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NEW MATH AT PRIMARY SCHOOLS IN WEST GERMANY

Summary: This paper seeks to analyse the influences on the emergence and development of the New Math, respectively Modern Mathematics, movement in primary schools in West Germany by examining a sample of exemplary textbooks and comparing their content and methods with the original reform ideas. It is expected that transfer and implementation into classroom concepts lead to adaptations, so-called recontextualizations, and that parts of the original ideas are lost in this way. Indeed, the results of a previous study show that this happened in this specific national case. However, the reasons for and influences on this development differed. The paper identifies the relevant actors who influenced the reform and brings the process in line with political and social circumstances that seem to have played a crucial role in the reform of primary mathematics education.

Keywords: history of mathematics education, New Math, Modern Mathematics, textbook analysis

Introduction

The New Math movement has been a subject in the history of mathematics education that has attracted increased interest for several years now. As it deals with an international reform, this contribution aims to serve as a further puzzle piece in the overall picture of New Math that – despite all previous efforts¹ – is far from being complete. This paper contributes to the topic by presenting some results of a study on the implementation of so-called Modern Mathematics in

¹ See *Modern Mathematics: An international movement?*, ed. by D. De Bock, Springer, Cham 2023 (History of Mathematics Education).

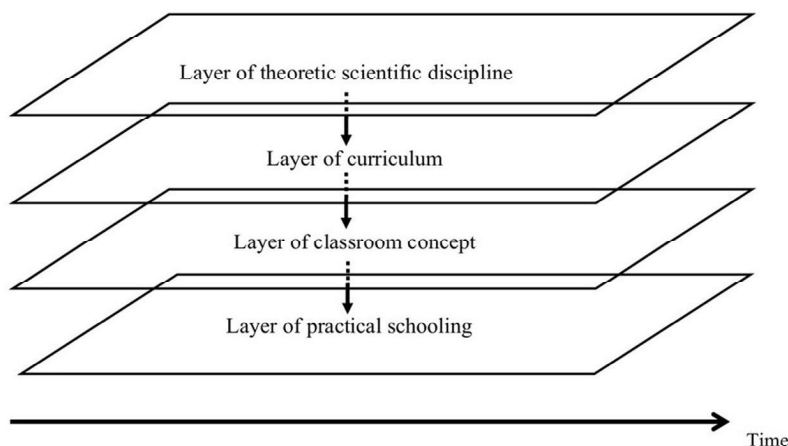


Fig. 1. Framework model used to describe and compare ideas on the layers of the educational system.

Note: Created by the author.

primary education in the former Federal Republic of Germany². Surely, the paramount aim would be to paint the bigger picture in a national (e.g., What was the role of New Math within the country's history of primary mathematics education?) as well as an international context (e.g., What differences can be identified regarding efforts in different countries, how can these be connected to national peculiarities, and what characteristics that were specific to the New Math reform itself can we possibly work out from this?). This, though, requires a thorough description of the national case first. The majority of hitherto existing studies on the reform focusses on secondary education while work on the implementation in primary schools is relatively scarce. Therefore, the focus of this paper will be on a description of the New Math movement in West Germany and its implementation in primary schools. To be more precise, the following questions lead the work: What were the conditions and driving forces that led to the reform? What were the conditions that influenced the reform process? Who were the protagonists at the different stages of reform?

Limitations were set by using a framework model (Fig. 1) based on and adapted from Fend³.

² The study is T. Hamann, *Die "Mengenlehre" im Anfangsunterricht: historische Darstellung einer gescheiterten Unterrichtsreform in der Bundesrepublik Deutschland*, universi, Siegen 2018 (Siegener Beiträge zur Geschichte der Geschichte und Philosophie der Mathematik, vol. 9), and the results presented in this paper are to large extent a summary of it. Only a selection of the literature used there is quoted here; for further evidence, see the original study.

