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Application of descriptor approaches in design of PD observer-based actuator fault estimation

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Stability analysis and design for continuous-time proportional plus derivative state observers is presented in the paper with the goal to establish the system state and actuator fault estimation. Design problem accounts a descriptor principle formulation for non-descriptor systems, guaranteing asymptotic convergence both the state observer error as fault estimate error. Presented in the sense of the second Lyapunov method, an associated structure of linear matrix inequalities is outlined to possess parameter existence of the proposed estimator structure. The obtained design conditions are verified by simulation using a numerical illustrative example.

Key words: PD observers, actuator fault estimation, descriptor system observation, convex optimization, linear matrix inequalities.

1. Introduction

As is well known, observer design is a hot research field owing to its particular importance in observer-based control, residual fault detection and fault estimation [1], where, especially from the stand point of the active fault tolerant control structures, the problem of simultaneous state and fault estimation is very eligible. In that sense various effective methods have been developed to take into account the faults effect on control structure reconfiguration and fault estimation [16], [20]. In particular, proportional plus derivative (PD) observers introduce a design freedom giving an opportunity for generating state and fault estimates with good sensitivity properties and improving the observer design performance [6], [18]. Since derivatives of the system outputs can be exploited in the fault estimator design to achieve faster fault estimation, a proportional multi-integral derivative estimators are proposed in [7], [22]. The requirement for fast adaptation reaction to faults means that the adaptive observers are exploited [3], [20].

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Although the state observers for linear and nonlinear systems received considerable attention, the descriptor design principles have not been studied extensively for non-singular systems. Modifying the descriptor observer design principle [13], the first result giving sufficient design conditions for linear time-delay systems can be found in [5]. Reflecting the same problems concerning the observers for descriptor systems, LMI methods were presented e.g. in [9] but a hint of this method can be found in [21], [23].

Adapting the approach to the observer-based fault estimation for descriptor systems as well as its potential extension [12], the main issue of this paper is to apply the descriptor principle in design of PD fault observers. Preferring LMI formulation, the stability condition proofs use standard arguments in the sense of Lyapunov principle to obtain the design conditions requiring to solve set of LMIs. This presents a method designing the PD observation derivative and proportional gain matrices such that the design is non-singular and ensures that the estimation error dynamics has asymptotical convergence. From viewpoint of application, although the descriptor principle is used, it is not necessary to transform the system parameter into a descriptor form or to use matrix inversions in design task formulation.

The paper is organized as follows. Situated after Introduction, Sec. 2 gives the problem statement for PD fault observer and Sec. 3 presents basic preliminaries to formulate a design problem in the descriptor form. A new LMI structure, describing the PD fault estimator design conditions, is theoretically explained in Sec 4. An example is provided to demonstrate the proposed approach in Sec. 5 and Sec. 6 draws some conclusions.

Used notations are conventional so that $\boldsymbol{x}^T$, $\boldsymbol{X}^T$ denote transpose of the vector $\boldsymbol{x}$ and matrix $\boldsymbol{X}$, respectively, $\boldsymbol{X} = \boldsymbol{X}^T > 0$ means that $\boldsymbol{X}$ is a symmetric positive definite matrix, the symbol $\boldsymbol{I}_n$ indicates the n-th order unit matrix, $\rho(\boldsymbol{X})$ and $rank(\boldsymbol{X})$ indicate the eigenvalue spectrum and rank of a square matrix $\boldsymbol{X}$, $\mathbb{R}$ denotes the set of real numbers and $\mathbb{R}^n$, $\mathbb{R}^{n\times r}$ refer to the set of all n-dimensional real vectors and $n \times r$ real matrices, respectively.

2. Problem statement

The systems under consideration are linear MIMO continuous-time dynamic systems represented in state-space form as

$$\dot{\boldsymbol{q}}(t) = \boldsymbol{A}\boldsymbol{q}(t) + \boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{F}\boldsymbol{f}(t)\,, \tag{1}$$

$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{q}(t)\,, \tag{2}$$

where $\boldsymbol{q}(t) \in \mathbb{R}^n$, $\boldsymbol{u}(t) \in \mathbb{R}^r$, $\boldsymbol{y}(t) \in \mathbb{R}^m$, $\boldsymbol{f}(t) \in \mathbb{R}^p$, are vectors of the state, input, output and fault variables, $\boldsymbol{A} \in \mathbb{R}^{n\times n}$, $\boldsymbol{B} \in \mathbb{R}^{n\times r}$, $\boldsymbol{C} \in \mathbb{R}^{m\times n}$, $\boldsymbol{F} \in \mathbb{R}^{n\times p}$ are real finite values matrices, $m, r, p < n$ and

$$\operatorname{rank}\begin{bmatrix} \boldsymbol{A} & \boldsymbol{F} \\ \boldsymbol{C} & \boldsymbol{0} \end{bmatrix} = n + p\,. \tag{3}$$

Moreover, it is supposed that $(\boldsymbol{A},\boldsymbol{C})$ is observable.

Focusing on fault estimation task for slowly-varying faults, a fault PD observer is considered in the following form [19]

$$\dot{\boldsymbol{q}}_e(t) = \boldsymbol{A}\boldsymbol{q}_e(t) + \boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{F}\boldsymbol{f}_e(t) + \boldsymbol{J}(\boldsymbol{y}(t) - \boldsymbol{y}_e(t)) + \boldsymbol{L}(\dot{\boldsymbol{y}}(t) - \dot{\boldsymbol{y}}_e(t)), \tag{4}$$

$$\boldsymbol{y}_e(t) = \boldsymbol{C}\boldsymbol{q}_e(t), \tag{5}$$

$$\dot{\boldsymbol{f}}_e(t) = \boldsymbol{M}(\boldsymbol{y}(t) - \boldsymbol{y}_e(t)) + \boldsymbol{N}(\dot{\boldsymbol{y}}(t) - \dot{\boldsymbol{y}}_e(t)), \tag{6}$$

where $\boldsymbol{q}_e(t) \in I\!R^n$, $\boldsymbol{y}_e(t) \in I\!R^m$, $\boldsymbol{f}_e(t) \in I\!R^p$ are estimates of the system states variables, output variables and augmented variables vectors, respectively, and $\boldsymbol{J},\boldsymbol{L} \in I\!R^{n\times m}$, $\boldsymbol{M},\boldsymbol{N} \in I\!R^{p\times m}$ is the set of observer gain matrices to be determined.

