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Energy-optimal current distribution in a complex linear electrical network with pulse or periodic voltage and current signals. Optimal control

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Abstract. The article presents that in the circuits of electrical signals belonging to the L^1-impulses space or periodic signals space, real distribution of electrical currents occurs which does not meet the principle of minimum energy losses. The paper presents a solution of this problem by using the control system in the form of current-dependent voltage sources entering it into a meshes set of a complex RLC network. It has been shown that the control is energy-neutral.

Key words: L^1-impulses, linear circuits, principle of minimum energy losses, operators, optimal control.

1. Introduction

The issues relating to the quality of electrical energy distribution and minimization of energy losses usually refer to minimization of some energy indicators, such as reactive power, or to obtain an optimal, on account of energy, currents distribution.

In the DC circuits there is the minimum energy principle, according to which the currents distribution in a complex network is such that the total energy loss is minimal [1, 2]. However, this rule usually no longer works in the sinusoidal current circuits. [3]

On the other hand, in non-sinusoidal signals domain, the term "reactive power" makes no sense, which means that this term should not be used during testing the quality of electrical energy distribution in the network [4, 5]. However, the compensation problems aimed at resetting the indicator of reactive power can be solved as optimization tasks consisting in minimizing energy losses in the network or as related tasks of minimizing the RMS value of currents [6, 7]. This publication comes out to meet this issue.

The article shows that in the circuits with the signals belonging to the linear L^1-impulses space, there actually occurring current distribution does not satisfy the principle of minimum energy losses. To make it so, it is necessary to use the control system. It was considered a complex network powered multicurrent and with a multidimensional current-voltage control. It has been shown that the system of controlled sources is energy neutral. Thus, the process of minimal energy controlling is energy neutral.

In Fig.1. the RLC network with power given as a vector of current signals i_0 is shown. Distribution of mesh currents within the network is determined by the vector of current signals i. The network is characterized by the so-called "internal operators matrix" $\mathbf{Z}$(s) (s = d/dt), and the matrix so-called "contact operators" $\mathbf{Z}_0$(s). The equations of network operator assume the form:

$$\begin{aligned} \mathbf{Zi} - \mathbf{Z_0 i_0} &= \mathbf{0}, \\ -\mathbf{Z}_0^{\mathrm{T}}\mathbf{i} + \mathbf{Z_{00} i_0} &= \mathbf{u_0} \end{aligned} \tag{1}$$

(**0** – zero vector (or zero operator), T – a sign of transposition)

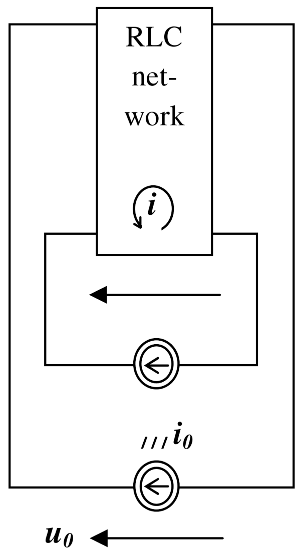

Fig. 1. The complex network with multicurrent power; i – internal mesh currents vector; i_0 external current vector

All impedance matrices have a distribution of: Hermitian **R** and skew-Hermitian **X** parts:

$$\mathbf{Z}(s) = \mathbf{R}(s) + \mathbf{X}(s) \tag{2}$$

i.e. such that

$$\mathbf{R}(-s) = \mathbf{R}(s); \qquad \mathbf{X}(-s) = -\mathbf{X}(s). \tag{3}$$

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In Fig. 2 the structure of system of equations (1) is illustrated. In this figure the sizes of the matrix and vectors are shown.