³ H. Fend, *Schule gestalten. Systemsteuerung, Schulentwicklung und Unterrichtsqualität*, VS Verl. für Sozialwissenschaften, Wiesbaden 2008, p. 30–33; for further details on the model,

Educational reforms take place in different places, on different layers, where on each layer there are different protagonists acting in their very own context under their own specific conditions and constraints. The framework model used differentiates the educational system into the layer of theoretical foundations, the layer of political and curricular documents, the layer of concepts and materials developed for classroom use, and, finally, the layer of school practice. The model proves helpful for describing ideas on each layer separately as well as following the ideas' development process in both directions, from one layer to another and throughout time. According to Fend, it is characteristic of educational reform processes that those acting on the different layers adapt the information they receive from another layer and bring it into accordance with the circumstances each of them faces in their own context. Fend calls these adaptations *recontextualizations*, a term that will be used here as well.

As we are dealing with a model, it is obvious that it is a simplified representation, where there is no complete separation between the layers. Among others, this is due to the fact that the same persons might have acted in different functions, as protagonists on different layers.

Thus, taking into account the framework, the specified central question of the study presented in this paper is: What were the basic ideas that were developed on the layer of theoretical scientific discipline, and how did they become recontextualized on the layer of classroom concept?

Sources⁴

Fig. 2 shows the sources that were exemplarily chosen for the description of ideas on each layer⁵.

The Royaumont seminar is taken as a starting point, while the year 1984 marks new curricula from which Modern Mathematics contents have largely been removed. The outcome of the Royaumont seminar serves at the same time as a source for ideas from mathematics as a scientific discipline. Piaget and Bruner were chosen as sources for psychological theories, as especially Piaget is explicitly named in many other sources. Finally, the concept of Zoltan P. Dienes is allocated

see T. Hamann, *New Math at primary schools in West Germany – a theoretical framework for the description of educational reforms*, [in:] *Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education*, ed. by U.T. Jankvist, M. van den Heuvel-Panhuizen, M. Veldhuis, Freudenthal Group & Freudenthal Institute, Utrecht University, Utrecht 2019.

⁴ All translations by the author.

⁵ Abbreviations used in this and in the following charts: KMK = *Kultusministerkonferenz* [conference of ministers of education], RRL = *Rahmenrichtlinien* [curricular guidelines], Handreichungen = *Handreichungen für den Mathematikunterricht in der Grundschule* [Handout for math lessons in primary school], N & S = Neunzig & Sorger, F & B = Fricke & Besuden, GS = *Grundschule* [primary school], MU = *Mathematikunterricht* [math lessons].

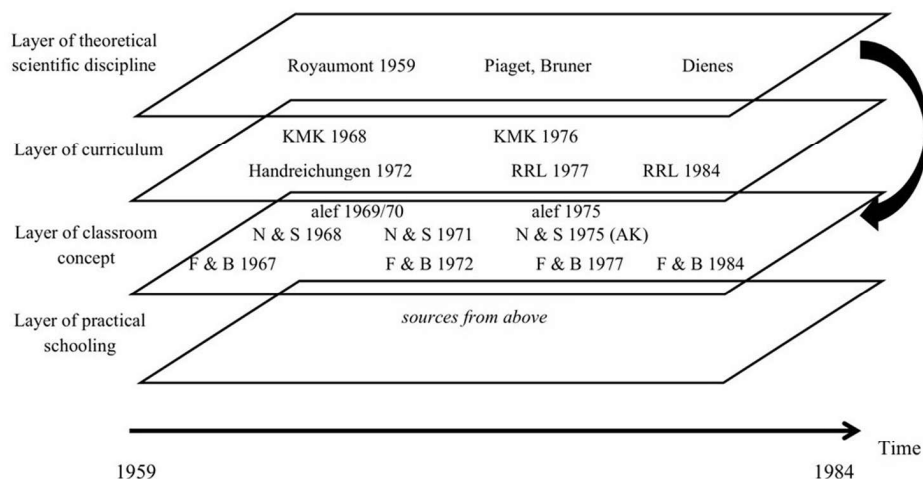


Fig. 2. Framework model filled with the sources used in the study.

Note: Created by the author.

to the layer of theory. Surely, he has practiced his suggestions for teaching math many times, but not on a regular basis in the mathematics classroom. His influence on reform efforts concerning primary mathematics education cannot be doubted and, indeed, he is named by many involved in the reform.