3. Basic preliminaries

To explain and concretize the obtained results the following well known lemma of Schur complement property is suitable.

Lemma 1 *(Schur Complement) Let* $\boldsymbol{S}$, $\boldsymbol{Q}=\boldsymbol{Q}^T$, $\boldsymbol{R}=\boldsymbol{R}^T$, $\det\boldsymbol{R} \neq 0$ *are real matrices of appropriate dimensions, then the next inequalities are equivalent*

$$\begin{bmatrix} \boldsymbol{Q} & \boldsymbol{S} \\ \boldsymbol{S}^T & -\boldsymbol{R} \end{bmatrix} < 0 \Leftrightarrow \begin{bmatrix} \mathbf{Q}+\mathbf{S}\mathbf{R}^{-1}\mathbf{S}^T & \mathbf{0} \\ \mathbf{0} & \mathbf{R} \end{bmatrix} < 0 \Leftrightarrow \mathbf{Q}+\mathbf{S}\mathbf{R}^{-1}\mathbf{S}^T < 0,\ \mathbf{R} > 0. \tag{7}$$

Proof (compare, e.g., [2], [4]) If the linear matrix inequality takes the composed form

$$\begin{bmatrix} \mathbf{Q} & \mathbf{S} \\ \mathbf{S}^T & -\mathbf{R} \end{bmatrix} < 0, \tag{8}$$

then using the Gauss elimination principle it yields

$$\begin{bmatrix} \mathbf{I} & \mathbf{S}\mathbf{R}^{-1} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{Q} & \mathbf{S} \\ \mathbf{S}^T & -\mathbf{R} \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{R}^{-1}\mathbf{S}^T & \mathbf{I} \end{bmatrix} = \begin{bmatrix} \mathbf{Q}+\mathbf{S}\mathbf{R}^{-1}\mathbf{S}^T & \mathbf{0} \\ \mathbf{0} & -\mathbf{R} \end{bmatrix}. \tag{9}$$

Since

$$\det \begin{bmatrix} \mathbf{I} & \mathbf{S}\mathbf{R}^{-1} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} = 1, \tag{10}$$

it is evident that this transform doesn't change negativeness of (8) and so (9) implies (7). This concludes the proof. □

Using the descriptor approach the basic descriptor system properties are presented, considering the descriptor system model of the form [10]

$$\boldsymbol{E}\dot{\boldsymbol{q}}(t) = \boldsymbol{A}\boldsymbol{q}(t) + \boldsymbol{B}\boldsymbol{u}(t)\,, \tag{11}$$

$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{q}(t)\,, \tag{12}$$

with singular matrix $\boldsymbol{E} \in I\!R^{n\times n}$, where $\mathrm{rank}(\boldsymbol{E}) = p < n$. This part is primarily limited on restrictions under which the descriptor system stability can be regularized and when this task is feasible.

Definition 1 [14]

(I) *The pair* $(\boldsymbol{E},\boldsymbol{A})$ *is regular if* $\det(s\boldsymbol{E}-\boldsymbol{A})$ *is not identically zero.*

(II) *For a regular pair* $(\boldsymbol{E},\boldsymbol{A})$ *the finite eigenvalues of* $(s\boldsymbol{E}-\boldsymbol{A})$ *are said to be the finite modes of* $(\boldsymbol{E},\boldsymbol{A})$*. If there is* $\boldsymbol{E}\boldsymbol{v}_1 = 0$*, then the infinite eigenvalues, associated with the generalized principal right vectors* $\boldsymbol{v}_i$ *satisfying* $\boldsymbol{E}\boldsymbol{v}_i = \boldsymbol{A}\boldsymbol{v}_{i-1}$ *for* $i = 2,3,4,\cdots$*, are impulsive modes of* $(\boldsymbol{E},\boldsymbol{A})$*.*

(III) *A pair* $(\boldsymbol{E},\boldsymbol{A})$ *is admissible if is regular and has neither impulsive modes nor unstable finite modes.*

Admissibility conditions for a descriptor system pair $(\boldsymbol{E},\boldsymbol{A})$ are given by the following lemma [15]:

Lemma 2 *(system pair admissibility) The pair* $(\boldsymbol{E},\boldsymbol{A})$ *is admissible if there exists* $\boldsymbol{X} \in I\!R^{n\times n}$ *such that*

$$\boldsymbol{X}^T\boldsymbol{A} + \boldsymbol{A}^T\boldsymbol{X} < 0\,, \tag{13}$$

$$\boldsymbol{X}^T\boldsymbol{E} = \boldsymbol{E}^T\boldsymbol{X} \geqslant 0\,. \tag{14}$$

Proof (compare [14]) Analyzing stability of the equilibrium point of the system (11), the following Lyapunov function candidate can be defined

$$v(\boldsymbol{q}(t)) = \boldsymbol{q}^T(t)\boldsymbol{E}^T\boldsymbol{X}\boldsymbol{q}(t) \geqslant 0\,, \tag{15}$$

where $\boldsymbol{X} \in I\!R^{n\times n}$ and $\boldsymbol{E}^T\boldsymbol{X} \geqslant 0$.
To obtain asymptotic stability condition then derivative of (15) with respect to time has to be

$$\dot{v}(\boldsymbol{q}(t)) = \dot{\boldsymbol{q}}^T(t)\boldsymbol{E}^T\boldsymbol{X}\boldsymbol{q}(t) + \boldsymbol{q}^T(t)\boldsymbol{E}^T\boldsymbol{X}\dot{\boldsymbol{q}}(t) < 0\,. \tag{16}$$