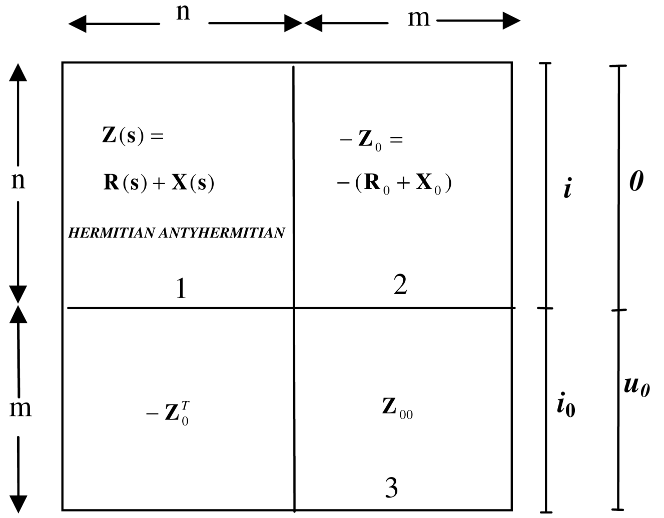


Fig. 2. Scheme the system of Eqs. (1); 1 – internal operators matrix, 2 – contact operators matrix, 3 – external operators matrix, **0** – vector (or operator) zero

The signals $i(t)$ – the coordinates of the current vector, belong to the $\mathbf{L}^1$ signal space, so-called the $\mathbf{L}^1$-impulses space:

$$\mathbf{L}^1 = \{x(t) : \int_{-\infty}^{\infty} |x(t)|dt < \infty\}$$

or to generated by it the T-periodic signals space $\mathbf{P_T}$ [8, 9]:

$$\mathbf{P_T} = \{\tilde{x}(t) \colon \tilde{x}(t) = \sum_{p=-\infty}^{\infty} x(t + pT); \;\; x(t) \in \mathbf{L}^1\}.$$

In these spaces the inner product is defined, in $\mathbf{L}^1$:

$$(u, i) = \int_{-\infty}^{\infty} u(t)i(t)dt$$

and in $\mathbf{P_T}$

$$(u, i) = \int_{o}^{T} u(t)i(t)dt.$$

All operators are convolutional type, i.e. in a time representation they have the form

$$Zi(t) = \int_{-\infty}^{+\infty} z(t - t')i(t')dt' \quad \text{in} \quad \mathbf{L}^1$$

or

$$Zi(t) = \int_{O}^{T} z(t\Theta t')i(t)dt' \quad \text{in} \quad \mathbf{P_T}$$

Θ – operation of subtraction modulo T.

In the notation using the Fourier transform:

$$ZI(s) = Z(s)I(s).$$

The impedance operator Z is decomposed into two components R and X [10, 11]:

$$Z = \frac{1}{2}(Z + Z^*) + \frac{1}{2}(Z - Z^*) = R + X,$$

i.e., operator

$$R = \frac{1}{2}(Z + Z^*),$$

which is the Hermitian operator (self-adjoined), ie: $R^* = R$, and the operator

$$X = \frac{1}{2}(Z - Z^*),$$

which is skew-Hermitian, ie: $X^* = -X$.

Operator Z^* is an adjoined operator relative to Z, i.e. such that for any signals x, y occurs $(Zx, y) = (x, Z^*y)$.

The R operator represents the active component of the impedance operator Z and the operator X is the passive component. This means that the following conditions for quadratic forms are fulfilled:

$$(Zi, i) = (Ri, i); \qquad (Xi, i) = 0$$

for any signal i.

It can be shown that the functions $Z(t)$ and $Z(s)$ determined by the convolution operators meet the conditions [12]:

$$Z^*(t) = Z(-t); \qquad Z^*(s) = Z(-s).$$

2. The principle of minimum energy losses in the electrical network at $\mathbf{L^1}$ and $\mathbf{P_T}$ spaces. Optimal control

