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Optimizing the minimum cost flow algorithm for the phase unwrapping process in SAR radar

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Abstract. The last three decades have been abundant in various solutions to the problem of Phase Unwrapping in a SAR radar. Basically, all the existing techniques of Phase Unwrapping are based on the assumption that it is possible to determine discrete "derivatives" of the unwrapped phase. In this case a discrete derivative of the unwrapped phase means a phase difference (phase gradient) between the adjacent pixels if the absolute value of this difference is less than π. The unwrapped phase can be reconstructed from these discrete derivatives by adding a constant multiple of 2π. These methods differ in that the above hypothesis may be false in some image points. Therefore, discrete derivatives determining the unwrapped phase will be discontinuous, which means they will not form an irrotational vector field. Methods utilising branch-cuts unwrap the phase by summing up specific discrete partial derivatives of the unwrapped phase along a path. Such an approach enables internally cohesive results to be obtained. Possible summing paths are limited by branch-cuts, which must not be intersected. These branch-cuts connect local discontinuities of discrete partial derivatives. The authors of this paper performed parametrization of the Minimum Cost Flow algorithm by changing the parameter determining the size of a tile, into which the input image is divided, and changing the extent of overlapping of two adjacent tiles. It was the basis for determining the optimum (in terms of minimum Phase Unwrapping time) performance of the Minimum Cost Flow algorithm in the aspect of those parameters.

Key words: Interferometry Synthetic Aperture Radar (IFSAR), minimum cost flow (MCF), phase unwrapping (PhU).

1. Introduction

Synthetic Aperture Radar (SAR) interferometry (InSAR) has already attracted because of its successful applications especially in deformation monitoring, [1–4] and topographic mapping [5]. Phase unwrapping (PhU) is considered to be a constrained minimization problem for many well-known algorithms [6–7]. At present a few methods utilizing various ways of applying branch-cuts onto a phase map are being developed, such as: tree, dipole, quality mask control and Minimum Cost Flow (MCF), or in other words discontinuities connected with the minimum cost [8–12]. The procedure of 2-Dimensional Phase Unwrapping in an IFSAR radar consists in removing 2π discontinuities located on the phase map. 2-Dimensional PhU is a stage of retrieving the actual value of phase difference, resulting from the difference in the distances travelled by the radar echo signals received by two IFSAR radar antennas, from the "wrapped" phase difference value. As the wavelengths are much shorter than this difference of distances, the signal phase within this time will change k times by 2π, before it has been measured [13]. It is the most important stage in the IFSAR processing. The optimum method for determining the number of full periods of the phase change is the phase unwrapping process [14]. Contrary to many two-dimensional methods of signal processing, such as the Fast Fourier Transform (FFT), the 2-Dimensional PhU process cannot be divided into one-dimensional Phase Unwrapping operations in lines and columns. To maintain the wave surface continuity, the process must be conducted simultaneously in the longitudinal and transverse planes [15]. The correct conducting of this process is still a subject of intensive scientific research. There are many problems, which should be solved in this process, and a number of methods, which have been developed for this purpose.

The PhU is a technique, where measured values of wrapped phases are used to eliminate 2π discontinuities placed on the phase map. The technique detects a 2π phase change and adds or subtracts the 2π total correction to or from the successive pixels. The implementation of this shift is based on an appropriate threshold mechanism. The threshold mechanism functions in this way that if the phase difference between two adjacent pixels on a path is greater than $+\pi$, then the 2π shift is subtracted from all subsequent pixels on the unwrapping path. The phase difference is calculated with the Eq. (1), where $\Phi(p_i)$ is the wrapped phase in the p_i pixel on the phase map.

$$\Delta\Phi(p_i) = \Phi(p_i) - \Phi(p_{i-1}). \tag{1}$$

However, if the phase difference is smaller than $-\pi$, the 2π shift is added to all subsequent pixels on the unwrapping path. Then, having all discontinuities located on the wrapped phase map, the phase of each pixel will be changed by an integral (k) multiple of the 2π shift, depending on the pixel position on the unwrapping path. It can be expressed by a special wrapping operator, which may be written as Eqs. (2), (3), where