Thus, to acquire a symmetric quadratic form of (16) (with respect to the left side of equation (11)), it is necessary that the constraints (14) has to be satisfied. This gives

$$\dot{v}(\boldsymbol{q}(t)) = \dot{\boldsymbol{q}}^T(t)\boldsymbol{E}^T\boldsymbol{X}\boldsymbol{q}(t) + \boldsymbol{q}^T(t)\boldsymbol{X}^T\boldsymbol{E}\dot{\boldsymbol{q}}(t) < 0\,. \tag{17}$$

Using (11) in the form reflecting the unforced system regime, it is easy to show that

$$\dot{v}(\boldsymbol{q}(t)) = \boldsymbol{q}^T(t)(\boldsymbol{A}^T\boldsymbol{X} + \boldsymbol{X}^T\boldsymbol{A})\boldsymbol{q}(t) < 0, \tag{18}$$

which implies (13).
If (13) is satisfied, then $\boldsymbol{X}$, $\boldsymbol{A}$ are nonsingular and it yields

$$(s\boldsymbol{E} - \boldsymbol{A}) = -s\boldsymbol{A}(-\boldsymbol{A}^{-1}\boldsymbol{E} + s^{-1}\boldsymbol{I}_n) = -s\boldsymbol{I}_n\boldsymbol{A}(s^{-1}\boldsymbol{I}_n - \boldsymbol{A}^{-1}\boldsymbol{E}). \tag{19}$$

Thus, since

$$\det(s\boldsymbol{E} - \boldsymbol{A}) = (-s)^n \det\boldsymbol{A} \det(s^{-1}\boldsymbol{I}_n - \boldsymbol{A}^{-1}\boldsymbol{E}), \tag{20}$$

this implies the regularity of $(\boldsymbol{E}, \boldsymbol{A})$.
Exploiting Krasovskii theorem [8] it can be set with respect to (13)

$$\boldsymbol{X}^T\boldsymbol{A} + \boldsymbol{A}^T\boldsymbol{X} \leqslant \boldsymbol{Z} < 0 \tag{21}$$

where $\boldsymbol{Z} \in \mathbb{R}^{n\times n}$ is a symmetric positive definite matrix. Because $\boldsymbol{Z}$ is nonsingular, satisfies the condition that $(\boldsymbol{E}, \boldsymbol{A}, \boldsymbol{Z}^{1/2})$ is observable [14]. This implies that $(\boldsymbol{E}, \boldsymbol{A})$ has neither impulsive nor unstable finite modes. This concludes the proof. □

Note, the inequality (13) is said to be Lyapunov inequality for a system given by (11) and (12) and Lemma 2 get the sufficient and necessary condition of descriptor systems to be regular, impulse free and stable by Lyapunov direct method.

To formulate the PD fault observer design conditions, a modification of the above given descriptor principle in the PD fault estimator stability analysis is applied in the following.

4. PD fault observer design conditions

If the observer errors between the system state vector and the observer state vector, as well as between the fault vector and the vector of its observer estimate, are defined as follows

$$\boldsymbol{e}_q(t) = \boldsymbol{q}(t) - \boldsymbol{q}_e(t), \tag{22}$$

$$\boldsymbol{e}_f(t) = \boldsymbol{f}(t) - \boldsymbol{f}_e(t), \tag{23}$$

then for slowly-varying faults it is reasonable to consider

$$\dot{\boldsymbol{e}}_f(t) = \boldsymbol{0} - \dot{\boldsymbol{f}}_e(t) = -\boldsymbol{M}\boldsymbol{C}\boldsymbol{e}_q(t) - \boldsymbol{N}\boldsymbol{C}\dot{\boldsymbol{e}}_q(t). \tag{24}$$

Note, since $\boldsymbol{f}_e(t)$ can be obtained as integral of $\dot{\boldsymbol{f}}_e(t)$, an adapting parameter matrix $\boldsymbol{G}$ can be computed interactively to set the amplitude of $\boldsymbol{f}_e(t)$, i.e.

$$\boldsymbol{f}_e(t) = \boldsymbol{G}\int_0^t \dot{\boldsymbol{f}}_e(\tau)\mathrm{d}\tau. \tag{25}$$

Giving notion about time derivative of the system state error $\boldsymbol{e}_q(t)$, the equations (1), (4) together with (2), (5) can be integrated as

$$\dot{\boldsymbol{e}}_q(t) = \boldsymbol{A}_e\boldsymbol{e}_q(t) + \boldsymbol{F}\boldsymbol{e}_f(t) - \boldsymbol{L}\boldsymbol{C}\dot{\boldsymbol{e}}_q(t)\,, \tag{26}$$

where

$$\boldsymbol{A}_e = \boldsymbol{A} - \boldsymbol{J}\boldsymbol{C} \tag{27}$$

and the PD observer system matrix is

$$\boldsymbol{A}_{PDe} = (\boldsymbol{I}_n + \boldsymbol{L}\boldsymbol{C})^{-1}\boldsymbol{A}_e = (\boldsymbol{I}_n + \boldsymbol{L}\boldsymbol{C})^{-1}(\boldsymbol{A} - \boldsymbol{J}\boldsymbol{C})\,. \tag{28}$$