For the layer of classroom concept, three textbook series have been chosen as sources. Each of them went through several editions, allowing not only to describe the didactical ideas they contain and compare them to the outcome from the theoretical scientific discipline layer, but also to compare them with each other to show their development over time.

Table 1. Choice of curricular documents used in the study.

| | Cross-curricular documents | | | Documents and dates concerning mathematics | | |
|------|--|---|-------------------------|---|-------------------------|-------------------------|
| | General, all types of school | Primary school, all FRG | Example of Lower Saxony | General, all types of school | Primary School, all FRG | Example of Lower Saxony |
| 1968 | | | | Richtlinien und Empfehlungen der KMK | | |
| 1970 | <i>Strukturplan des Bildungsrats</i> [Structural plan of the council of education] | <i>Empfehlungen zur Arbeit in der GS</i> [Recommendations for work in primary school] | | | | |
| 1972 | | | | Start of implementation | | <i>Handreichungen</i> |

| | | | | | | |
|------|--|--|-----|--|---|-----|
| 1975 | | | RRL | | | |
| 1976 | | | | | <i>Richtlinien und Empfehlungen der KMK</i> | |
| 1977 | | | | | | RRL |
| 1984 | | | | | | RRL |

Table 2. Sample of 1st grade textbooks used in the study.

| | <i>alef 1</i> / Bauersfeld et al. | <i>Wir lernen Mathematik I</i> / Neunzig & Sorger | <i>Mathematik in der GS I</i> / Fricke & Besuden |
|-------------|-----------------------------------|---|--|
| 1967 | | | Ed. A |
| 1968 | | 1. ed. | |
| 1969 | 1. ed. | | |
| | | | |
| 1971 | | 2. ed. | |
| 1972 | | | Ed. B |
| | | | |
| 1974 | | Task cards (2 ed.) | |
| 1975 | 2. ed. | | |
| 1976 | | | |
| 1977 | | | Ed. C |
| | | | |
| 1984 | | | Ed. D |

Table 1 and Table 2 show the curricular documents and the textbooks used, along with their chronological position. The German New Math reform started officially in October 1968 with the *Empfehlungen und Richtlinien zur Modernisierung des Mathematikunterrichts* [Recommendations and guidelines for the modernization of mathematics education] issued by the *Kultusministerkonferenz* (conference of ministers of education, i.e., the group of the federal states' ministers of education)⁶, a document prescribing the implementation of Modern Mathematics in all types of school from 1972 onwards. Table 1 shows, among others, the dates of issue of all the relevant curricular documents concerning mathematics education in primary schools for the federal state of Lower Saxony⁷. Several things become apparent when taking a closer look at the table. One thing is the short time span between the *Empfehlungen und Richtlinien* and the start of implementation, which is very likely

⁶ KMK [Kultusministerkonferenz], *Empfehlungen und Richtlinien zur Modernisierung des Mathematikunterrichts an den allgemeinbildenden Schulen. Beschluß der Kultusministerkonferenz vom 3.10.1968*, KMK 1968 (Sammlung der Beschlüsse der Ständigen Konferenz der Kultusminister der Länder der Bundesrepublik Deutschland, 611).

⁷ The focus on the federal state of Lower Saxony is due to the availability of sources. Of course, national documents are relevant for the federal state's documents, as well; that is why Table 1 includes both types of sources.

to have led to practical problems. The other thing is the rather huge gap that one sees up to the first curricular guidelines that were specifically meant for mathematics in primary school, and that were not issued until 1976⁸.

Moving a column further to the right, one finds the syllabi for Lower Saxony. The *Handreichungen für den Mathematikunterricht in der Grundschule* [Hand-out for math lessons in primary school] from 1972⁹ were only meant as a temporary document, a helping hand, or rather some kind of first aid, for primary school teachers, to be used until the release of the ‘proper’ curriculum. However, this proper syllabus was only released in 1977, so that in the end, the *Handreichungen* were the only curricular document that existed for the first five years of Modern Mathematics reform in the federal state of Lower Saxony. It is obvious that, for classroom purposes, this gap had to be filled, and it does not seem too hard to guess that it would have been filled by already existing textbooks. This again makes the books very relevant sources, for classroom concepts as well as – at least partly – for classroom practice.

