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CONTROL, INFORMATICS AND ROBOTICS

The fast convergence controller design for a class of Markov jump delay systems with partially known transition rates

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Abstract. Fast convergence is one of the core pursuit goals of modern high-performance control systems. This article investigates the fast convergence control problem of Markov jump systems. In order to ensure the feasibility of the fast convergence controller design method, the Lyapunov-Krasovskii functional is constructed based on Lyapunov stability theory in the derivation process of the algorithm. Based on a time-varying proportional function, a distributed control algorithm is designed and the fast convergence controller design method is proposed for the considered systems. The variation of controller gain with time is discussed in sections, and the boundary of controller output is given to make the designed controller more practical for real-world engineering applications. Finally, two simulation examples are given to demonstrate the effectiveness of the proposed method in this paper.

Keywords: fast convergence control; input saturation; time delay; partially unknown transition rates.

1. INTRODUCTION

In many practical systems, such as aerospace systems, network systems, production systems [1, 2], etc., due to various random factors, the structural parameters of the system may undergo sudden changes, and the above systems can be described as Markov jump systems. In the past few decades, significant progress has been made in this field, including filter design [3], H_∞ control [4–8], fuzzy control [9]. For example, the reliable $L_2 - L_\infty$ filter was derived to deal with the nonlinearity of the considered system in [10], and the observer-based $L_2 - L_\infty$ controller design issue has been discussed for networked SPSMJSs in [11]. In [12], the quantized control was studied to deal with singular perturbation and nonlinearity. Based on event triggering theory, the authors designed a controller to handle sudden situations where state measurement and control channels are both disrupted in [13]. In [14], a novel resilient hybrid learning scheme for discrete-time Markov jump cyber–physical systems with malicious attacks was developed.

As is well known, the existence of time delay can increase the difficulty of controller design and even make the system unstable. In view of this, the research on time-delay systems is necessary [15–18]. To mention a few, for a class of mixed time-delay systems, the existence, uniqueness and boundedness of guaranteed system solutions were established in [16]. In order to improve the effectiveness of robot-assisted collaborative rehabilitation training, a new variable admittance delay control method was proposed in [17].

In practical systems, the input and output of the actuator are limited within a certain range due to physical limitations. The controllers where actuator saturation poses a critical constraint are widely applied in practical engineering systems with stochastic parameter jumps and time delays, such as in industrial motor drives (to compensate for torque or current saturation in precision motion), robotic manipulators (handling joint torque saturation during trajectory tracking), and power electronics (managing voltage/current limits in converters). In theoretical research, nonlinearity caused by saturation may lead to system instability. Therefore, designing controller subject to actuator saturation is absolutely necessary [19–22]. In order to address the nonlinearity caused by saturation, certain limitations need to be placed on the initial conditions during the control design process. For example, the fault-tolerant control was studied for satellite systems to deal with actuator failures and input saturation in [23]. For a class of intervention autonomous vehicle system with input saturation and output constraints, a higher-order controller was designed to handle the disturbances and uncertainties in [24].

On the other hand, as the state deviation gradually decreases, the control quantity of the state feedback controller also approaches zero, which leads to a longer convergence time. Therefore, how to guarantee fast or even fixed-time convergence remains a critical challenge worthy of further investigation [25–30]. From literature review, it can be seen that fixed-time convergence is mainly focused on multi-agent systems, and there is relatively little research on Markov jump systems. Meanwhile, in order to improve convergence speed, the gain of the controller may approach infinity at the set time, which is difficult to achieve in practical systems.

In view of this, for Markov jump systems where parameters randomly jump with modal changes, the primary motivation of

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the paper is to avoid the excessively large control output required by fixed-time control while ensuring rapid stabilization, i.e., how to achieve fast convergence under actuator saturation. Therefore, fast convergence control problem for a class of Markov jump delay systems is studied in this work, and the controller gain is discussed in segments over time, in order to provide a boundary for the controller output and make the controller closer to practical engineering. The main contributions of this work are as follows:

1. To address the nonlinearity introduced by saturation, a tuning parameter is introduced, and a novel saturation lemma incorporating a piecewise function is proposed;
2. In order to achieve faster convergence speed than the general Lyapunov stability, the variation of controller gain with time is discussed in sections, and the distributed control algorithm is designed to ensure the fixed-time convergence of the Markov jump systems;
3. Further considering the time-delay and partially unknown transition rates, the controller of this work is designed to achieve rapid convergence of the controlled object, while avoiding excessively large controller output values during the process to align with practical engineering requirements.