To eliminate matrix inversion, (24), (26) can be rewritten in the following composed form

$$\begin{bmatrix} \dot{\boldsymbol{e}}_q(t) \\ \dot{\boldsymbol{e}}_f(t) \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_e & \boldsymbol{F} \\ -\boldsymbol{M}\boldsymbol{C} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{e}_q(t) \\ \boldsymbol{e}_f(t) \end{bmatrix} - \begin{bmatrix} \boldsymbol{L}\boldsymbol{C} & \boldsymbol{0} \\ \boldsymbol{N}\boldsymbol{C} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{e}}_q(t) \\ \dot{\boldsymbol{e}}_f(t) \end{bmatrix}, \tag{29}$$

and using the following notations

$$\boldsymbol{e}^{\circ T}(t) = \begin{bmatrix} \boldsymbol{e}_q^T(t) & \boldsymbol{e}_f^T(t) \end{bmatrix}, \tag{30}$$

$$\boldsymbol{A}^{\circ} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{F} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}, \ \boldsymbol{J}^{\circ} = \begin{bmatrix} \boldsymbol{J} \\ \boldsymbol{M} \end{bmatrix}, \ \boldsymbol{L}^{\circ} = \begin{bmatrix} \boldsymbol{L} \\ \boldsymbol{N} \end{bmatrix}, \tag{31}$$

$$\boldsymbol{I}^{\circ} = \begin{bmatrix} \boldsymbol{I}_n & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I}_p \end{bmatrix}, \ \boldsymbol{C}^{\circ} = \begin{bmatrix} \boldsymbol{C} & \boldsymbol{0} \end{bmatrix}, \tag{32}$$

where $\boldsymbol{A}^{\circ}, \boldsymbol{I}^{\circ} \in \mathbb{R}^{(n+p)\times(n+p)}$, $\boldsymbol{J}^{\circ}, \boldsymbol{L}^{\circ} \in \mathbb{R}^{(n+p)\times m}$, $\boldsymbol{C}^{\circ} \in \mathbb{R}^{m\times(n+p)}$, then (29) can be written as

$$(\boldsymbol{I}^{\circ} + \boldsymbol{L}^{\circ}\boldsymbol{C}^{\circ})\dot{\boldsymbol{e}}^{\circ}(t) = (\boldsymbol{A}^{\circ} - \boldsymbol{J}^{\circ}\boldsymbol{C}^{\circ})\boldsymbol{e}^{\circ}(t)\,, \tag{33}$$

$$\boldsymbol{A}_e^{\circ}\boldsymbol{e}^{\circ}(t) - \boldsymbol{D}_e^{\circ}\dot{\boldsymbol{e}}^{\circ}(t) = \boldsymbol{0}\,, \tag{34}$$

respectively, with

$$\boldsymbol{A}_e^{\circ} = \boldsymbol{A}^{\circ} - \boldsymbol{J}^{\circ}\boldsymbol{C}^{\circ}, \quad \boldsymbol{D}_e^{\circ} = \boldsymbol{I}^{\circ} + \boldsymbol{L}^{\circ}\boldsymbol{C}^{\circ}. \tag{35}$$

Introducing the equality

$$\dot{\boldsymbol{e}}^{\circ}(t) = \dot{\boldsymbol{e}}^{\circ}(t)\,, \tag{36}$$

then (36), (34) can be written as

$$\begin{bmatrix} \boldsymbol{I}^{\circ} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{e}}^{\circ}(t) \\ \ddot{\boldsymbol{e}}^{\circ}(t) \end{bmatrix} = \begin{bmatrix} \dot{\boldsymbol{e}}^{\circ}(t) \\ \boldsymbol{0} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I}^{\circ} \\ \boldsymbol{A}_e^{\circ} & -\boldsymbol{D}_e^{\circ} \end{bmatrix} \begin{bmatrix} \boldsymbol{e}^{\circ}(t) \\ \dot{\boldsymbol{e}}^{\circ}(t) \end{bmatrix}. \tag{37}$$

Thus, denoting

$$\boldsymbol{E}^{\bullet} = \begin{bmatrix} \boldsymbol{I}^{\circ} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}, \; \boldsymbol{A}_e^{\bullet} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I}^{\circ} \\ \boldsymbol{A}_e^{\circ} & -\boldsymbol{D}_e^{\circ} \end{bmatrix}, \; \boldsymbol{e}^{\bullet}(t) = \begin{bmatrix} \boldsymbol{e}^{\circ}(t) \\ \dot{\boldsymbol{e}}^{\circ}(t) \end{bmatrix}, \tag{38}$$

the obtained descriptor form to PD fault observer is

$$\boldsymbol{E}^{\bullet}\dot{\boldsymbol{e}}^{\bullet}(t) = \boldsymbol{A}_e^{\bullet}\boldsymbol{e}^{\bullet}(t)\,, \tag{39}$$

where $\boldsymbol{A}_e^{\bullet}, \boldsymbol{E}^{\bullet} \in \mathbb{R}^{2(n+p)\times 2(n+p)}$.

The following solvability theorem is proposed to design PD fault observer in the structure given in (4)-(6).