Table 2 gives an overview of the existing editions of the grade 1 textbooks that were used in the study. Again, some peculiarities attract one’s attention. The textbook series *alef* by Heinrich Bauersfeld¹⁰ originated from the Frankfurt project, the biggest, not to say the only big classroom project in the Federal Republic of Germany during this time. In the course of this project, more than 40 school classes had been taught with specifically developed teaching materials throughout their complete primary school years. After several evaluations and extensive adaptations, the experimental materials were turned into the *alef* program. There were two editions of *alef*, the second edition of the 1st grade material was released in 1975 and thus before the issue of the first nationwide primary school mathematics curriculum.

The example of *Wir lernen Mathematik* [We learn mathematics] by Walter Neunzig and Peter Sorger¹¹ is even more striking. After the second edition of the

⁸ KMK [Kultusministerkonferenz], *Empfehlungen und Richtlinien zum Mathematikunterricht in der Grundschule. Beschluß der KMK-Konferenz vom 3.12.1976*, [in:] *Der Mathematikunterricht in der Primarstufe. Ziele, Inhalte, Prinzipien, Beispiele*, by G. Müller, E. Wittmann, Vieweg, Braunschweig 1977, p. 159–164.

⁹ Der Niedersächsische Kultusminister, *Handreichungen für den Mathematikunterricht in der Grundschule*, Niedersächsisches Kultusministerium, Hannover 1972.

¹⁰ H. Bauersfeld, H. Gnirk, U. Görner, G. Homann, U. Lubeseder, H. Radatz, K. Rickmeyer, *Alef 1. Wege zur Mathematik, Arbeitsblätter für den Schüler*, Schroedel, Hannover 1969; H. Bauersfeld, H. Gnirk, U. Görner, G. Homann, U. Lubeseder, H. Radatz, K. Rickmeyer, *Alef 1. Wege zur Mathematik, Handbuch zum Lehrgang, Teil 1 und 2*, Schroedel, Hannover 1970; H. Bauersfeld, H. Gnirk, G. Homann, U. Lubeseder, U. Mitsos-Görner, H. Radatz, K. Rickmeyer, *Alef 1. Wege zur Mathematik, Überarbeitete Fassung, Arbeitsheft für den Schüler*, Schroedel, Hannover 1975; H. Bauersfeld, H. Gnirk, G. Homann, U. Lubeseder, U. Mitsos-Görner, H. Radatz, K. Rickmeyer, *Alef 1. Wege zur Mathematik, Überarbeitete Fassung, Handbuch zum Lehrgang*, Schroedel, Hannover 1975.

¹¹ W. Neunzig, P. Sorger, *Wir lernen Mathematik I. Erstes Schuljahr*, Herder, Freiburg 1968 (Programm Moderne Mathematik); W. Neunzig, P. Sorger, *Wir lernen Mathematik I. Erstes Schul-*

textbook, the series was only augmented by a second edition of task cards. The second and therefore final edition of the book was issued in 1971, before full implementation of New Math had even started. The first edition, though, was released already in March 1968 and therefore even before the issue of the 1968 guidelines.

As for *Mathematik in der Grundschule* [Mathematics in primary school] by Arnold Fricke and Heinrich Besuden¹², the case is slightly different. This textbook series saw its first edition released already in 1967. Even though the authors' explicit aim was to modernize primary education by integrating traditional numeracy with mathematics, they based their work on their very own concept of the operative method. By that, their work depended far less on political decisions and guidelines.