2. PROBLEM STATEMENTS AND PRELIMINARIES

Considering the Markov jump systems as follows:

$$\begin{aligned}&\dot{x}(t) = A(r(t))x(t) + A_d(r(t))x(t-\tau) + B(r(t))(u(t)),\\&x(t) = \eta(t), \quad t \in [-\tau,\ 0], \quad r_t = r_0,\end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$ and $\eta(t) \in L_2^n[-\tau,\ 0]$ are the state, input and initial function of systems, respectively. $\{r(t)\}$ represents a Markovian process and take values in the finite set $S = \{1, 2, \ldots, \mathcal{N}\}$, and with

$$\Pr\left\{r(t+\Delta) = j \middle| r(t) = i\right\} = \begin{cases}\pi_{ij}\Delta + o(r) & i \neq j,\\ 1 + \pi_{ii}\Delta + o(r) & i = j,\end{cases}$$

where $\Delta > 0$ is an infinitesimal quantity. The transition rate $\pi_{ij_{j\neq i}} \geq 0$, and with

$$\sum_{j\in S} \pi_{ij} = 0. \tag{2}$$

In this paper, we consider the system with the following:

$$\Pr = \begin{bmatrix}\pi_{11} & ? & \pi_{13} & \cdots & \pi_{1n}\\ \pi_{21} & ? & \pi_{23} & \cdots & ?\\ \vdots & \vdots & ? & \ddots & \vdots\\ \pi_{n1} & ? & \pi_{n3} & \ldots & ?\end{bmatrix},$$

where "?" is the unknown transition rates, and for $\forall i \in S$, the set S^i denotes:

$$S^i = S_k^i \bigcup S_{uk}^i, \tag{3}$$

where S_k^i and S_{uk}^i are known and unknown part of transition rates, respectively.

For each $r(t) = i \in S$, we define $A_i = A(r(t))$, $A_{di} = A_d(r(t))$, $B_i = B(r(t))$. Designing the controller as:

$$u(t) = -H(t)k(r(t))x(t), \tag{4}$$

where $H(t) = \left(\alpha + \beta\dfrac{\dot{f}(t)}{f(t)}\right)$,

$$f(t) = \begin{cases}\dfrac{\kappa^h}{(\kappa - t)^h}, & t \in [0,\ \kappa),\\ 1 & t \in [\kappa,\ \infty),\end{cases}$$

and

$$\dot{f}(t) = \begin{cases}\dfrac{h}{\kappa} f^{1+\frac{1}{h}}, & t \in [0,\ \kappa),\\ 0 & t \in [\kappa,\ \infty).\end{cases}$$

Since $H(t)$ approaches infinity as $t \longrightarrow \kappa$, this scenario is impractical. Therefore, saturation limits are introduced to system (1) as follows. Based on (1) and (4), we have

$$\dot{x}(t) = (A_i - H(t)B_i k_i)x(t) + A_{di}x(t-\tau) + B_i\varphi(u(t)), \tag{5}$$

where $sat(u(t))$ is the bounded output of controller, and $\varphi(u(t)) = sat(u(t)) - u(t)$.

Before giving the main results, the following lemmas should be given.

Lemma 1. If $x(t)$ of system (5) is in the following set $D(u_o)$:

$$\begin{aligned}D(u_o) = \Big\{&x(t) \in \mathbb{R}^n;\ -H(t)u_{0(k)} \leq (-H(t)k_{i(k)}\\ &+ L_{i(k)})x(t) \leq H(t)u_{0(k)},\ \ u_{0(k)} > 0\Big\},\end{aligned}$$

where $L_i \in \mathbb{R}^{m\times n}$ are the given appropriate matrix, L_{ik} is the $k-th$ line element of L_i, and

$$H(t) = \begin{cases}H(t), & t \in [0,\ \kappa - \kappa_1),\\ H(\kappa - \kappa_1), & t \in [\kappa - \kappa_1,\ \kappa),\\ \alpha, & t \in [\kappa,\ \infty),\end{cases}$$

then one can obtain:

$$\varphi(u(t))^T \kappa_i(\varphi(u(t)) - L_i x(t)) \leq 0,$$

where $\kappa_i > 0$ are any diagonal positive matrix.