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$\Psi(p_i)$ is the unwrapped phase in the p_i pixel on the phase map and Z is the integer set.

$$W\left[\Psi(p_i)\right] = \Psi(p_i) + 2\pi k(p_i), \qquad k(p_i) \in Z, \tag{2}$$

$$-\pi \leq W\left[\Psi(p_i)\right] \leq +\pi. \tag{3}$$

With this operator the correct phase gradient $\widehat{\nabla}\Phi(p_i)$ between the subsequent pixels on the unwrapping path may be determined, as presented in the Eq. (4).

$$\widehat{\nabla}\Phi(p_i) = W\left[\Phi(p_i) - \Phi(p_{i-1})\right]. \tag{4}$$

The PhU is a process of integrating (summing up) the phase, which may be carried out in two ways. From the local perspective the Phase Unwrapping (summing up) runs from a pixel to a pixel, while from the global perspective the values of the unwrapped phases in pixels are obtained as a result of operations concerning the whole image area. It is also possible to combine these two methods into a single hybrid algorithm. The local summing (integration) techniques are called "path-following methods", whereas the methods representing the global approach are called "minimum-norm methods" [16–17] The problem of correct drawing of these lines has not been solved by the general approach yet, which may substantially limit the capabilities of these methods [18].

2. MCF algorithm approach

The Minimum Cost Flow algorithm utilizing the "Hungarian algorithm" has been developed by Harold Kuhn. The MCF is focused on minimisation of discontinuities. This method was for the first time presented by Constantini [11]. It utilises a flow network, which determines the position of each section of branch-cuts based on the cost coefficient and the general minimization strategy, as described by the Eqs. (5), (6), where $\min E$ is the total number of minimum errors (discontinuities) on the unwrapped phase map, $\Phi_{i,j}$ is the measured wrapped phase in a pixel with the coordinates $[i,j]$, $c^x_{i,j}$ and $c^y_{i,j}$ is the weight coefficients or the cost function and $\min\{\cdot\}$ symbol represents the minimization operation.

$$\min E = \sum_{i=1}^{M}\sum_{j=1}^{N} c^x_{i,j}\left|k^x_{i,j}\right| + \sum_{i=1}^{M}\sum_{j=1}^{N} c^y_{i,j}\left|k^y_{i,j}\right|, \tag{5}$$

$$\begin{aligned} & k^y_{i,j+1} - k^y_{i,j} - k^x_{i+1,j} + k^x_{i,j} \\ &= \frac{-1}{2\pi}\left[\Phi^y_{i,j+1} - \Phi^y_{i,j} - \Phi^x_{i+1,j} + \Phi^x_{i,j}\right]. \end{aligned} \tag{6}$$

The value $k^x_{i,j}$ given by Eq. (7) is the total number of discontinuities in the x direction, and value $k^y_{i,j}$ given by Eq. (8) is the total number of discontinuities in the y direction, where $\Psi_{i,j}$ is the estimated unwrapped phase in a pixel with the coordinates $[i,j]$, $\nabla\Phi^x_{i,j}$ is the wrapped phase gradient in the x direction, $\nabla\Phi^y_{i,j}$ is the wrapped phase gradient in the y direction, according to Eqs. (9)–(11).

$$k^x_{i,j} = Int\left(\frac{\Psi_{i,j} - \Psi_{i-1,j} - \widehat{\nabla}\Phi^x_{i,j}}{2\pi}\right), \tag{7}$$

$$k^y_{i,j} = Int\left(\frac{\Psi_{i,j} - \Psi_{i,j-1} - \widehat{\nabla}\Phi^y_{i,j}}{2\pi}\right), \tag{8}$$

$$\nabla\Phi^x_{i,j} = W\left[\Phi_{i+1,j} - \Phi_{i,j}\right], \tag{9}$$

$$\nabla\Phi^y_{i,j} = W\left[\Phi_{i,j+1} - \Phi_{i,j}\right], \tag{10}$$

$$\begin{aligned} & W\left[\Phi_{i,j}\right] = \Phi_{i,j} + 2\pi k \\ \text{for} \quad & k \in Z \;\; i \quad -\pi \leq W[\Phi_{i,j}] \leq +\pi. \end{aligned} \tag{11}$$