Theorem 1 *The PD fault observer* (4)-(6) *is stable if for given positive scalar* $\delta \in \mathbb{R}$ *there exist a symmetric positive definite matrix* $\boldsymbol{P}_1^{\circ} \in \mathbb{R}^{(n+p)\times(n+p)}$*, a regular matriz* $\boldsymbol{P}_3^{\circ} \in \mathbb{R}^{(n+p)\times(n+p)}$ *and matrices* $\boldsymbol{Y}^{\circ} \in \mathbb{R}^{(n+p)\times m}$*,* $\boldsymbol{Z}^{\circ} \in \mathbb{R}^{(n+p)\times m}$ *such that*

$$\boldsymbol{P}_1^{\circ} = \boldsymbol{P}_1^{\circ T} > 0\,, \tag{40}$$

$$\begin{bmatrix} \boldsymbol{A}^{\circ T}\boldsymbol{P}_3^{\circ} + \boldsymbol{P}_3^{\circ T}\boldsymbol{A}^{\circ} - \boldsymbol{Y}^{\circ}\boldsymbol{C}^{\circ} - \boldsymbol{C}^{\circ T}\boldsymbol{Y}^{\circ T} & * \\ \boldsymbol{V}_{21}^{\circ} & \boldsymbol{V}_{22}^{\circ} \end{bmatrix} < 0\,, \tag{41}$$

$$\boldsymbol{V}_{21}^{\circ} = \boldsymbol{P}_1^{\circ} - \boldsymbol{P}_3^{\circ} + \delta\boldsymbol{P}_3^{\circ T}\boldsymbol{A}^{\circ} - \delta\boldsymbol{Y}^{\circ}\boldsymbol{C}^{\circ} - \boldsymbol{C}^{\circ T}\boldsymbol{Z}^{\circ T}\,, \tag{42}$$

$$\boldsymbol{V}_{22}^{\circ} = -\delta\boldsymbol{P}_3^{\circ} - \delta\boldsymbol{P}_3^{\circ T} - \delta\boldsymbol{Z}^{\circ}\boldsymbol{C}^{\circ} - \delta\boldsymbol{C}^{\circ T}\boldsymbol{Z}^{\circ T}\,. \tag{43}$$

If the above conditions hold, the set of the extended observer gain matrices is given by the equations

$$\boldsymbol{J}^{\circ} = (\boldsymbol{P}_3^{\circ T})^{-1}\boldsymbol{Y}^{\circ}, \quad \boldsymbol{L}^{\circ} = (\boldsymbol{P}_3^{\circ T})^{-1}\boldsymbol{Z}^{\circ} \tag{44}$$

and the matrices $\boldsymbol{J}$*,* $\boldsymbol{L}$ $\boldsymbol{M}$*,* $\boldsymbol{N}$ *can be separated with respect to* (31).

Proof Defining the Lyapunov function of the form

$$v(\boldsymbol{e}^{\bullet}(t)) = \boldsymbol{e}^{\bullet T}(t)\boldsymbol{E}^{\bullet T}\boldsymbol{P}^{\bullet}\boldsymbol{e}^{\bullet}(t) > 0\,, \tag{45}$$

where, considering (14), it yields

$$\boldsymbol{E}^{\bullet T}\boldsymbol{P}^{\bullet} = \boldsymbol{P}^{\bullet T}\boldsymbol{E}^{\bullet} \geqslant 0\,, \tag{46}$$

then the time derivative along a trajectory of (39) becomes, using the constraints (46),

$$\dot{v}(\boldsymbol{e}^{\bullet}(t)) = \dot{\boldsymbol{e}}^{\bullet T}(t)\boldsymbol{E}^{\bullet T}\boldsymbol{P}^{\bullet}\boldsymbol{e}^{\bullet}(t) + \boldsymbol{e}^{\bullet T}(t)\boldsymbol{P}^{\bullet T}\boldsymbol{E}^{\bullet}\dot{\boldsymbol{e}}^{\bullet}(t) < 0\,. \tag{47}$$

Thus, substituting (39) into (47), it yields

$$\dot{v}(\boldsymbol{e}^{\bullet}(t)) = \boldsymbol{e}^{\bullet T}(t)(\boldsymbol{P}^{\bullet T}\boldsymbol{A}_e^{\bullet} + \boldsymbol{A}_e^{\bullet T}\boldsymbol{P}^{\bullet})\boldsymbol{e}^{\bullet}(t) < 0\,, \tag{48}$$

which implies

$$\boldsymbol{P}^{\bullet T}\boldsymbol{A}_e^{\bullet}+\boldsymbol{A}_e^{\bullet T}\boldsymbol{P}^{\bullet}<0. \tag{49}$$

Defining the Lyapunov matrix

$$\boldsymbol{P}^{\bullet}=\begin{bmatrix}\boldsymbol{P}_1^{\circ} & \boldsymbol{P}_2^{\circ}\\ \boldsymbol{P}_3^{\circ} & \boldsymbol{P}_4^{\circ}\end{bmatrix}, \tag{50}$$

then with respect to (38) it has to be

$$\begin{bmatrix}\boldsymbol{I}^{\circ} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\boldsymbol{P}_1^{\circ} & \boldsymbol{P}_2^{\circ}\\ \boldsymbol{P}_3^{\circ} & \boldsymbol{P}_4^{\circ}\end{bmatrix}=\begin{bmatrix}\boldsymbol{P}_1^{\circ T} & \boldsymbol{P}_3^{\circ T}\\ \boldsymbol{P}_2^{\circ T} & \boldsymbol{P}_4^{\circ T}\end{bmatrix}\begin{bmatrix}\boldsymbol{I}^{\circ} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\geqslant 0, \tag{51}$$

which gives

$$\begin{bmatrix}\boldsymbol{P}_1^{\circ} & \boldsymbol{P}_2^{\circ}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}=\begin{bmatrix}\boldsymbol{P}_1^{\circ T} & \mathbf{0}\\ \boldsymbol{P}_2^{\circ T} & \mathbf{0}\end{bmatrix}\geqslant 0. \tag{52}$$