Lemma 2. [25] Define a differentiable function $y : \mathbb{R} \longrightarrow \mathbb{R}_{\geq 0}$ with $a, b > 0$, if the following holds:

$$\dot{y}(t) \leq -\left(a + b\frac{\dot{f}(t)}{f(t)}\right)y, \quad t \in [0,\ \infty), \tag{6}$$

we have

$$y(t)\begin{cases}\leq f(t)^{-b}e^{-at}y(t_0), & t \in [0,\ \kappa),\\ \equiv 0, & t \in [\kappa,\ \infty).\end{cases}$$

3. MAIN RESULTS

Theorem 1. For the given positive scalars, λ, λ_2, λ_4, α, β, κ, κ_1, h, and given matrix L_i, system (5) with $A_{di} = \mathbf{0}$ is said to be locally fast convergent with controller gain $k_i = \bar{k}_i P_i$, if there exists matrix $\bar{k}_i$, $X_i = P_i^{-1} > 0$ and diagonal matrix $G_i = \kappa_i^{-1} > 0$, and the initial state belongs to the set $\varepsilon(P_i,\ 1)$, such that

$$-(\bar{k}_i B_i + B_i \bar{k}_i) + \lambda_2 X_i < 0, \tag{7}$$

$$\lambda_4 + \pi_{ii} - \lambda_2 \alpha < 0, \tag{8}$$

$$\begin{bmatrix} \Lambda & B_i G_i + X_i L_i^T & \Pi \\ * & -2G_i & \mathbf{0} \\ * & * & \Theta_i \end{bmatrix} < 0, \tag{9}$$

$$\begin{bmatrix} X_i & * & * \\ -\bar{k}_{i(k)} & -\dfrac{\lambda}{\Upsilon^2} + u_0^2(k) & * \\ L_{i(k)} X_i & 0 & \lambda I \end{bmatrix} \geq 0, \tag{10}$$

$$\begin{bmatrix} X_i & * \\ -\Upsilon_1 \bar{k}_{i(k)} + L_{i(k)} X_i & \Upsilon_1^2 u_0^2(k) \end{bmatrix} \geq 0, \tag{11}$$

$$\begin{bmatrix} X_i & * \\ -\alpha \bar{k}_{i(k)} + L_{i(k)} X_i & \alpha^2 u_0^2(k) \end{bmatrix} \geq 0, \tag{12}$$

where $k = 1, \ldots, m$ and

$$\Lambda = -\lambda_4 X_i + X_i A_i^T + A_i X_i\,,$$
$$\Pi = \left[\sqrt{\pi_{i1}} X_i\ \sqrt{\pi_{i2}} X_i\ \cdots\ \sqrt{\pi_{ij}} X_i\right],$$
$$\Theta_i = \mathrm{diag}(-X_1,\ -X_2,\ \ldots,\ -X_j),$$

and $\Upsilon = \alpha + \beta \dfrac{h}{\kappa}$, $\Upsilon_1 = H(\kappa - \kappa_1)$, the set $\varepsilon(P_i,\ 1)$ means that $x^T(t) P_i x(t) \leq 1$.