This method minimises the total sum of integral multiples $\pm 2\pi$ added to the original gradient value for each pixel before starting the Phase Unwrapping process. The *branch-cut* line distribution is determined by the cost function value (weight coefficients). Indeed, finding the best possible distribution of *branch-cut* lines helps the minimization criterion to achieve the minimum total value. If the costs in this method, or the values of weight coefficients, are constant, then the minimum flow cost minimises the total length of the *branch-cut* lines [19]. However, the costs in this method are usually defined by the user defining weights or quality maps for a specific image using such parameters as coherence, correlation and pseudo-correlation coefficient, phase gradient change, maximum gradient, residue density, flatness or smoothness of the unwrapped phase [11]. The solution in this method is obtained based upon a network flow algorithm (the technique derived from the graph theory and network programming). In recent years this very method is becoming most widely used. It may be implemented with the use of general purpose programming environments. However, it requires hardware with enormous calculation capacities and memory resources. Lack of the optimum method for determining weight coefficients, which are necessary in this method, still poses a big problem.

3. Specification of input data for MCF

The Phase Unwrapping process was examined on the basis of the Minimum Cost Flow algorithm for the Constantini method. The input data for the algorithm examined was a 2D (two-dimensional) table containing the wrapped phase value [in radians]. The dimensions examined in the above mentioned table had the sizes, respectively: 50, 75, 100, 125, 150, 200, 250 [pixel-by-pixel]. Parametrization of the MCF algorithm enabled it to be modified by changing:

- maximum Tile Size ($T_{S\max}$) – the coefficient that defines the size of tiles, into which the input image is divided, i.e. the 2D table containing the wrapped phase value. $T_{S\max}$ parameter default equal to [125];

- Tile Overlapping Size (T_{OS}) coefficient – a scalar parameter in the form of a decimal fraction, which determines the extent of overlapping of two adjacent tiles. T_{OS} parameter default equal to [0.25].

3.1. The MCF algorithm procedure execution. As a result of the MCF algorithm operation in the PhU process, a 2D table of the unwrapped phase and the minimum network flow calculated with the MatLab LINPROG procedure, were obtained. In order to analyse the influence of the MCF algorithm, the actual measurement results obtained from a Pol-In SAR radar were used as the input data. A 6-by-6 coherence matrix was obtained. A term (T_{14}), containing the complex phase difference for each pixel and amplitude, was extracted from this coherence matrix. The measured phase values have a table binary structure with the size 1300-by-1200 pixels, in accordance with the Eqs. (7), (8).

$$\overline{\mathbf{T}}_{14} = \mathrm{Re}\,\mathrm{T}_{14} + \mathrm{Im}\,\mathrm{T}_{14}, \tag{12}$$

$$\Phi = \mathrm{arctg}\,\frac{\mathrm{Im}\,\mathrm{T}_{14}}{\mathrm{Re}\,\mathrm{T}_{14}}. \tag{13}$$

The obtained graphic image for this SAR radar is presented in Fig. 1, (source: DLR Portal – Microwaves and Radar Institute: DLR Site at Oberpfaffenhofen, Germany- Pol-In SAR Radar Image). It will be used as the original of the obtained image in order to compare results of MCF algorithm processing.

Fig. 1. An example of graphic image from the Pol-In SAR radar

4. Optimizing MCF algorithm parameters

The examination of the MCF algorithm in the SAR radar PhU process was performed on the basis of a change in the maximum tile size and the tile overlapping value. This was the basis for determining the optimum (in terms of Phase Unwrapping minimum time) performance of this algorithm in the aspect of the above parameters. The obtained results are presented in Figs. 2–4 below in the form of a graphic image of the real and imaginary part of the phase and its unwrapping with the MCF method.

