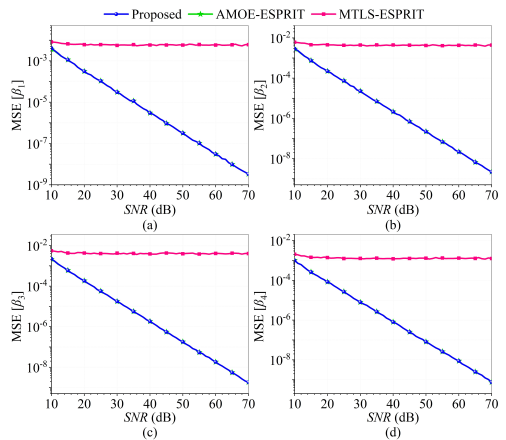Fig. 6. MSEs of the estimated frequencies versus SNR for $x_2(t)$.

Fig. 7. MSEs of the estimated damping factors versus SNR for $x_2(t)$.

4.3. Computational Burden Analysis

The computational burden of the proposed method is also analysed using simulations in MATLAB R2023a running on a computer with 16-GB RAM and a 2.3-GHz processor. In such simulations, test signals with a length of 10s are considered, *i.e.*, $N = 299$. The total execution time of 2,000 runs is reported in Table 3. Although the proposed method is heavier than AMOE-ESPRIT, it is lighter than MTLS-ESPRIT. Moreover, the average execution time of the proposed method is still an acceptable value at 13.529 ms. This indicates that it is still suitable for field applications. Compared to AMOE-ESPRIT, the proposed method sacrifices computational efficiency but improves robustness in the presence of complex disturbances. Specifically, the proposed method can avoid false modes in the estimation results and provides the smallest reconstruction error.

Table 3. Total Execution Time of 2000 Runs in Seconds.

Signal/Method	MTLS-ESPRIT	AMOE-ESPRIT	Proposed
$x_1(t)$	26.611 s	15.600 s	21.850 s
$x_2(t)$	32.996 s	18.993 s	27.057 s

5. Field PMU Measurements Experiment

To validate the effectiveness of the method, field PMMU measurements including actual events are analysed. These events occurred in the Independent System Operator-New England System (ISO-NE), a North-Eastern part of the Eastern Interconnection in the United States (its peak load being about 26,000 MW). The measurements are from the Test Cases Library [35] and the investigation was carried out using MATLAB R2019b.

The event happened on October 3, 2017 (Case 2 of [35]) when an issue in the governor of a large generator outside the ISO-NE created a multi-frequency process that lasted 5 minutes. In the case described, the active power flows through the transmission line Sub2-Ln3 that are shown in Fig. 8. The PMU reporting rate is 30 frames/s. Two acquisitions (Window 1 and Window 2) lasting 10 seconds each (*i.e.*, 301 samples), as shown in Fig. 8, were used to validate the robustness of the proposed method. Windows 1 and 2 respectively correspond to data obtained from 144.1993 to 154.1993 s, and 248.7318 to 258.7318 s. The estimation of dominant modes of the oscillation signals corresponding to the two windows was evaluated by using the proposed method, the AMOE-ESPRIT method [28] and the MTLS-ESPRIT method [19]. The frequencies, damping ratios, rough model order M_R, final model order M_{final}, and RQI$_{final}$ obtained through both methods are reported in Table 4.

Modal analysis results for both windows are reported in Table 4 and the reconstructed PMU signals are shown in Fig. 9. For Window 1, the proposed method and the AMOE-ESPRIT method behave in the same way. They extracted the four dominant modes from PMU measurements and offered the highest reconstruction quality among the three methods. This is because: i) the model order estimation methods adopted in both methods offer the same result under the Window 1 condition; ii) the proposed method did not detect overestimation and a fake mode with zero frequency in such a situation. As for the MTLS-ESPRIT method, it not only provides an overestimation of the six modes, but also has the worst signal reconstruction quality indicator with an RQI = 4.90.