Proof. For the considered system (5), we define the following Lyapunov-Krasovskii function:

$$V(x(t), i) = x^T(t) P_i x(t),$$

and it is easy to have

$$\begin{aligned}\dot{V} &= x^T(t)(A_i^T P_i + P_i A_i + \pi_{ii} P_i - H(t)(k_i^T B_i^T P_i + P_i B_i k_i))x(t) \\ &\quad + \Sigma_{j\in S, j\neq i} \pi_{ij} x^T(t) P_j x(t) + 2\varphi^T(u(t)) B_i^T P_i x(t).\end{aligned}$$

Based on Lemma 1, one has

$$\begin{aligned}\dot{V} \leq\ & x^T(t)\big(A_i^T P_i + P_i A_i + \pi_{ii} P_i \\ & - H(t)(k_i^T B_i^T P_i + P_i B_i k_i)\big)x(t) \\ & + \Sigma_{j\neq i\in S} \pi_{ij} x^T(t) P_j x(t) + 2\varphi^T(u(t)) B_i^T P_i x(t) \\ & + 2x^T(t) L_i^T \kappa_i \varphi(u(t)) - 2\varphi^T(u(t)) \kappa_i \varphi(u(t)).\end{aligned} \tag{13}$$

Post and pre-multiply condition (7) with P_i, one can obtain the following:

$$-(k_i^T B_i^T P_i + P_i B_i k_i) < -\lambda_2 P_i\,.$$

Since that $H(t) > 0$, then we have

$$\begin{aligned}\dot{V} \leq\ & (\lambda_4 + \pi_{ii} - H(t)\lambda_2)V \\ & + x^T(t)(A_i^T P_i + P_i A_i - \lambda_4 P_i)x(t) \\ & + \Sigma_{j\neq i\in S} \pi_{ij} x^T(t) P_j x(t) + 2\varphi^T(u(t)) B_i^T P_i x(t) \\ & + 2x^T(t) L_i^T \kappa_i \varphi(u(t)) - 2\varphi^T(u(t)) \kappa_i \varphi(u(t)).\end{aligned} \tag{14}$$

Define

$$\dot{V}_1 = \dot{V} - \left(\lambda_4 + \pi_{ii} - H(t)\lambda_2\right)V, \tag{15}$$

post and pre-multiply condition (9) with $\mathrm{diag}\{P_i,\ \kappa_i,\ I\}$, and by using the Schur complement, then we have $\dot{V}_1 < 0$ and

$$\begin{aligned}\dot{V} &\leq \left(\lambda_4 + \pi_{ii} - H(t)\lambda_2\right)V \\ &= -\left(\lambda_2 \alpha - \lambda_4 - \pi_{ii} + \lambda_2 \beta \frac{\dot{f(t)}}{f(t)}\right)V.\end{aligned} \tag{16}$$

Based on condition (8) and Lemma 2, the considered system (5) with $A_{di} = \mathbf{0}$ is locally fixed-time convergent.

Consider that $H(t) \geq \Upsilon \Rightarrow -\dfrac{1}{H^2(t)} \geq -\dfrac{1}{\Upsilon^2}$, $t \in [0,\ \kappa - \kappa_1)$, then we have

$$\begin{bmatrix} X_i & * & * \\ -\bar{k}_{i(k)} & -\dfrac{\lambda}{H^2(t)} + u_0^2(k) & * \\ L_{i(k)} X_i & 0 & \lambda I \end{bmatrix} \geq (10) \geq 0. \tag{17}$$

Post and pre-multiply (17) with $\mathrm{diag}\{P_i,\ H(t)I,\ I\}$, one has

$$\begin{bmatrix} P_i & * & * \\ -H(t) k_{i(k)} & -\lambda + H^2(t) u_0^2(k) & * \\ L_{i(k)} & 0 & \lambda I \end{bmatrix} \geq 0. \tag{18}$$

Based on the Schur complement lemma, one can further achieve

$$\begin{aligned} &\begin{bmatrix} P_i - \dfrac{1}{\lambda} L_{i(k)}^T L_{i(k)} & * \\ -H(t) k_{i(k)} & -\lambda + H^2(t) u_0^2(k) \end{bmatrix} \geq 0, \\ \Rightarrow &\begin{bmatrix} P_i & * \\ -H(t) k_{i(k)} + L_{i(k)} & H^2(t) u_0^2(k) \end{bmatrix} \geq 0, \end{aligned} \tag{19}$$

then we have

$$\begin{bmatrix} P_i & * \\ -H(t) k_{i(k)} + L_{i(k)} & H^2(t) u_0^2(k) \end{bmatrix} \geq 0. \tag{20}$$