The current functional

$$f(\mathbf{i}) = [\mathbf{i}^T, \mathbf{i}_0^T] \begin{bmatrix} \mathbf{R} & -\mathbf{R}_0 \\ -\mathbf{R}^T & \mathbf{R}_{00} \end{bmatrix} \begin{bmatrix} \mathbf{i} \\ \mathbf{i}_0 \end{bmatrix}, \tag{4}$$

has a value that is equal to the energy losses in the electrical network. Equation (4) creates an operator-matrix inner product, ie. if

$$\mathbf{A} = [A_{pq}]; \;\; \mathbf{X} = [x_q], \;\; \mathbf{Y} = [y_q],$$

the bilinear form takes a form

$$\begin{aligned} \mathbf{x^T A y} &= \sum_{p,q} (x_p, A_{pq} y_q) \\ &= \sum_{p,q}\sum \int x_p(t) \left[\int A_{pq}(t - t')y_q(t')dt'\right] dt \\ &= \sum_{p,q}\sum \iint A_{pq}(t - t')x_p(t)y_q(t')dtdt'. \end{aligned} \tag{5}$$

Integrals in the formula (5) are taken in the interval $(-\infty, +\infty)$ or in [0,T], depending on which space $\mathbf{L^1}$ or $\mathbf{P_T}$ is used. Depending on this, the operators are linear or cyclic convolutions.

The principle of minimum requires the functional (4), which value is equal to the total energy losses in the network, to reach a minimum. To find this minimum, variation of a functional is required:

$$\begin{aligned} df(\mathbf{i}) &= f(\mathbf{i} + d\mathbf{i}) - f(\mathbf{i}) \\ &= \left[d\mathbf{i}^T, \mathbf{0}^T\right] \begin{bmatrix} \mathbf{R} & -\mathbf{R}_0 \\ -\mathbf{R}_0^T & \mathbf{R}_{00} \end{bmatrix} \begin{bmatrix} \mathbf{i} \\ \mathbf{i}_0 \end{bmatrix} \\ &+ \left[\mathbf{i}^T, \mathbf{i}_0^T\right] \begin{bmatrix} \mathbf{R} & -\mathbf{R}_0 \\ -\mathbf{R}_0^T & \mathbf{R}_{00} \end{bmatrix} \begin{bmatrix} d\mathbf{i} \\ \mathbf{0} \end{bmatrix} \\ &+ \left[d\mathbf{i}^T, \mathbf{0}^T\right] \begin{bmatrix} \mathbf{R} & -\mathbf{R}_0 \\ -\mathbf{R}_0^T & \mathbf{R}_{00} \end{bmatrix} \begin{bmatrix} d\mathbf{i} \\ \mathbf{0} \end{bmatrix}, \end{aligned}$$

where $\mathbf{0}$ – vector-zero signal, $\mathbf{R}$ – the Hermitian part of the $\mathbf{Z}$ operator matrices.

After other transformations variation takes the following form:

$$\begin{aligned} df(\mathbf{i}) &= d\mathbf{i}^T\mathbf{R}\mathbf{i} - d\mathbf{i}^T\mathbf{R}_0\mathbf{i}_0 \\ &+ \mathbf{i}^T\mathbf{R}d\mathbf{i} - \mathbf{i}_0^T\mathbf{R}_0^T d\mathbf{i} + d\mathbf{i}^T\mathbf{R}d\mathbf{i} \end{aligned}$$

but occurs:

$$\mathbf{x}^T\mathbf{R}\mathbf{y} = \mathbf{y}^T\mathbf{R}\mathbf{x}$$

because

$$\mathbf{x}^T\mathbf{R}\mathbf{y} = \left(\mathbf{x}^T\mathbf{R}\mathbf{y}\right)^T = \mathbf{y}^T\mathbf{R}^T\mathbf{x}$$

but

$$\mathbf{R}^T = \mathbf{R}$$

thus:

$$\begin{aligned} df(\mathbf{i}) &= 2d\mathbf{i}^T\mathbf{R}\mathbf{i} - 2d\mathbf{i}^T\mathbf{R}_0\mathbf{i}_0 + d\mathbf{i}^T\mathbf{R}d\mathbf{i} \\ &= 2d\mathbf{i}^T(\mathbf{R}\mathbf{i} - \mathbf{R}_0\mathbf{i}_0) + d\mathbf{i}^T\mathbf{R}d\mathbf{i}. \end{aligned} \tag{6}$$