Fig. 2. A graphic image of the real part of the signal analysed phase

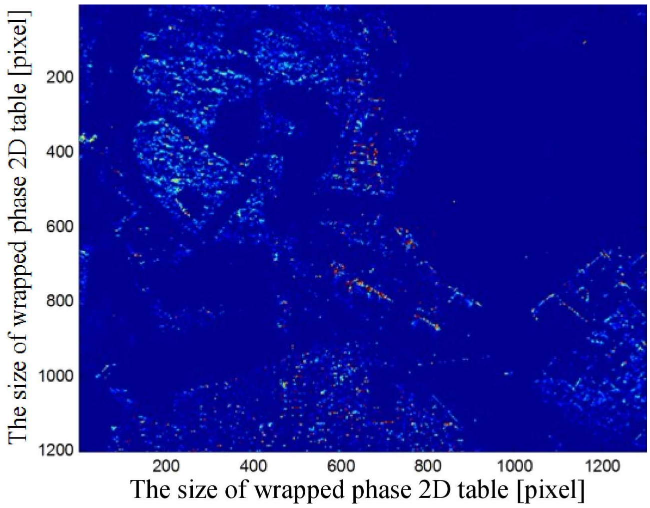

Fig. 3. A graphic image of the imaginary part of the phase

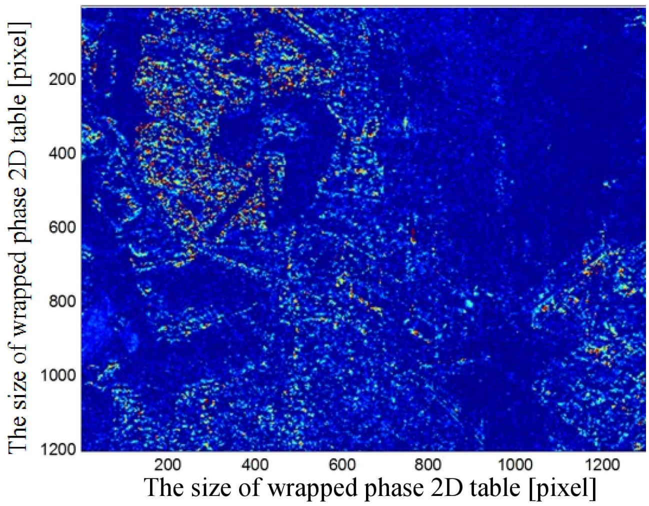

Fig. 4. A graphic image of the unwrapped phase with the MCF method

5. Parameter examination results

The process of the MCF algorithm examination determined the impact of the maximum Tile Size coefficient and the Tile Overlapping Size coefficient on the speed of this algorithm operation. The T_{OS} influence on the speed of the MCF algorithm operation in the PhU process was examined on the basis of a change in the T_{OS} coefficient, in accordance with the following values, set as the initial values, i.e.: 0.10; 0.15; 0.20; 0.25; 0.30; 0.40 and 0.50. The calculation phase unwrapping time results (T_{PhU}) are presented in Table 1. The $T_{S\,\max}$ influence on the speed of MCF algorithm operation in the PhU process was examined on the basis of a change in the $T_{S\,\max}$ coefficient, i.e.: 50, 75, 100, 125, 150, 200 and 250 [pixel-by-pixel]. The calculation T_{PhU} results for the algorithm are presented in Table 1.

5.1. The results of the PhU time for the set values of T_{OS} and $T_{S\,\max}$ coefficient. The algorithm "work" results are presented in Table 1. Based on the obtained values, the graphic image was made in the form of bar charts for the Phase Unwrapping time, for T_{OS} and $T_{S\,\max}$ coefficients (see exemplary Figs. 5–8).