One can further derive

$$x^T(t) P_i x(t) - \frac{((-H(t) k_{i(k)} + L_{i(k)}) x(t))^2}{H^2(t) u_0^2(k)} \geq 0.$$

Due to that $\dot{V} < 0$, one has $x^T(t)P_ix(t) < x^T(0)P_ix(0) < 1$, and

$$\begin{aligned} x^T(t)P_ix(t) &\geq \frac{((-H(t)k_{i(k)} + L_{i(k)})x(t))^2}{H^2(t)u_0^2(k)} \\ \Rightarrow \ 1 &\geq \frac{((-H(t)k_{i(k)} + L_{i(k)})x(t))^2}{H^2(t)u_0^2(k)} \\ \Rightarrow \ H^2(t)u_0^2(k) &\geq ((-H(t)k_{i(k)} + L_{i(k)})x(t))^2, \end{aligned}$$

so as to obtain that $\varepsilon(P_i, 1) \in D(u(0))$. According to conditions (11) and (12), similar results can be obtained when $t \in [\kappa - \kappa_1, +\infty)$.

Remark 1. Obviously, when κ_1 approaches 0, the output of the controller will become very large. If $\kappa_1 = 0$, then fixed-time convergence can be achieved at this point. Adjusting κ_1 can reduce the convergence time of the system and ensure that $H(t)$ is bounded, which makes the design of the controller feasible and closer to practical engineering.

Theorem 2. For the given positive scalars, λ, λ_2, λ_4, α, β, κ, κ_1, h, and given matrix L_i, system (5) with controller gain $k_i = \bar{k}_iP_i$ is said to be locally fast convergent, if there exists matrix $\bar{Q} = X_iQX_i > 0$, $X_i = P_i^{-1} > 0$ and diagonal matrix $G_i = \kappa_i^{-1} > 0$, and the initial state belongs to $\varepsilon(P_i, Q, 1)$, such that the conditions (7), (8), (10), (11), (12) and the following holds

$$\begin{bmatrix} \Lambda & A_{di}X_i & B_iG_i + X_iL_i^T & \Pi \\ * & -\bar{Q} & \mathbf{0} & \mathbf{0} \\ * & * & -2G_i & \mathbf{0} \\ * & * & * & \Theta_i \end{bmatrix} < 0, \tag{21}$$

where

$$\begin{aligned} \Lambda &= -\lambda_4X_i + X_iA_i^T + A_iX_i + \bar{Q}, \\ \Pi &= \left[\sqrt{\pi_{i1}}X_i \ \sqrt{\pi_{i2}}X_i \ \cdots \ \sqrt{\pi_{ij}}X_i\right], \\ \Theta_i &= \mathrm{diag}(-X_1, -X_2, \cdots, -X_j), \end{aligned}$$

and the set $\varepsilon(P_i, Q, 1)$ means that

$$x^T(t)P_ix(t) + \int_{t-\tau}^{t} x^T(s)Qx(s)\,\mathrm{d}s \leq 1.$$

Proof. In this case, we choose a Lyapunov-Krasovskii function as follows:

$$V(x(t),i) = x^T(t)P_ix(t) + \int_{t-\tau}^{t} x^T(s)Qx(s)\,\mathrm{d}s,$$

one has

$$\begin{aligned} \dot{V} = {} & x^T(t)(A_i^TP_i + P_iA_i + \pi_{ii}P_i \\ & - H(t)(k_i^TB_i^TP_i + P_iB_ik_i))x(t) \\ & + \Sigma_{j\in S, j\neq i}\pi_{ij}x(t)^TP_jx(t) + 2\varphi^T(u(t))B_i^TP_ix(t) \\ & + 2x^T(t-\tau)A_{di}^TP_ix(t) \\ & + x^T(t)Qx(t) - x^T(t-\tau)Qx(t-\tau). \end{aligned} \tag{22}$$