For the energy reasons the last component of the expression (6) is positively determined quadratic form (this is the energy loss within the network). Thus, the condition of minimum energy functional (4), i.e.

$$\bigwedge_{d\mathbf{i}} df(\mathbf{i}) > 0$$

takes the form:

$$\mathbf{R}\mathbf{i} - \mathbf{R}_0\mathbf{i}_0 = \mathbf{0}$$

or the system of operator equations form

$$\mathbf{R}\mathbf{i} = \mathbf{R}_0\mathbf{i}_0. \tag{7}$$

The system of Eq. (7) must be reconciled with the system of Eq. (1):

$$\mathbf{Z}\mathbf{i} = \mathbf{Z}_0\mathbf{i}_0 \tag{8}$$

the solution of which is the real distribution of mesh currents inside the network. The systems of Eqs. (7) and (8) have the same structure, but the system (7) is Hermitian type, part of, the system of Eq. (8).

The solution of the system of Eq. (7) is energy optimal distribution of mesh currents minimizing energy losses within the network:

$$\mathbf{i}^{opt} = \mathbf{R}^{-1}\mathbf{R}_0\mathbf{i}_0 \tag{9}$$

called the optimal distribution. Whereas the solution system of Eq. (8):

$$\mathbf{i} = \mathbf{Z}^{-1}\mathbf{Z}_0\mathbf{i}_0 \tag{10}$$

gives the actual distribution of mesh currents in the network called "current divider" distribution.

For DC currents the optimal distribution matches the distribution of the current divider, but it is not only in this case. However, these distributions do not match in general.

This means that the optimal distribution is achieved by using a current-voltage control system:

$$\begin{aligned} \mathbf{e}^{st} &= \mathbf{Z}\mathbf{i}^{opt} - \mathbf{Z}_0\mathbf{i}_0 \\ &= \mathbf{R}\mathbf{i}^{opt} - \mathbf{R}_0\mathbf{i}_0 + \mathbf{X}\mathbf{i}^{opt} - \mathbf{X}_0\mathbf{i}_0 \\ &= (\mathbf{X}\mathbf{R}^{-1}\mathbf{R}_0 - \mathbf{X}_0)\mathbf{i}_0. \end{aligned} \tag{11}$$

Equation (11) gives the voltage signal sources which must be plugged into internal network meshes to induce energy-optimal current distribution. That formula is written in the form

$$\mathbf{e}^{st} = \mathbf{X}^{st}\mathbf{i}_0, \tag{12}$$

where

$$\mathbf{X}^{st} = \mathbf{X}\mathbf{R}^{-1}\mathbf{R}_0 - \mathbf{X}_0. \tag{13}$$

Is the skew-Hermitian, matrix control operator.

3. Deviation operator

The optimal distribution of current (9) and distribution of the current divider (10) generally do not match. This is because of the operator

$$\mathbf{\Delta} = \mathbf{R}^{-1}\mathbf{R}_0 - \mathbf{Z}^{-1}\mathbf{Z}_0 \tag{14}$$

called then *deviation operator*. It is related to the control operator as follows:

$$\mathbf{\Delta} = \mathbf{Z}^{-1}(\mathbf{Z}\mathbf{R}^{-1}\mathbf{R}_0 - \mathbf{Z}_0) = \mathbf{Z}^{-1}(\mathbf{X}\mathbf{R}^{-1}\mathbf{R}_0 - \mathbf{X}_0),$$

where, (see. Eq. (13)):

$$\mathbf{\Delta} = \mathbf{Z}^{-1}\mathbf{X}^{st}. \tag{15}$$

From formulas (12), (13) and (15) arises the equivalence of the following conditions:

$$\begin{matrix} \mathbf{\Delta} = \mathbf{0} & & \mathbf{X}\mathbf{R}^{-1}\mathbf{R}_0 = \mathbf{X}_0 \\ \Updownarrow & \Leftrightarrow & \text{or} \\ \mathbf{X}^{st} = \mathbf{0} & & \mathbf{R}\mathbf{X}^{-1}\mathbf{X}_0 = \mathbf{R}_0 \end{matrix} \tag{16}$$

Then current distributions: optimum and current divider overlap without control. Such a network is called *naturally energy-optimal*. It is, of course, the DC network, where the operators Z and R overlap, but it is also an infinite number of networks which meet the conditions of equivalence (16).