Table 1
A comparison of Phase Unwrapping time for MCF for the set initial values of the maximum Tile Size and Tile Overlapping Size coefficients

maximum Tile Size [pixel-by-pixel]	$T_{S\,\max}$						
	50	75	100	125	150	200	250
T_{PhU} [s] for T_{OS} = 0.10	16	27	40	42	51	53	53
T_{PhU} [s] for T_{OS} = 0.15	23	28	38	40	26	26	25
T_{PhU} [s] for T_{OS} = 0.20	25	30	23	22	47	46	46
T_{PhU} [s] for T_{OS} = 0.25	27	32	44	53	24	23	23
T_{PhU} [s] for T_{OS} = 0.30	35	44	64	48	25	26	24
T_{PhU} [s] for T_{OS} = 0.40	44	42	53	86	88	79	78
T_{PhU} [s] for T_{OS} = 0.50	63	42	59	77	104	93	90

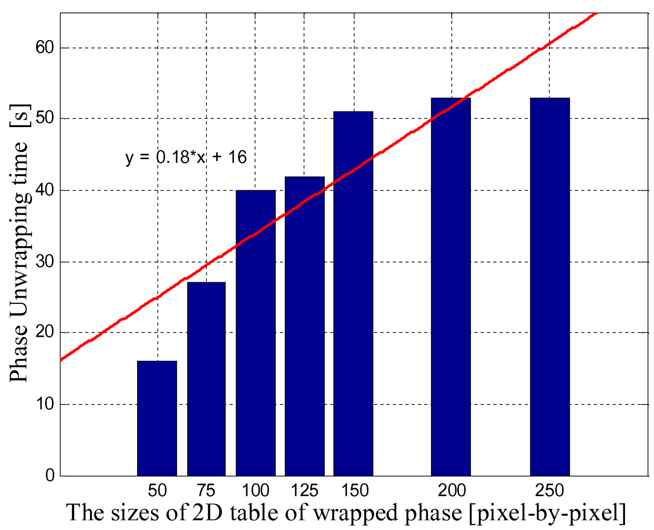

Fig. 5. The MCF graph of PhU time for coefficient $T_{OS} = 0.10$

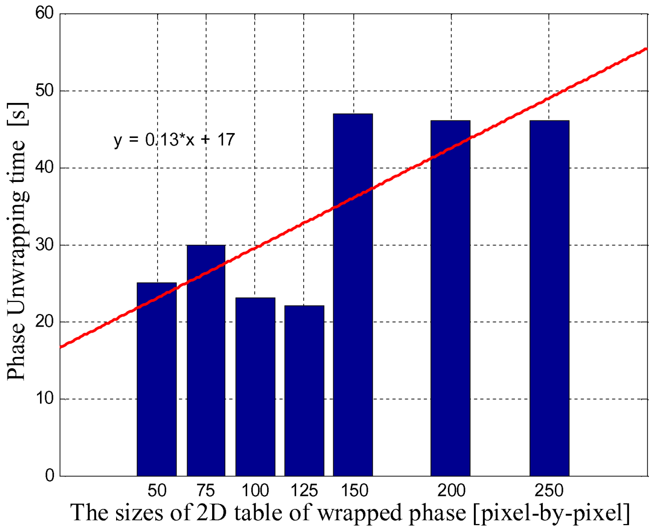

Fig. 6. The MCF graph of PhU time for coefficient $T_{OS} = 0.20$

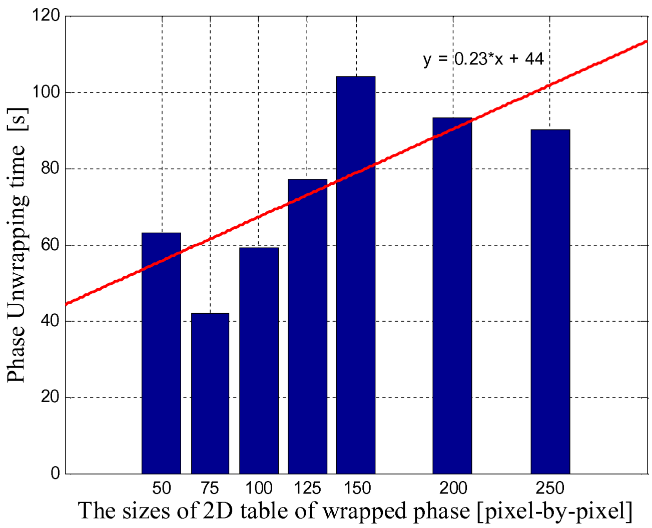

Fig. 7. The MCF graph of PhU time for coefficient $T_{OS} = 0.50$

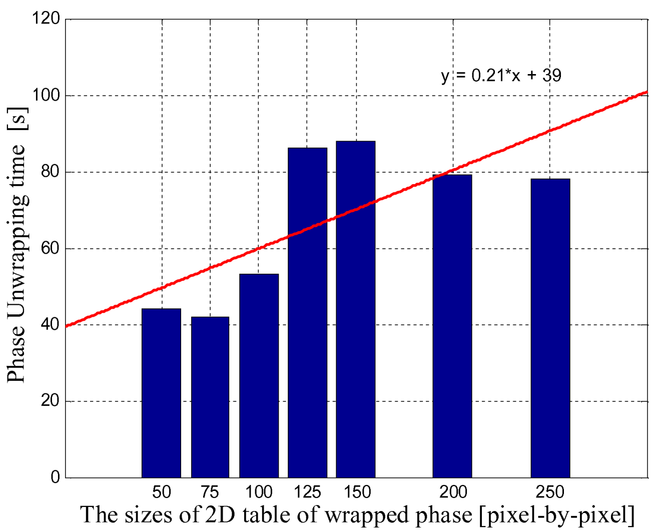

Fig. 8. The MCF graph of PhU time for coefficient $T_{OS} = 0.40$

For the obtained values of PhU time (T_{PhU}) depending on $T_{S\,\max}$ and T_{OS} coefficients, a linear regression line was determined, marked by red in Figs. 5–8. The analysis of the slopes of the obtained red lines indicates that the PhU time increases, when the T_{OS} coefficient increases also. The maximum Tile Size considerably influences the calculation time of the algorithm in the PhU process. It can be noticed that an increase in the aforesaid parameter causes an increase in the PhU time (see exemplary Figs. 5–8).