Findings from the textbook sources

It shows that the *alef* program is a course that brings together the different basic reform ideas and connects them to form a coherent concept and curriculum. Elementary mathematics education is restructured and reorganised from the ground up. *Alef* clearly shows influences from Piaget and Dienes, but also takes up bits from the holistic approach to arithmetic education as developed by Johannes Wittmann in the 1930s¹³. Obviously, Bauersfeld did not limit his work to 'modern', contemporary concepts, but grounded it on a broad base. This included pedagogic, social, and language issues to a great extent. In fact, Bauersfeld did not primarily build his concept on subject-matter arguments, but in his opinion, mathematics is above all a good means to reach his pedagogic and social goals. A comparison of the two editions brings up one substantial difference. In the first edition, numbers are not explicitly addressed until the

jahr, 2nd ed., Herder, Freiburg 1971 (Programm Moderne Mathematik); W. Neunzig, P. Sorger, *Wir lernen Mathematik I. I. Schuljahr; Lehreranleitung*, Herder, Freiburg 1968 (Programm Moderne Mathematik); W. Neunzig, P. Sorger, *Wir lernen Mathematik I. I. Schuljahr; Lehreranleitung*, Herder, Freiburg 1971 (Programm Moderne Mathematik).

¹² A. Fricke, H. Besuden, *Mathematik in der Grundschule 1. Operatives Rechnen mit farbigen Stäben*, Klett, Stuttgart 1967; A. Fricke, H. Besuden, *Mathematik in der Grundschule 1. Operatives Rechnen mit farbigen Stäben, Lehrerheft*, Klett, Stuttgart 1967; A. Fricke, H. Besuden, *Mathematik in der Grundschule 1. Ausgabe B, Lehrerband*, Klett, Stuttgart 1972; A. Fricke, H. Besuden, *Mathematik in der Grundschule 1. Ausgabe B, Grundbuch*, Klett, Stuttgart 1972; A. Fricke, H. Besuden, *Mathematik in der Grundschule 1. Ausgabe C, Regionalausgabe 1, Grundbuch*, Klett, Stuttgart 1977; A. Fricke, H. Besuden, *Mathematik in der Grundschule 1. Ausgabe C, Regionalausgabe 1, Lehrerband*, Klett, Stuttgart 1977; A. Fricke, H. Besuden, *Mathematik in der Grundschule. Neu, 1. Schuljahr*, Klett, Stuttgart 1984; A. Fricke, H. Besuden, *Mathematik in der Grundschule. Neu, 1. Schuljahr; Lehrerband*, Klett, Stuttgart 1985.

¹³ See, e.g., H. Bauersfeld, H. Gnirk, G. Homann, U. Lubeseder, U. Mitsos-Görner, H. Radatz, K. Rickmeyer, *Alef 1. Wege zur Mathematik, Handbuch zum Lehrgang*, p. 74–75.

very end of school year one. This happens much earlier in edition two. Bauersfeld himself names the reason for this stronger focus on arithmetic explicitly: It was politics that forced this quite massive change to the concept – against the authors’ conviction¹⁴. Without an earlier inclusion of numbers, the book would not have been admitted for official use in school. So, we find in *alef* an example of political influence on the course of reform, while we can only guess what might have been the underlying reasons. We may assume, though, that long-standing numeracy tradition played a crucial role here, wherefrom we would have to widen the purely political to an overall societal influence. There is yet another side of political-social influence on the *alef* concept. When the Frankfurt project started, it had hardly been 20 years since the end of World War II and the Nazi regime. The 1960s saw an increasing engagement in West Germany with the country’s recent history. This was related to questions about teaching democracy. What and how would children have to be taught to prevent them from supporting another dictatorship? So, controversies in education in the 1960s and 1970s cannot be separated from questions of democracy, equal opportunities, and emancipation. In the case of *alef*, these considerations resulted among others in a strong focus on differentiation.