Following the proof process of Theorem 1, one has $\dot{V} < 0$. If the $H(t)$ is changed to be $H = \Upsilon$, $t \in [0, \kappa - \kappa_1)$, we have

$$\begin{aligned} \dot{\bar{V}} = {} & x^T(t)(A_i^TP_i + P_iA_i + \pi_{ii}P_i \\ & - H(k_i^TB_i^TP_i + P_iB_ik_i))x(t) \\ & + \Sigma_{j\in S, j\neq i}\pi_{ij}x^T(t)P_jx(t) + 2\varphi^T(u(t))B_i^TP_ix(t) \\ & + 2x^T(t-\tau)A_{di}^TP_ix(t) \\ & + x^T(t)Qx(t) - x^T(t-\tau)Qx(t-\tau). \end{aligned} \tag{23}$$

From (22) and (23), we derive the following:

$$\dot{V} - \dot{\bar{V}} = x^T(t)(-H(t) + \Upsilon)(k_i^TB_i^TP_i + P_iB_ik_i)x(t).$$

From condition (7), it is easy to get $k_i^TB_i^TP_i + P_iB_ik_i > 0$, and consider that $H(t) \geq \Upsilon$, then we have $\dot{V} - \dot{\bar{V}} \leq 0$. Compared with the fixed gain controller, the designed controller in this paper can achieve faster convergence. Following the proof process of Theorem 1, one can obtain

$$\begin{aligned} & x^T(t)P_ix(t) - \frac{((-H(t)k_{i(k)} + L_{i(k)})x(t))^2}{H^2(t)u_0^2(k)} \geq 0 \\ \Rightarrow \ & x^T(t)P_ix(t) \leq \frac{((-H(t)k_{i(k)} + L_{i(k)})x(t))^2}{H^2(t)u_0^2(k)} \\ \Rightarrow \ & x^T(t)P_ix(t) + \int_{t-\tau}^{t} x^T(s)Qx(s)\,\mathrm{d}s \\ \leq \ & \frac{((-H(t)k_{i(k)} + L_{i(k)})x(t))^2}{H^2(t)u_0^2(k)}, \end{aligned}$$

so as to derive that $\varepsilon(P_i, Q, 1) \in D(u(0))$.

Theorem 3. For the same scalars and matrix given in Theorem 1 and 2 and partially unknown transition rates, system (5) is said to be locally fast convergent with controller gain $k_i = \bar{k}_iP_i$ and initial conditions belonging to $\varepsilon(P_i, Q, 1)$, if there exists matrix $\bar{W}_i = X_iW_iX_i > 0$, such that the conditions (10), (11), (12) and the following holds

$$-\left(\bar{k}_iB_i + B_i\bar{k}_i\right) + \lambda_2X_i < 0, \tag{24}$$

$$\lambda_4 + \pi_{ii} - \lambda_2\alpha < 0, \quad i = j \in S_k^i, \tag{25}$$

$$\lambda_4 - \lambda_2\alpha < 0, \quad i = j \in S_{uk}^i, \tag{26}$$

$$X_i - \bar{W}_i \geq 0, \quad i = j \in S_{uk}^i, \tag{27}$$

$$\begin{bmatrix} -\bar{W}_i & X_i \\ X_i & -X_j \end{bmatrix} \geq 0, \quad i \neq j \in S_{uk}^i, \tag{28}$$

$$\begin{bmatrix} \Lambda & A_{di}X_i & B_iG_i + X_iL_i^T & \Pi \\ * & -\bar{Q} & \mathbf{0} & \mathbf{0} \\ * & * & -2G_i & \mathbf{0} \\ * & * & * & \Theta_i \end{bmatrix} < 0, \tag{29}$$

where

$$\Lambda = -\lambda_4 X_i + X_i A_i^T + A_i X_i + \bar{Q} + \Sigma_{j\in S_k^i} \pi_{ij} \bar{W}_i ,$$

$$\Pi = \left[\sqrt{\pi_{i1}} X_i \ \sqrt{\pi_{i2}} X_i \ \cdots \ \sqrt{\pi_{ij}} X_i\right]_{i\neq j\in S_k^i} ,$$

$$\Theta_i = \mathrm{diag}\left(-X_1,\ -X_2,\ \ldots,\ -X_j\right)_{i\neq j\in S_k^i} .$$