At the same time an influence of the Tile Overlapping coefficient on the algorithm effect speed may be observed. For a constant value of the tile size, with an increasing T_{OS} value (within the range examined of 0.10–0.50), an increase in the PhU time can be observed. The above conclusion is confirmed by the analysis of the PhU time with linear regression (see the slopes of red lines in Figs. 5–8). The obtained linear regression line equations indicate an increase in the PhU time with an increase in the T_{OS} coefficient, generally.

5.2. The results of the PhU mean time for the set values of T_{OS} and $T_{S\,\max}$ coefficients. In the above-mentioned analysis the mean time of PhU ($T_{meanPhU}$) for the MCF algorithm was determined depending on T_{OS} and $T_{S\,\max}$ coefficients. The conducted calculations yielded the following results, presented in Table 2 and graphic images of the obtained results, shown in Figs. 9, 10.

Table 2

The Phase Unwrapping mean time for the MCF for the set value of the Tile Overlapping and maximum Tile Size coefficients

	Tile Overlapping coefficient T_{OS}						
	0.10	0.15	0.20	0.25	0.30	0.40	0.50
mean time of PhU ($T_{meanPhU}$) [s]	40.28	29.42	34.14	32.28	38.0	67.14	75.42
	maximum Tile Size coefficient $T_{S\,\max}$						
	50	75	100	125	150	200	250
mean time of PhU ($T_{meanPhU}$) [s]	33.28	35.0	45.85	52.57	52.14	49.42	48.42

The analysis of results of the PhU mean time ($T_{meanPhU}$) for the set values of coefficients T_{OS} and $T_{S\,\max}$ indicates an increase $T_{meanPhU}$ when the T_{OS} parameter increases, for a constant value of the $T_{S\,\max}$ (see Table 2 and Fig. 9). At the same time, when $T_{S\,\max}$ increases (for a constant T_{OS} value), the PhU mean time also increases (see Table 2 and Fig. 10). It can be noticed that the T_{OS} parameter for values in the range 0.15–0.25 gives the best results (the shortest PhU time), while the minimum PhU time is obtained for the value of $T_{S\,\max}$ in the size range 50–75 [pixel-by-pixel] of the signal wrapped phase input table (see Fig. 10).