The proposed method outperforms the other two methods under the Window 2 condition. It can be observed that the proposed method has the highest signal reconstruction quality of
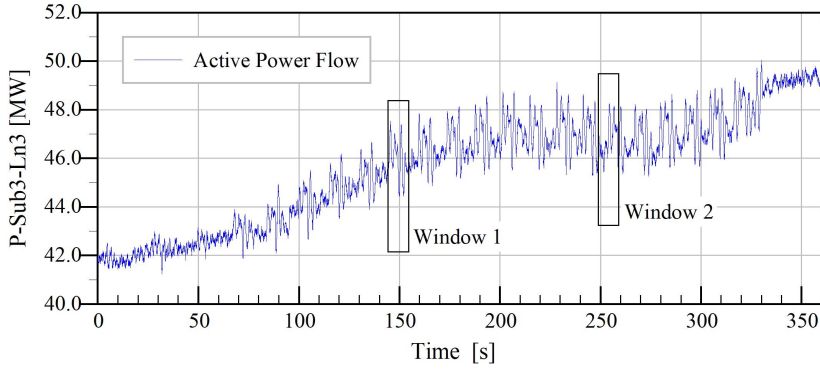
Fig. 8. Active Power Flow in MW during power oscillation.

Table 4. Estimated Modes Corresponding to Two Windows.

Estimated modes corresponding to Analysis Window 1					Estimated modes corresponding to Analysis Window 2					
Methods	M_R	M_{final}	RQI_{final}	$\hat{f}_i$ (Hz)	$\hat{\gamma}_i$ (%)	Methods	M_R	M_{final}	RQI_{final}	$\hat{f}_i$ (Hz)	$\hat{\gamma}_i$ (%)
MTLS-ESPRIT	–	6	4.90	0.1034	16.34	MTLS-ESPRIT	–	8	2.73	0.1123	4.782
				0.3539	−3.460					0.3032	−22.12
				0.5744	−4.300					0.4862	−1.750
				0.5373	−43.67					0.5770	5.567
				0.7928	−1.080					0.8149	−2.328
				1.0763	−2.580					0.9792	2.942
				–	–					1.1176	−1.766
				–	–					4.7927	0.066
AMOE-ESPRIT	–	4	−11.9	0.1027	13.99	AMOE-ESPRIT	–	5	−5.14	0.0000	100.0
				0.3605	−1.800					0.1228	9.161
				0.5499	−3.690					0.3659	−12.46
				0.6798	−2.810					0.5274	−5.082
				–	–					0.9078	3.683
Proposed	4	4	−11.9	0.1027	13.99	Proposed	5	4	−10.83	0.1156	5.395
				0.3605	−1.800					0.3188	−17.53
				0.5499	−3.690					0.5130	−4.917
				0.6798	−2.810					0.8186	6.982

RQI = −10.83. Meanwhile, the reconstructed signal of the proposed method, as shown in Fig. 9, provides the highest coincidence with the original PMU measurements compared to other two methods. Although a fake mode with zero frequency occurred during the rough estimation stage, *i.e.*, $M_R = 5$, the proposed correction process (in Sec III, B) successfully detected this overestimation and offered a better selection of $M_{\text{final}} = 4$. However, the AMOE-ESPRIT method and the MTLS-ESPRIT method provide neither overestimation detection nor fake mode identification. The proposed method, which is based on the RQI values of the reconstructed signals, greatly improves the robustness of the modal analysis results. This feature allows the proposed method to be more suitable for field PMU data subject to complex interferences compared to existing methods.
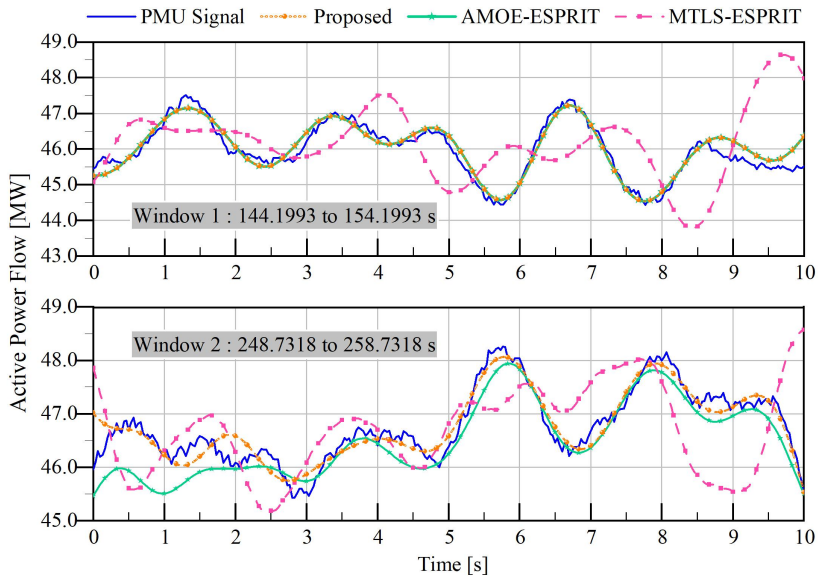
Fig. 9. Recorded and reconstructed signals for two analysis windows.