It is evident that (52) can be satisfied only if

$$\boldsymbol{P}_1^{\circ}=\boldsymbol{P}_1^{\circ T}>0, \quad \boldsymbol{P}_2^{\circ}=\boldsymbol{P}_2^{\circ T}=\mathbf{0} \tag{53}$$

and, using (38) and (50) with (53) in (49), it yields

$$\begin{bmatrix}\mathbf{0} & \boldsymbol{A}_e^{\circ T}\\ \boldsymbol{I}^{\circ} & -\boldsymbol{D}_e^{\circ T}\end{bmatrix}\begin{bmatrix}\boldsymbol{P}_1^{\circ} & \mathbf{0}\\ \boldsymbol{P}_3^{\circ} & \boldsymbol{P}_4^{\circ}\end{bmatrix}+\begin{bmatrix}\boldsymbol{P}_1^{\circ} & \boldsymbol{P}_3^{\circ T}\\ \mathbf{0} & \boldsymbol{P}_4^{\circ T}\end{bmatrix}\begin{bmatrix}\mathbf{0} & \boldsymbol{I}^{\circ}\\ \boldsymbol{A}_e^{\circ} & -\boldsymbol{D}_e^{\circ}\end{bmatrix}<0. \tag{54}$$

After simple algebraic matrix manipulations, (54) can be expressed in the following form

$$\begin{bmatrix}\boldsymbol{U}_1^{\bullet} & \boldsymbol{U}_2^{\bullet T}\\ \boldsymbol{U}_2^{\bullet} & \boldsymbol{U}_3^{\bullet}\end{bmatrix}<0, \tag{55}$$

where, with (35),

$$\boldsymbol{U}_1^{\bullet}=(\boldsymbol{A}^{\circ}-\boldsymbol{J}^{\circ}\boldsymbol{C}^{\circ})^T\boldsymbol{P}_3^{\circ}+\boldsymbol{P}_3^{\circ T}(\boldsymbol{A}^{\circ}-\boldsymbol{J}^{\circ}\boldsymbol{C}^{\circ}), \tag{56}$$

$$\boldsymbol{U}_2^{\bullet}=\boldsymbol{P}_1^{\circ}-\boldsymbol{P}_3^{\circ}+\boldsymbol{P}_4^{\circ T}(\boldsymbol{A}^{\circ}-\boldsymbol{J}^{\circ}\boldsymbol{C}^{\circ})-\boldsymbol{C}^{\circ T}\boldsymbol{L}^{\circ T}\boldsymbol{P}_3^{\circ}, \tag{57}$$

$$\boldsymbol{U}_3^{\bullet}=-\boldsymbol{P}_4^{\circ}-\boldsymbol{P}_4^{\circ T}-\boldsymbol{P}_4^{\circ T}\boldsymbol{L}^{\circ}\boldsymbol{C}^{\circ}-\boldsymbol{C}^{\circ T}\boldsymbol{L}^{\circ T}\boldsymbol{P}_4^{\circ}. \tag{58}$$

Setting

$$\boldsymbol{P}_4^{\circ}=\delta\boldsymbol{P}_3^{\circ}, \quad \boldsymbol{Y}^{\circ}=\boldsymbol{P}_3^{\circ T}\boldsymbol{J}^{\circ}, \quad \boldsymbol{Z}^{\circ}=\boldsymbol{P}_3^{\circ T}\boldsymbol{L}^{\circ}, \tag{59}$$

where $\delta>0$, $\delta\in\mathbb{R}$, then (55)-(58) imply (41)-(43). This concludes the proof. □

Note, though the form seems to be complicated in Theorem 1, it is easily to get the solution when they is applied.

Remark 1 Writing (58) as follows

$$\boldsymbol{U}_3^{\bullet} = -(\boldsymbol{P}_4^{\circ T}(\boldsymbol{I}^{\circ} + \boldsymbol{L}^{\circ}\boldsymbol{C}^{\circ}) + (\boldsymbol{I}^{\circ} + \boldsymbol{L}^{\circ}\boldsymbol{C}^{\circ})^T \boldsymbol{P}_4^{\circ}) = -\boldsymbol{R}^{\bullet} \tag{60}$$

and comparing (7) and (55), then, if (40)-(43) is satisfied, the Schur complement property implies that $\boldsymbol{R}^{\bullet}$ is positive definite. Since $\boldsymbol{P}_4^{\circ}$ is regular, $(\boldsymbol{I}^{\circ} + \boldsymbol{L}^{\circ}\boldsymbol{C}^{\circ})$ is also regular and so $\boldsymbol{A}_{PDe}$ given by (28) exists.

Theorem 2 *(LMIs with one symmetric slack matrix) The PD observer* (4)-(6) *is stable if for given positive scalar $\delta \in \mathbb{R}$ there exist a symmetric positive definite matrix $\boldsymbol{Q}^{\circ} \in \mathbb{R}^{(n+p)\times(n+p)}$ and matrices $\boldsymbol{Y}^{\circ} \in \mathbb{R}^{(n+p)\times m}$, $\boldsymbol{Z}^{\circ} \in \mathbb{R}^{(n+p)\times m}$ such that*

$$\boldsymbol{Q}^{\circ} = \boldsymbol{Q}^{\circ T} > 0\,, \tag{61}$$

$$\begin{bmatrix} \boldsymbol{A}^{\circ T}\boldsymbol{Q}^{\circ} + \boldsymbol{Q}^{\circ}\boldsymbol{A}^{\circ} - \boldsymbol{Y}^{\circ}\boldsymbol{C}^{\circ} - \boldsymbol{C}^{\circ T}\boldsymbol{Y}^{\circ T} & * \\ \boldsymbol{W}_{21}^{\circ} & \boldsymbol{W}_{22}^{\circ} \end{bmatrix} < 0\,, \tag{62}$$

$$\boldsymbol{W}_{21}^{\circ} = \boldsymbol{P}^{\circ} - \boldsymbol{Q}^{\circ} + \delta\boldsymbol{Q}^{\circ}\boldsymbol{A}^{\circ} - \delta\boldsymbol{Y}^{\circ}\boldsymbol{C}^{\circ} - \boldsymbol{C}^{\circ T}\boldsymbol{Z}^{\circ T}\,, \tag{63}$$

$$\boldsymbol{W}_{22}^{\circ} = -2\delta\boldsymbol{Q}^{\circ} - \delta\boldsymbol{Z}^{\circ}\boldsymbol{C}^{\circ} - \delta\boldsymbol{C}^{\circ T}\boldsymbol{Z}^{\circ T}\,. \tag{64}$$