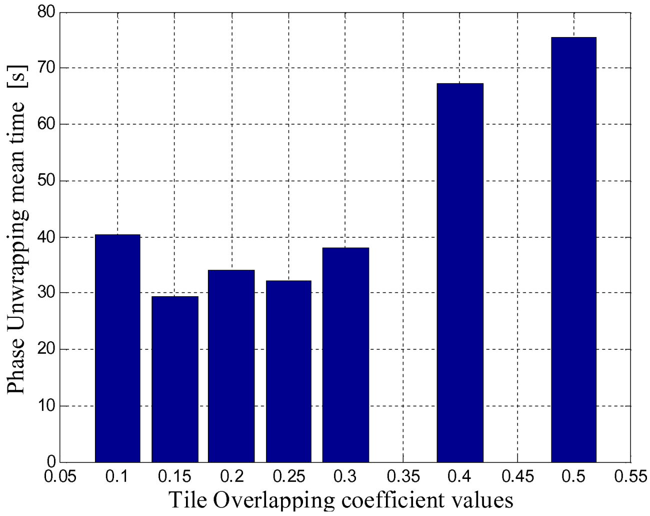

Fig. 9. PhU mean time graph for a constant values of the Tile Overlapping Size coefficient

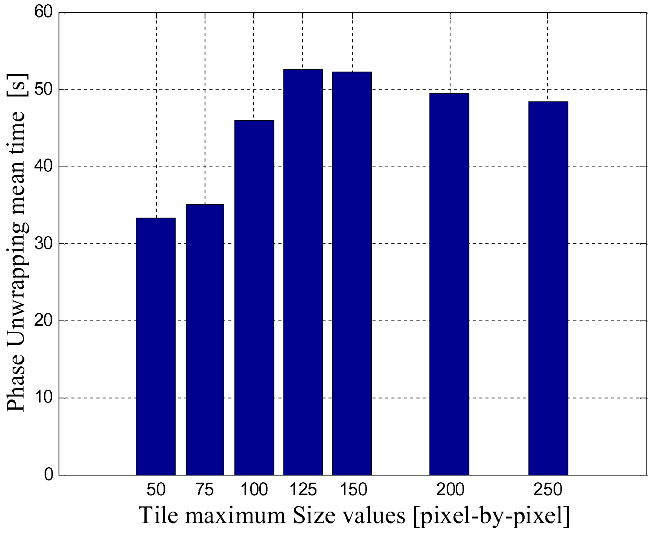

Fig. 10. PhU mean time graph for a constant values of maximum Tile Size coefficient

6. Conclusions

The PhU process was examined on the basis of the Minimum Cost Flow algorithm for the Constantini method. The input data to the algorithm examined was a 2D table of the wrapped phase value. The algorithm parametrization allowed to modify its behaviours by changing the maximum Tile Size coefficient and the Tile Overlapping Size coefficient. The algorithm operation in the PhU process resulted in a 2D table of the unwrapped phase (see Fig. 4).

The PhU time (T_{PhU}) was analysed with reference to the T_{OS} and T_{Smax}coefficients, where the input data was the table of actual measurement data of the wrapped signal phase with the size 250-by-250 [pixel]. Graphic images in the form of bar charts, presented in this article, were obtained. Based on the conducted research on the PhU process using the MCF algorithm, one can find that the above mentioned method is very sensitive to the size of the input data (i.e. the size of the 2D table of the signal wrapped phase input data), where the calculation time is considerably longer when the input data table increases. In these circumstances, the optimisation of the MCF algorithm operation in terms of Phase Unwrapping time minimization should base on the ”optimum” selection of the maximum Tile Size and the Tile Overlapping coefficient, which was confirmed in this article by determining the variability areas of the above mentioned parameters in order to minimize the Phase Unwrapping time.

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