6. Conclusions

In this article, a robust ESPRIT-based method, *i.e.*, RTSS-ESPRIT, has been presented for the analysis of LFOs occurring in power systems. The contribution is fourfold: firstly, the singular energy spectrum has been proposed as the basis for estimating the model order of LFOs, which is more suitable for model order estimation than the original singular spectrum. Secondly, a tailored Rauch–Tung–Striebel smoother has been introduced at the model order estimation stage, which improves the success rate of the model estimation method under complex interference and low SNR working conditions. Thirdly, the reconstruction quality indicator has been proposed to quantify the reconstruction error of modal analysis. Fourthly, a supplementary procedure has been designed to detect and correct overestimation and fake mode. Compared to the existing ESPRIT-based methods, the proposed solution not only provides the highest success rate for model order estimation, but also provides the optimal modal analysis results with the highest reconstruction quality. The effectiveness of the solution has been verified by simulations and actual data provided by a field PMU in North America. Results reveal that the proposed solution can effectively estimate dominant modes from PMU measurements, even if the PMU data contains mixed density modes and strong/impulse noises. It should be also noted that the limitations of RTSS-ESPRIT are twofold. The first is that the RTS smoother cannot be applied to estimate the parameter of the mode whose frequency is larger than 3 Hz. The second is that the computational burden of RTSS-ESPRIT is greater than that of AMOE-ESPRIT.

References

[1] Zamani, H., Karimi-Ghartemani, M., & Mojiri, M. (2018). Analysis of Power System Oscillations from PMU Data Using an EPLL-Based Approach. *IEEE Transactions on Instrumentation and Measurement*, *67*(2), 307–316. https://doi.org/10.1109/tim.2017.2777538

[2] Song, J., Zhang, J., Kuang, H., & Wen, H. (2023). Dynamic Synchrophasor Estimation Based on Weighted Real-Valued Sinc Interpolation Method. *IEEE Sensors Journal*, *23*(1), 588–598. https://doi.org/10.1109/jsen.2022.3224365

[3] Morshed, M. J., & Fekih, A. (2019). A Probabilistic Robust Coordinated Approach to Stabilize Power Oscillations in DFIG-Based Power Systems. *IEEE Transactions on Industrial Informatics*, *15*(10), 5599–5612. https://doi.org/10.1109/tii.2019.2901935

[4] Sun, Z., Cai, G., Yang, D., Liu, C., Wang, B., & Wang, L. (2019). A Method for the Evaluation of Generator Damping During Low-Frequency Oscillations. *IEEE Transactions on Power Systems*, *34*(1), 109–119. https://doi.org/10.1109/tpwrs.2018.2868717

[5] Song, J., Zhang, J., & Wen, H. (2021). Accurate Dynamic Phasor Estimation by Matrix Pencil and Taylor Weighted Least Squares Method. *IEEE Transactions on Instrumentation and Measurement*, *70*, 1–11. https://doi.org/10.1109/tim.2021.3066187

[6] Khalilinia, H., & Venkatasubramanian, V. M. (2015). Modal Analysis of Ambient PMU Measurements Using Orthogonal Wavelet Bases. *IEEE Transactions on Smart Grid*, *6*(6), 2954–2963. https://doi.org/10.1109/tsg.2015.2410138

[7] Zhang, J., Song, J., Li, C., Xu, X., & Wen, H. (2024). Novel Frequency Estimator for Distorted Power System Signals Using Two-Point Iterative Windowed DFT. *IEEE Transactions on Industrial Electronics*, 1–12. https://doi.org/10.1109/tie.2023.3347846

[8] Hwang, J. K., & Liu, Y. (2016). Identification of interarea modes from ringdown data by curve-fitting in the frequency domain. *IEEE Transactions on Power Systems*, *32*(2), 842-851. https://doi.org/10.1109/Tpwrs.2016.2588478

[9] Rueda, J. L., Juarez, C. A., & Erlich, I. (2011). Wavelet-Based Analysis of Power System Low-Frequency Electromechanical Oscillations. *IEEE Transactions on Power Systems*, *26*(3), 1733–1743. https://doi.org/10.1109/tpwrs.2010.2104164

[10] Avdakovic, S., Nuhanovic, A., Kusljugic, M., & Music, M. (2012). Wavelet transform applications in power system dynamics. *Electric Power Systems Research*, *83*(1), 237–245. https://doi.org/10.1016/j.epsr.2010.11.031