Example 1. For the network of branches RLC (Fig. 3), operators type $\mathbf{Z}$ have the form:

$$\begin{aligned} \mathbf{Z}(\mathbf{s}) &= \mathbf{r} + \mathbf{s}\mathbf{L} + \mathbf{s}^{-1}\mathbf{\Sigma} = \mathbf{r} + \mathbf{X}(\mathbf{s}), \\ \mathbf{Z}_0(\mathbf{s}) &= \mathbf{r}_0 + \mathbf{s}\mathbf{L}_0 + \mathbf{s}^{-1}\mathbf{\Sigma}_0 = \mathbf{r}_0 + \mathbf{X}_0(\mathbf{s}), \end{aligned}$$

where $\mathbf{r}$, $\mathbf{r}_0$ – resistance matrices, $\mathbf{L}$, $\mathbf{L}_0$ – inductance matrices, $\mathbf{\Sigma}$, $\mathbf{\Sigma}_0$ – elastance matrices (the inverse of the capacity).

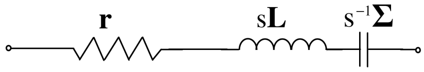


Fig. 3. Structural construction of RLC branch

Conditions (16) of naturally energy-optimal networks then take the following form:

$$(s\mathbf{L} + s^{-1}\mathbf{\Sigma})\mathbf{r}^{-1}\mathbf{r}_0 = s\mathbf{L}_0 + s^{-1}\mathbf{\Sigma}_0$$

for each s where

$$\mathbf{r}^{-1}\mathbf{r}_0 = \mathbf{L}^{-1}\mathbf{L}_0 = \mathbf{\Sigma}^{-1}\mathbf{\Sigma}_0.$$

Example 2. For the distribution of current signal *01* into two parallel branches 1, 2 (Fig. 4), determine the *current-voltage optimal control operator* $X^{st}i_{01}$.

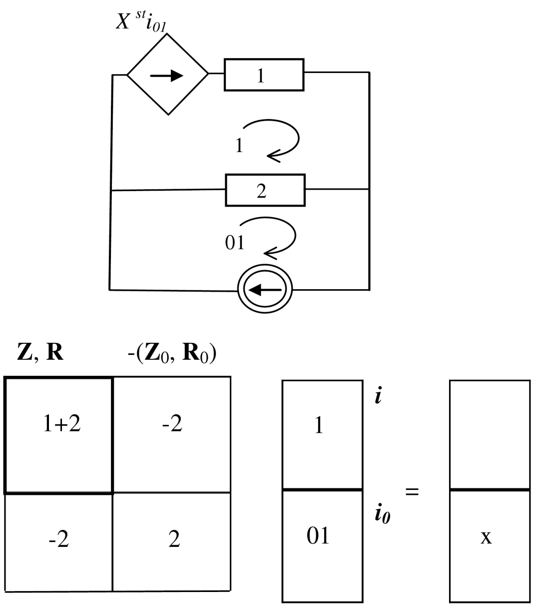
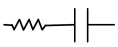


Fig. 4. Energy-optimal distribution of the current signal *01* into two parallel branches 1, 2; below there is the structure of Eqs. (8) and (7)