If the above conditions are affirmative, the extended observer gain matrices are given by the equations

$$\boldsymbol{J}^{\circ} = \boldsymbol{X}^{-1}\boldsymbol{Y}^{\circ}, \quad \boldsymbol{L}^{\circ} = \boldsymbol{X}^{-1}\boldsymbol{Z}^{\circ}. \tag{65}$$

Proof Since there is no restriction on the structure of $\boldsymbol{P}_3$ it can be set

$$\boldsymbol{P}_1^{\circ} = \boldsymbol{P}^{\circ} > 0, \quad \boldsymbol{P}_3^{\circ} = \boldsymbol{P}_3^{\circ T} = \boldsymbol{Q}^{\circ} > 0 \tag{66}$$

and the conditioned structure of $\boldsymbol{P}_4^{\circ}$ (with respect to $\boldsymbol{P}_3^{\circ}$ and $\boldsymbol{A}_e^{\circ}$) can be taken into account as

$$\boldsymbol{P}_4^{\circ} = \delta\boldsymbol{P}_3^{\circ} = \delta\boldsymbol{Q}^{\circ}, \tag{67}$$

where $\delta > 0$, $\delta \in \mathbb{R}$. If these conditions are incorporated into associated elements of (56)-(58), then

$$\boldsymbol{P}_3^T\boldsymbol{A}_e^{\circ} = \boldsymbol{Q}^{\circ}(\boldsymbol{A}^{\circ} - \boldsymbol{J}^{\circ}\boldsymbol{C}^{\circ}) = \boldsymbol{Q}^{\circ}\boldsymbol{A}^{\circ} - \boldsymbol{Y}^{\circ}\boldsymbol{C}^{\circ}, \tag{68}$$

$$\boldsymbol{P}_4^{\circ T}\boldsymbol{L}^{\circ}\boldsymbol{C}^{\circ} = \delta\boldsymbol{P}_3^{\circ T}\boldsymbol{L}^{\circ}\boldsymbol{C}^{\circ} = \delta\boldsymbol{Q}^{\circ}\boldsymbol{L}^{\circ}\boldsymbol{C}^{\circ} = \delta\boldsymbol{Z}^{\circ}\boldsymbol{C}^{\circ}, \tag{69}$$

where

$$\boldsymbol{Y}^{\circ} = \boldsymbol{Q}^{\circ}\boldsymbol{J}^{\circ}, \quad \boldsymbol{Z}^{\circ} = \boldsymbol{Q}^{\circ}\boldsymbol{L}^{\circ}. \tag{70}$$

and with these modifications then (55)-(58) imply (62)-(64). This concludes the proof. □

Note, the design conditions formulated in Theorem 2 give potentially more conservative solutions.

5. Illustrative example

The considered system is represented by the model (1), (2) with the model parameters [11]

$$A = \begin{bmatrix} 1.380 & -0.208 & 6.715 & -5.676 \\ -0.581 & -4.290 & 0.000 & 0.675 \\ 1.067 & 4.273 & -6.654 & 5.893 \\ 0.048 & 4.273 & 1.343 & -2.104 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.000 & 0.000 \\ 5.679 & 0.000 \\ 1.136 & -3.146 \\ 1.136 & 0.000 \end{bmatrix}, \quad C = \begin{bmatrix} 4 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

To consider single actuator faults it was set $E = B$, and the matrix variables P_1°, P_3°, Y°, Z° satisfying (40)-(43) for $\delta = 0.1$ were as follows

$$P_1^\circ = \begin{bmatrix} 1.1656 & -0.0292 & 0.1399 & -0.0330 & 0.0540 & 0.1956 \\ -0.0292 & 0.7469 & 0.0382 & 0.0930 & -0.3023 & -0.0165 \\ 0.1399 & 0.0382 & 0.6756 & 0.1090 & -0.0627 & 0.2747 \\ -0.0330 & 0.0930 & 0.1090 & 1.1578 & -0.0744 & -0.0321 \\ 0.0540 & -0.3023 & -0.0627 & -0.0744 & 1.1850 & -0.0067 \\ 0.1956 & -0.0165 & 0.2747 & -0.0321 & -0.0067 & 1.1826 \end{bmatrix},$$

$$P_3^\circ = \begin{bmatrix} -0.4070 & -0.1151 & 0.7466 & -0.7338 & 0.0774 & 0.5150 \\ 0.0578 & 0.6332 & -0.2863 & 0.1368 & 0.0928 & -0.1483 \\ -0.0743 & 0.3681 & 0.7681 & -0.7179 & -0.2233 & 0.1232 \\ 0.1152 & 0.1302 & -0.2963 & 0.2091 & -0.6558 & 0.0281 \\ -0.1147 & -0.6971 & 0.0402 & 0.0067 & 1.2193 & 0.0645 \\ -0.7134 & 0.1249 & 0.2635 & -0.2000 & -0.0920 & 1.1889 \end{bmatrix},$$

$$Y^\circ = \begin{bmatrix} 0.0636 & -0.0189 \\ -0.0260 & 0.3146 \\ 0.0029 & 0.6994 \\ -0.0345 & 0.0126 \\ 0.0364 & -0.1243 \\ 0.2618 & -0.0972 \end{bmatrix}, \quad Z^\circ = \begin{bmatrix} 0.4497 & -0.2263 \\ 0.0208 & -0.3275 \\ -0.1868 & 0.4742 \\ 0.2276 & 0.9275 \\ 0.0060 & 0.5989 \\ -0.0765 & 0.1860 \end{bmatrix},$$

where the SeDuMi package [17] was used to solve given set of LMIs.