Neunzig and Sorger claim to offer a schoolbook that realizes the concept by Zoltan P. Dienes, making it suitable for West German primary schools¹⁵. But even though the influence of Dienes is clearly visible in the inclusion of set theory, the use of Logic Blocks, and the effort to follow the principle of multiple embodiment, the concept of *Wir lernen Mathematik* shows major differences from fundamental reform ideas. We see a lack of differentiation, no inclusion of geometry, hardly any approach to relations and only very few efforts towards open learning. The course is basically an arithmetic course augmented by set theory. The reason for this becomes clear when considering the textbook’s release date before October 1968. To be admitted for use in classroom practice, schoolbooks have to be in line with current curricula. Before the issue of New Math curricula, elementary mathematics education was all about arithmetic and numeracy. Therefore, an alleged improvement of arithmetic skills was all that was there to be used as a basis for argumentation in favour of new contents. Surely, certain notions from elementary set theory are necessary to found the concept of number. The inclusion of set theory could be justified with the aim to enhance abilities in numeracy. So we find here another example of political influence, this time related to curricular circumstances. Neunzig and Sorger’s recontextualization from the layer of theoretical scientific discipline to the layer of classroom concept came with a massive

¹⁴ Ibidem, p. 10.

¹⁵ W. Neunzig, P. Sorger, *Einstieg in die Mathematik. Aufriß eines systematischen Weges für die Grundschule*, Herder, Freiburg 1969 (Programm Moderne Mathematik), p. 29–31.

loss of original conceptual ideas¹⁶. On the one hand, it was not possible for them to consider the curriculum layer. As ideas from the layer of theoretical scientific discipline had not yet been taken up there at all, they could not refer to it. On the other hand, this still missing transfer proved crucial in this case, preventing them from sticking any closer to the original theoretical concepts. A comparison of the two textbook editions shows an earlier approach to numbers with the newer edition, as well. We do not know anything about the reasons for this, though.

Fricke and Besuden present a course that is very coherent, consequently incorporating Piaget's theory of operations by underpinning the whole program with the operative method. Due to this clear focus, it seems they chose to implement strictly those original conceptual ideas that were in line with Piaget's theories. Piaget names sets, numbers, and relations as basic notions that intelligent thinking builds upon. The suggested conceptions of including set theory, though, met Fricke and Besuden's explicit disapproval. Nevertheless, curricula from 1968 on forced them to include set theory in their textbook. Comparison of the editions shows that they limited these changes to a minimum, not giving too much room to the engagement with sets. Instead, the schoolbook is characterized by the use of Cuisenaire rods, as had already been the case with the first edition. The changes in content still fit the overall concept due to the role Piaget attributes to sets. Anyway, we find in *Mathematik in der Grundschule* yet another example of curricular, and therefore political influence on the New Math reform that does not take into account any empirical evidence or experience with an already existing classroom concept for teaching mathematics in West German primary schools.

Some conclusions

So, what were the conditions and driving forces that led to the reform of primary school mathematics education in the FRG, and that we can retrace in classroom concepts? New psychological insights, especially those gained by Jean Piaget, clearly had an influence and were transferred from the layer of theoretical discipline to the layer of classroom concept. Recontextualization was a necessary part of this process. Piaget himself was neither a mathematician nor an educationalist, and he did not make suggestions on practical implications from his findings, leaving this task to the authors of textbooks. As we can tell from looking at a sample of 1st grade textbooks, the extent to which Piaget's theories found their way into these concepts varied considerably.

Zoltan P. Dienes and his teaching concept serve as another obvious reference point for classroom concepts. Dienes became well-known and popular in

¹⁶ Sorger himself remarked that the book was full of compromises, see E. Hollmann, *Moderne Mathematik in der Grundschule. Bericht über eine Studienwoche mit Z. P. Dienes in Rinteln/Weser*, Herder, Freiburg 1969, p. 59.

the FRG, having visited the country several times, largely at the invitation of his German publisher Herder. His influence is made especially explicit by Neunzig and Sorger, who take up his ideas for their schoolbook series. In this case, though, we see that the recontextualization of the original conceptual ideas to the context of West German primary schools in the late 1960s goes along with considerable shortcomings. Basically, Dienes' enormous stock of ideas was constrained to introducing set theory by working with Logic Blocks.