It is obtained for branches 1, 2 having the structure *RL*: 

$$\mathbf{X}^{st}(s) = s(\mathbf{L}\mathbf{r}^{-1}\mathbf{r}^0 - \mathbf{L}^0)$$
$$= s\left(\frac{L_1 + L_2}{r_1 + r_2}r_2 - L_2\right) = s\frac{L_1 r_2 - L_2 r_1}{r_1 + r_2}$$

or for branches 1, 2 *RC* type: 

$$\mathbf{X}^{st}(s) = s^{-1}(\mathbf{\Sigma}\mathbf{r}^{-1}\mathbf{r}^0 - \mathbf{\Sigma}^0)$$
$$= s^{-1}\left(\frac{\Sigma_1 + \Sigma_2}{r_1 + r_2}r_2 - \Sigma_2\right) = s^{-1}\frac{\Sigma_1 r_2 - \Sigma_2 r_1}{r_1 + r_2}$$
$$= s^{-1}\frac{r_2 C_2 - r_1 C_1}{C_1 C_2 (r_1 + r_2)}.$$

These are appropriately: the differential operators (for *RL* type of branches) and the integral operators (for *RC* type of branches).

Example 3. Determine the conditions (16) of naturally energy-optimal networks in the case of the distribution of current signal *01* into two parallel branches (*1-2* circuit) and three parallel branches (*1-2-3* circuit). Suitable circuits are shown in Fig. 5.

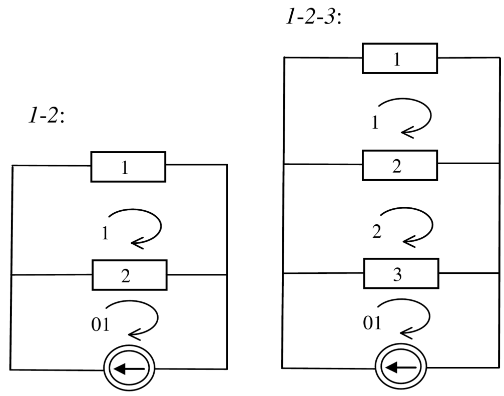
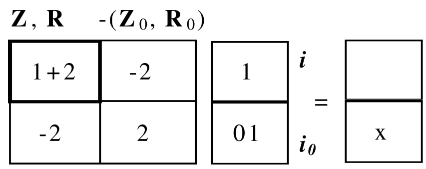


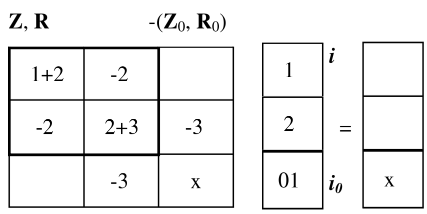
Fig. 5. Energy-optimal distribution of the current *01* for two and three parallel branches

Structures "**Z, R**" of operator Eqs. (7) and (8) are as follows:

(*1-2*):

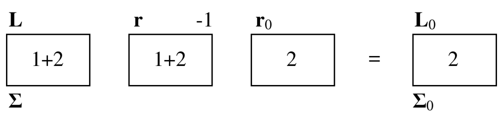


(*1-2-3*):

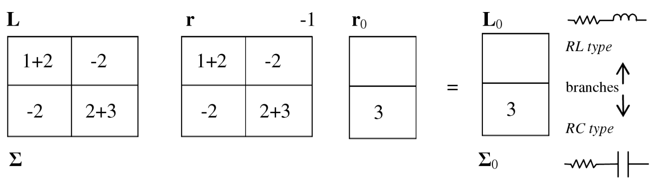


Appropriate conditions (16) for the network (*1-2*) and (*1-2-3*) take the form of matrix equations:

(*1-2*):



(*1-2-3*):



From equation (*1-2*) it is obtained:

$$\frac{L_1+L_2}{r_1+r_2} r_2 = L_2, \quad \text{or} \quad \frac{L_1}{r_1} = \frac{L_2}{r_2}.$$