Proof. Choosing the same Lyapunov-Krasovskii function as Theorem 2, and defining $W_i > 0$, it is easy to have $-\Sigma\pi_{ij}W_i = 0$. One has

$$\begin{aligned}
\dot{V} &= x^T(t)(A_i^T P_i + P_i A_i - H(t)(k_i^T B_i^T P_i + P_i B_i k_i))x(t) \\
&\quad + \Sigma_{j\in S}\pi_{ij} x^T(t) P_j x(t) + 2\varphi^T(u(t)) B_i^T P_i x(t) \\
&\quad + 2x^T(t-\tau) A_{di}^T P_i x(t) - x^T(t-\tau) Q x(t-\tau) \\
&\quad + x^T(t) Q x(t) - \Sigma\pi_{ij} x^T(t) W_i x(t) \\
&= V_0 + \pi_{ii} x^T(t)(P_i - W_i)x(t) \\
&\quad + \Sigma_{j\neq i\in S_k^i} \pi_{ij} x^T(t)(P_j - W_i)x(t) \\
&\quad + \Sigma_{j\neq i\in S_{uk}^i} \pi_{ij} x^T(t)(P_j - W_i)x(t).
\end{aligned} \tag{30}$$

If $\pi_{ii} < 0$ is unknown, condition (26) ensures that $\pi_{ii} x^T(t)(P_i - W_i)x(t) < 0$. Based on the same method, condition (27) can ensure that $\Sigma_{j\neq i\in S_{uk}^i} \pi_{ij} x^T(t)(P_j - W_i)x(t) < 0$. Following the proof process of Theorem 1 and 2, one has $\dot{V} < 0$.

4. NUMERICAL EXAMPLES

Example 1. Choosing the following two subsystems for system (1):

$$A_1 = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix}, \quad B_1 = B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

In this example, we define that $\alpha = 0.5$, $\beta = 1.5$, $\kappa = 6$, $h = 2$, and give the following matrix of transition rate:

$$\pi_{ij} = \begin{bmatrix} -0.5 & 0.5 \\ 0.8 & -0.8 \end{bmatrix}.$$

The steps to solve the feedback gain parameter by using Theorem 1 are as follows:

Step 1. Define matrices and variables: define $\bar{k}_i$, $X_i = P_i^{-1} > 0$ and diagonal matrix $G_i = \kappa_i^{-1} > 0$ as the decision variables for the LMIs;

Step 2. Formulate the LMIs: construct the linear matrix inequalities (7)–(12) for each mode $i \in \mathscr{S}$;

Step 3. Solve the convex optimization problem: utilize an LMI solver in MATLAB LMI Toolbox to find a feasible solution for the matrix variables $\bar{k}_i$, X_i, G_i;

Step 4. Extract the controller gains: once a feasible solution is obtained, recover the controller gain matrix k_i from the solutions of $\bar{k}_i$ and X_i using the relations $k_i = \bar{k}_i X_i^{-1}$.

According to the above steps, it can be concluded that

$$K_1 = \begin{bmatrix} 0.1 & 1.5 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 0.1 & 2.9 \end{bmatrix}.$$

Example 2. In this case, we choose $\alpha = 0.5$, $\beta = 2.5$, $\kappa = 10$, $h = 2$, $\tau = 0.5$ and give the following two subsystems for system (1):

$$A_1 = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix},$$

$$A_{d1} = \begin{bmatrix} -1 & 0.5 \\ 0.1 & -0.2 \end{bmatrix}, \qquad A_{d2} = \begin{bmatrix} 0.1 & 0.2 \\ 0 & -0.1 \end{bmatrix},$$

$$B_1 = B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \qquad \pi_{ij} = \begin{bmatrix} ? & ? \\ 0.5 & -0.5 \end{bmatrix}.$$

According to similar solving steps in Example 1 and Theorem 3, we have

$$K_1 = \begin{bmatrix} 0.21 & 2.51 \end{bmatrix}, \qquad K_2 = \begin{bmatrix} 0.11 & 3.51 \end{bmatrix}.$$