[11] Hosseini, S. A., Amjady, N., & Velayati, M. H. (2015). A Fourier Based Wavelet Approach Using Heisenberg's Uncertainty Principle and Shannon's Entropy Criterion to Monitor Power System Small Signal Oscillations. *IEEE Transactions on Power Systems*, *30*(6), 3314–3326. https://doi.org/10.1109/tpwrs.2014.2377180

[12] Wang, K., Wang, J., Song, J., Tang, L., Shan, X., & Wen, H. (2023). Accurate DFT Method for Power System Frequency Estimation Considering Multi-Component Interference. *IEEE Transactions on Instrumentation and Measurement*, *72*, 1–11. https://doi.org/10.1109/tim.2023.3322493

[13] Song, J., Shan, X., Zhang, J., & Wen, H. (2024). Parameter Estimation of Power System Oscillation Signals under Power Swing Based on Clarke–Discrete Fourier Transform. *Electronics*, *13*(2), 297. https://doi.org/10.3390/electronics13020297

[14] Wies, R. W., Pierre, J. W., & Trudnowski, D. J. (2003). Use of ARMA block processing for estimating stationary low-frequency electromechanical modes of power systems. *IEEE Transactions on Power Systems*, *18*(1), 167–173. https://doi.org/10.1109/tpwrs.2002.807116

[15] Zhou, N., Pierre, J. W., Trudnowski, D. J., & Guttromson, R. T. (2007). Robust RLS Methods for Online Estimation of Power System Electromechanical Modes. *IEEE Transactions on Power Systems*, *22*(3), 1240–1249. https://doi.org/10.1109/tpwrs.2007.901104

[16] Ning Zhou, Trudnowski, D. J., Pierre, J. W., & Mittelstadt, W. A. (2008). Electromechanical Mode Online Estimation Using Regularized Robust RLS Methods. *IEEE Transactions on Power Systems*, *23*(4), 1670–1680. https://doi.org/10.1109/tpwrs.2008.2002173

[17] Hauer, J. F., Demeure, C. J., & Scharf, L. L. (1990). Initial results in Prony analysis of power system response signals. *IEEE Transactions on Power Systems*, *5*(1), 80–89. https://doi.org/10.1109/59.49090

[18] Hauer, J. F. (1991). Application of Prony analysis to the determination of modal content and equivalent models for measured power system response. *IEEE Transactions on Power Systems*, *6*(3), 1062–1068. https://doi.org/10.1109/59.119247

[19] Tripathy, P., Srivastava, S. C., & Singh, S. N. (2011). A Modified TLS-ESPRIT-Based Method for Low-Frequency Mode Identification in Power Systems Utilizing Synchrophasor Measurements. *IEEE Transactions on Power Systems*, *26*(2), 719–727. https://doi.org/10.1109/tpwrs.2010.2055901

[20] Philip, J. G., & Jain, T. (2018). Analysis of low frequency oscillations in power system using EMO ES-PRIT. *International Journal of Electrical Power & Energy Systems*, *95*, 499–506. https://doi.org/10.1016/j.ijepes.2017.08.037

[21] Chen, J., Jin, T., Mohamed, M. A., & Wang, M. (2019). An Adaptive TLS-ESPRIT Algorithm Based on an S-G Filter for Analysis of Low Frequency Oscillation in Wide Area Measurement Systems. *IEEE Access*, *7*, 47644–47654. https://doi.org/10.1109/access.2019.2908629

[22] Pande, P. W., Chakrabarti, S., Srivastava, S. C., & Sarkar, S. (2020). A Clustering-Based Approach for Estimation of Low Frequency Oscillations in Power Systems. *IEEE Transactions on Power Systems*, *35*(6), 4666–4677. https://doi.org/10.1109/tpwrs.2020.2992125

[23] Jin, T., Liu, S., & Flesch, R. C. C. (2017). Mode identification of low-frequency oscillations in power systems based on fourth-order mixed mean cumulant and improved TLS-ESPRIT algorithm. *IET Generation, Transmission & Distribution*, *11*(15), 3739–3748. https://doi.org/10.1049/iet-gtd.2016.2131

[24] Song, J., & Wen, H. (2020, May). Dynamic phasor estimations by using matrix pencil and Taylor least squares method. In *2020 IEEE International Instrumentation and Measurement Technology Conference (I2MTC)* (pp. 1–6). IEEE. https://doi.org/10.1109/I2MTC43012.2020.9128420