While the condition (*1-2-3*) turns into:

$$\frac{1}{\delta}\begin{bmatrix} L_1+L_2 & -L_2 \\ -L_2 & L_2+L_3 \end{bmatrix}\begin{bmatrix} r_2+r_3 & r_2 \\ r_2 & r_1+r_2 \end{bmatrix}\begin{bmatrix} \\ r_3 \end{bmatrix} = \begin{bmatrix} \\ L_3 \end{bmatrix}$$

where $\delta = r_1 r_2 + r_1 r_3 + r_2 r_3$

thus: $\dfrac{L_1}{r_1} = \dfrac{L_2}{r_2} = \dfrac{L_3}{r_3}$

and after the transformation *RL*→*RC*:

$$L \to \Sigma \to C^{-1}:$$

$$r_1 C_1 = r_2 C_2 = r_3 C_3$$

Example 4. Energy-optimal distribution of the current signals *01*, *02* in the circuit of the ladder structure (Fig. 6).

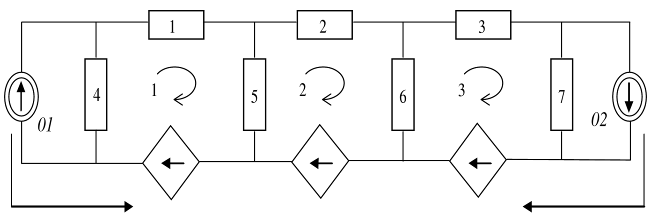
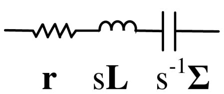


Fig. 6. Energy-optimal distribution of the currents *01*, *02* in the ladder circuit

Structures "**Z**, **R**" of operator equations: current divider (8) and the optimal one (7) have the following form:

$$\underset{\mathbf{Z},\,\mathbf{R}}{\begin{bmatrix} 1+4+5 & -5 & \\ -5 & 2+5+6 & -6 \\ & -6 & 3+6+7 \end{bmatrix}} \underset{\mathbf{i}}{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}} = \underset{\mathbf{Z}_0,\,\mathbf{R}_0}{\begin{bmatrix} 4 & \\ & \\ & 7 \end{bmatrix}} \underset{\mathbf{i}_0}{\begin{bmatrix} 01 \\ 02 \end{bmatrix}}$$

For the branch structure of RLC type:



$$\mathbf{Z}(s) = \mathbf{R}(s) + \mathbf{X}(s) = \mathbf{r} + s\mathbf{L} + s^{-1}\mathbf{\Sigma}$$

optimal control operators (11)–(13) have the form:

$$\mathbf{X}^{st}(s) = s(\mathbf{L}\mathbf{r}^{-1}\mathbf{r}_0 - \mathbf{L}_0)$$

or $\mathbf{X}^{st}(s) = s^{-1}(\mathbf{\Sigma}\mathbf{r}^{-1}\mathbf{r}_0 - \mathbf{\Sigma}_0)$,

or $\mathbf{X}^{st}(s) = s(\mathbf{L}\mathbf{r}^{-1}\mathbf{r}_0 - \mathbf{L}_0) + s^{-1}(\mathbf{\Sigma}\mathbf{r}^{-1}\mathbf{r}_0 - \mathbf{\Sigma}_0)$.

Whereas, the condition (16), which the naturally energy-optimal networks must meet, takes the form of the following matrix structure:

$$\underset{\mathbf{L}\ (\mathbf{\Sigma})}{\begin{bmatrix} 1+4+5 & -5 & \\ -5 & 2+5+6 & -6 \\ & -6 & 3+6+7 \end{bmatrix}} \underset{\mathbf{r}^{-1}}{\begin{bmatrix} 1+4+5 & -5 & \\ -5 & 2+5+6 & -6 \\ & -6 & 3+6+7 \end{bmatrix}^{-1}} \underset{\mathbf{r}_0}{\begin{bmatrix} 4 & \\ & \\ & 7 \end{bmatrix}} = \underset{\mathbf{L}_0\ (\mathbf{\Sigma}_0)}{\begin{bmatrix} 4 & \\ & \\ & 7 \end{bmatrix}}$$

Figure 6 shows also optimal control implemented using voltage sources controlled by the currents *01*, *02* distributed in the meshes of the ladder circuit.