Besides these personalized ideas, we find that general considerations and reflections on education, especially aiming at democracy, emancipation, and equality, led to arguments in favour of a reform at all and influenced the conceptual side from the beginning. Therefore, altogether, we have retraced driving forces from psychology (with Piaget in the lead), didactics of mathematics (as represented by Dienes here), pedagogy, and social sciences. We have not found any evidence that developments from mathematics as a scientific discipline had an immediate influence on the start of the Modern Mathematics movement within primary education. It follows from this that the Royaumont seminar does not serve as a reference point for those acting on the layer of classroom concept.

At the beginning of the reform, the original ideas became recontextualized rather directly from the theoretical layer to the layer of classroom concept, bypassing the layer of curriculum. Where we can still see an impact of curricula then in force, it is a negative one, when missing curricular content keeps the authors of textbooks from taking up more theoretical content.

The role of the protagonists in the curriculum layer changed considerably in the course of the reform. As we can see from all three textbook examples, all of them were undergoing changes in content or structure. The cases may differ, but all of them were subject to certain curricular and political demands. In contrast, we can hardly trace any further impact from scientific findings. The handling of the *alef* program may serve as a paradigm for the latter. At the end of the Frankfurt project, Bauersfeld and his associates assessed the children's arithmetic competencies at the end of grade 4 showing they were just as good as those of children who had not taken part in the project. Although this assessment empirically proved that the postponement of the introduction of numbers did not do any harm to computational abilities, there were political actors forcing the course to be changed.¹⁷ At the same time, not all primary mathematics educationalists supported Modern Mathematics; some of them strictly defended the rather traditional

¹⁷ V. Weis, H. Bauersfeld, *Neue Mathematik und Rechenfertigkeit: Ergebnisse aus dem "Frankfurter Projekt"*, "Westermanns pädagogische Beiträge" 1973, vol. 25, no. 3, p. 127–135. This assessment was actually a requirement for the approval of the project by the ministry of the federal state of Hessen, which obviously seriously feared a decline in numeracy. With this in mind, the forced changes seem even more inconsistent.

numeracy and offensively opposed New Math¹⁸ and it is not clear how far these people influenced the reform process. However, the opponents lacked empirical evidence on the advantages of the traditional curriculum.

We must consider, though, that politics cannot be seen as acting independently from society as a whole, and influence from this side was multifaceted. In 1974, the German magazine “Der Spiegel” asked on its front page ‘Macht Mengenlehre krank?’ (Sickened by set theory?), providing an expressive source for the social mood inside the Federal Republic. The story related gives an impression of diverse arguments from people that were, in a large part, completely outside the subject area and far from factual and unbiased discourse. The possibly most significant citation is from a ‘set theory opponent’ who expresses his fear of loss of credit when his children find him ignorant and helpless¹⁹. We have seen before that tendencies towards the strengthening of emancipation and a democratic attitude have been driving forces for the reform of mathematics education, and neither the further process nor the decline of Modern Mathematics in West Germany can be isolated from this context. West German post-war society was, in a large part, conservative. Now, trusted hierarchies threatened to shift and dissolve; people’s traditional body of knowledge and education seemed to become devaluated; group work and children playing with colourful blocks in school looked like the end of all serious work and overcome (Prussian) discipline. For a considerable part of society, all of this must have caused fear of the deterioration of what they were familiar with and made their members disapprove of an educational reform that seemed to stand for the decay of society as a whole. And yet on top of this, the early Federal Republic was characterized by a ‘lasting effect of anti-intellectualism from the Nazi era’²⁰ not acknowledging the value of higher or academic education.

Now, who were the protagonists at the different stages of the reform, who were those actors putting their mark on the progress of New Math at West German primary schools? Weiss²¹ has worked out that the reform of secondary math education was decisively promoted by secondary school teachers who also acted as textbook authors. This was not the case with primary education. Teachers did not play a decisive role in the beginning. This should come as no surprise in view

¹⁸ A prominent example is found in H. Karaschewski, *Irrwege der modernen Rechendidaktik*, Dürr, Bonn 1969. The author was an advocate of traditional arithmetic education; the title, which can be translated to ‘Modern numeracy didactics on the wrong track’, clearly shows Karaschewski’s disapproval of the changes applied to the subject.