The PD observer extended matrix gains are then computed using (44) as

$$\boldsymbol{J}^{\circ} = \begin{bmatrix} -0.1164 & -3.5011 \\ 0.0759 & -2.3994 \\ 0.2602 & 2.0855 \\ 0.7696 & -3.8495 \\ 0.5644 & -1.5565 \\ 0.2142 & 1.2266 \end{bmatrix}, \quad \boldsymbol{L}^{\circ} = \begin{bmatrix} -2.0570 & -7.1432 \\ -0.5649 & -5.1745 \\ 1.8857 & 2.7134 \\ 1.9169 & -7.8482 \\ 1.7384 & -2.6444 \\ 0.4862 & 2.8783 \end{bmatrix},$$

which imply the PD fault estimator parameters

$$\boldsymbol{J} = \begin{bmatrix} -0.1164 & -3.5011 \\ 0.0759 & -2.3994 \\ 0.2602 & 2.0855 \\ 0.7696 & -3.8495 \end{bmatrix}, \quad \boldsymbol{L} = \begin{bmatrix} -2.0570 & -7.1432 \\ -0.5649 & -5.1745 \\ 1.8857 & 2.7134 \\ 1.9169 & -7.8482 \end{bmatrix},$$

$$\boldsymbol{M} = \begin{bmatrix} 0.5644 & -1.5565 \\ 0.2142 & 1.2266 \end{bmatrix}, \quad \boldsymbol{N} = \begin{bmatrix} 1.7384 & -2.6444 \\ 0.4862 & 2.8783 \end{bmatrix}.$$

Verifying the PD observer system matrix eigenvalue spectrum, the results were

$$\rho(\boldsymbol{A}_e) = \rho(\boldsymbol{A} - \boldsymbol{J}\boldsymbol{C}) = \left\{ -6.4616,\ -7.7123,\ 3.2804 \pm 1.7596\,\mathrm{i} \right\},$$

$$\rho(\boldsymbol{A}_{PDe}) = \rho\left((\boldsymbol{I}_n + \boldsymbol{L}\boldsymbol{C})^{-1}(\boldsymbol{A} - \boldsymbol{J}\boldsymbol{C})\right) = \\ = \left\{ -0.4875,\ -0.6138,\ -4.1538 \pm 3.0874\,\mathrm{i} \right\},$$

that means the PD fault estimator is stable although its ”P” part stay unstable.

Of course, also the descriptor form (33) of the PD fault estimator is stable, where

$$\rho\left((\boldsymbol{I}^{\circ} + \boldsymbol{L}^{\circ}\boldsymbol{C}^{\circ})^{-1}(\boldsymbol{A}^{\circ} - \boldsymbol{J}^{\circ}\boldsymbol{C}^{\circ})\right) = \\ = \{-0.3117,\ -0.4654,\ -0.8185 \pm 1.9964\,\mathrm{i},\ -4.0034 \pm 5.0494\,\mathrm{i}\}\,.$$

Note, solving (61)-(64), it is not possible to obtain for given system a stable ”P” part of the fault estimator. If it would be desirable for a reason to set up stable also this ”P” part, it is needed to look a more conservative solution from (61)-(64), e.g., for $\delta = 1$.

Setting the adapting parameter matrix $\boldsymbol{G}$ in (25) as follows

$$\boldsymbol{G} = \begin{bmatrix} 12.0000 & 0.1000 \\ 0.1000 & 12.0000 \end{bmatrix},$$

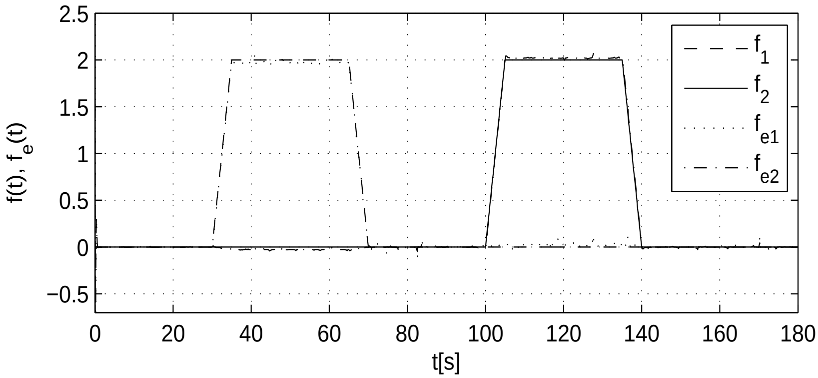

Figure 1: Single actuator faults and their estimates.

the results are illustrated in Fig. 1. This figure present the fault signals, as well as their estimates, reflecting a single actuator fault in the first actuator, starting at the time instant $t = 30s$ and applied for $40s$ and then the fault of the first actuator is introduced in the time instant $t = 100s$ and lasts for $40s$. The presented simulation was carried out in the system autonomous mode, practically the same results were obtained for forced regime of the system.

The adapting parameter $\boldsymbol{G}$ and the tuning parameter δ were set experimentally considering the maximal value of fault signal amplitude and fault observer dynamics. The faults considered in simulation do not cause closed-loop instability and it can be seen that there exist small differences between the signals reflecting single actuator faults and the observer approximate ones for piecewise constant actuator faults.

6. Concluding remarks

Based on the descriptor system approach a new PD fault observer design method for continuous-time linear systems and slowly-varying actuator faults is introduced in the paper. Presented version is derived in terms of optimization over LMI constraints using standard LMI numerical procedures to manipulate the fault observer stability and fault estimation dynamics. Formulated in the sense of the second Lyapunov method, expressed through LMI formulation, design conditions guaranty the asymptotic convergence of the state as well as fault estimation errors. The numerical simulation results show good estimation performances.

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