[25] Song, J., Mingotti, A., Zhang, J., Peretto, L., & Wen, H. (2022). Accurate Damping Factor and Frequency Estimation for Damped Real-Valued Sinusoidal Signals. *IEEE Transactions on Instrumentation and Measurement*, *71*, 1–4. https://doi.org/10.1109/tim.2022.3220300

[26] Song, J., Mingotti, A., Zhang, J., Peretto, L., & Wen, H. (2022). Fast Iterative-Interpolated DFT Phasor Estimator Considering Out-of-Band Interference. *IEEE Transactions on Instrumentation and Measurement*, *71*, 1–14. https://doi.org/10.1109/tim.2022.3203459

[27] Pande, P. W., Kumar, B. R., Chakrabarti, S., Srivastava, S. C., Sarkar, S., & Sharma, T. (2020). Model order estimation methods for low frequency oscillations in power systems. *International Journal of Electrical Power & Energy Systems*, *115*, 105438. https://doi.org/10.1016/j.ijepes.2019.105438

[28] Wen, L., Song, J., Liu, Y., Zhang, J., & Wen, H. (2021, September). Power system low frequency oscillations analysis based on adaptive ESPRIT method. In *2021 IEEE 11th International Workshop on Applied Measurements for Power Systems (AMPS)* (pp. 1–6). IEEE. https://doi.org/10.1109/amps50177.2021.9586039

[29] Manolakis, D. G., Ingle, V.K., & Kogon, S. M. (2000). *Statistical and Adaptive Signal Processing*. Artech House Publishers.

[30] Song, J., Zhu, L., Mingotti, A., Peretto, L., & Wen, H. (2023, May). Adaptively Determination of Model Order of SVD-based Harmonics and Interharmonics Estimation. In *2023 IEEE International Instrumentation and Measurement Technology Conference (I2MTC)* (pp. 1–5). IEEE.

[31] Netto, M., & Mili, L. (2018). Robust Data Filtering for Estimating Electromechanical Modes of Oscillation via the Multichannel Prony Method. *IEEE Transactions on Power Systems*, *33*(4), 4134–4143. https://doi.org/10.1109/tpwrs.2017.2775063

[32] Rauch, H. E., Tung, F., & Striebel, C. T. (1965). Maximum likelihood estimates of linear dynamic systems. *AIAA Journal*, *3*(8), 1445–1450. https://doi.org/10.2514/3.3166

[33] Jain, S. K., & Singh, S. N. (2012). Exact Model Order ESPRIT Technique for Harmonics and Interharmonics Estimation. *IEEE Transactions on Instrumentation and Measurement*, *61*(7), 1915–1923. https://doi.org/10.1109/tim.2012.2182709

[34] Banerjee, P., & Srivastava, S. C. (2015). An Effective Dynamic Current Phasor Estimator for Synchrophasor Measurements. *IEEE Transactions on Instrumentation and Measurement*, *64*(3), 625–637. https://doi.org/10.1109/tim.2014.2349232

[35] Maslennikov, S., Wang, B., Zhang, Q., & Litvinov, E. (2016, July). A test cases library for methods locating the sources of sustained oscillations. In *2016 IEEE Power and Energy Society General Meeting (PESGM) (pp. 1–5)*. IEEE. https://doi.org/10.1109/pesgm.2016.7741772

Hanjun Deng received his B.Sc. degree from China Three Gorges University. He is currently a senior engineer at the State Grid Hunan Electric Power Limited Company Power Supply Service Center (Metrology Center). His research activity focuses on electrical measurement technology and power marketing.

Minqi Yu received her M.Sc. degree in electrical engineering from the College of Electrical and Information Engineering, Changsha University of Science and Technology, Changsha, China, in 2018. He is currently a senior engineer at the State Grid Hunan Electric Power Limited Company Power Supply Service Center (Metrology Center). His research activity focuses on electric energy metering and power consumption information collection.

Mouhai Liu received his M.Sc. degree in electrical engineering from the College of Electrical and Information Engineering, Changsha University of Science and Technology, Changsha, China, in 2016. He is currently a senior engineer at the State Grid Hunan Electric Power Limited Company Power Supply Service Center (Metrology Center). His research activity focuses on electric energy metering and power consumption information collection.

Dezhi Xiong received his Ph.D. degree from Wuhan University of Technology in 2018. He is working as a senior engineer in the State Grid Hunan Electric Power Limited Company Power Supply Service Center (Metrology Center). His research interests include Intelligent distribution networks and electrical measurement technology.