¹⁹ *Mengenlehre*: “ $3 + 5 = 5 + 3$ ”, “Der Spiegel” 25.03.1974, vol. 28, no. 13, p. 63.

²⁰ G. Picht, *Die deutsche Bildungskatastrophe: Analyse und Dokumentation*, Walter, Olten 1964, p. 66.

²¹ Y. Weiss, *West German Neue Mathematik and some of its protagonists*, [in:] *Modern Mathematics: An international movement?*, ed. by D. De Bock, Springer, Cham 2023 (History of Mathematics Education), p. 103–125.

of the fact that primary school teachers were not academically educated back then. Instead, the textbooks were written by educationalists from institutions of higher education who thereby had a massive influence on the course of the reform. In the context of textbook production and publication, another protagonist comes into view who has not yet been talked about. We have identified the gap in curricula up to the year 1976, presumably leading to textbooks taking the role of a syllabus in school practice. At the same time, textbook production in a time of such extensive reform promised considerable profit for publishers. The first to grasp the chance was the publisher Herder. From 1965 on, Herder published the works by Z. P. Dienes, and throughout the years they built a huge series around Dienes' books, named *Programm Moderne Mathematik* [Program Modern Mathematics]. The textbook by Neunzig and Sorger was part of this program. The author of this paper has not been able to find out who first initiated the publication of the schoolbook, but a mutual interest is likely. In any case, Herder published the book, and it's obvious that they did so out of commercial interest. And the commercial aim might very well have been the reason for this notably early issue of *Wir lernen Mathematik*. The timing, alongside a huge promotional effort²², led to a wide dissemination of the schoolbook. At the same time, in the light of missing curricula and the need for new educational materials, we may assume that other textbook authors sought guidance by looking at already existing works and, in that regard, Neunzig and Sorger clearly had an advantage. But, as we have seen before, recontextualization of the Dienes concept to the context of traditional numeracy curricula went along with considerable shortcomings. The product of recontextualization, though, became paradigmatic and style-forming for teaching set theory in West German primary schools. In other words, Herder's commercial goals might have played a crucial role in the replication of these shortcomings, which would indeed make Herder a relevant actor in the history of New Math.

We have already identified the decisive role of politics in the progress of the reform. In the end, politicians were the ones to reduce and take back the innovations. Their decisions, though, are the result of social currents, so we must include all those who communicated their disapproval more or less openly. Those people could be found everywhere. There were parents who disliked not being able to help with their children's homework²³, there were teachers insufficiently educated who simply continued teaching the way they had always done. And there were those general conservative parts of society struggling with all the changes

²² Hollmann's report on a so-called study week that took place on the initiative of Herder can be taken as a paradigmatic source here. This event was one of the many occasions that Herder invited Dienes to come to Germany and promote his ideas, E. Hollmann, *Moderne Mathematik in der Grundschule*.

²³ See, e.g., J. Lauter, *Der Mathematikunterricht in der Grundschule. Didaktisch-methodische Hilfen für die Unterrichtspraxis*, Auer, Donauwörth 1977, p. 24–25.

that came along with the overall tendencies towards emancipation and more left-wing politics that had stirred the FRG since the late 1960s.

Outlook

As has been said in the introduction, this paper as well as the study it is based on wishes to serve as a puzzle piece of the big picture of New Math. To find out common features in different countries as well as developments that were conditioned by national peculiarities comparisons are necessary. The results of this study, though, suggest that comparing processes, practices and protagonists can only be a starting point but will not suffice. Instead studies are required that approach the history of Modern Mathematics as part of general educational and general political history and bring it in line with a country's then social trends and developments. Studies of such a kind are complex and yet scarce. The work by Zelbo²⁴ though, suggests that the story of New Math in the USA was bound to general political-societal tendencies, as well, and that further work in this direction might support the alleged link between teaching Modern Mathematics and teaching towards democracy.

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²⁴ S. Zelbo, *Ideology and Public Opposition to the 'New Math' Reform Movement in the United States (1960 to 1980)*, "Analecta. Studies and Materials for the History of Science" 2025, this issue.

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