4. Summary

The study showed that in the complex *RLC* network, besides the currents flows arising from the normal laws of Kirchhoff-called *current divider*, through appropriate controls may also be received other distributions of current, resulting from certain optimization criteria.

This paper examined the distribution that meets the condition of the minimum energy losses within the network, calling it the *energy-optimal distribution*. A current divider is described by the system of operator equations (8):

$$\mathbf{Z}(s)\mathbf{i} = \mathbf{Z}_0(s)\mathbf{i}_0$$

and energy-optimal current distribution also meets the system of operator equations (7):

$$\mathbf{R}(s)\mathbf{i} = \mathbf{R}_0(s)\mathbf{i}_0.$$

Matrices of impedance $\mathbf{Z}(s)$ and $\mathbf{R}(s)$ type of network are related in the way that:

$$\mathbf{Z}(s) = \mathbf{R}(s) + \mathbf{X}(s),$$

where

$$\mathbf{R}(-s) = \mathbf{R}(s); \qquad \mathbf{X}(-s) = -\mathbf{X}(s)$$

what makes this distribution a unique one:

$$\mathbf{R}(s) = \frac{1}{2}[\mathbf{Z}(s) + \mathbf{Z}(-s)],$$

$$\mathbf{X}(s) = \frac{1}{2}[\mathbf{Z}(s) - \mathbf{Z}(-s)].$$

In this way the systems of Eqs. (7) and (8) are a matrix identical, but in an operator way the Eq. (7) is the Hermitian variant of Eq. (8).

Optimal distribution itself is not as reachable as the distribution of a current divider, but in order to trigger it off, it is necessary to use *optimal control* carried out by the control operator $\mathbf{X}^{st}(s)$ (13), generating an appropriately distributed signal of the voltage source $\mathbf{e}^{st}$ (12):

$$\mathbf{e}^{\mathbf{st}} = \mathbf{X}^{\mathbf{st}}(s)\mathbf{i}_0,$$

$$\mathbf{X}^{\mathbf{st}}(s) = \mathbf{X}(s)[\mathbf{R}(s)]^{-1}\mathbf{R}_0(s) - \mathbf{X}_0(s).$$

It also appears that distributions: *energy-optimal* (7) and the *current divider* (8) can be the same without control when the deviation operator disappears (14):

$$\mathbf{\Delta}(s) = [\mathbf{R}(s)]^{-1}\mathbf{R}_0(s) - [\mathbf{Z}(s)]^{-1}\mathbf{Z}_0(s)$$

which is related to the *optimal control operator* by the formula (15):

$$\mathbf{X}^{st}(s) = \mathbf{Z}(s)\mathbf{\Delta}(s).$$

Networks meeting this condition are called *naturally energy-optimal networks*. As shown, for them (16) must occur:

$$\mathbf{X}(s)[\mathbf{R}(s)]^{-1}\mathbf{R}_0(s) = \mathbf{X}_0(s)$$

or equivalently (16):

$$\mathbf{R}(s)[\mathbf{X}(s)]^{-\mathbf{1}}\mathbf{X_0}(s) = \mathbf{R_0}(s).$$

The optimal control operator $\mathbf{X}^{st}(s)$ is skew-Hermitian, i.e. $\mathbf{X}^{st}(-s) = -\mathbf{X}^{st}(s)$ which makes that skew-Hermitian is also an operator:

$$[\mathbf{R}_0(s)]^T[\mathbf{R}(s)]^{-1}\mathbf{X}^{st}(s)$$

and so disappears quadratic form

$$(\mathbf{i}^{opt})^T\mathbf{e}^{st} = \mathbf{i}_0^T\mathbf{R}_0^T\mathbf{R}^{-1}\mathbf{X}^{st}\mathbf{i}_0.$$

Thus, the controlled sources $\mathbf{e}^{st}$ do not produce energy – the optimal control is *energy-neutral*.

The study presents several examples, with particular emphasis on networks consisted of branches with a serial structure of RLC elements.

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