Figures 1 and 2 are the open-loop and closed-loop state response curves of Example 1, respectively. Figure 3 is the bounded output of controller. Figures 4 and 5 are state response of Example 2. The $x(fk)$ and $x(k)$ represent the state responses with time-varying or fixed controller gains, respectively. From Figs. 1 and 2, the controllers of this work can ensure that the closed-loop system is locally fast convergent. Compared with the traditional controller with fixed gain, the time-varying gain

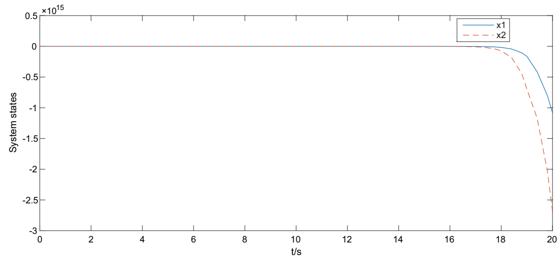

Fig. 1. The open-loop state of the system of Example 1

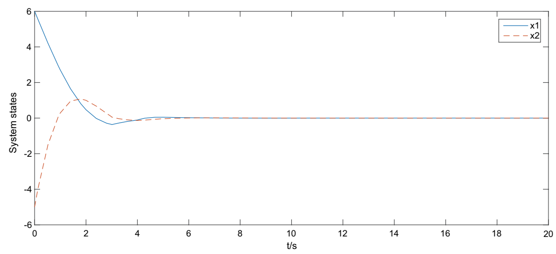

Fig. 2. The closed-loop state of the system of Example 1

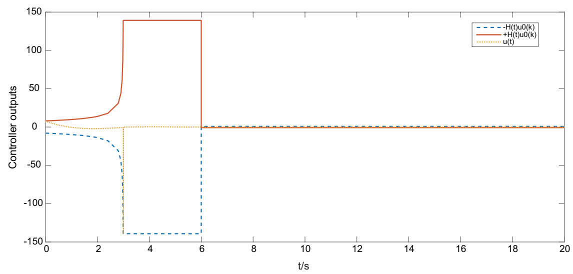

Fig. 3. The bounded output of Example 1

controller designed in this paper can achieve faster convergence, as shown in Figs. 4 and 5. From Fig. 3, the designed controller achieves fast convergence while its output is closer to practical engineering.

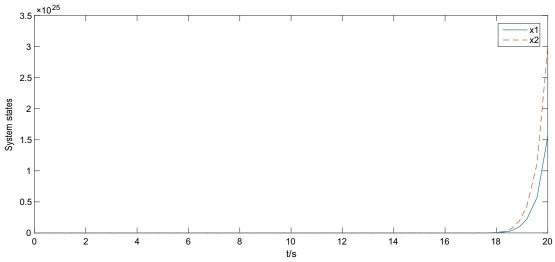

Fig. 4. The open-loop state of the system of Example 2

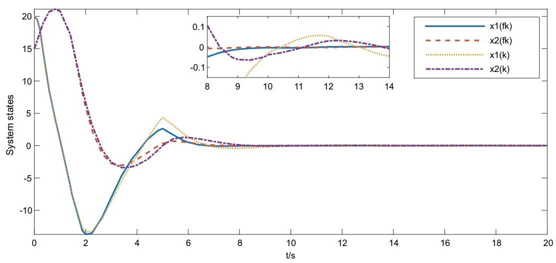

Fig. 5. The closed-loop state of the system of Example 2

Remark 2. Compared with [6], the algorithm in this paper can achieve rapid convergence through adjustable parameter settings, while the algorithm in [6] can only guarantee general Lyapunov stability and convergence. Compared to the fixed time control method in [30], the system addressed in this paper, due to the presence of controller saturation constraints, can avoid excessive controller output, thereby enhancing the practical applicability of the method.

5. CONCLUSIONS

The fast convergence control method is proposed to ensure the local stability of the considered Markov jump delay systems with partially known transition rates in the work. The controller gain is discussed in segments over time, and the boundary for the controller output is given to ensure the feasibility of the controller design method. Future research directions include extending the proposed method to singular Markov jump systems and further extending it to nonlinear systems.

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